Manufacturing Systems Management
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Lecture – 16
Reducing Intercell Moves

In this lecture, we continue the discussion on minimizing intercell moves.

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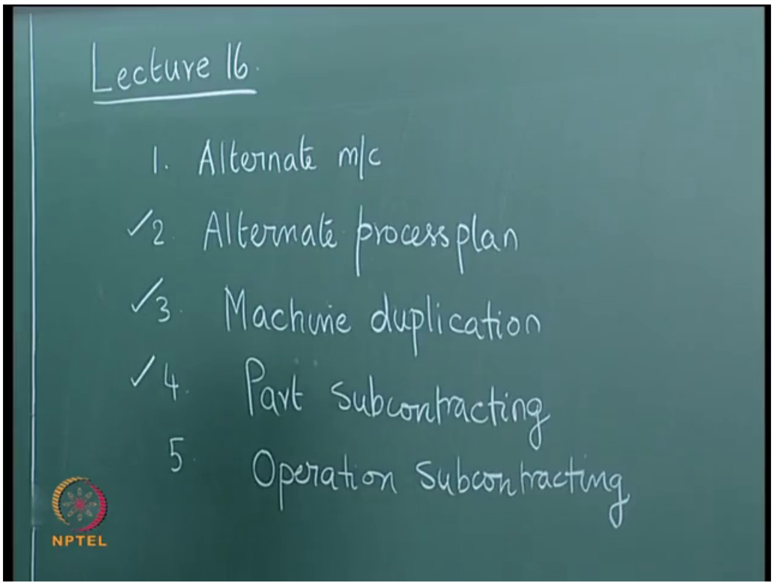

In previous lectures, we have seen that minimizing intercell moves can be done in multiple ways 1 is Alternate machines, Alternate process plan, Machine duplication, Part Sub contracting, Operation Sub contracting etcetera, out of these 5, we will concentrate on the alternate process plan machine duplication and part sub-contracting. Knowing fully well that from the solution to the cell formation problem, which could come out through any of the algorithms that we have seen it is always possible to find out whether a particular operation can be done on an alternate machine or which are the operations to be looked at so that they can be done on alternate machines within the cell plus operation subcontracting.

The final solution also gives us a feel of what are the components for which we have to look at alternate process plans as well as what are the components are parts that can be subcontracted and machines that can be duplicated, but we will look at these 3 in a little more detail and try to formulate problems that will address the issues of alternate process plan machine duplication and part subcontracting.

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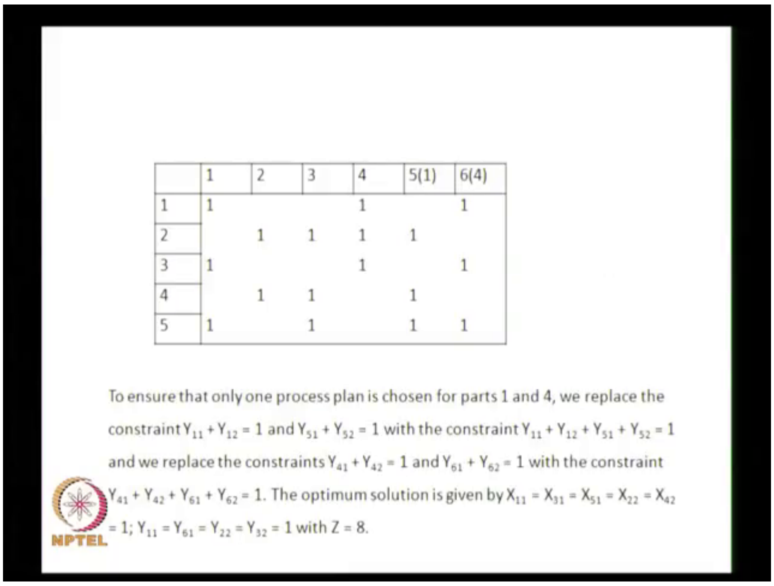

So, we take this example this is a 5 machine 4 part incidence matrix you actually see 6 columns, but the first 4 columns represent 4 parts, the 5 rows represent 5 machines. The columns 5 and 6 where we have shown 1 and 4 in brackets mean that this is an alternate process plan to do part 1 and this is an alternate process plan to do part 4. So, part 1 can be manufactured using a process plan that follows a sequence or that visits machines 1 3 and 5 or part 1 can also be done by visiting machines 2 4 and 5 as an alternate process plan. Similarly column 6 or process plan 6 is an alternate process plan to make part 4.

So, now we want to group the machines and parts such that we minimize intercell moves, at the same time making sure that every part is assigned to a group based on 1 process plan. If we take these 6 parts part numbers 2 and 3 have only 1 process plans while part numbers 1 and 4 have to process plans. So, we have to choose 1 of them and then assign the part to a group based on the chosen process plan. So, let us look at formulations of this problem and let us see how we modify the Boctors formulation as well as the P median formulation to include alternate process plans in our analysis.

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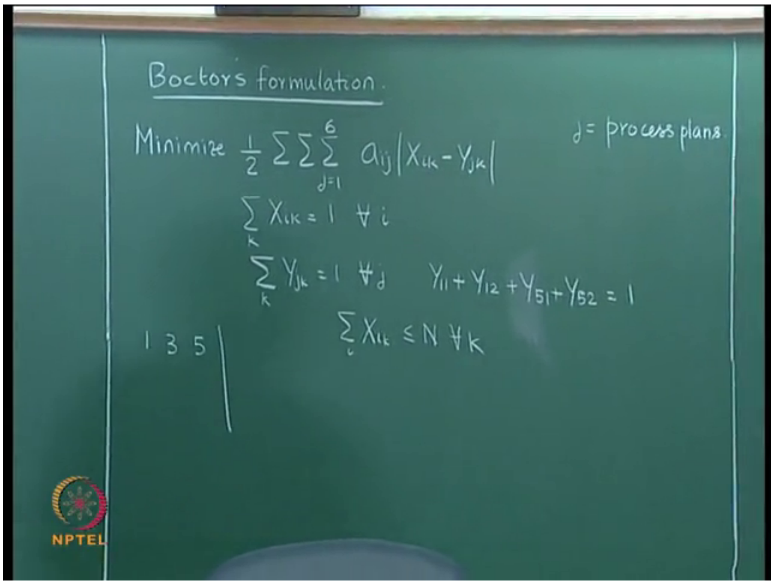

Now, let us look at the Boctors formulation; Boctors formulation, but this formulation; now the Boctors formulation will minimize which represent the inter cell moves X i k equal to 1, if machine i goes to cell number k or and why j k equal to 1 if part j goes to cell number k. We have already seen that this minimizes the intercell move and since it is counted twice the half is also brought now the only (Refer Time: 06:22) here is j will be from 1 to 6 because there are 6 process plans. So, we calculate j equal to 1 to 6.

Subject to the condition sigma X i k equal to 1 summed over k for every i each machine goes to only 1 group Y j k equal to 1 for every j summed over k each process plan goes to only 1 group here j does not represent parts j represents process plans, j represent process plans and does not represent parts. So, each process plan will go to only 1 group, now if we look at parts containing alternate process plans that are part number 1 and part number 4.

Ordinarily part number 1 will this constraint for part number 1 will be Y 1 1 plus Y 1 2 equal to 1, saying that part number 1 will go to either group number 1 or group number 2. Now if we do this then what will happen is we will end up assigning both part number 1 as well as part number 5 a process plan number 1 or process plan number 5, we do not want to do that we want to choose only 1 out of the process plans. Therefore, this particular constraint is modified and written as Y 1 1 plus Y 1 2 plus Y 5 1 plus Y 5 2 equal to 1.

So, only 1 of these 4 will be chosen which means 1 of these 4 will be chosen which clearly means either 1 will be chosen or 5 will be chosen both will not be chosen. So, only 1 out of the many alternate process plans for part number 1 will be chosen and that will either be assigned to group number 1 or group number 2.

So, here there will be only 4 constraints the first constraint will be like this the second constraint will be Y 2 1 plus Y 2 2 equal to 1, the third constraint will be Y 3 1 plus Y 3 2 equal to 1, the fourth constraint will be Y 4 1 plus Y 4 2 plus Y 6 1 plus Y 6 2 equal to 1. So, that process plan 2 is chosen definitely and goes to either group 1 or group 2 3 is chosen it will go to either 1 or 2 as far as part number 1 is concerned either process plan 1 is chosen or process plan 5 is chosen and the chosen process plan can go to either group number 1 or group number 2, in a similar manner for part number 6 also either process plan 4 our process plan 6 is chosen for part 4 and the chosen process plan will either go to group number 1 or go to group number 2, plus we need to have a restriction sigma X i k summed over i is less than or equal to sum N for every k unless we do that it will put all the machines into a single group. So, we will have to add this for the Boctors formulation.

Now let us see what happens to the solution now here we have said that the optimum solution is given by X 1 1 equal to X 3 1 equal to X 5 1 X 2 2 equal to now the optimum solution is given by machines 1 3 and 5 forming 1 group and machines 2 and 4 forming 1 group.

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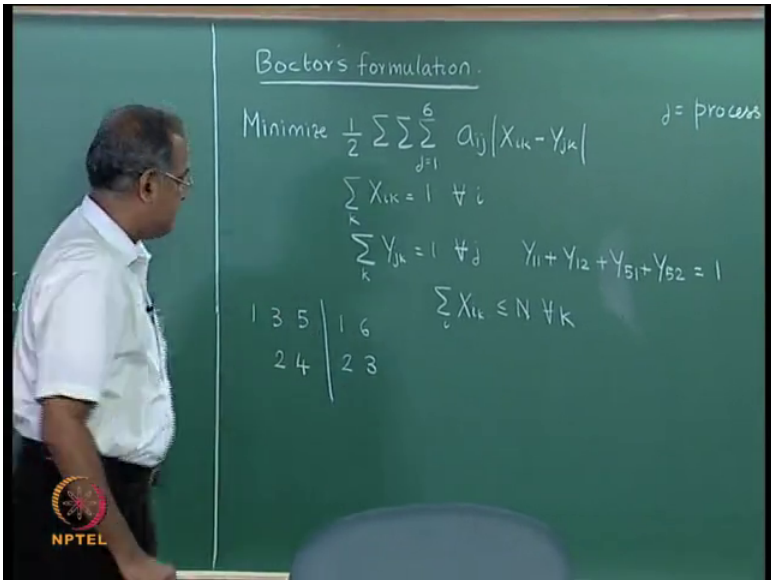

With parts Y 1 1 Y 6 1 so 1 and 6 are chosen and 2 and 3 are chosen, now let us try and compute the intercell moves process plan 1 goes to 1 3 and 5 no intercell move, process plan 2 goes to 2 and 4 no intercell move, process plan 3 goes to 2 4 and 5.

So, process plan 3 goes to 2 4 and 5 there is 1 intercell move, there is 1 intercell move 4 is not chosen 5 is not chosen 6 is chosen with 1 3 and 5 there is no intercell move.

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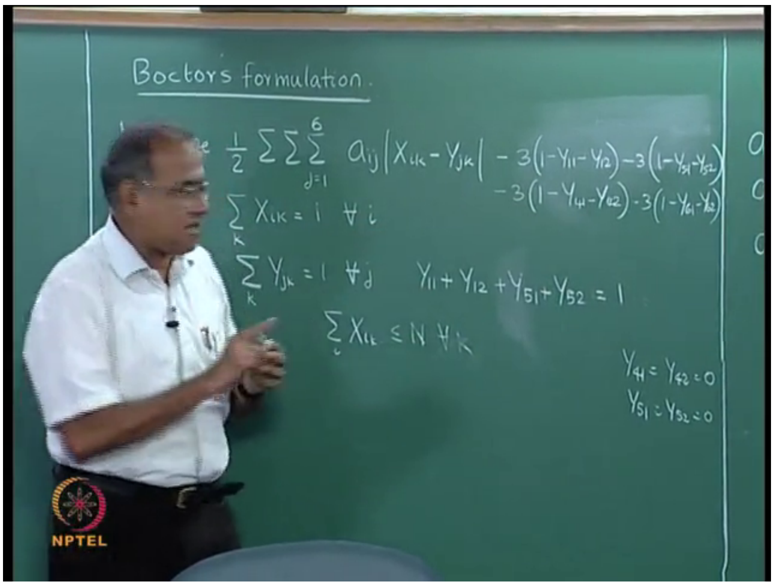

But let us go back to this formulation as far as this formulation is concerned both now in these solution process plans 4 and 5 are not chosen so in this solution Y 4 1 equal to Y 4 2 equal to 0 Y 5 1 equal to Y 5 2 equal to 0.

Now the way this objective function will be calculated is since Y 4 1 is equal to 0 process plan number 4 requires visits of machines 1 2 and 3. So, we will have a situation where there is a visit a 1 4 from this matrix there is a visit a 1 4 X 1 1 is 1 Y 4 1 is 0, because Y 4 1 is 0. So, this will contribute to an intercell move based on the objective function because the way the objective function will calculate the moves it will do summation a i j absolute value of X i k minus Y j k.

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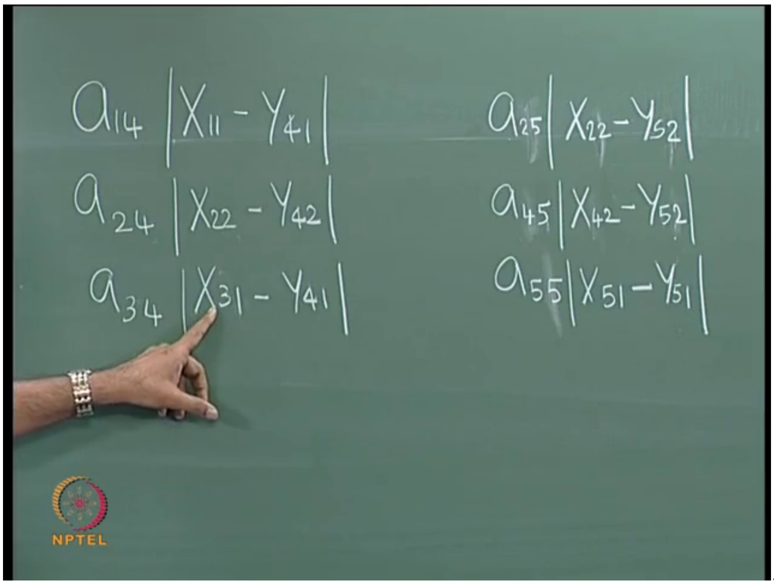

So, somewhere it will look at a term where a 4 1 is 1. So, a 4 1 is 1 and it will look at a 4 1 into absolute value of X 4 1 minus X 1 1 a 1 4 equal to 1 part family number 4 requires mes machine number 1 from this matrix and then it will have X 1 1 will be 1 Y 4 1 will be 0. So, this will calculate 1 intercell move here, it will calculate 2 more intercell moves a 2 4 into absolute value of X 2 2 minus Y 4 2 and a 3 4 into absolute value of X 3 1 minus Y 4 1.

Now, these 3 moves come because process plan number 4 that is not chosen has 3 elements in the corresponding column. Similarly for the process plan 5 that is not chosen it will compute 3 intercell moves a 2 5 in to absolute value of X 2 2 minus 5 Y 2 Y 5 2 now a 2 5 is 1 from the matrix X 2 2 is 1 from the solution Y 5 2 is 0 because process plan 5 is not chosen.

So, there are 2 more intercell moves that it will calculate. So, for the 2 process plans that are not chosen it will effectively calculate 3 plus 3 6 moves 3 comes because each of these process plan has exactly 3 elements in the corresponding column. So, this objective function computation is an over estimate of the computation of the intracell moves the actual intercell moves is not this is an over estimation of the actual intercell moves.

So, we should do minus in this case 3 times mine now we know that either process plan 1 is chosen our process plan 4 is chosen we are not going to choose both. So, we should have 1 minus Y 1 1 minus Y 1 2 minus Y 4 1 1 minus they will have 3 times 1 minus Y 1 1 1 minus Y 1 2 minus 3 times 1 minus Y 4 1 minus Y 4 2 minus 3 times 1 minus Y Y 5 1 Y 5 2 1 minus Y 4 1 minus Y 4 2 1 and 5 4 and 6 minus 3 times 1 minus Y 6 1 minus Y 6 let me explain this addition.

Now, between process plans 1 and 5 we are going to choose only 1 of them, now this constraint will ensure that 1 of them is chosen either 1 is chosen or 5 is chosen if 1 is chosen 1 of these 2 put together will take a value 1 this is put together we will take a value 0. So, if you look at this if 1 is chosen process plan 1 is chosen as in the example if process plan 1 is chosen 1 of these 2 terms will be 1 and this thing will become 0 now 5 is not chosen therefore, both of these will be 0 and this will contribute a 3 to the other 1 the 3 is contributed because process plan 5 requires 3 machines and those 3 this will consider.

So, you should remove the 3 from that as the intercell move i said they are not intercell moves, similarly between process plan 4 and process plan 6 only 1 of them will be chosen. So, in our example 6 is chosen which means 1 1 of these 2 is 1 the other is 0. So, this term will be 0 whereas, 4 is not chosen. So, both are 0. So, this will contribute a minus 3 because by not choosing the process plan 4 we are actually adding 3 that comes here, I explain now how each 1 will add 2 times. So, there will be 6 contributions this half will ensure that it is 3 and that 3 is taken away. So, this is how the boctors formulation has to be modified if we are using alternate process plans an optimum solution to this problem would give us the correct choice.

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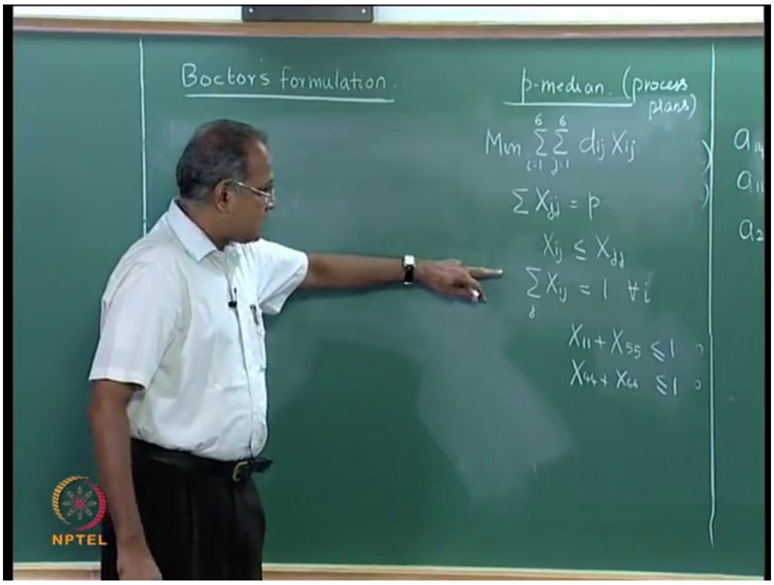

Now, in order to modify the p median problem, now we look at we consider this matrix now there are 6 process plans. So, we are doing a p-median problem for process plans not for parts for process plans. So, it becomes the distance matrix or a dissimilarity matrix becomes a 6 by 6 matrix. So, minimize d i j X i j subject to the condition let us go back to the p-median formulations sigma X j j equal to p X i j less than or equal to X j j and sigma X i j equal to 1 summed over i or summed over j for every i.

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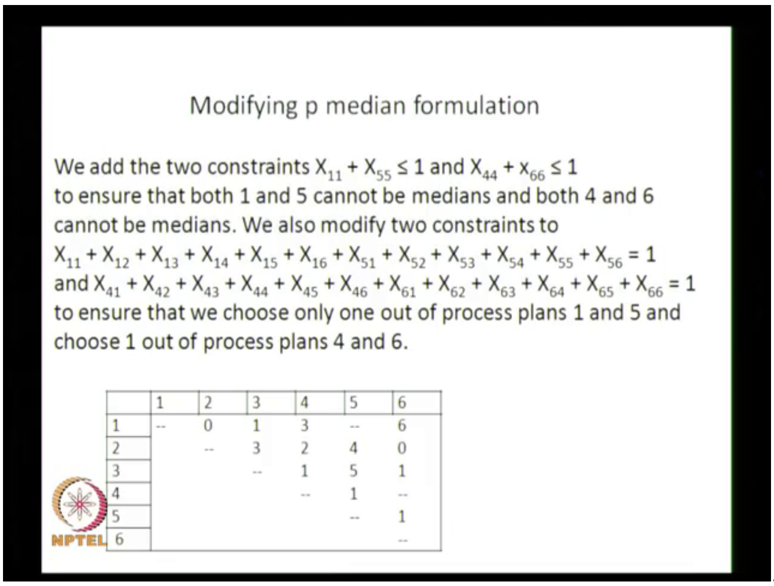

Now, we will compute a 6 by 6 distance matrix which is like this which is shown here now we they are going to choose either (Refer Time: 22:46) we are going to choose 2 medians out of 1 2 3 4 5 6 at the same time we have to make sure that 1 and 5 are alternate process plans. So, 1 and 5 should not become 2 medians only 1 of them has to be a medium.

So, we will add constraint X 1 1 plus X 5 5 is equal to 1 will be equal to 1 similarly X 4 4 plus X 6 6 is equal to 1 both cannot be chosen as medians. Now or In fact, less than or equal to 1. So, that we can have a situation by which both are also not medians we could have in this case 2 and 3 coming in as medians. So, to ensure that both cannot be medians we choose X 1 1 plus X 5 5 less than or equal to 1 and X 4 4 plus X 6 6 less than or equal to 1. So, these 2 constraints will ensure that both 1 and 5 are not medians.

Now, we have 6 constraints that are here now these 6 constraints ordinarily would be for example.

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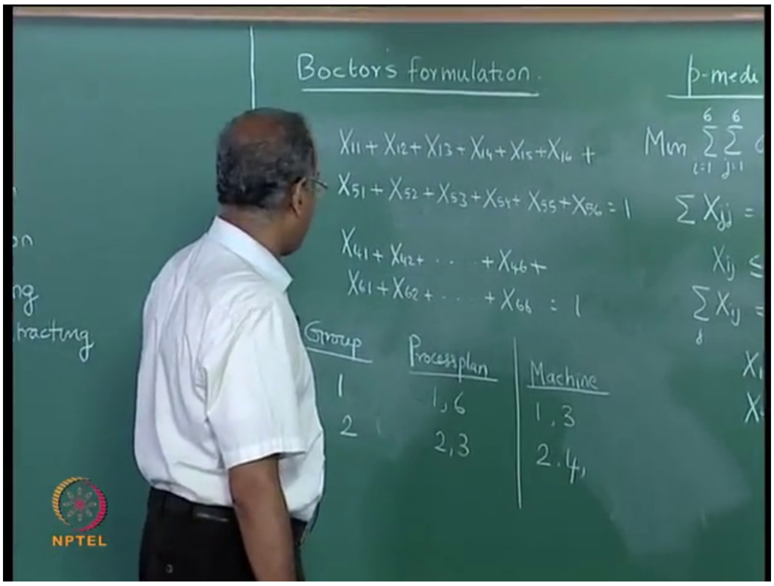

It will be like X 1 1 plus X 1 plus X 1 3 plus X 1 4 plus X 1 5 plus X 1 6 equal to 1 for the first process plan, in the original p median formulation. Now there will be 1 for process plan number 5 also which will look like X 5 1 plus X 5 2 plus X 5 3 plus X 5 4 plus X 5 5 plus X 5 6 is equal to 1, there will be 1 4 2 3 2 3 4 and 6 also. Now let us take the case for process plant 1 and process plant 5, because process plans 1 and 5 are alternate plans we are going to use only 1 of them we are not going to use both.

So, what we do is we simply put a plus and say the whole thing should be equal to 1 which means between points 1 and 5 in the p median formulation there are 2 medians I will take 1 either point 1 or point 5 and try to put it to the nearest median because I am minimizing a d i j X i j. So, this will also make sure that if 1 is assigned to a median 5 cannot be assigned to a median if you take point number 1 point number 1 can either be a median or will be assigned to another median. So, it will be a median if X 1 1 is 1 that will automatically ensure that X 5 5 is 0 which will satisfy this. So, 5 cannot become a median if 1 is a median similarly 5 becomes a median 1 cannot be a median. Now if 1 and 5 are not medians then they have to be assigned to another median.

So, this will make sure that there will be assigned to another median, but both will not be assigned because the entire summation is equal to 1 will ensure that either 1 will go to a median or 5 will go to a median such that the distance is minimized. So, a similar constraint is written for 4 and 6. So, that will be written as X 4 1 plus X 4 2 plus X 4 6 plus X 6 1 plus X 6 2 plus 6 3 plus 6 4 plus 6 5 plus 6 6 is equal to 1. The other 4 constraints will remain like this the other the other 2 constraints for 2 and 3 will remain only up to this part. So, that is how these 6 constraints will now become 4 constraints X i j less than or equal to X j j will stay intact except that for all 6 we will do that, X j j equal to p will also stay intact this will ensure that alternate process plans both do not become median.

It will get little more complicated than for a particular part there are 3 alternate process plans, now if there are 3 alternate process plans then they will become a additional columns and now instead of these 2 for example, if we had for part 1 the process plans were 1 5 and another 7, then this will be 1 1 plus 5 5 plus 7 7 less than or equal to 1 and here there will be a third edition for 7 which will ensure that it does not go.

So, this is how the p-median formulation can be modified and once again solved optimally to get the solution with alternate process plans.

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	1	6	2	3
1	1	1		
3	1	1		
5	1	1		1
2			1	1
4			1	1

We would get the same solution here when we actually do that for this instance where the same set of solutions given earlier with machines 1 3 and 5 forming 1 group 2 and 4 forming the other group, process plans 1 and 6 chosen and 2 and 3 chosen into the other. The other possibility also is by using the distance matrix or dissimilarity matrix we can also have a heuristic algorithm which will do that. For example, we will now look at the distances are dissimilarities for the part families. So, let us look at the original 1 this is the original 1. So, let us look at between 1 and 2 the distance is 0 between process plan 1 and process plan 2 the distance is 0. So, we look at this. So, this represents a distance matrix or a dissimilarity matrix.

So, we can use a similarity coefficient based method or a distance based method. So, where there is a 0 we can ensure that these 2 will belong to when the distance between 2 process plans is we look at this process plan 1 has visits machines 1 3 and 5 process plan 2 visits machines 2 and 4. So, the similarity is 0 the distance or dissimilarity is 5 now you can see that similarity is 0 because there are 5 pairs all of them are 1 0 or 0 1 pairs 1 0 0 1 1 0 0 1 1 0.

So, similarity is 0 distance is 5; therefore, this matrix is a similarity matrix. So, when we use P median on this matrix we will maximize S i j X i j and not minimize d i j X i j. So, now, since it is a similarity matrix we will go back to this matrix and find out which are the process plans that have maximum similarity.

So, 1 and 6 have maximum similarity with 6 from this. So, 1 and 6 now this is process plans this is group 1 process plan 1 and 6 are given to group number 1, next highest is for 3 and 5. So, next highest is for process plans 3 and 5, but then we know that 1 and 5 are alternate process plans and 1 has already been assigned to this. So, 5 will not be considered.

So, our next highest will be as we see is 2 and 4 2 and 5 with similarity equal to 4, now once again we realized that 5 is an alternate process plan for this therefore, it does not go the next highest is 1 and 4 once again it does not go because 1 is already chosen then we look at 2 and 3 and we now put 2 and 3 into the second.

So, the reasoning behind this is we have implicitly assumed that there will be a maximum of 2 process plans in a group, otherwise we could have looked at some other way for example, if we had we had chosen 1 and 6 here if we go by this the highest similarity was 6 which gave us process plans 1 and 6 here next highest similarity is 5 which is for 3 and 5 3 and 5 have 5.

Now, we know that 1 and 5 are alternate process plans, but we have chosen 1 therefore, we cannot choose 5, we cannot add a 3 here because 1 and 5 are the same they are not the same they are alternate process plans we either use this or use (Refer Time: 33:16) we cannot have both. So, we will not look at the similarity coefficient of 5 between 3 and 5.

We look at the next 1 which is 4 between 2 and 5 for the same reason we will not look at that because 5 has to be left out, next highest similarities between 1 and 4 once again 4 and 6 are alternate process plans. So, that is left next highest happens to be for 2 and 3. So, either we could put 2 and 3 here or here because we have a restriction that we will have a maximum of 2 process plans in a group 2 and 3 will come into another group. Now once the part families or process plan families are chosen you can apply the machine allocation rule to choose to allocate machines based on the chosen process plans. So, now, this will we will look at process plan number 1 requires or we go to machines machine 1 does 1 4 and 6 actually 4 and 6 are alternatives.

So, 4 should not be looked at. So, it does 1 and six. So, machine 1 will go here machine 2 does 2 3 4 and 5, machine 2 has 2 3 4 and 5 4 is not to be looked at because 6 is chosen. So, it has 2 3 and 5. So, machine 2 will come here with 1 intercell move, machine 3 has 1

4 and 6 once again 4 has to be left out because 6 is chosen. So, 1 and 6 so machine 3 will come here machine for does parts 2 3 and 5 machine 4 does 2 3 and 5. So, it will look at 2 3 and 5. So, machine for will come here and machine 5 does 1 3 5 and 6. So, 1 and 5 are alternate process plans. So, machine 5 looks at 3 5 and 6 sorry 1 3 and 6. So, machine 5 will come here.

Once again machine 5 has 1 3 5 and 6 now out of these 1 and 5 are alternate process plans 1 is chosen. So, 5 has to be left out it does 1 3 and 6. So, it goes to group number (Refer Time: 36:27). So, this is how the machine groups are also formed through this. So, this is how we bring in alternate process plans into the formulation stage and either we can modify the Boctors formulation to do it or modify the p median formulation to do it or have a similarity coefficient based approach which considers alternate process plans.

So, we will now look at the next 1 which is the problem to duplicate machines, this picture shows the old example the familiar example where there are 6 machines and 8 parts and this is the solution obtained using the earlier p median model or the Boctors formulation acceptable. So, from this we know that if we were to duplicate the machine we will be duplicating machine number 6 because it has 3 intercell moves from the first group.

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	1	4	5	6	2	3	7	8
1	1	1	1	1				
3	1	1		1				
4	1		1	1				
2				1	1		1	1
5			1		1	1	1	1
6	1	1		1	1	1	1	1

Since there are two groups, we add the constraint $Y_{11} + Y_{12} + Y_{21} + Y_{22} + Y_{31} + Y_{32} + Y_{41} + Y_{42} + Y_{51} + Y_{52} + Y_{61} + Y_{62} + Y_{71} + Y_{72} + Y_{81} + Y_{82} \leq 7$

The optimal solution to the problem is given by $X_{11} = X_{31} = X_{41} = X_{22} = X_{52} = X_{62} = 1$.

$Y_{11} = Y_{41} = Y_{51} = Y_{22} = Y_{32} = Y_{72} = Y_{82} = 1$ with $Z = 11$.

We also have $Z_{161} = Z_{262} = Z_{361} = Z_{461} = Z_{551} = Z_{552} = Z_{611} = Z_{612} = Z_{641} = Z_{642} = Z_{662} = 1$ giving an objective function value of 11.

So, 1 way to capture machine duplication is by looking at the block diagonal structure itself and from the block diagonal structure trying to find out which is the machine that is attracting a lot of intercell moves and that will be the first candidate to duplicate, but then we have to quickly say that this block diagonal structure has been obtained assuming that it has been obtained using an optimization. So, 1 way is to look at an optimum solution without considering machine duplication and from that solution try and find out which are the machines to be duplicated, the other way is write at the optimization stage itself can I bring in the idea of machine duplication and simply solve 1 problem which at the end of which will give me a solution with machines duplicate.

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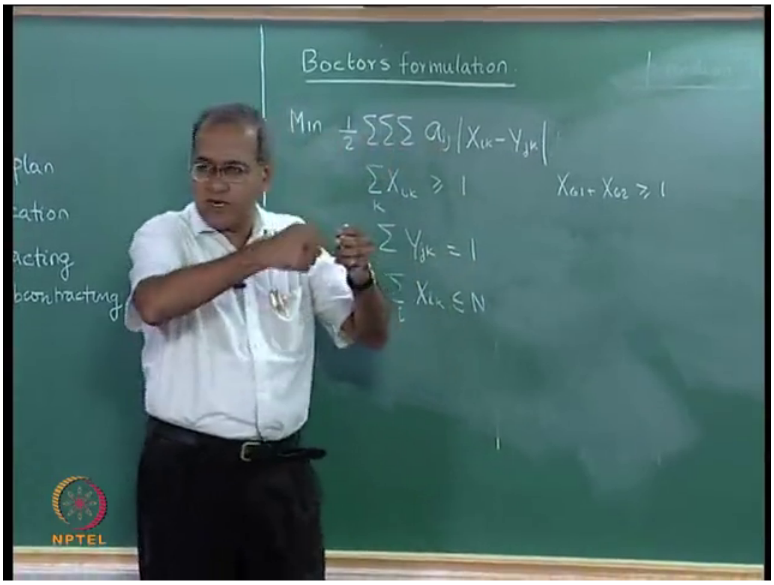

So, that is the purpose of revisiting these formulations by considering machine duplications and part subcontracting and so on, so we go back to the boctors formulation. So, intercell moves again will be minimize half into a i j into X i k minus Y j k, this will be the intercell moves subject to the condition that each machine is assigned to only 1 group summed over k each part is also assigned to only 1 group by j k and a restriction on the cell size i k summed over i is less than or equal to M for every k this is the boctors formulation original formulation of course, we have seen how to linearize the term and so on.

Now if we allow machine duplication where is the modification this constraint permits a machine to be attached to only 1 cell this permits a machine to be attached only 1 cell and this does not allow a machine to go to 2 cells for example, if we are looking at 2 groups we will have a constraint which is X 6 1 plus X 6 2 equal to 1 for this table and that will force machine 6 either to group number 1 or 2 group number 2.

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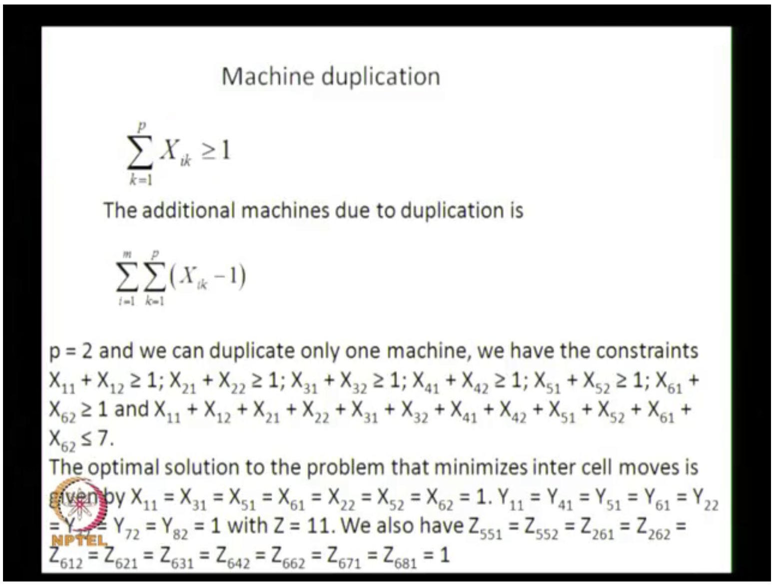

So, the only thing that we have to do is we could say greater than or equal to 1 X i k X j k greater than or equal to 1 that we have, but X i k is also binary which means we are basically allowing a constraint of the type X 6 1 plus X 6 2 greater than or equal to 1.

We are also allowing X 5 1 plus X 5 2 greater than equal to 1 and. So, on when we write this, but this is going to help particularly in duplicating machine number 6 in both of them, but again we restrict that we could have 1 machine of 6 in the first group and 1 in the second group they are still binary. So, this will not have more than 1 machine of 6 in 1 group. So, these will this is enough to do this at the same time for each of these machines we write this.

So, when we write this for each of these machines and solve if we look at this particular solution which is the final block diagonal form if we duplicate machine 6 alone there would still be 2 intercell moves if we duplicate machine number 5 there will be 1 intercell move and we duplicate machine number 2 there will be 0 intercell moves. So, the actual solution with 0 intercell moves is when group 1 has 1 2 3 4 5 6 and group 2 has 2 5 6, but you also know that the moment group 1 has 1 2 3 4 5 6 you can always have a solution where all parts go to group 1 and no part goes to group 2.

Therefore it will at the end give you a trivial solution where all of them go to only 1 group the other group does not attract anything. So, in order to prevent that we have this this will ensure that all of them cannot go to 1 group because this N will be less than or equal to 6. So, we need to fix this N very very carefully. So, that all of them do not go to the same group in the process we are also duplicating machines and the number of machines that are duplicated is given by sigma X i k minus 1.

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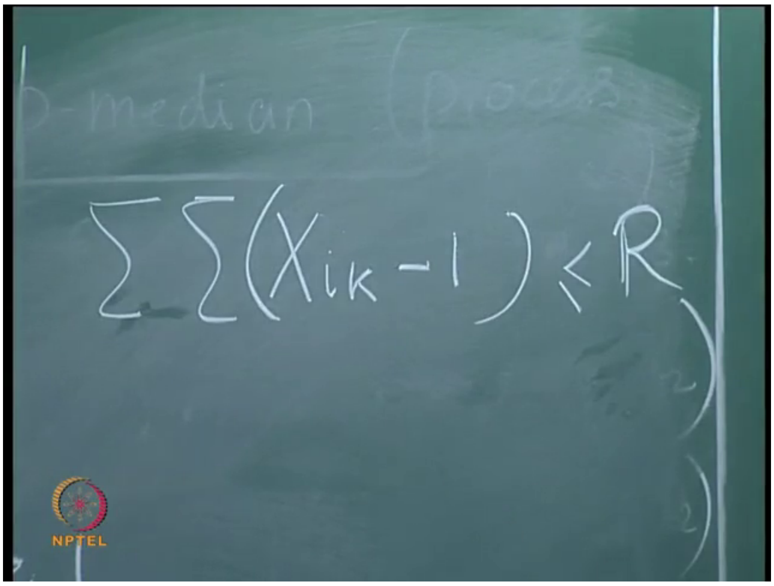

For example, if we are adding a 6 to the first group and to the second group. So, X 6 1 and X 6 2 will be 1. So, sigma X 6 will be 2 2 minus 1 1 will ensure that there is 1 extra machine of 6 which is duplicate.

So, similarly the extra machines that are duplicated is this. So, we could also put a restriction that the extra machines that are duplicated is less than or equal to sum R are some number, I do not want to duplicate more than 3 or 4 machines in a large sized example. So, this is not a absolutely necessary constraint, but this would also help us to ensure that it does not throw up a solution where a large number of machines are duplicate, the reason is machine duplication involves the cost explicitly we are not bringing in the cost of the machine at this stage. The other way to look at it is we assume that all machines are equally costly and therefore, we say that we will minimize the number of machines that are duplicate.

We may use this we may not use this, but this has to be done very judiciously such that all the machines do not go to the same group. So, this is how the Boctors formulation is modified; now the Boctors formulation can be modified to this problem and if we allow a cell size of 4 let us say then we will get the solution where this machine number 6 is duplicated to the first group also. So, first group will have 1 3 4 and 6 second group will have to 5 and 6 if we allow for here if we allow 3 it would still have the same solution even though we have allowed for machine duplication. So, M has to be chosen very carefully.

Now, let us look at minimizing part subcontracting or look at the case of what we do with respect to part subcontracting.

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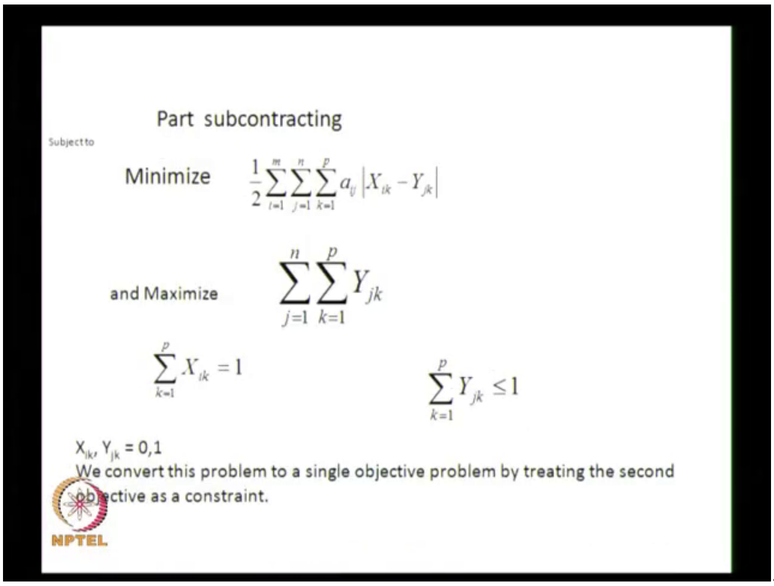

Now, let us look at the same solution which is obtained through a cell formation algorithm now there are 4 interce 5 intercell moves and we want to reduce the number of intercell moves. So, we said 1 of them is to try and do part subcontracting now if we were to choose 1 part that we will subcontract, which means we will take it out of the incidence matrix then that will be part number 6 which is now the fourth column of the table, because if we take out part number 6 2 intercell moves are going, but if we take out part number 6 actually 5 moves are effectively going 5 moves are effectively going if we take out part number 6.

So, the part sub-contracting problem is to say that, amongst these parts which are the parts that are going to go into the cell which are the parts that are going to be sub contract such that we minimize the intercell moves.

So, now in the earlier case we will have Y j k equal to 1.

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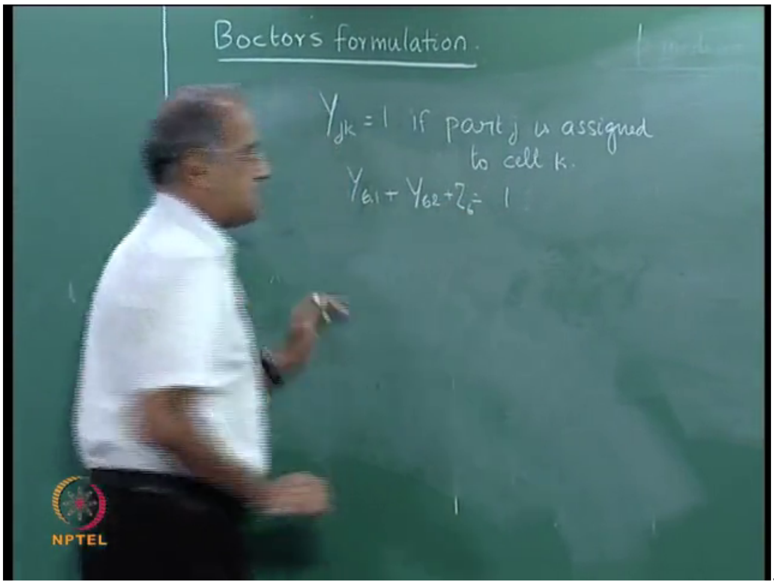

If part j is assigned to cell k and if there are 2 cells we would have a constraint Y 1 1 or let us say part number 6. So, we would say Y 6 1 plus Y 6 2 is equal to 1, now we have an alternative that the part can also be subcontracted. So, Y 6 1 plus Y 6 2 plus plus some z 6 equal to 1 if that 6 is part 6 is subcontracted. So, the formulation has to be modified accordingly for subcontracting. So, how the Boctors formulation can be modified for part subcontracting we will see in the next lecture.