Manufacturing Systems Management
Prof. G. Srinivasan
Department of Management Studies
Indian Institute of Technology, Madras

Lecture - 18
Product based cells

In this lecture we continue the discussion on Product Based Cells.

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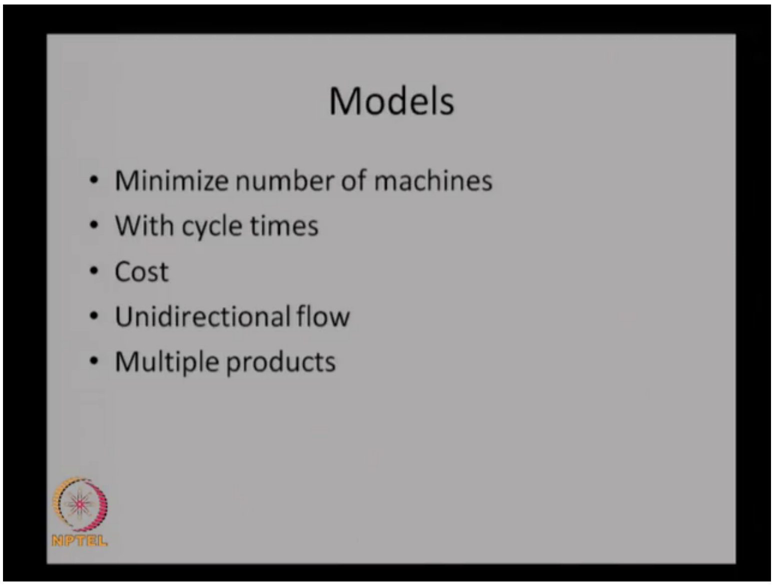

Under product based cell we will be looking at various models which are shown in the slide the first model would be to minimize the number of machines such that all operations can be carried out; we saw this model and its solution to the example in the previous lecture.

Other models would include minimizing the number of machines considering a cycle time restriction minimizing number of machines considering cost and cost as well as cycle time restrictions building models that take into account unidirectional flow and extending this model to multiple products. So, these will be the topics or aspects that we will be looking at under product based cells.

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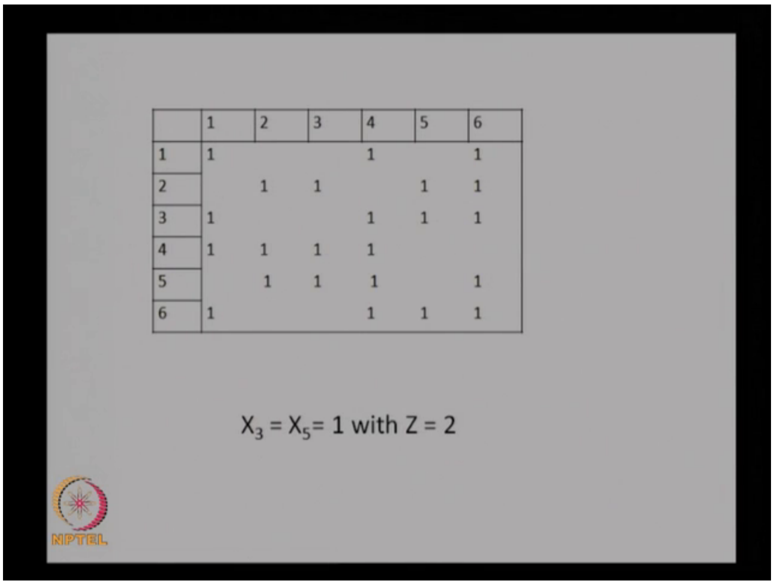

In the previous lecture, we saw the first model that minimizes the number of machines to carry out the operations related to the product the incidence data or the data related to product based cells is also given in the form of a binary matrix 1 set matrix is shown in the picture here again we have 6 rows and 6 columns; the 6 columns represent 6 operations that have to be performed to complete a project the 6 rows represent the machines, but with a slight difference.

Here, we would say that the first column represents operation one and the operation number one can be carried out either on machine one or on machine 3 or on mission 4 or on machine 6 unlike the earlier incidence matrix where the operation to do that particular part have to be carried out on all the machines that are shown in the column. So, it is enough to choose one of them. So, the problem is to choose minimum number of machines such that all the operations can be carried out.

So, we saw the first model that would minimize sigma X i; if X i equal 1; if machine i is chosen and we saw a solution with X 3 equal to 1 and X 5 equal to 1 with Z equal to 2 that X 3 equal to 1 means machine number 3 is chosen X 5 equal to 1 means machine number 5 is chosen. And we can now see that all the 6 operations can be carried out either on machine 3 or on machine 5 because machine 3 can carry your operations 1, 4, 5 and 6 while machine 5 can carry out 2, 3, 4 and 6.

We actually have more than one optimal solution. In fact, a straight forward optimal solution would be to choose 4 and 6, 4 can do 1, 2, 3, 4 and 6 can do 4, 5, 6, there are several optima, but one of the optima is shown in this solution we now move to the second model where we try to have a cycle time restriction.

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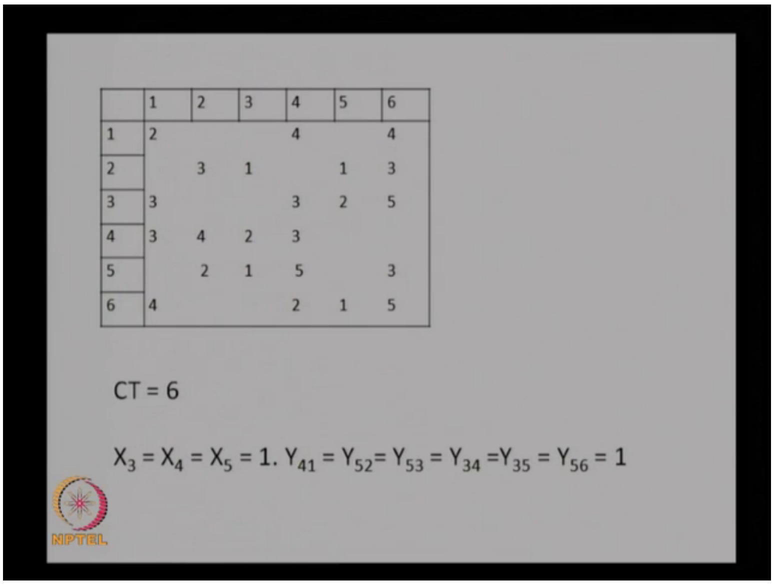

	1	2	3	4	5	6
1	2			4		4
2		3	1		1	3
3	3			3	2	5
4	3	4	2	3		
5		2	1	5		3
6	4			2	1	5

CT = 6

$X_3 = X_4 = X_5 = 1.\ Y_{41} = Y_{52} = Y_{53} = Y_{34} = Y_{35} = Y_{56} = 1$

Now, the extension here is that there are again 6 operations to be performed to complete the project once again there are 6 machines the incidence matrix is similar in terms of its size and in terms of its character in the sense that it is enough to choose one machine out of the many to do that the only difference is that there is another number inside which represents the processing time. For example, if operation one can be is done in machine number one it would take 2 minutes say while it will take 3 minutes to do it on machine number 3 and 4 and would take 4 minutes to do on machine number 6, but it is still enough to take it to one of these machines and complete the operation.

Now the decision is which are the machines to be chosen or what is the minimum number of machines to be chosen such that all operations are performed or carried out using these machines and a cycle time or time taken on each of the machines is less than or equal to 6.

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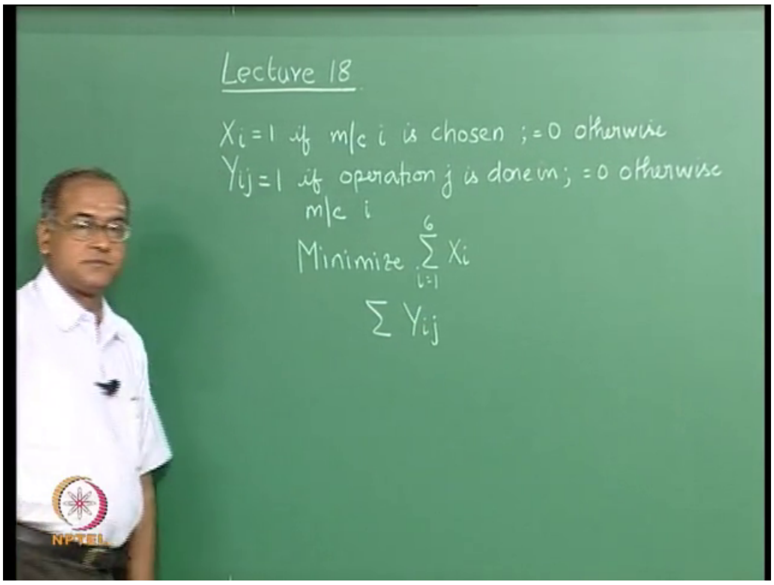

So, this model would be like this. So, we will have X i equal to 1 if machine i is chosen and we now have to allocate the operations to the machines. Therefore, Y ij equal to 1, if operation i if operation j is done in machine i. Now we want to choose minimum number of machines such that all these operations are carried out. So, the objective function would be to minimize X 1 plus 2 plus 3 plus 4 plus 5 plus 6 X rays are binary.

So, we will let us say equal to 1; if machine i is chosen equal to 0, otherwise similarly equal to 0 otherwise. Now the cycle time on each machine on each chosen machine should be less than or equal to 6; now each operation is assigned to only one machine.

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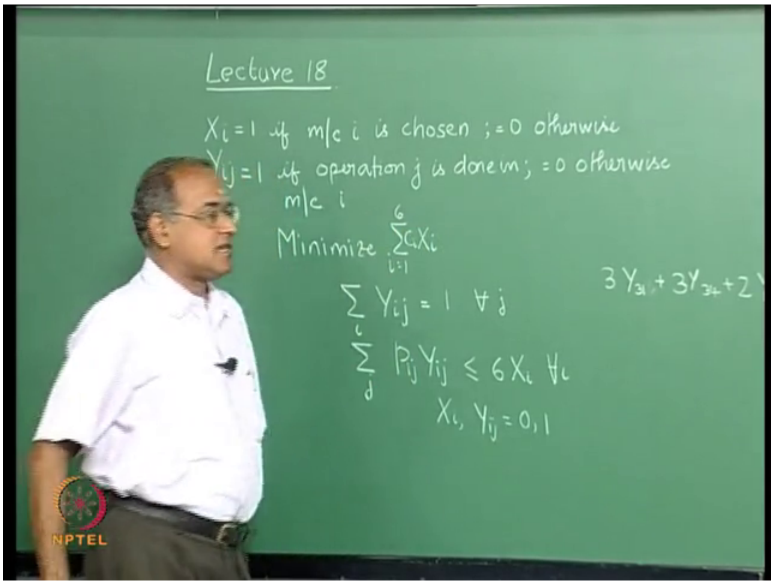

So, if we take the first operation sigma Y ij summed over i is equal to 1 for every operations j. So, if I take the first operation, then I will have Y 11 plus Y 31 plus Y 41 plus Y 61 is equal to 1 indicating that operation number one goes to only one machine out of machine number 1, 3, 4 and 6.

So, this takes care of the fact that each operation is assigned to only one machine we also have to ensure that the cycle time restriction is met. So, summation P ij Y ij summed over j is less than or equal to 6 X i, let me explain this let us take one of the machines let us take machine number 3 to illustrate this; so 3 into Y 31 plus 4 into Y plus 3 into Y 34 plus 2 into Y 35 plus 5 into Y 36.

Let me write the constraint one of the constraints out of the set corresponding to machine number 3. So, this is part namely i.

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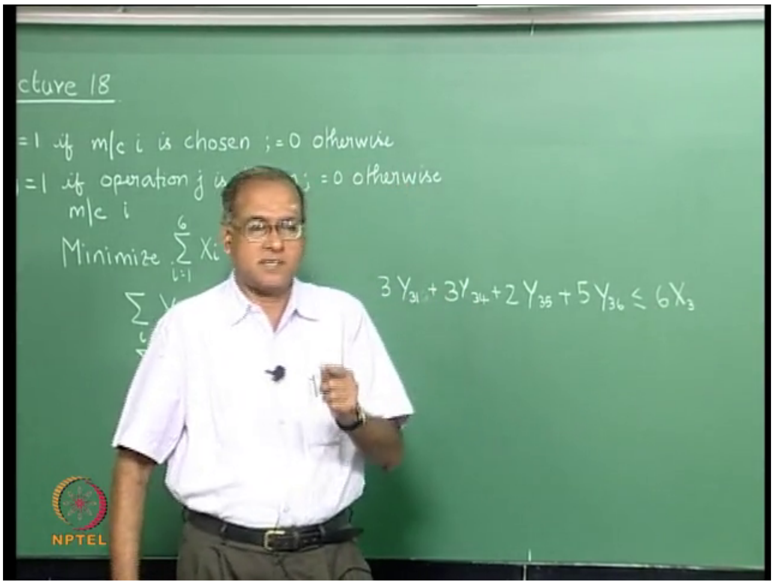

So, corresponding to machine number 3, it would look like 3 Y 13 plus 3 Y 31 plus 3 4 plus 2 Y 3 5 plus 5 Y 3 6 less than or equal to X 3 now if operation one is assigned to machine number 3, it will contribute to 3. Similarly, if any of these values take one which means the corresponding operation is assigned to that machine it will take up so much of time.

So, the left hand side is the time taken on machine number 3 the right hand side is the time available on machine number 3 maximum which is 6, but importantly it has to be 6 X 3 because only if this machine is chosen we can assign operations to that machine. So, only if X 3 equal to 1, then 6 units of time is available on machine 3 if X 3 equal to 0, then 6 machines; 6 units of time is not available only 0 units of time is available. And therefore, we will not be able to allocate any operation on a machine that is not chosen. So, this would contain for every j in our case 6 constraints corresponding to each of the columns and this would have another 6 constraints corresponding to each of the rows or of the machines and then we will have X i Y ij equal to 0 1 or binary.

So, we can solve this optimization problem to try and get a solution X 3 equal to X 4 equal to X 5 equal to 1 which is shown again here for a cycle time of 6. Now 3 machines X 3, X 4 and X 5 are chosen, we can also the operation assignment in that solution.

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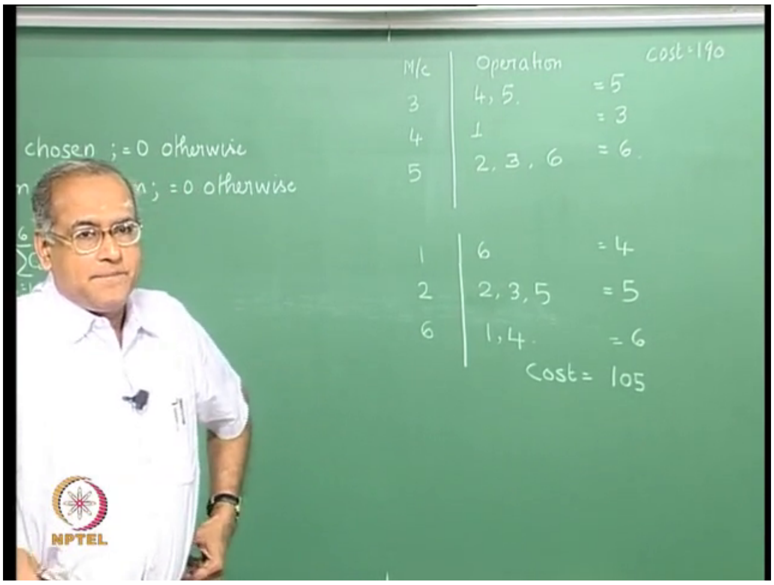

So, the solution has machine number 3 chosen, machine number 4 chosen, machine number 5 chosen; now operate X 4 1 is equal to 1. So, operation one goes to machine number 4. So, operation one goes to machine number 4 5 2. So, operation this notation says that 2 and 3 go to machine number 5 5 6 also goes to machine number 5 and 4 and 5 go to machine number 3.

So, the total times required are if we go to machine number 3, 4 and 5 go to machine number 3. So, time equal to 5 here machine one operation one goes to machine number 4, so another 3 units for machine number 5, 2, 3 and 6, 2 plus 1, 3 plus 3, 6. So, for each machine the cycle time is less than or equal to 6. Now this is the assignment that we have now for example, if we go back to the previous solution which has X 3 and X 5.

Now we clearly we can say that if we add this operation one onto machine 3 the time will exceed 6. So, X 3 X 5 is not a solution. So, if we put a cycle time restriction of 6, then we will need a solution which has 3 machines with maximum cycle time 6 given here. So, this model helps us to analyze to find out the minimum number of machines that can carry out the setup operations to complete the product such that there is a cycle time restriction.

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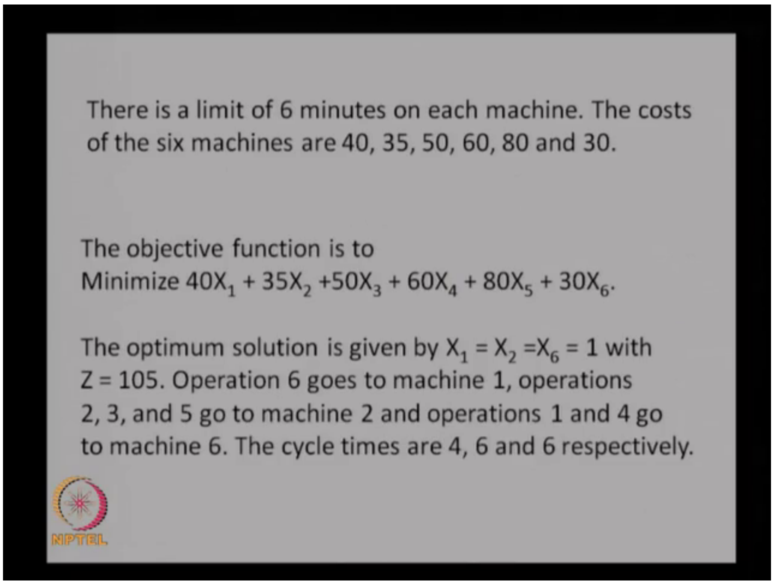

The next model is if we assume that these machine are all bought out machine that these machines are going to be bought. So, instead of minimizing the number of machines that we need it is now necessary to minimize the total cost of buying these machines if these machines are bought from outside. So, what should be the money that we should be spending such that we buy enough machines and those machines are enough to make or produce this with a cycle time restriction of 6.

So, when we look at such a problem essentially everything is the same except that the objective function instead of minimizing sigma X i will now minimize Ci X i where ci is the cost of each of these machines. So, the objective function will change to 40 X 1 plus 35 X 2 plus 50 X 3 plus 60 X 4 plus 80 X 5 plus 30 X 6. So, this will be the objective function if the costs of these machines are 40, 35, 50, 60, 80 and 30.

So, these costs are given the objective function is to minimize the total cost. So, when we look at a solution that minimizes the total cost and solve this optimally we now have a solution which has X 1 equal to X 2 equal to X 6 equal to 1 operation 6 goes to machine one operations 2, 3 and 5 go to machine 2 and operations 1 and 4 go to machine 6. Now let us look at this figure and find out that the cycle time operation 6 goes to machine one. So, cycle time equal to 4 that comes from operation 6 go into machine number one cycle time is 4 operations 2, 3 and 5 go to machine 2, 3 plus 1 plus 1 which is 5 and operations 1 and 4 go to machine number operations 1 and 4 go to machine number 6 which is 4 plus 2 6. So, here operation 6 goes to machine number one which is given by this 4, 2, 3 and 5 go to machine 2 are given by these and operations 1 and 4 are given by these.

So, we have a cycle time equal to 6 in this case the cost is equal to X 1 plus X 2 plus X 6 that would give us 40 plus 35; 75 plus 30; 105. Now the previous solution shows again 3 machines again has a cycle time of 6, but when we computed this solution we did not minimize the total cost here we minimize the total cost object to this. So, this is actually an alternate optima to the other one.

But then if we evaluate the cost of 3, 4 and 5 the cost would be 50 plus 60; 110 plus 80; 190; therefore, this is not optimal to the other one even though this has a cycle time of 6 and so on.

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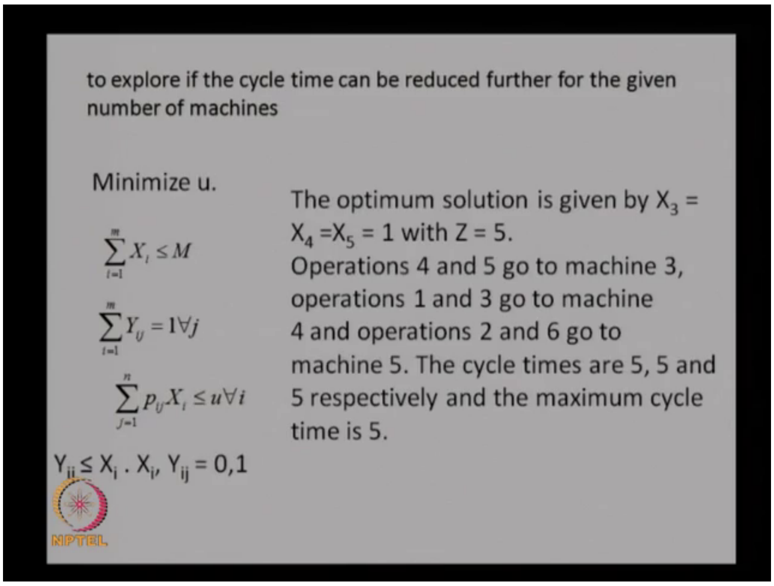

So, we are now seeing a model where the cost of buying these machines has also been incorporated into the solution the next one is this. Now if we look at this solution this solution did not consider cost of buying these machines this solution was for the problem that is given here where there is no cost given. So, we minimize the total number of machines that are needed with a cycle time restriction of 6 and each operation is assigned to one machine there is a cycle time restriction of 6 and we minimize the total number of machines that are required.

Next thing would be to see that now this gave us a solution with 3 machines. So, this gives rise to 2 interesting problems one thing that we know is if we reduce this cycle time further instead of 6 if I had given 5, then I may require 3 machines or I may require 4 machines, if on the other hand, I increase the cycle time I may still require 3 or I may require 2, but a related problem would be if I am choosing 3 machines, I have a solution with cycle time equal to 6.

Now, if I decide to choose 3 machines can I have another solution with cycle time equal to 5. So, there are 2 ways of solving this problem one is to see one is to change the cycle time to 5 and solve this and see whether I am getting 3 machines the other is to say that I am going to choose 3 machines what is the minimum cycle time that I can achieve if I choose 3 machines. So, that is another problem. So, we will now look at this problem to explore if cycle time can be reduced further for a given number of machines not the same machines, but for a given number of machines with 3 machines can I reduce the cycle time further.

So, if I do that I have I have the formulation has given there; now sigma Y ij equal to 1 for every j; now this means every operation is assigned to only one machine; now this says that every operation is assigned to one machine.

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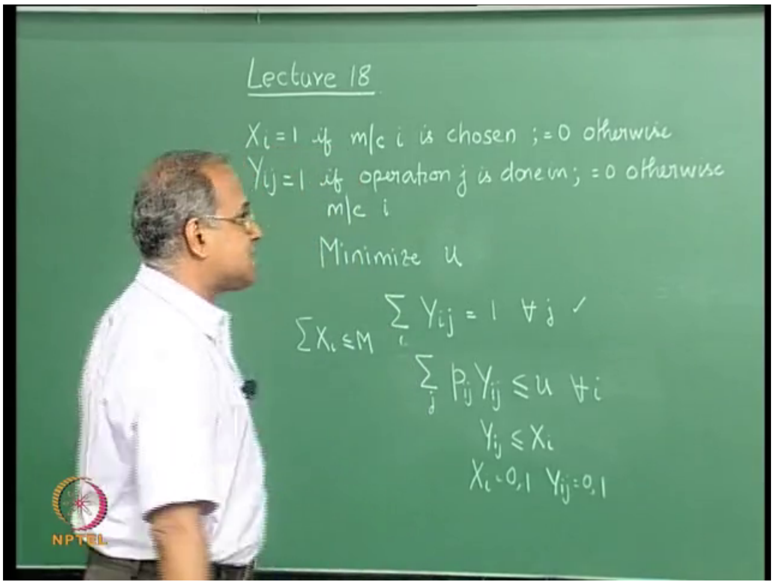

Now, the next one is if machine i is chosen if machine i is chosen, then the workload on machine i would be sigma P ij X j if machine i is chosen then the workload on that machine would be P ij X j P ij Y ij Y ij equal to 1 if operation j is assigned to this machine. So, this is the cycle time summed over j for machine, if I take a particular machine i the load on this machine would be P ij Y ij if operation j is assigned to this machine, then P ij Y ij summation is the workload on this particular machine. Now this should be less than or equal to sum u where u is the maximum workload u is the maximum workload and I can assign an operation j onto machine i only if machine i is chosen. So, Y ij less than or equal to X i and each of these workload should be less than or equal to u. So, u would represent the maximum workload on all of these machines and we want to minimize that. So, minimize u.

So, the formulation there should read minimize u subject to sigma X i less than equal to M sigma Y ij equal to 1 P ij Y ij less than or equal to u and Y ij less than or equal to Xi; X i binary X i equal to 0 1 Y ij is also equal to 0 1. So, this is the correct formulation which now says if Y ij equal to 1, then this is workload on each of these machines for every I and I can assign operation j to machine i only if machine is chosen. So, less than or equal to X i and then we also have another restriction that sigma X i is less than or equal to m.

So, I should have a minimum of or a maximum of only 3 machines. So, with maximum of 3 machines how do I allocate this such that the workload on each of the machine is less than or equal to u and that u we try to minimize. So, we solve this we get a solution same solution X 3 equal to X 4 equal to X 5.

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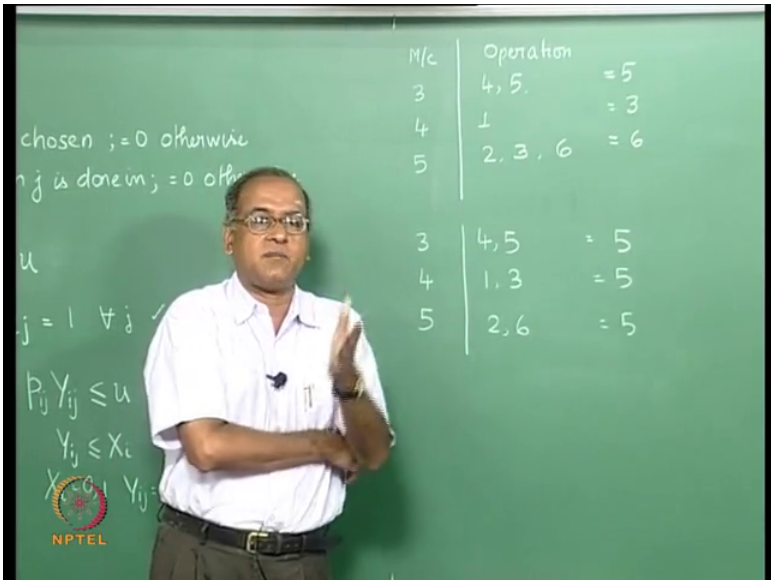

So, we have the solution X 3, X 4, X 5 operations 4 and 5 go to machine 3 1 and 3 go to machine 4 and 2 and 6 go to machine 5 that is the only difference.

We get the same solution; so except the distribution of operations to machines have now changed. So, 4 and 5 go to machine 3. So, machine 3 is here operations 4 and 5 gives a cycle time equal to 5 machine 4 is here operations one and 3 one and 3 gives us a cycle time of 5 and machine 5 is here 2 and 6 would give 2 and 6 also gives a cycle time of 5.

We actually have the same solution except that the operation distribution has changed now here it gave a solution with cycle time equal to 6, because the cycle time restriction was 6. And therefore, it did not bother to further optimize it and bring it down to the maximum of 5 unless that is explicitly asked for by the formulation.

Now in this case or in this formulation for which this is a solution, we have explicitly asked to minimize the cycle time which means you minimize the maximum of the cycle time by distributing the operations in such a manner here we gave a cycle time restriction of 6. So, it gave us a solution with 6, it did not bother whether there is a solution where all of them are less than or equal to 5.

So, when that was explicitly asked it gave us this kind of a solution. So, we have now seen 2 important aspects actually 3 minimum number of machines minimum cost and cycle time. So, we have seen a model where given a cycle time we minimize the number of machines given a cycle time we minimize the total cost given the number of machines we minimize the cycle time. Now we could also build similar models were given a budget restriction we minimize the cycle time that is possible give a budget restriction we also minimize the number of machines that can be bought and so on.

So, those models are logically built, but the basic idea is the same whenever you try to minimize a cycle time you have a minimize u or t or whatever variable we keep each cycle time is less than or equal to u. If there is a budget restriction C j X j sigma less than to the budget restriction either the given cycle time restriction, then each of these will be less than or equal to the given cycle time there we try to minimize the total cost.

So, we could use some of these and try to formulate different problems depending on what the objective is one such possibility is to minimize the number of machines as the objective subject to a given cycle time restriction and a given budget restriction another problem would be to minimize the cycle time given a certain budget restrictions. So, here we minimize the cycle time subject to number of machines just. So, this problem can everything else will be the same except that sigma Ci X i is less than equal to b means that is a budget restriction. So, that would lead us to another optimization problem that we can solve.

So, now coming back to the earlier slide now we have seen 3 aspects of what has been shown here which are minimizing the number of machines and then considering cycle times and considering costs we now move to unidirectional flow.

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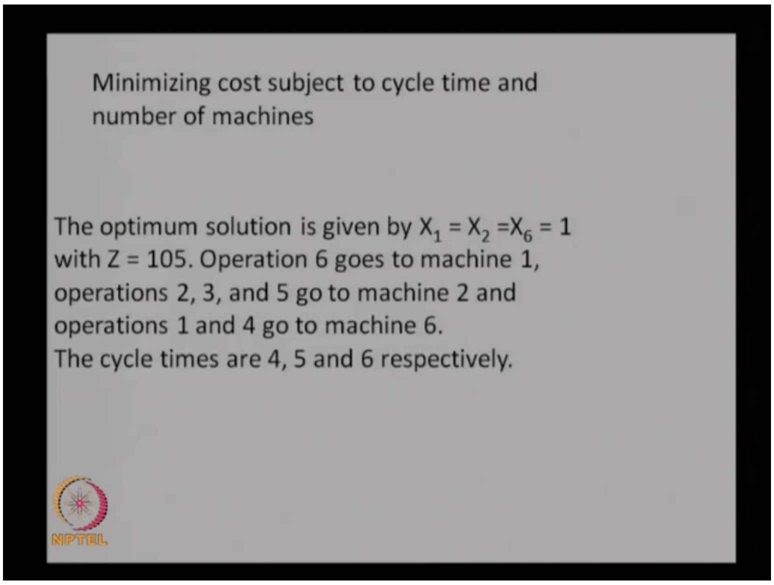

Now, if we look at this solution we look at any of these solutions if we look at any of these solutions let us take this one. Now if we assume that this product is made after by carrying out 6 operations and the 6 operations are to be carried out following the unidirectional flow.

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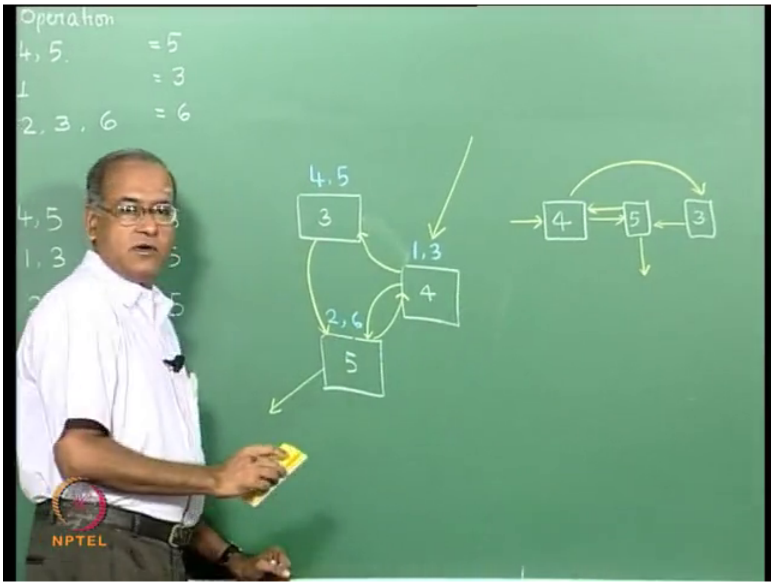

So, if we for example, build a cell with machines 3 4 and 5 now if we say that our cell will be like this machine 3 machine 4 machine 5 and if we trace the direction of flow one and 3. So, one and 3 happen here 4 and 5 happen here 2 and 6 happen here.

Now, if we trace the product flow for this solution we will have something like this the product comes from somewhere one then it goes to machine for operation 2, this is operation one then from here it goes back to 3 goes back to 4 back 2. So, it does 4 and 5 here and then it comes back to 6 and it leaves. So, this will be the direction of flow this is not unidirectional because right here it follows and comes back, then it goes in the opposite direction and so on or this can be rewritten as I start with 4 because that is where the first operation comes.

Then I go to 5 and then I go to 3 if I do this then I start here for the first operation, then I go to 5 for the second operation then I come back to this for the third operation, then I go to this for 4 and 5 and then I come back to this and then I leave. So, I do not have unidirectional flow well unidirectional flow is extremely important for the sake of control.

So, now let us do a formulation where we explicitly consider unidirectional flow here.

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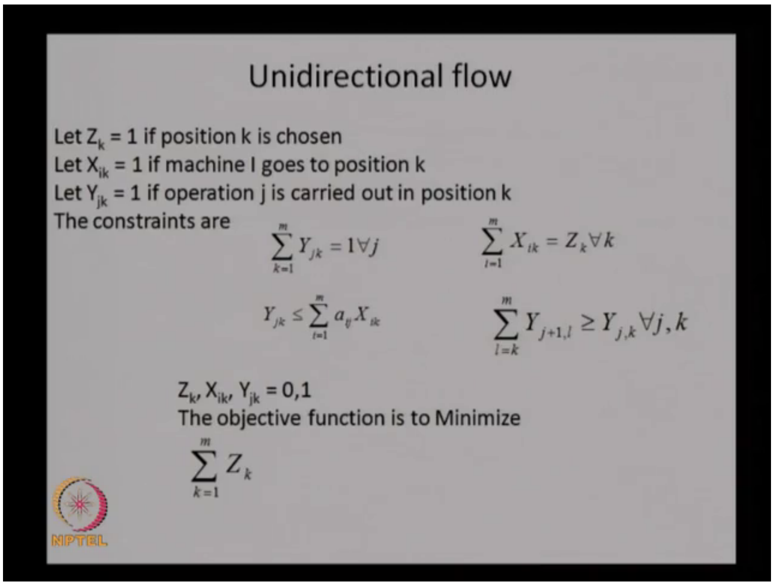

Now, when we explicitly bring the unidirectional flow we define variables Z k equal to 1.

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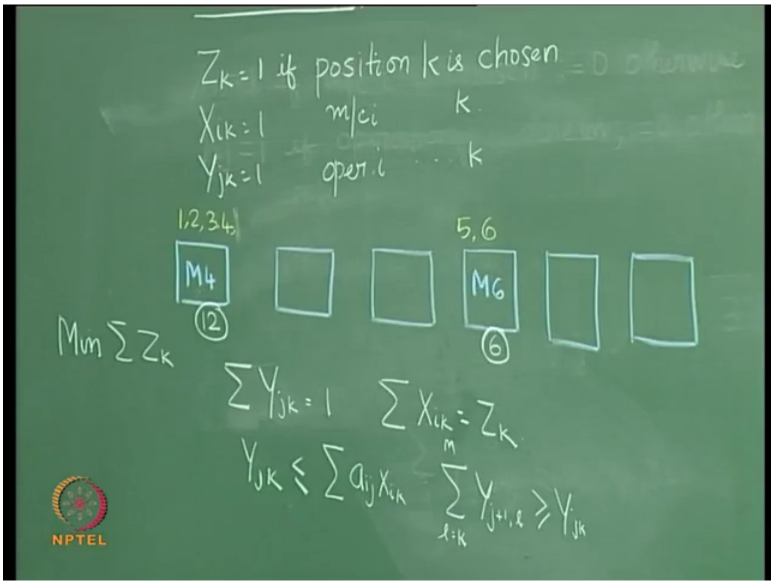

If position k is chosen X ik is equal to 1, if machine i goes to position k and Y ik equal to 1 if operation i is done in position k.

So, what are we trying to do here in this formulation there are 6 machines. So, we assume that there are 6 positions. So, we call these 6 positions as it is it is something like I have 6 positions. So, this is 1, 2, 3, 4, 5, 6 positions.

So, depending on the number of machines I choose I will choose as many positions as the number of machines. So, Z k equal to 1 if position k is chosen X ik equal to 1 if machine i goes to position k. So, if I choose 3 positions then it is like saying I choose the first 3 or some positions let us say if I choose finally, 3 out of this 6 Z k's are one then it is like saying any 3 of these 6 Z k's can be 1. So, these 3 can be 1 and I choose 3 machines and I find out which machine will go to this position which machine will go to this which machine will go to this and then which operation will go to which of these machines such that we get unidirectional flow.

Now, the first one is this sigma Y ik equal to 1. So, each operation will go to only one position that shown here sigma Y ik equal to 1 sigma X ik equal to Z k; Y ik is less than or equal to sigma A ij X ik and the last one sigma Y j plus 1; l is greater than or equal to Y ik l equal to k to M Y ik and the objective is to minimize Z k.

So, let me again explain these constraints and the objective function. So, the objective function minimizes the number of positions chosen which means it minimizes the number of machines because each position will have exactly one machine now minimize the number of positions is given here this ensures that each selected position will have exactly one machine if this position is chosen then Z one is one. So, this will force one of the 6 machines to come here this is if Z one is chosen then X 1 one plus X 2 one plus X 3 one plus X 4 one plus X 5 one plus X 6 one equal to 1. So, one of the machines will have to come and sit here one of the machines will have to come and sit here

If this position is not chosen then no machine will come here and sit here. So, this will take care of the fact that if the position is chosen then the machine will come and sit here sigma X ik I equal to 1 to M for every k for every chosen position one machine will come and sit here. So, this is taken care this is also taken care what does this do now when I assign an operation to opposition then I should have a machine in that position and that machine should be capable of doing the operation.

So, if I assign operation one to this position then this position should be chosen that is the first one which is taken care by Z k and you should have a machine that is capable of doing that there. So, because this position is chosen this will force one machine to sit here and if operation one is assigned here then that machine should be capable of doing operation one so; that means, you will look at operation one what are the machines than can do one 3 4 and six. So, this is that A ij. So, this will mean one into X 111 into X 131 into X 14. So, one of them, it will have to do and this is anyway binary. So, there is no problem the summation is all right.

So, it should do one of these. So, this will make sure that when an operation is chosen and put in a particular position the position has a machine which is capable of doing that operation that capability comes from the A ij which itself is a 01; it is a it does not processing time it; it is a binary A ij is a binary parameter which says whether machine I can do operation j or not. So, this will ensure that we are putting the right machine in and the machine is capable of doing the operation the last one will ensure unidirectional flow. So, if I am putting operation fourth operation let us say in the second position then I have to ensure their operations 1, 2 and 3 are already put in positions one and two. So, that unidirectional flow is taken care; for example, that would prevent if operation 6 is going to this position then it means all the other earlier operations have to come to this position then you may not find machines capable of doing all of that. So, this takes care of the unidirectional flow.

The real unidirectional flow comes only from this particular constraint Y j plus 1; l sigma is greater than or equal to Y jk. So, this ensures that the unidirectional flow is maintained for example, if the previous one if I; if the forth operation is put here then the fifth operation can be put either here or here or here or here if the fifth operation is put here then the sixth operation can be put here or here or here cannot be put there that is where that summation for j plus 1 comes greater than or equal to Y jk. So, that will ensure unidirectional flow.

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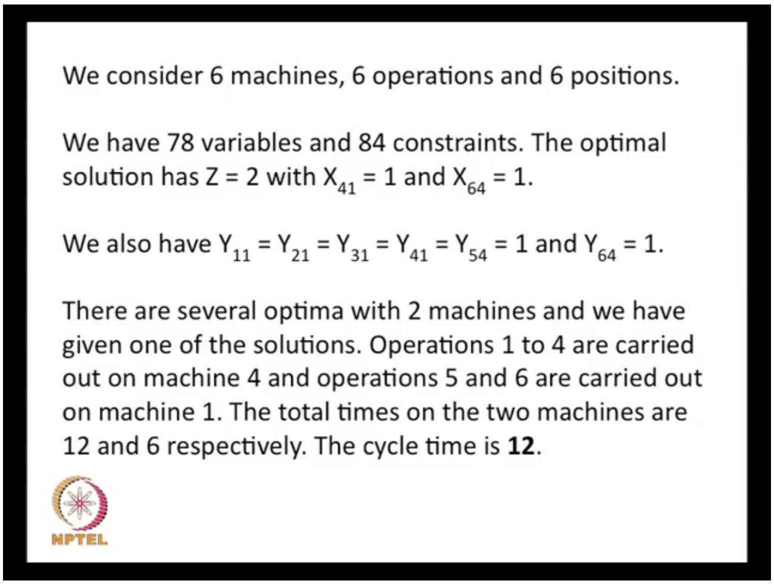

So, now the all these are binary we now try to minimize the number of positions. So, we go back if consider 6 machines, 6 operations and 6 positions, then we have 78 variables and 84 constraints; the optimum solution has Z equal to 2 with X 4 1 and X 6 4 is equal to 1. So, the optimum solution will be like machine 4 goes to position one and machine 6 goes to position 4 these are the machines now what are the operation Y 11, Y 21, Y 31, Y 41, Y 51 equal to 1 and Y 64 equal to 1 is another solution. So, 1, 2, 3, 4, 5 and 6 these are the operation assignments.

Now, let us go back to this M 4 sorry. So, this would give; would come here solution is written incorrectly. So, this would come here. So, operations 1, 2, 3 and 4 will go to M 4 which will give us a cycle time of 3 plus 4; 7 plus 2; 9 plus 3; 12 is the cycle time here and 5 and 6 will go with cycle time equal to 6. So, total cycle time will be equal to 12. So, this is the solution. So, now, what happens to all of these positions nothing happens no machine is chosen because we try to minimize the number of machines.

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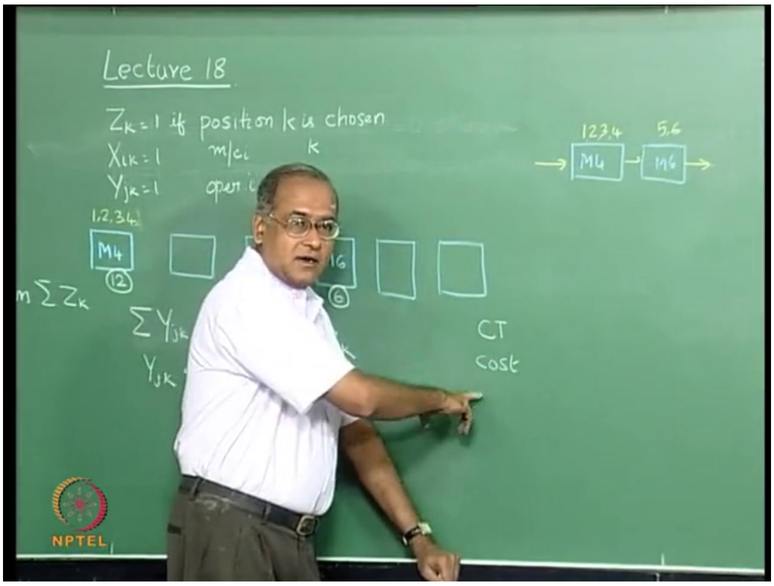

So, finally, on implementation what we will do is we will only have 2 cells with or 2 machines with M 4 here and M 6 here, it will enter do 1, 2, 3, 4 come over here do 5, 6, and it will go out. So, this is how we bring in unidirectional flow into the model suppose there are a few things that we have not done explicitly in this model we have considered unidirectional flow, but we have not put the cycle time restriction they did not have a constraint with said that cycle time should be less than or equal to something.

So, it gave us a solution with cycle time equal to 12 cycle time is the maximum between 12 and 6. So, it gave us a solution with cycle time equal to 12 now we put another cycle time restriction constraint which would say sigma Pjj Y ij less than or equal to some cycle time restriction, then that would forces to operate to allocate fewer operations to opposition which would force us pick an extra position.

And then put an extra machine on that we could also have a cost restriction we could have another constraint which will say that yes I minimize the number of machines, but the total money that I spend on the machine should be less than or equal to something. So, that might force me to buy a machine that is little less expensive than the machine that I can put in and in the process it may force me to use another slot which is because every constraint that you bring in is only going to make the solution slightly worse. So, we have seen only one model which is a basic model for unidirectional flow this is the famous unidirectional unidirectionality constraint.

Now, we could add depending on the situation we could add cycle time, we could add cost of the machines and then we could minimize the number of machines sometimes we simply replace this by the positions and be a little careful that we could have an objective function that minimizes cost subject to fixing certain number of positions. So, we could have minimize C j X j and then we would have something like sigma Z k less than or equal to 3 which means I choose 3 or less positions.

So, you could minimize a total cost subject to number of machines subject to cycle time you could minimize the cycle time subject to a budget restriction subject to number of machines and you could minimize the cycle time or you could minimize the number of machines and add cycle time restriction and cost restrictions on this. So, the unidirectional flow model is one of the most important models in cellular manufacturing where we constraint the problem or we make sure that unidirectional flow is maintained, but sometimes the solution with unidirectional flow could be costlier than the other solution that we have seen which did not have unidirectional flow, but unidirectional flow is the way by which we maintain control in a cell in addition to the ownership and responsibility that is associated with the cell.

So, we have seen models that minimize number of machines we have seen models that consider cycle times, we have seen models that consider cost of the machines and we have seen models that consider unidirectional flow now within unidirectional flow we have seen a certain type of an optimization model which we have seen and we have given the solution.

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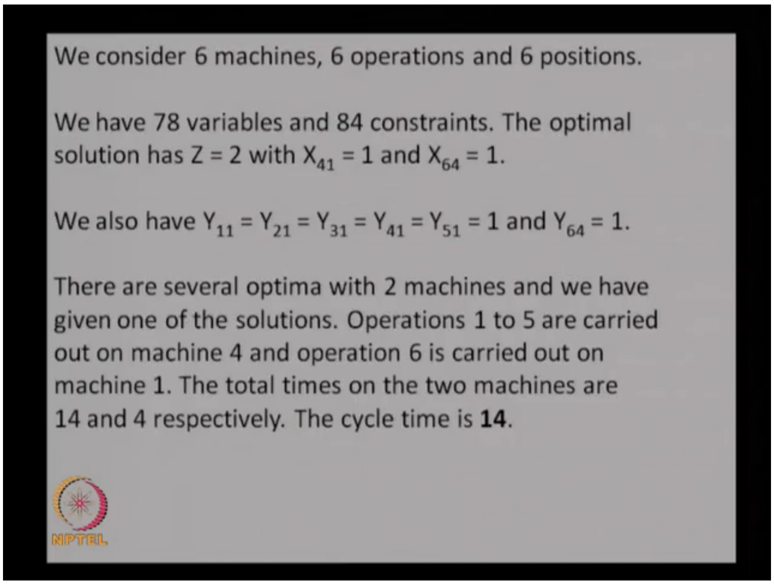

We also observed here that for a problem which is as simple and small as a one containing 6 machines, 6 candidate, machines 6 operations we have a slightly large integer programming problem with 78 binary variables and 84 constraints. So, as the problem size increases solving such a problem optimally using perhaps a solver would become a little more difficult. So, in such cases, we can resort to heuristic solutions heuristic solutions are quick and fast, but heuristic solutions have the disadvantage that they do not provide us with the optimal solution.

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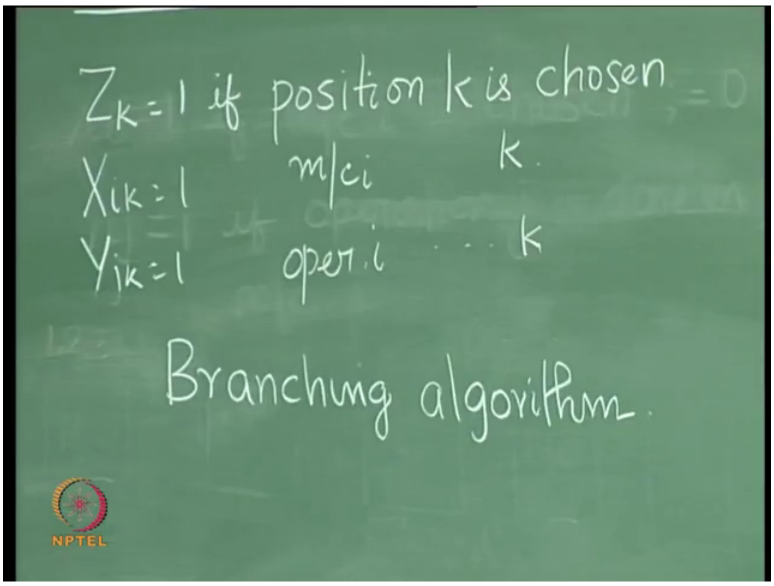

Now, the next thing is can we look at some kind of an algorithm which does not explicitly solve the ip per say, but it can give us an optimum solution and then we could think in terms of what is called a branching algorithm a branching algorithm to solve this particular problem.

So, this branching algorithm will not solve the problem using objective functions and constraints, but it would progressively start building solutions which will take care of these constraints. And finally, try to give us an optimum solution now one such branching algorithm we will see in the next lecture.