

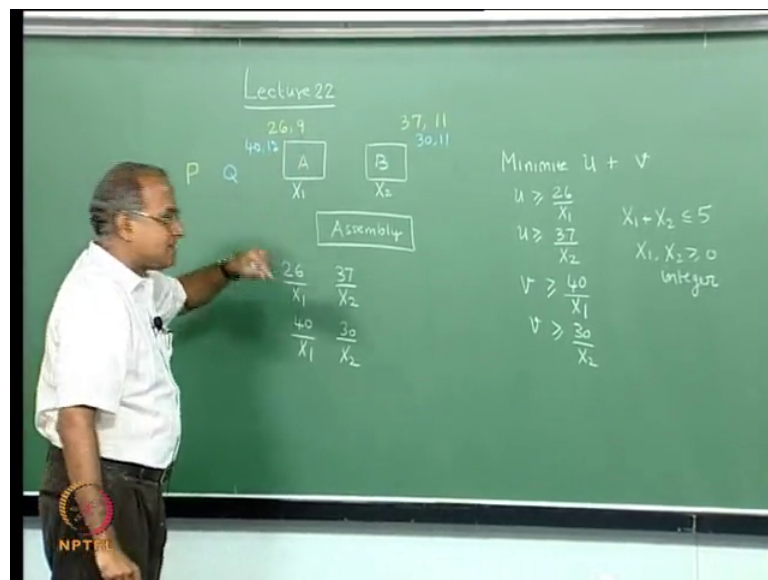
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**Lecture - 22**

**Static and dynamic Operator allocation, Multiple products and incremental cells**

In this lecture, we continue the discussion on static and dynamic operator allocation problems to cells. We look at situation where there are multiple cells and multiple products.

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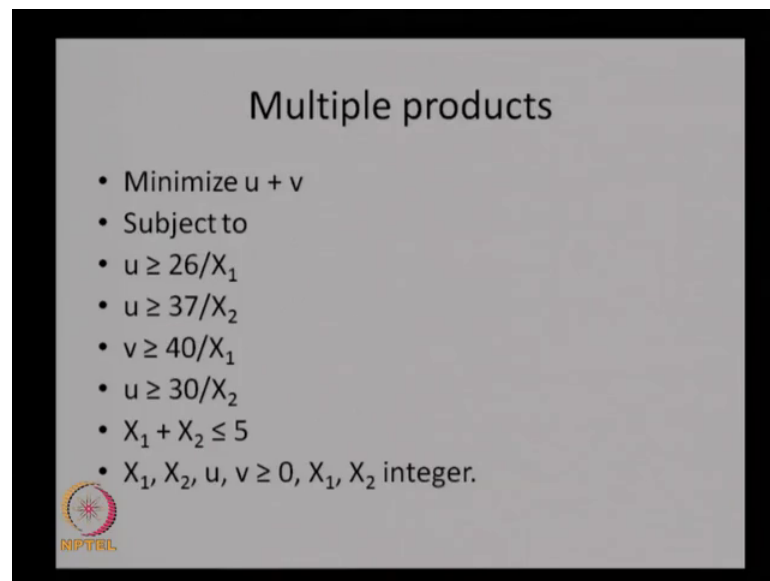
So, we look at 2 cells A and B that feed to the assembly. And we assume that there are 2 products, which we call as P and Q. Now we will assume that the cell A makes one part or component for P 1, for P and it also makes another one for Q. Similarly, this one would make one for P, and then one for Q. So, we could call we could say that this makes P 1 and P 2, both come here. And after the assembly P comes out. Similarly, this makes Q 1 and Q 2, which come to the assembly, and then Q comes out of the assembly.

So, this is the problem setting that we have. Now in cell A P 1 takes 26 comma 9, which means the total time taken on all the machines is 26. The maximum time called bottleneck time is 9. So, this means that if we have one operator, then the output will be 26. 2 operators the output will be 13. 3 operators the output will be 9. 26 by 3 is 8.66 which is smaller than 9. So, for 3 operators, the output will be 9. For the second cell it

will be 37 and 11. Once again saying that for 1 it is 37. For 2 it is 18.5, for 3 it is 12.33 and 4 for it is 11.

Now, as far as the second product is concerned. This would be 40 and 12. And this would be 30 and 11. How should operators be assign, such that we maximize the output or sum of the outputs of both the products?

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**Multiple products**

- Minimize  $u + v$
- Subject to
- $u \geq 26/X_1$
- $u \geq 37/X_2$
- $v \geq 40/X_1$
- $u \geq 30/X_2$
- $X_1 + X_2 \leq 5$
- $X_1, X_2, u, v \geq 0, X_1, X_2$  integer.

So, let us assume that  $X_1$  and  $X_2$  are the number of operators assigned to this cell as well as this cell. So, once again we would have the output of this as 26 by  $X_1$ , and the output as 37 by  $X_2$ . Now we want to minimize the maximum of these 3. So, minimize  $u$ , where  $u$  is greater than or equal to 26 by  $X_1$ . And  $u$  is greater than or equal to 37 by  $X_2$ .

Now, when we assign  $X_1$  and  $X_2$  operators to this A and B, as far as the second product is concerned, it will be 40 by  $X_1$  and 30 by  $X_2$ . So, again we want to minimize the maximum of this. So, we call that as  $v$ , and then we say  $v$  is greater than or equal to 40 by  $X_1$ , and  $v$  is greater than or equal to 30 by  $X_2$ .

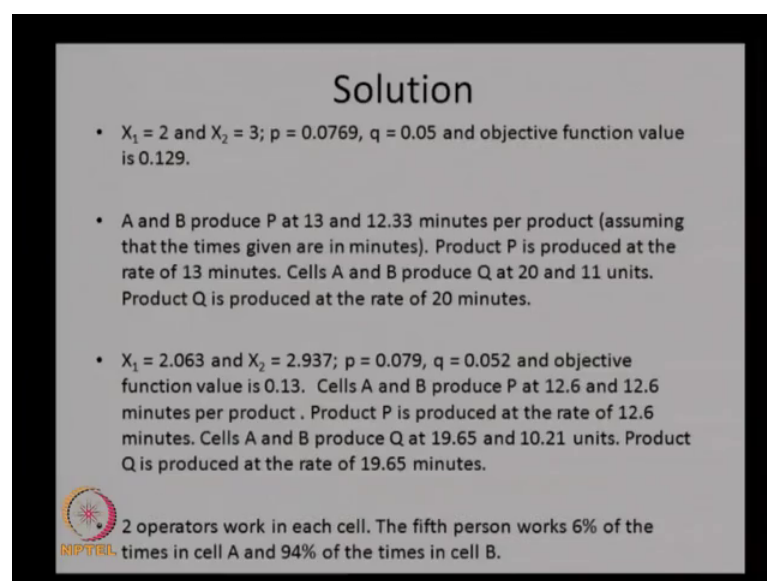
Now, we are going to say that we want to minimize the sum of these. So, we simply add which is a kind of an approximation that we make. In the sense that since there are 2 products, we want to minimize or we want to maximize the productivity of both taken together, which means we would like to minimize the maximum time taken for each product on the cells.

Then we have a restriction that  $X_1 + X_2$  is less than or equal to 5 assuming that there are 5 operators.  $X_1, X_2$  greater than or equal to 0 and integer.

So, as mentioned or as carried out in the previous lecture, we now see a certain non-linearity coming in. So, we would write this as  $X_1$  greater than or equal to  $26/u$ ,  $X_2$  greater than or equal to  $37/u$  and so on. And then write  $1/u$  as some small  $P$ , and  $1/v$  as some small  $Q$ . So, we end up actually minimizing  $1/P + 1/Q$  which would be to maximize  $P$  and  $Q$ , and then the problem can be made as a linear problem. So, this problem is very similar to the case where we have. Single product multiple cells feeding to the assembly except that, we have one more term for the second product. There we had only one term because there was a single product.


So, a second product brings in another term into the solution. As mentioned again in the previous lecture, there are some approximations in this formulation that for example, if we take  $X_1$  equal to 3, then this will assume that the output is  $26 \times 3$  whereas, the actual output is not  $26 \times 3$ , but it is 9. Which is more than  $26 \times 3$ . That would bring in a little more complication into the integer programming formulation. And at this stage, I am not bringing that additional complication into the formulation. Simply making an assumption that it is  $26 \times X_1$ , and depending on the answer we can do the final computation, and find out what is best for the cells.

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### Solution

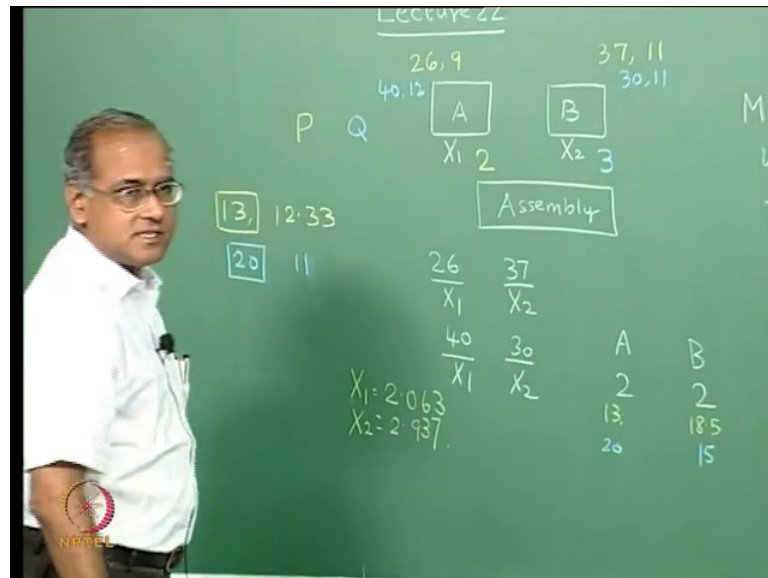
- $X_1 = 2$  and  $X_2 = 3$ ;  $p = 0.0769$ ,  $q = 0.05$  and objective function value is 0.129.
- A and B produce P at 13 and 12.33 minutes per product (assuming that the times given are in minutes). Product P is produced at the rate of 13 minutes. Cells A and B produce Q at 20 and 11 units. Product Q is produced at the rate of 20 minutes.
- $X_1 = 2.063$  and  $X_2 = 2.937$ ;  $p = 0.079$ ,  $q = 0.052$  and objective function value is 0.13. Cells A and B produce P at 12.6 and 12.6 minutes per product. Product P is produced at the rate of 12.6 minutes. Cells A and B produce Q at 19.65 and 10.21 units. Product Q is produced at the rate of 19.65 minutes.

 2 operators work in each cell. The fifth person works 6% of the times in cell A and 94% of the times in cell B.

So, when we solve this integer programming problem, they have a solution.  $X_1$  equal to 2 and  $X_2$  equal to 3. So, we have a solution  $X_1$  equal to 2 and  $X_2$  is equal to 3.

So now what happens when  $X_1$  equal to 2 and  $X_2$  equal to 3.

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When  $X_1$  equal to 2 product P will be made at the rate of 13 per unit 26 by 2. Q will be made at the rate of 37 by 3 which is 12.33 minutes assuming that these are minutes. So, the product P will come out of the assembly at the rate of 13 which is the maximum of the 2 values. Similarly, product Q at  $X_1$  equal to 2 will be 20 40 divided by 2 20, and 30 divided by 3 which is 10. At the same time, it will take a value 11. So, the output will be at the rate of 20 at the rate of 20. But if we go back to the solution, the LP solution would be  $X_1$  is equal to 2.063. And  $X_2$  is equal to 2.937. So, this would give us we can once again do the usual reallocation. And then we would we would again come back to the solution, which is actually 2 and 3. When we do this allocation, this we start by we can we can do this we can give each of them their lower integer value.

So, we would give 2 and 2 to both these cells, this is A, this is B. So, we give 2 and 2 to both of them. Because the lower integer value is 2 lower integer value is 2. So, that would give us 26 by 2 13, 37 by 2 is 18.5. And 2 and 2 4 this would give us 20. And the value here is 15 30 by 2 is 15.

Now, if we went by the previous model of static allocation, we would ordinarily do this 20 plus 13, 30 3 15 plus 18.5; 33.5. So, the third one will go here. So, we will make it 2 and 3, 2 and 3 which is our solution the first operator will actually go here. 2 and 3 will be the solution. But let us look at it a little carefully, when we do 2 and 3 as the solution is we fix 2 and 3, if we fix 2 and 3 as the solution which we did here.

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A	B	$X_1$	$X_2$
2	3	30	
13	12.33	3	2
20	11	13.33	15

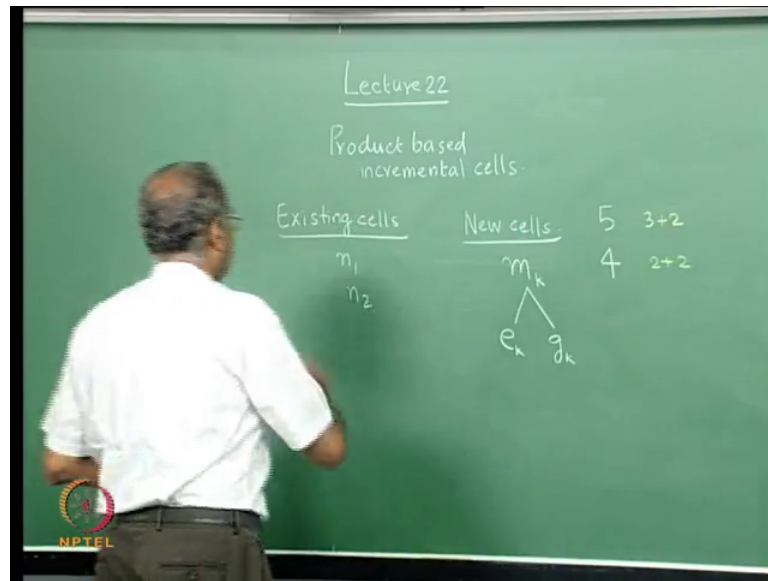
The first product will be made here at 13, first product will be made at 12.33. So, we are fine we can do this. But when we do it for the second product, this is going to be 20, and this is going to be 11.

So now we realize that there is a fair amount of imbalance, when we assign 2 and 3. Here 2 and 3 seems to be for the first product, because these times are balanced. So, there is no need to even actually shift at any point. Because it is very, very close to each other. So, when we do the first product we can assign 2 and 3 here. So, this will be doing at the rate of 13 this will be doing at 12.33. So, effectively 13. Very small amount of inventory buildup here. Or if we control the material, a very small amount of underutilization. But when we do this for the second product, when we do this for the second product, we realize that it is 20 and 11. So, there is a fair amount of imbalance, this is flow. Well, this is reasonably fast. So, what we can do is after a while let both of them work at their full pace.

So, this is the slower one. This will have an inventory buildup. Because this is 11. Then after some time there is an inventory buildup here. So, after some time let the third operator go here. So, that it becomes 3 and 2 after some time. Particularly, when we do the second product. So, that if it is 3 and 2 on the second product, then the outputs will be 26 by 3 will be 9, and this will be for 2, this will be sorry, 40 by 3 is 12.33, 12 point 40 by 3 is 13.33. When we have 3 operators, and we have 2 operators there will be 30 by 2 is 15. So, we should be having 3 and 2 for the second one. And we should not consistently use 2 and 3. Other way to do is use 2 and 3 buildup inventory bring it back shift it to 3 and 2 and do this.

So, when we actually do multiple products, this needs a certain relook. The reason this needs a certain relook is because, we are making this little assumption of  $u + v$ . Because we did  $u + v$  it gave us a solution with 3 3 and 2, or it gave us a solution with 2.063 and 2.937. Then we actually have multiple products. We have to do it product wise, and not do it together. And then depending on the solution that we have, for example we may have; 2 and 3 for the first product and 3 and 2 for the second product, and then we say after some time one operator will move from this cell to the other cell to carry out the second product. So, the solution to this will be 2 and 3 for the first product, and 3 and 2 for the second product. So, this is how we work on operator allocation when we do multiple products and multiple cells feeding to the assembly. So, we will have what is called a dynamic operator allocation because, the operator allocation will change with the product. It will not be the same irrespective of the product.

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Now, let us look at operator allocation in the context of what are called product based cells. Or what we call as incremental cells.

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### Operator allocation to new cells

- Let  $n_1$  be the number of experienced operators available
- $n_2$  be the number of new operators available
- $m_k$  be the number of operators required in new cell  $k$  ( $k = 1, \dots, K$ )
- Let  $e_k$  be the number of experienced operators required in cell  $k$ .
- Let  $g_k$  be the number of new operators required in cell  $k$ .
- Let  $P_{ik}$  be the preference given by operator  $i$  to work in cell  $k$ .
- Let  $X_{ik} = 1$  if operator  $i$  is allotted to new cell  $k$



Now, what happens is; let us assume that we have some existing cells, and we are creating some new cells. So, we have existing cells, and we have some new cells; obviously, the new cells require new operators. When a cell is created we need operators to work. Now we cannot have entirely very new people working in the new cell. So, we need to balance the new cell, by having some people taken from this pool the existing

pool and made to work in the new cells at the same time. We also require new operators who have to start working in the new cells and get trained in the process. So, this talks about this model talks about; how do we balance this number of experienced operators and new operators to work in a new cell. So, that is the primary focus of this model. So, let  $n_1$  be the number of experienced operators available. We assume that people who are working in the existing cells are experienced operators. Let  $n_2$  to be the number of new operators available. This slide shows the notation and the definition of the variables. Let  $m_k$  be the number of operators required an new cell  $k$ .

So, there are  $k$  new cells and  $m_k$  is a number of operators required in the new cell. Let we will also now say that this  $m_k$  is now divided into 2 sets  $e_k$  which is the number of experienced people that I need here in that. And let  $g_k$  be the number of new people that I need here. For example, if there is a if there are 2 new cells, and it requires 5 and 4 operators we could say that this 5 could be 3 plus 2 and this 4 could be 2 plus 2; which means  $m_1$  is 5,  $m_2$  is 4,  $e_1$  is 3,  $e_2$  is 2,  $g_1$  is 2,  $g_2$  is 2. Then we say that we ask all the operators, the  $n_1$  plus  $n_2$  operators. To give preference to work in the new cell. So, let  $P_{ik}$  be the preference given by the operator to work in the new cell.

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Operator	Old cell	New cell
1	8	7
2	4	4
3	7	9
4	6	8
5	5	6
6	7	8

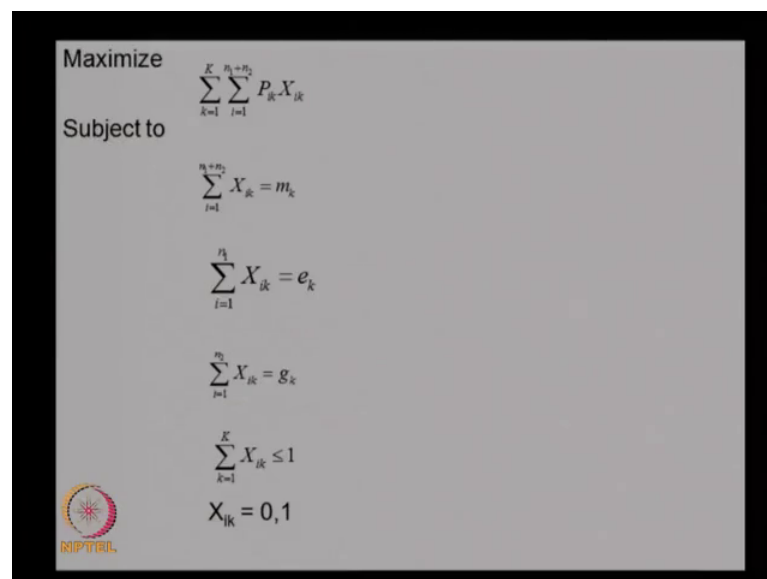
The optimum solution to the binary ILP is  $X_{11} = X_{32} = X_{42} = X_{51} = X_{61} = 1$  with  $Z = 37$ . In this solution experienced operators 1 and 3 are allotted to the old and new cell respectively. New operators 5 and 6 are allotted to the existing cell and new operator 4 goes to the new cell. Operator 2 who worked in the existing cell is not assigned to any cell.

So, for example, we will have something like this. So, we have 6 operators. So, here we are going to assume that the first 3 operators are experienced operators, the next 3 operators are new operators. So, we ask all of them to give some kind of a preference.



So, the objective function is to maximize the operator preference. So, we have this  $P_{ik}$ .  $P_{ik}$  is the preference given by operator  $i$  to work in cell  $k$ . Now here again we have old cell and new cell. So, there are 2 cells, one cell is an old cell the other cell is a new cell. So, we will have this. Right now, in this example I have said that there are 2 new cells, but in the problem, that we are going to do there is only one new cell. So, we could have a number which is like this there is a  $m_1$  which is 5 the  $e_1$  could be 3 I want 3 experience people to new people and so on.

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Maximize 
$$\sum_{k=1}^K \sum_{i=1}^{n_1+n_2} P_{ik} X_{ik}$$

Subject to


$$\sum_{i=1}^{n_1+n_2} X_{ik} = m_k$$

$$\sum_{i=1}^{r_1} X_{ik} = e_k$$

$$\sum_{i=1}^{n_2} X_{ik} = g_k$$

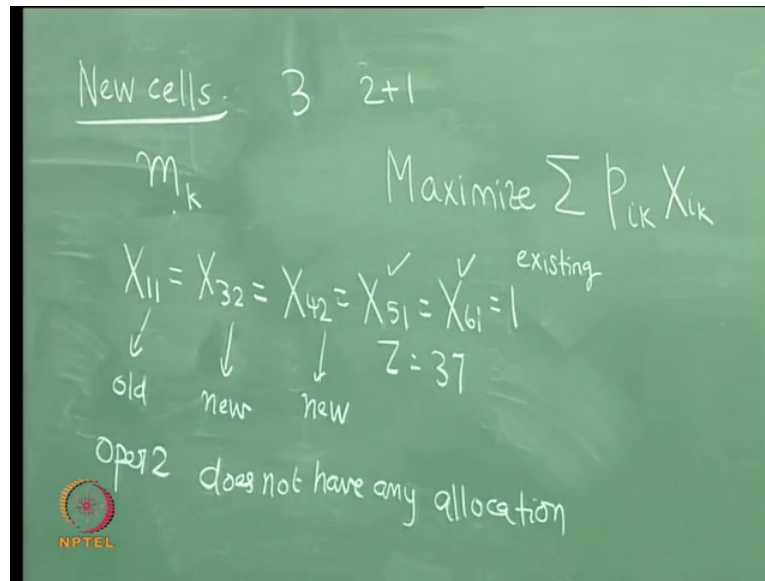
$$\sum_{k=1}^K X_{ik} \leq 1$$

$$X_{ik} = 0, 1$$



So, the formulation will be like this. So, maximize  $P_{ik} X_{ik}$  maximize the operator preference.

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Maximize  $\sum p_{ik} X_{ik}$ . Some of the operators is equal to  $m_k$ . Some of the operators is equal to  $m_k$ . So, in this case we have said that;  $m_k$  is the total number of operators required in new cell  $k$ . So, we assume that 3 operators are required in new cell  $k$ . So, in this example 3 operators are required in new cell  $k$ . And I think it is a 2 plus 1 that we have insisted. So, 3 operators are required in new cell  $k$ .

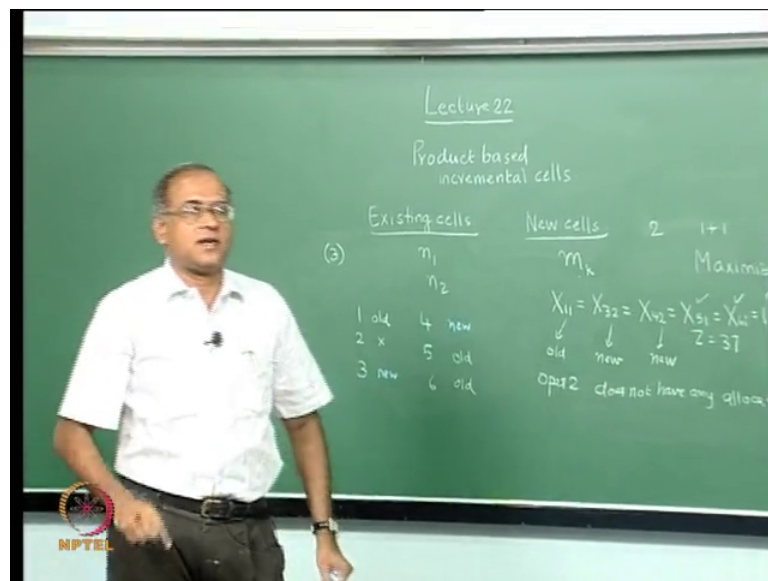
So, some of the operators allocated total should be equal to  $m_k$ . Now there are 3 experienced operators available, out of which 2 have to be given to the new cell. There are 3 new operators available, one has to be given to the new cell, and an operator may be allocated may not be allocated. That is the constraint of this particular problem. So, there are 6 operators, the 3 of them are experienced, 3 of them are new. We are now trying to find out which of the experienced operators are going to go to the new cell, which of the new operators are going to go to the new cell, such that we maximize the preference. And we limit the number of experienced and new operators going to the new cell. So, when we solve this we have a solution  $X_{11}$  is equal to  $X_{32}$  is equal to  $X_{42}$  is equal to  $X_{51}$  is equal to  $X_{61}$  equal to 1. So,  $X_{11}$  equal to  $X_{32}$  equal to  $X_{42}$  equal to  $X_{51}$  equal to  $X_{61}$  equal to 1 with  $z$  equal to 37. So, experienced operators 1 and 3 are allotted to old and new cell respectively.

So, this is old cell, this is new cell. New operators 5 and 6 are allotted to existing cell. So, new operators 5 and 6 go to existing cell old cell. A new operator 4 goes to new cell.

If you look at the formulation, you will see that the objective function maximizes the sum of preferences  $k$  equal to 1 to  $k$  and  $i$  equal to 1 to  $n_1$  plus  $n_2$ ; which means in this example there are 6 operators and 2 cells, and there are exactly 3 people who are if we see here operator 2 who worked in the existing cell is not assigned to any cell. Now this has happened because when we worked with the preferences. When we worked with the preferences, we observe that the new operators 4, 5 and 6 have given large preference for the new cell, while operator 2 has not given a very large preference. Therefore, what this model did is this model has picked all the 3 new operators, and it has said that this new operator will work in the new cell. And these 2 new operators will work in the old cell, one of the persons who is working in the old cell comes to the new cell.

So, there is one gap here one from the old cell comes to the new cell. So, these 3 people are there, and 2 out of the new people go back to the old cell, and operator number operator number 2 does not have any allocation.

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So, if we see carefully, 1, 2 and 3 are old ones, 4 5 and 6 are new ones. So, from this solution, this person is with the old, this person does not get anything, this person goes to new now 4 2. So, this person goes to new, this person 5 1 and 6 1. So, these 2 people go to old. So, we now say that there are 2 operators who are assigned to the new cells, operator 2 does not have any one person. So, the new cell requires only 2 operators,


which is a 1 plus 1. The old cell has 3 people, and one person from the old goes to the new.

One person does not go anywhere. While 2 people who are new, come and replace. The old one. So, this is one type of operator allocation problem, but then at the end of this kind of an assumption, we will have to ask a different question as to what do we do with this person. So, this formulation now has to be redefined, such that all the old people either work in the old cell or in the new cell. And as many people who go from the old cells to the new cells, so many new people will come and replace them. So, we cannot have a situation where an experienced operator who is here is left out. So, this formulation has to be fine-tuned to take care of the fact that; all the old operators are retained the new one some of them are picked to work on old cells and new cells.

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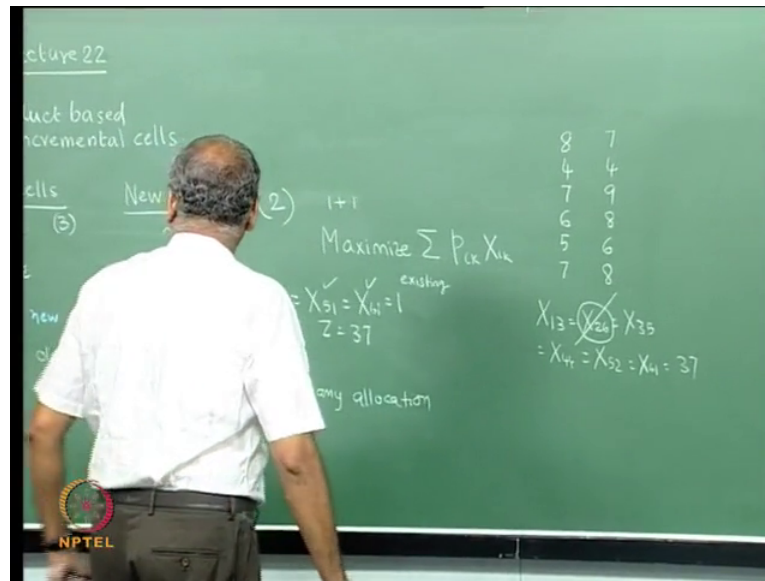
Cell	1	1	1	2	2	2
Operator						
1	-M	8	8	-M	7	0
2	-M	4	4	-M	4	0
3	-M	7	7	-M	9	0
4	6	6	-M	8	-M	0
5	5	5	-M	6	-M	0
6	7	7	-M	8	-M	0

$X_{13} = X_{26} = X_{35} = X_{44} = X_{52} = X_{61} = 1$  with  $Z = 37$ .



So, we will do that, but before we get into that we also show that this can be modeled as an assignment problem. Now this can be modeled as an assignment problem, under the assumption that; there are 6 operators, and there are 2 cells. Now one of them, there are 3 people that are who have to work in the existing cell. There are 2 people who work in the new cell. So, the 6th one will be a dummy, we where you have a 0 coming in the column that represents a dummy.

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The given values are 8 4 7 6 5 7, 8 4 7 6 5 7 and 7 4 9 8 6 8 are the given preferences. So, the way the assignment is constructed is as follows.

Now, we have 3 experienced operators, and we want one of the experienced operators to definitely go to the new cell. So, there are 3 experienced operators. So, we assume that this person who is so one of them will go to the new cell. So, what happens is; the first column talks about the 3 experienced operators having a minus M, and the new operators having this 6 5 and 7. Now what will happen is when one experienced operator goes to the new cell. See here we have a situation where the new cell has one experienced operator, and one new operator should go to the new cells. So, those are given by the columns 4 5 and 6, 6 is a dummy. So, one of the experienced operator should go. So, 1 2 and 3 these operators have preference 7 4 and 9, while the other one is minus M minus M minus M. So, that only one experienced operator goes. Similarly, only one new operator goes. So, for operators 4 5 and 6. One of the columns is 8 6 8 which is this preference, to work in the new cell. The other one is minus M minus M and minus M.

So, that is how the columns number 4 5 and 6, which stand which are actually columns 5 6 and 7, these 3 columns. These 3 columns are explained this way. The requirement in the new cell is only 2. So, only these 2 columns have entries this becomes a dummy. Now these 2 represent the 2 operators for the new cell, these 3 represent the 3 operators for the old cell. So, one of them, one of this has to be a new operator. So, the new

operator preference are listed here. The old operator preference are given as minus M this is a maximization problem. So, it is given as a minus M. So, that the allocation happens in one of this. Similarly, it requires exactly one experienced operator. So, the experienced operator preference is given here, while the new operator preference is given as minus M.

So, that it has definitely one experienced operator now we come to these 3 positions. We come to these 3 positions. These 3 columns; what we say is that now the 2 people that are required here is a 1 plus 1 that is taken care of, whereas, the 3 people that are required here is we say that this should have at least one experienced operator should remain with this cell. Since one experienced operator should remain with this cell, we have one column that has definitely 8 4 7 6 5 7.

So, one experienced operator should remain would mean that this 8 4 7 comes, the minus M minus M minus M comes here, because this will force one of the experienced operators to remain with this cell. This one will force one new operator to come into this cell, because I have a minus M for the experienced. And this will give the third position to either an experienced person or a new person. Which is what this formulation did. So, when we solve this assignment problem, we have the assignment solution which said  $X_{13}$  equal to  $X_{26}$ .  $X_{35}$  equal to  $X_{44}$  equal to  $X_{52}$  equal to  $X_{61}$  equal to 37. Now we go back and observe that while the integer programming had only 5 variables, the assignment has 6 variables. The reason the assignment has 6 variables is; if we look at this 2 comma 6 very carefully this represents operator number 2.

Operator number 2 does not figure here. That is the person was being left out. Now 2 6 implies that there is an allocation at the dummy. Which means this operator does not have an allocation. So, 2 6 simply means this operator does not have an allocation. Which is what is given here, which is what is shown here. Now we look at the rest of them  $X_{13}$ . So, operator one goes to the old one,  $X_{35}$  operator 3 goes to the new one.  $X_{44}$  operator goes to the new one,  $X_{52}$  and  $X_{61}$  they go back to the old ones. So, the same solution is given by the assignment model. So, this particular integer programming problem that we solved here, can be modeled as the assignment problem.

So, the assumptions are for in this numerical example there is only one old cell and one new cell, the new cell requires exactly 2 operators. And exactly one experienced operator


and one new operator. The old cell will have 3 operators, it has 3 experienced people, but at the end of reallocation we have to insist that the old one has at least one experienced person, at least one. So, the solution gave us a situation where 2 new people are coming in, because of the problem assumption it allowed 2 new people to come in, which meant that one existing person does not have any place to go. So, this has to be modified. And we look at a new modified operator allocation problem, where we look at this.

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### Replacing existing operators

- Let  $n_1$  be the number of experienced operators available
- $n_2$  be the number of new operators available
- $m_k$  be the number of operators required in new cell  $k$  ( $k = 1, \dots, L$ )
- Let  $e_k$  be the minimum number of experienced operators required in cell  $k$ .
- Let  $g_k$  be the minimum number of new operators required in cell  $k$ .
- Let  $P_{ik}$  be the preference given by operator  $i$  to work in cell  $k$ .
- Let  $a_k$  be the number of experienced operators available in cell  $k$  ( $k = 1, \dots, K$ )

Let  $X_{ik} = 1$  if operator  $i$  is allotted to new cell  $k$



So, again the new model will look at let  $n_1$  be the number of experienced operators.  $n_2$  be the number of new operators again  $m_k$  is the total number of operators required in the new cell. Again,  $e_k$  be the minimum number of experienced operators required a new cell, and  $g_k$  be the minimum number of new operators required in cell  $k$ . Minimum number of experienced operators required in cell  $k$ , and  $g_k$  is the minimum number of new operators required in cell  $k$ . Now cell  $k$  can be an old cell or a new cell.  $P_{ik}$  is the preference  $a_k$  be the number of experienced operators available in cell  $k$ , now  $X_{jk}$  equal to 1.

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Maximize  $\sum_{k=1}^{K+L} \sum_{i=1}^{n_1+n_2} P_{ik} X_{ik}$

Subject to

$$\sum_{i=1}^{n_1+n_2} X_{ik} = m_k \quad \forall k = 1, \dots, L$$

$$\sum_{i=1}^{n_1} X_{ik} = e_k \quad \forall k = 1, \dots, L$$

$$\sum_{i=1}^{n_2} X_{ik} = g_k \quad \forall k = 1, \dots, L$$

$$\sum_{k=1}^{K+L} X_{ik} \leq 1$$

$$\sum_{i=1}^{n_1} \sum_{k=1}^L X_{ik} = \sum_{i=1}^{n_2} \sum_{k=1}^K X_{ik} \quad X_{ik} = 0, 1$$

Now, we look at this formulation. This formulation is very similar to the earlier formulation. Except for one constraint which is the last constraint.  $X_{ik}$  summation here is equal to  $X_{ik}$  summation here. Whereas, if we go back to the earlier formulation we did not have that constraint. So, this last constraint will ensure something. It will ensure that we have 3 experienced people here. We have 3 new operators who are coming in, now it will ensure that as many number of experienced operators who go from here to the other.

So, many new operators will only come here; which means the differences, if we look at the old solution out of these 3. Only one person went to the new cell. Therefore, only one person will come one new person will come to the experienced cell. This person will remain here. This person will remain here. Whereas, in the earlier model we said that, we need a minimum number of this many experienced person which is 1, we said the other 2 positions can go to 2 new operators. Which actually happened the other 2 positions went to 2 new operators. Who came back into these 2 new operators came back to the old cell. Now the difference that we are going to show here is that if there are 3 if one of them is going to a new cell.


One new operator will come to the old cell. Which means; all the experienced operators will remain either with their own old cells, or they will go to new cells. From the new operator set some of them will go to the new cell. Some of them will replace the



experienced people who have gone from here to here. And some of them will not be allocated at all. We will not have a situation where an experienced person is left out. Which happened in the earlier formulation. So, that is what the present formulation will now do. So, when we solve this present formulation.

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Consider the situation where 6 operators are available. Operators 1 to 3 are experienced and operators 4 to 6 are new. Their preferences to work in an existing cell and a new cell are given in Table 7.7. Each new cell and existing cell should have at least one experienced operator and one new operator. Find the operator allocation to maximize preferences? The new cell requires 2 operators and the operators taken from the existing cells have to be replaced by new operators




So, once again we have the same data that is given here. 6 operators are available 1 2 3 are experienced 4 to 6 are new and so on. This is the same operator preferences that we used.

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### Formulation

- The ILP is to Maximize  $\sum_{k=1}^2 \sum_{i=1}^6 P_{ik} X_{ik}$
- Subject to
- $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} = 3$
- $X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} = 3$
- $X_{12} + X_{22} + X_{32} \geq 1$
- $X_{42} + X_{52} + X_{62} \geq 1$
- $X_{i1} + X_{i2} \leq 1$
- $X_{12} + X_{22} + X_{32} - X_{41} - X_{51} - X_{61} = 0$
- $X_{ij} = 0, 1$




But then if we put an additional restriction, and then we have a formulation which will look like this, minimize maximize the sum of preferences.  $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} = 3$ . So, at the end of the allocation, I should have 3 people working in the old cell. In this case I have 3 people working in the new cell. The new cell should have at least one experienced operator. The new cell should have at least one new operator, which comes from this constraint. The new cell should have at least one new operator, the new cell should have at least one experienced operator. Each operator can go to a maximum of one cell, somebody can be left out. The last one is important,  $X_{11} + X_{22} + X_{12} + X_{22} + X_{32}$ . This quantity represents, the number of experienced operators, who are going to go to new cell. Now that many number has to be replaced by new operators coming to old cell.

So,  $X_{12} + X_{22} + X_{32}$ , should be equal to  $X_{41} + X_{51} + X_{61}$ . So, as many experienced operators go to the new cell, so many new operators come back to the old cell. So, we get into this integer programming problem.

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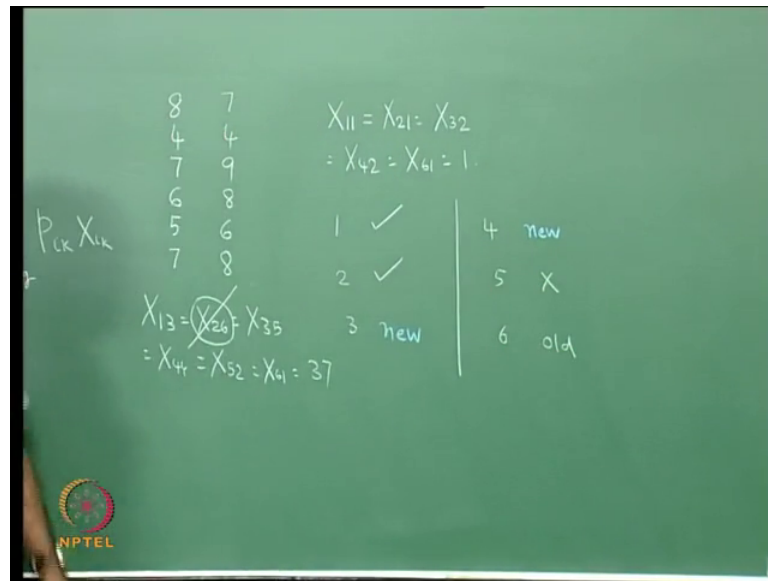
### Solution

- The optimal solution to the Integer linear programming problem is given by  $X_{11} = X_{21} = X_{32} = X_{42} = X_{61} = 1$  with  $Z = 36$ . Operator 3 goes to the new cell and is replaced by operator 6. Operator 4 is assigned to the new cell. Operator 5 is not assigned to any cell.
- This solution is different from the solution in Illustration 7.10 where existing operator 2 was left out. Here all existing operators are retained while a new operator is left out.
- It is also observed that the LP relaxation gives the same optimal solution. In this case the problem can be formulated as a minimum cost flow problem and solved optimally.



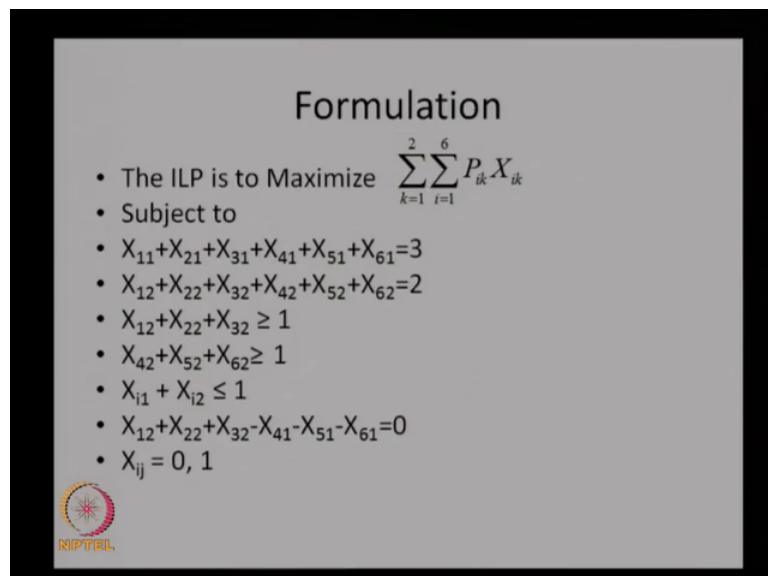
And the optimum solution is given by  $X_{11} = X_{21} = X_{32} = X_{42} = X_{61} = 1$  with  $z = 36$ . So, there is a minor correction that this is not 3, this is 2, this is 2. So, we need 2 people coming here.

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So, the solution is  $X_{11}$ , solution for the new case will be  $X_{11}$  equal to  $X_{21}$  equal to  $X_{32}$ , equal to  $X_{42}$ , equal to  $X_{61}$  equal to 1. So, if we look at operators 1 2 3, which are old operators 4 5 6 which are new operators. Now  $X_{11}$  so, this person remains  $X_{21}$  this person remains,  $X_{32}$  this person goes to the new one,  $X_{42}$  this person goes to the new one. 5 does not have an allocation. So, the reason is this one is still 2 and not 3 this is 2. So, this is 2 and not 3.

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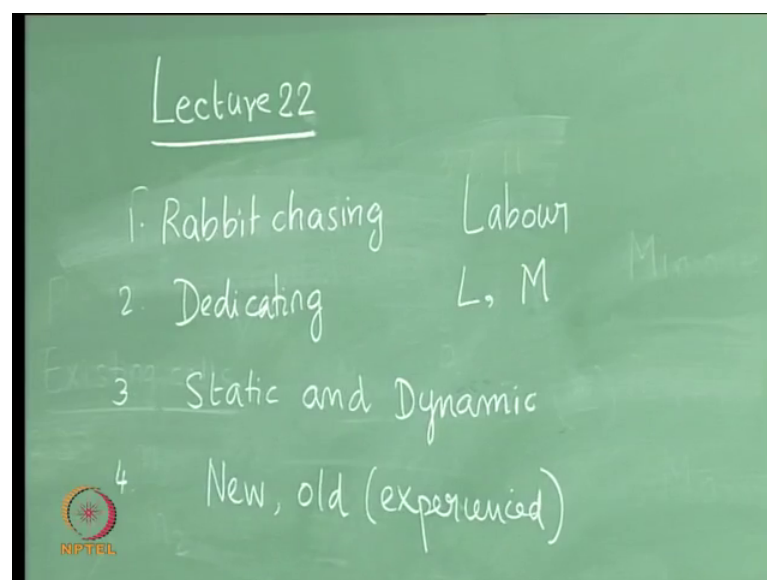


So, this person has no allocation, and  $X_{61}$  is 1. So, this person goes to old.

So, one of this person comes to the new cell, and that person is replaced by a new operator going to the old cell. Out of the 3 new operators this person is chosen to work in the new cell. So, the new cell will have this person and this person. The old one will have this this and this and this person is left out. The objective function comes down because it is a maximization problem, with an additional constraint the objective function value becomes poorer in the case of a maximization problem. So, the next question that we address is somewhere here, this is different which is what I have explained.

Now I P relaxation gives the same optimal solution. In the earlier case we were able to show, that this can be modeled as an assignment problem, now here when we solved as an integer programming problem we got this solution. Now binary I P, now if we solve as a linear programming problem it still will give us integer valued solutions. Which simply means that there is some uni modularity character in the formulation, and on subsequent analysis, one can show that this problem can be formulated as a minimum cost flow problem and can be solved optimally. So, we are not showing the minimum cost formulation, but we are only showing the integer programming, but also saying that when solved as a I P, it would still give us integer valued solutions. So, this kind of brings us to the end of our discussion on operator allocation problems.

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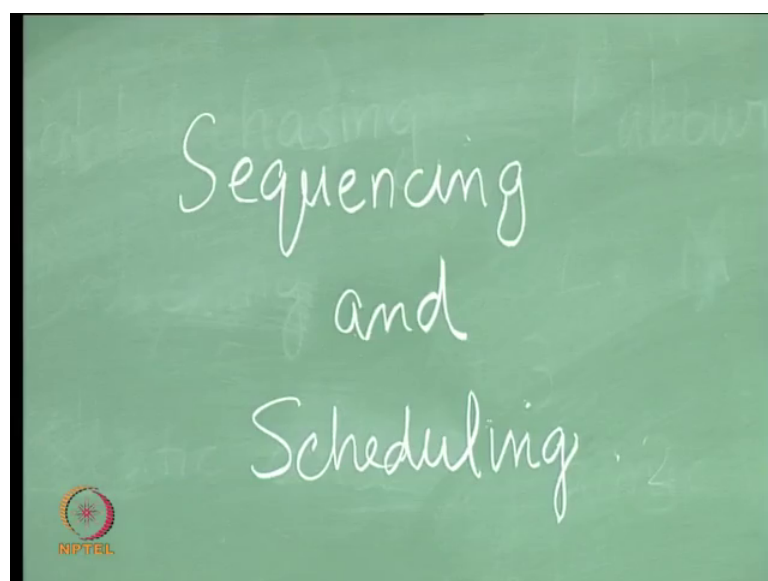
So, a very quick summary of this would say that we started with rabbit chasing. In the context of labor intensive cells, then we looked at dedicating, in the context of both

labor-intensive cells, and in the context of machine intensive cells. Now we looked at very specific models, here we separated the time into loading time unloading time and processing time, and then we showed that operator working cycle versus operation time cycle. We also said here as well as here that the operator paths should not cross, and we could do it for multiple operators. Then we looked at static and dynamic allocation problems, static and dynamic allocation problems.

We looked at case where we have a single product multiple cells feeding to assembly, we also looked at multiple products and multiple cells feeding to assembly. Then we looked at an aspect of new and old operators, or new and experienced operators. Who are to be assigned to cells? We looked at 2 cases. There was one case where we said that this many experienced and new people minimum are required for the new cell. We can have a situation where even in the old cell an existing operator can be replaced. Then we put a further restriction that in the old cell experienced operators should not be replaced. So, how many ever-experienced operators go to the new cell that many new operators come back to the old cell. New operators can also go to the new cell.

So, we put that additional restriction and then we solved it. And we also were able to show that it can be modeled as a; we mentioned that it can be modeled as a minimum cost flow problem. Well the first one can be modeled as a traditional assignment problem with a maximization object.

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So, the next topic that we will actually be seeing is sequencing and scheduling in the sense, if we look at traditional aspects of sequencing and scheduling, sequencing and scheduling are extremely important. Particularly to find out the order in which jobs are parts that are there will have to be taken up for processing. Traditional sequencing and scheduling models or models applied to flow shop type of scheduling situations, and job shop type of scheduling situations. Job shop type of scheduling situations are relevant particularly to the functional layout.

So, the first question that; we will have to look at is what is the relevance of these 2 types of scheduling models and algorithms. Which are abundantly available in the literature are they applicable? In the context of cellular manufacturing, or any other modern manufacturing methodology. In addition, how are they to be modified? Or how can they be modified to fit into the context of cellular manufacturing, or any other modern manufacturing methodology? The third question is; are there very specific sequencing and scheduling ideas which are different from what we see from basic flow shop and job shop scheduling? Which can again be brought into the context of this? 4th and a far more important question is; this scheduling the key thing in areas like cellular manufacturing, or is it line balancing, line balancing in the sense where by dedicating equipment to operators.

We are essentially doing some kind of a dynamic line balancing. Plus, in every case in the in the early part of this lecture, and in the previous lecture by allocating operators to cells we were trying to bring down or we were trying to minimize the maximum time taken; which is like a line balancing problem. Now we will have to look at the relevance of scheduling and sequencing, also in the context of the line balancing idea. So, we will see the relevance of traditional algorithms for scheduling and sequencing, in the context of cellular manufacturing, and minor modifications that one can think of we will address all these issues in the next lecture.