Manufacturing Systems Management
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Lecture - 15
Algorithm considering cell load data, alternate process plans

In this lecture we consider cell formation by incorporating cell load data into the machine component incidence matrix.

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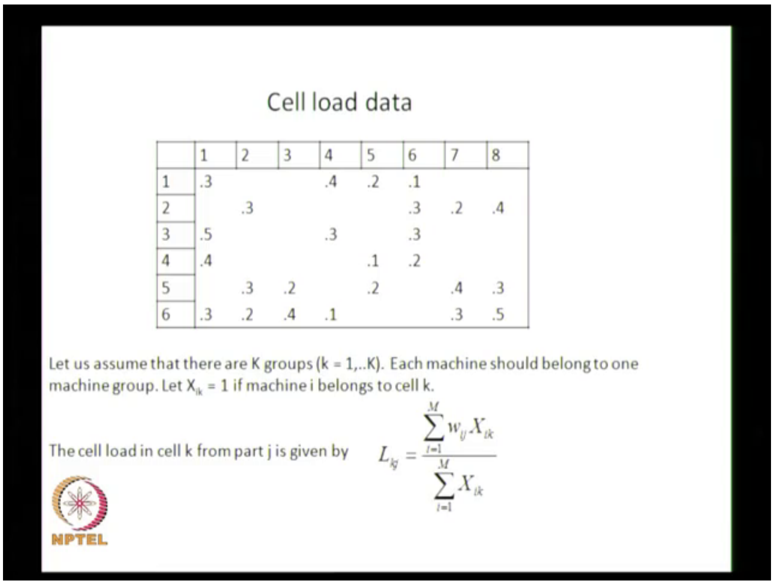

So, the example of an incidence matrix that has cell load data is given in this slide. So, again we use a 6 by 8 incidence matrix, knowing that in real life the incidence matrices are much larger in size and a 6 by 8 is used only for the sake of illustration. Now here the incidence data is captured in the form of a decimal between 0 and 1. For example: part 1 visits machine 1 and has a value 0.3 now that 0.3 we have already seen how the cell load data is calculated. Is a very quick recap if the volume of production for part 1 in a given time period say 1 month is V 1 if the unit processing time for part 1 on machine 1 is p 1 1.

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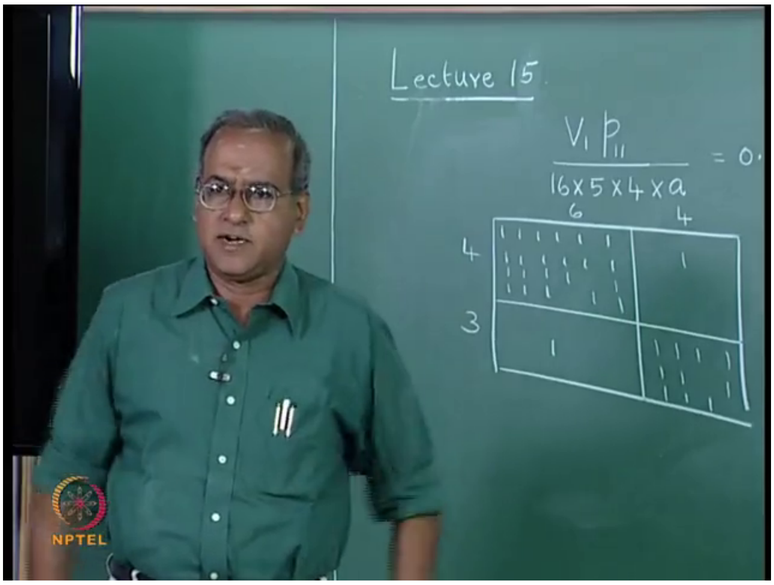

So, this much time is required on machine 1 by part number 1. Now the machine availability for the same 11 month period if we assume that we work for 16 hours a day and 5 days a week. So, it will be 16 hours a day into 5 days a week into 4 weeks in a month. Now this will give us a fraction or a decimal or invariably a number between 0 and 1. We also have to factor in availability which we have seen in a previous lecture. So, we will say this multiplied by some factor a which would represent availability; it could be 0.8 it could be 0.7 it could be anything. So, you can factor an availability into it, and then we say all this is now represented as some 0.3

Now, in an earlier lecture we used this cell load calculation primarily to find out the capacity required in itself. After the production flow analysis we have grouped machines into cells and parts into families and then we checked the capacity of each machine or the load or demand for each machine in each cell and then we said that if it exceeds 1 then it might be necessary to add one more machine into the cell. Now here we are going to look at cell load data in a slightly different way. Now if we look at a block diagonal matrix here, let us say we have a block diagonal matrix like this. Now let us say we have 4 machines here we have 3 machines here, we have 6 parts here we have 4 parts here.

What I wish to point out is if we had a very large matrix, we would have more multiple machine cells and part families and they all need not be of the same size. In the 6 by 8 example invariably our solution gave us 2 machine cells each having 3 machines and 2 part families each having 4 parts. When we actually work it out on larger problems, we will not have cells with equal size or part families of equal size. In fact, the larger instance that we have seen, which is the 16 machine 43 part, gave us something like this where there were 5 groups some of them had more machines some of them had less machines. Some of them handle large number of parts some of them handle fewer number of parts.

Now let us go back to the starting point where we said that cellular manufacturing is a way by which we bring in ownership responsibility and control into manufacture in an early lecture. We have also seen that extent of cellularization was one of the ways by which one could look at a performance of a cellular manufacturing system where we defined extent of cellularization as the percentage of total business that came out of cells. Now let us go into this extent of cellularization further.

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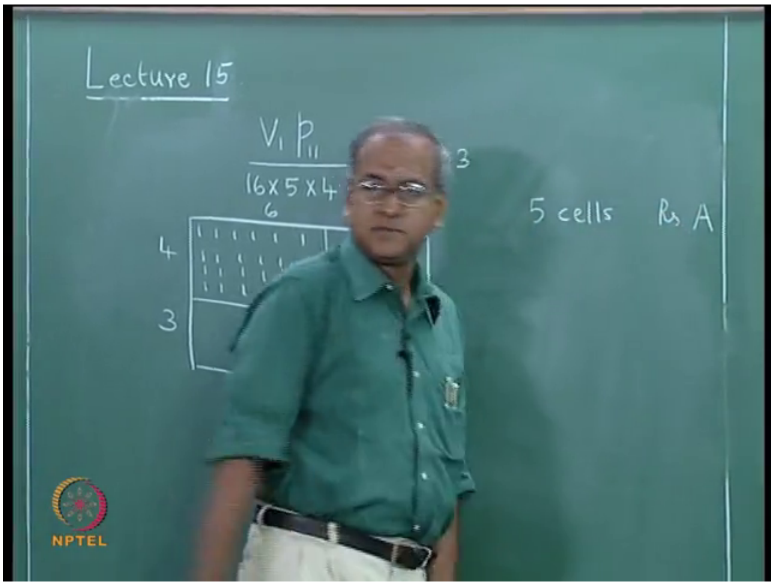

now if there are 5 cells and let us say that these 5 cells put together, through their production and sale is generating rupees A. We know that each one will not contribute exactly to a by 5, some cells will contribute more some cells will contribute less. Now if we start looking at each cell as a profit center or a profit making entity by itself. Then the first question is during cell formation should we ensure that each cell contributes more or less equally to the resource generation or should we leave it and say at the end we leave the cell formation. You do a production flow analysis or any other algorithm. We may end up having cells that some of them will be bigger, some of them will be smaller; there is not much of a need for us to ensure that the revenue generation should be equally balanced.

So, we normally do not try to look at that, as I as a way of as a input to cell formation. Even though if for example, cell incentive is a function of the total revenue generated by the cell, there will be an imbalance when people attached to smaller cells or cells that contribute less to the revenue versus cells that contribute more to the revenue may end up having a higher incent, that will happen only when the cell in incentive depends on the output of the cell, but if the cell incentive is common across all cells that issue does not happen. Now researchers looked at cell load data and cell load balancing amongst the load balancing amongst the cells in a slightly different way. What they said or what they did is if there is a part 1 as we have shown here, which requires 4 machines 1 3 4 and 6.

If they are able to bring all the 4 machines into one cell, now there is a certain load taken by this cell and they want to find out the variance; and that and there is a load taken by another cell for this part and find out the variance in that. And if we try to minimize the sum of these variances, we will invariably end up creating solutions such that majority of the work associated with the part will go to oneself. So, minimizing the cell load variation particularly in the context of cell load data was taken as an objective and further analyzed to create machine cells and part families. Now let us understand what this cell load variation is through an exampled first and then we look at a formulation that minimizes the cell load variation.

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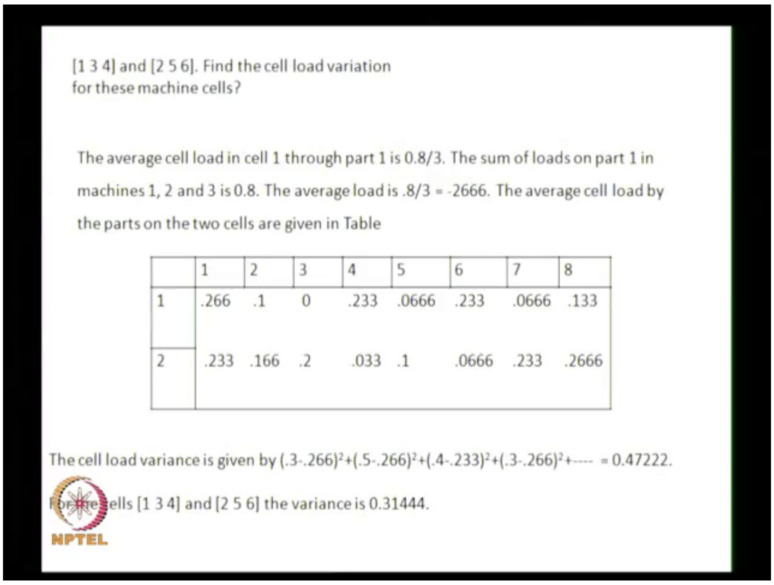

Now, let us assume that this is the incidence matrix that we have, this is the machine component incidence matrix that we have and let us assume that we have 2 cells which are 1 2 3 and 4 5 6 let us take 2 cells 1 2 3.

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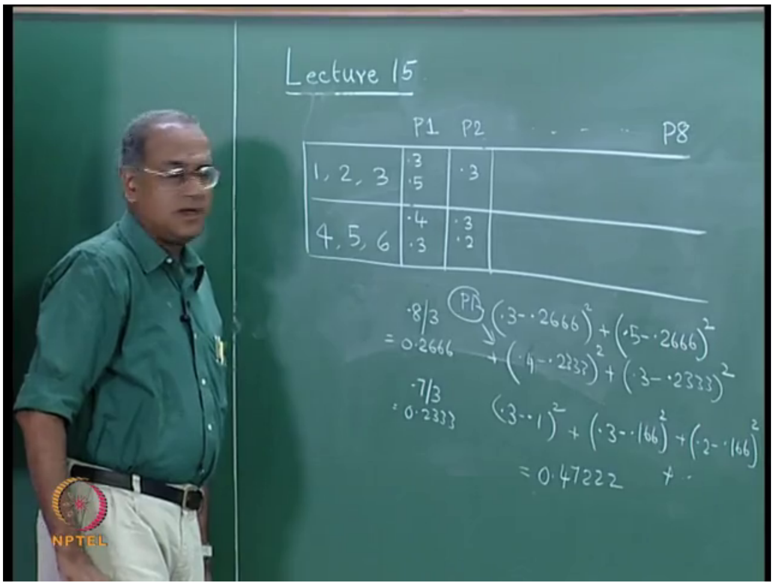

So, let us take 1 2 3 and 4 5 6 as 2 cells, let us take part 1 now. So, if we take 1 2 3 and 4 5 6 part 1 requires 0.3 plus 0.5. So, 0.3 and 0.5 here and it requires 0.4 and 0.3 here. Part 2 will require 0.3 here and 0.3 and 0.2 here and so on. Now there are 3 machines in this cell the total load on this cell through part P 1 is 0.8. So, the average load is 0.8 by 3 it is not pointed by 2 it is taken as 0.8 by 3. Ideally if this self-configuration is a very good configuration then P1 will visit all the machines and the cell therefore, the load is now taken as a per machine average it is not taken as by 2 its taken as by 3.

So, 0.8 by 3 would give us 0.2666, 20 by 3 2 6 6 6 cell load on this cell is 0.7. So, the average is 0.7 by 3 which is 0.2333. Now what is the cell load variation? The cell load variation is given by for this its 0.3 minus 0.2666 square plus 0.5 minus 0.2666 square plus 0.3 minus 0.2 3, 0.433 square plus 0.3 minus 0.2 3 3 3 square is the contribution of P 1 this is the contribution of P1. Now what is the contribution of P 2 to the cell load variation contribution will be 0.3 minus 0.1 square because total is 0.3 by 3 0.1 square plus 0.3 minus.

This is 0.5 0.5 by 3 is 0.1666 square plus 0.2 minus 0.166 square like that this is p twos contribution. So, like that we can add p ones contribution p twos contribution up to p 8 contribution and find out the total cell load variation.

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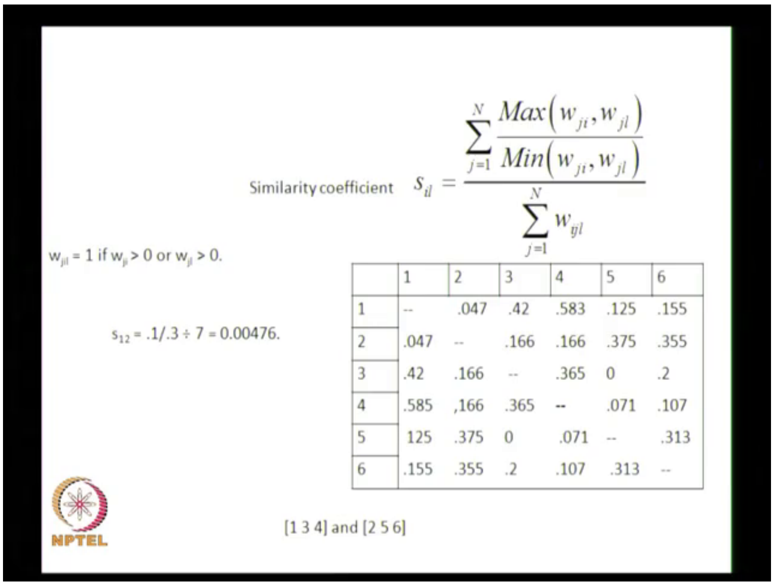

And then we do that computation the total cell load variation comes to 0.47222. So, it comes to 0.47222 this is for the configuration 1 2 3 4 5 6.

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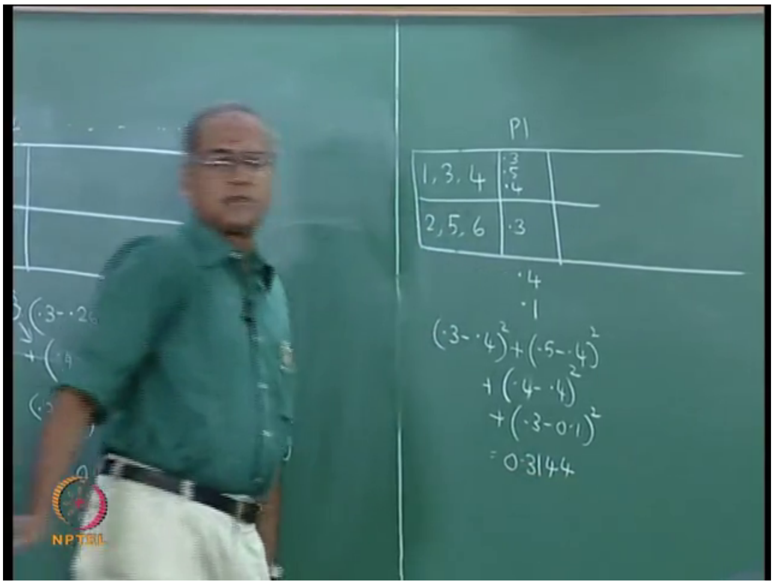

Now, if we have the configuration 1 3 4 2 5 6 if we have this configuration, then the cell loads will be 1 3 4 0.3, 0.5, 0.4; 0.3 0.5 0.4 2 5 6 is 0.3.

So, the average is 0.4 for this and average is 0.1 for this. So, the contribution of p 1 will be 0.3 minus 0.4 square plus 0.5 minus 0.4 square plus 0.3 minus 0.5 4 minus 0.4 square plus 0.3 minus 0.1 square. Now when use when you realize that with respect to P 1 1 3 4 2 5 6 is a better configuration than 1 2 3 4 5 6 the variation actually comes down whereas, the variation here will be large. So, when we compute this whole thing, the final load variation for this is given by 0.3 1 4 4. 3 1 4 4 which is given here I am not shown the entire computation, but I have shown to you that this is what the number comes.

So, based on this computation one could say that this is a better configuration than this configuration because the cell load variation is less therefore, we can find the optimal configuration which minimizes the cell load variation. So, that is formulated as a optimization problem and let us see that. So, cell load variation will be L k j is given by summation over all machines work load into X i j if it is there divided by the number of machines in that cell, which is what we did when we did this work load into those machines which are here which is 0.3 plus 5 0.8 divided by is the by 3 is what we did here as the average.

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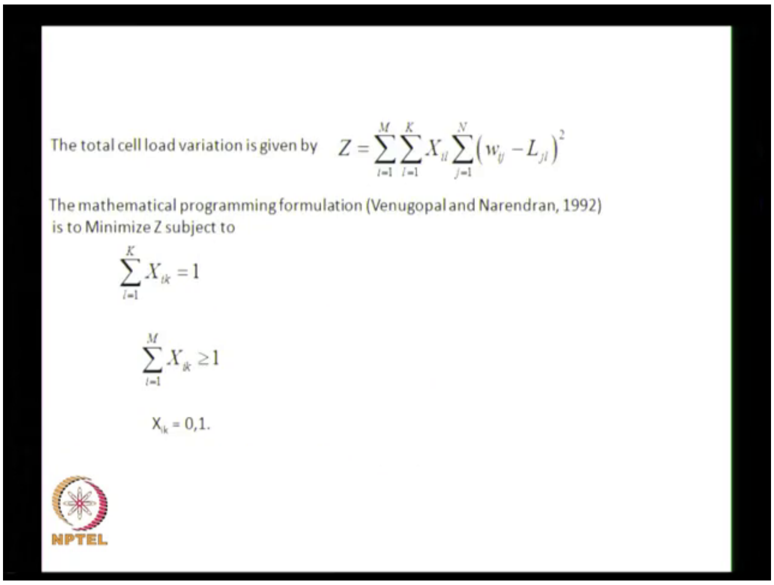

And then we did the square term. The variation was calculated as w i j minus L j l square which is how we calculated the variation into X i l.

So, this objective function Z equal to double sigma, sigma I equal to 1 to M k equal to l is equal to 1 to k whatever is given X i l summation w i j minus L j l whole square minimizes the total cell load variation. So, the objective is to minimize the total cell load variation subject to some constraints. The obvious constraint is one machine should go to exactly one cell, which also implies that the number of cells is a given prerequisite to this problem here capital K is a number of cells.

So, each machine should go to exactly one cell which is a constraint which is shown here minimize z minimize the cell load variation subject to the constraint that each machine goes to only one cell, then we get into the peculiar situation. It may not happen that if we fix 2 cells, it may not give us a solution where all the machines and all the parts go to 1 cell with minimum very variance. It can happen that if there is a solution with 2 cells and the solution with 1 cell all machines going to 1 cell and all machines going to 2 cells.

Now, it can happen that the solution with where machines go to 2 cells can have less variance than all machines going to 1 cell. Just to prevent; however, just to prevent the situation that all machines go to 1 cell we can now put either an upper limit on the number of machines in a cell or we can put a lower limit on the number of machines in a cell. In this case there is a lower limit that is put on the number of machines in a cell which meant that each cell should attract at least one machine and X i k is binary. So, this optimization problem will now allocate machines to cells such that the cell load variation amongst the cells is minimized. When I say load variation amongst the cell I do not mean to say that if there are 3 cells.

If a part is completely processed in one cell and does not visit the other 2 cells. So, there will be a load on the first cell there will be a 0 on the second cell there will be a 0 on the third cell. We are not trying to minimize the variance between that load 0 and 0 we are not doing that. What we are trying to do is within that said how much of load goes to each cell and if more load goes to one cell the variation amongst the load within the cell is minimized. So, this would automatically ensure that more load of a part will go to oneself thereby minimizing inter cell moves indirectly.

So, one should not think that we are actually trying to minimize the variation of the load amongst the cells let me repeat it again. If there is a part which is completely processed by cell 1 and let us say it has a total load of l. It has 0 load on cell 2 and 0 load on cell 3. We are not minimizing the variance between l 0 and 0 we are trying to minimize how that l is distributed within that cell, the variance of that how this 0 has distributed.

So, this will contribute a variance of 0 this will contribute third will contribute a variance of 0. The first one will contribute a small variance because the load l is distributed amongst many machines in one cell. So, if parts are us if machines are assigned to cells such that all the requirement of the parts are carried out within one cell the cell load variation is expected to be minimum. Parts come indirectly because the cell load variation involves computation of the part loads. After the machine loads machine cells are configured the parts can be again assigned based on our part assignment rule. So, this optimization problem can actually help in creating cells considering cell load data and to minimize the cell load variation. The apparent disadvantage or which is an obvious disadvantage is the fact that this problem is not a linear problem.

This problem is quadratic because the objective function has a quadratic term w i j is known it is the workload, but L j l depends on the X i ks not only does it depend on X i ks it depends on X i ks both in the numerator and in the denominator. So, it is not only quadratic it is some form of a fractional program, where the variables come both in the numerator and in the denominator plus it is quadratic.

So, even though this formulation is very nice and special it is not easy to solve this formulation to optimality comfortably. Therefore, the researchers who proposed this formulation use the heuristic approach and they used an approach based on simulated annealing to solve this problem. Now we are not going to look at the simulated annealing algorithm here, but we will look at another heuristic to solve this problem which is again based on a similarity coefficient and this similarity coefficient captures cell load data which is what we will see here.

Now, in there is a remember 0.1 by 0.3 divided by 0.7 correct and there is a both the max by man.

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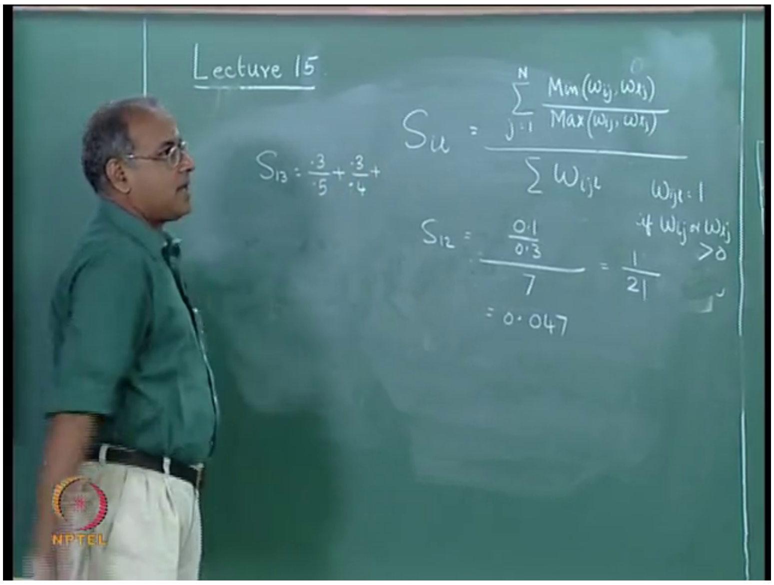

The similarity coefficient is given by between machines i and l is given by summation j equal to 1 to N number of parts minimum of w i j w l j by maximum of w i j w l j divided by w i j l summation. Now w i j l is equal to 1 if w i l or w i j or w l j is greater than 0. Now let us explain the similarity computation by considering machines 1 and 2 from this.

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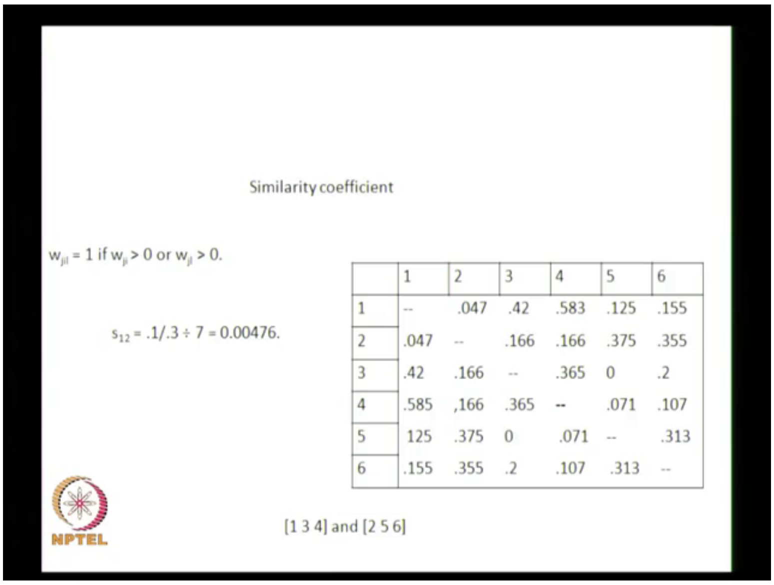

So, if we take machines 1 and 2 the numerator is summed over all. So, if you see carefully only the common parts will contribute to the numerator the numerator of this fraction not this numerator, but this one because when a part visits only one of these the minimum will be 0 and therefore, it will not contribute to the numerator.

Only common parts that visit both will contribute to the numerator. So, the only common part is 6. So, the loads are 0.1 and 0.3. So this numerator part alone is equal to 0.1 by 0.3. Denominator is contributed by parts that visit both the machines and parts that visit one of the machines. So, the denominator contribution comes from parts 1 2 4 5 6 7 8 3 does not visit. So, 1 2 4 5 6 7 8 the contribution of the denominator this will be 7. So, this coefficient will be 1 by 3 by 7 which is 1 by 21 which will give us 0.047 which is shown here.

Let us also explain 1 to 3 through this if we take 1 and 3. So, this is for S 1 and 2 we take 1 and 3 we will get part 1 is common part 4 is common part 6 is common. So, part 1 will contribute 0.3 by 0.5 minima by maximum part 4 is common it will contribute 0.3 by 0.4 and part 6 is common which will contribute 0.1 by 0.3. Denominator contribution comes from parts that visit either 1 or 3 or both. So, that will happen with part 1, part 4 and part 6. We do not have any other part which visits part 1, part 4, part 5 and part 6.

So, there are 4 parts that visit 3 come from these 3 plus 1.

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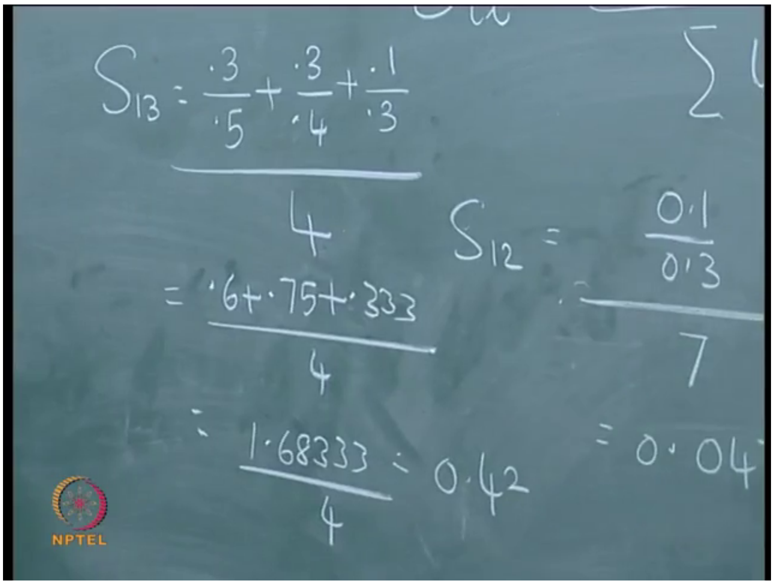

So, this gives us a coefficient of 0.6 plus 0.75 plus 0.333 divided by 4, 1.35 plus 0.333 is 1.68333 divided by 4 which is 0.4208 something which has shown as 0.42 in this 1 2 3 is shown as 0.42. Now the nice thing about the similarity coefficient is again its like the jacquards similarity coefficient with a slight difference, the numerator part the contribution comes from the common parts, which is similar to jacquards the difference of course, is that each term itself has its own numerator and denominator, which comes from minimum by maximum.

So, what is each of these terms is less than one and only the common parts will contribute which is a very nice thing in this similarity coefficient. Now each of this is less than 1 denominator is 3 the number the number of terms in the numerator plus something. So, the similarity coefficient is between 0 and 1. Secondly, if all the parts visiting these 2 machines visit both; in our example there was an additional 5 that came here there additional part 5 resulted in the denominator of 4 which pulled down the similarity coefficient. If I did not visit then the similarity coefficient will be higher because you are dividing this by 3. So, the more it is packed the higher the similarity coefficient which is what you want. Denominator part as in jacquards coefficient also is contributed by parts that visit both the machines are either of the machines.

So, the denominator is exactly what is there in the jacquards similarity coefficient. Since we are also trying to minimize the cell load variation and the variation comes in the variation of the terms for example, it comes if we look at the coefficient between 1 and 3, the variation when we actually compute comes out of some arithmetic mean and the variation of the individual loads. So, in a way the maximum and minimum contribute to the variation. So, if the loads are comparable for example, if this is 0.3 this is also 0.3 then minimum by maximum is close to 1. Therefore, this similarity coefficient will be higher and in a very ideal situation where all loads are the same this similarity coefficient will be 1 therefore, it is a good similarity coefficient which captures almost everything that we want to capture.

Now, with this similarity coefficient matrix, we can now form cells and now we realize that 1 2 3 is 0.4 to 1 to 4 is 0.5833 to 4 is 0 0.365. So, we will begin with 1 2 4 which is the highest. So, we begin with 1 2 4 0.5 8 3 then we add 3 that 0.4 2 we add. So, 1 3 and 4 will become 1 cell and as we move down. In fact, you realize as you move down you realize 2 5 and 6 have 0.375, 0.355 and 0.313. So, we will pick 2 2 5 first and then we will pick 5 to 6. So, 1 3 4 will form 1 group 2 5 6 will form another go. So, this way using the similarity coefficient we can capture cell load variation data and once again we have a way by which we minimize the inter cell moves indirectly by minimizing the cell load variation.

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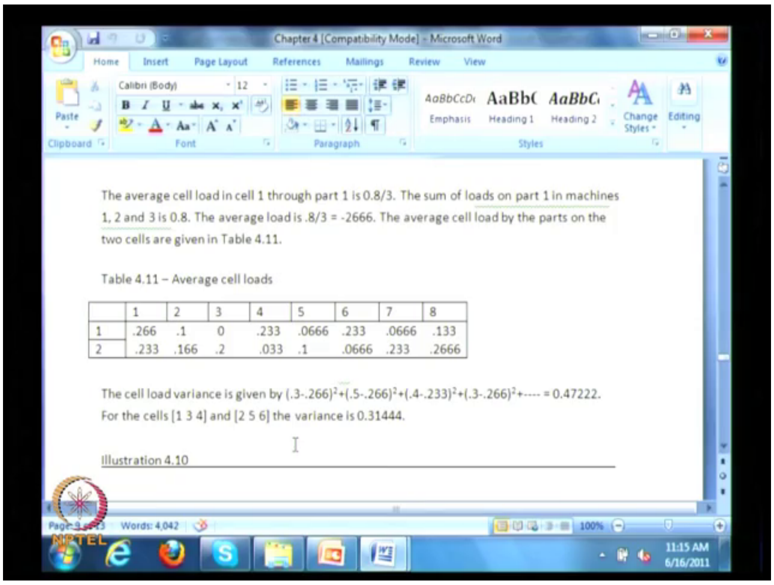

The average cell load in cell 1 through part 1 is 0.8/3. The sum of loads on part 1 in machines 1, 2 and 3 is 0.8. The average load is .8/3 = -2666. The average cell load by the parts on the two cells are given in Table 4.11.

Table 4.11 – Average cell loads

	1	2	3	4	5	6	7	8
1	.266	.1	0	.233	.0666	.233	.0666	.133
2	.233	.166	.2	.033	.1	.0666	.233	.2666

The cell load variance is given by $(.3-.266)^2+(.5-.266)^2+(.4-.233)^2+(.3-.266)^2+$---- = 0.47222.
For the cells [1 3 4] and [2 5 6] the variance is 0.31444.

Illustration 4.10

No so, far what we have seen in cell formation is, we have seen cell formation with binary data, we have seen cell formation with sequence data and we have seen cell formation with workload date. Now let us go back and try to look at a few other things. In all the 3 the common thread is to minimize the inter cell moves which was also what we try to do in production flow analysis.

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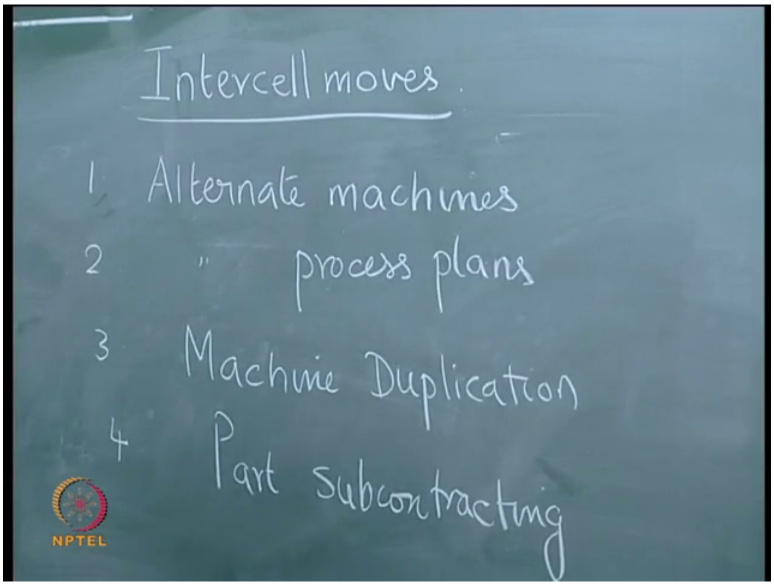

So, at the end we have intercell moves. Now in a previous lecture I had indicated that there are some ways by which we can try and minimize the intercell moves and we looked at 4 different ways of doing it: one is to look at alternate machines; two, to look at alternate process plans; three, machine duplication; and four, part subcontracting.

Now, let us look at all these 4 aspects through a particular example that we have been consistently following.

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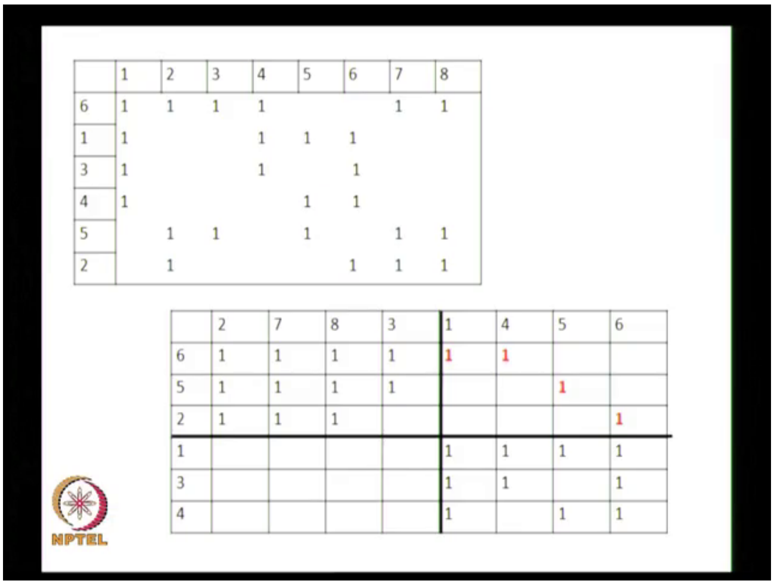

	1	2	3	4	5	6	7	8
6	1	1	1	1			1	1
1	1			1	1	1		
3	1			1		1		
4	1				1	1		
5		1	1		1		1	1
2		1				1	1	1

	2	7	8	3	1	4	5	6
6	1	1	1	1	1	1		
5	1	1	1	1			1	
2	1	1	1					1
1					1	1	1	1
3					1	1		1
4					1		1	1

So, we look at the same example that we have done here we have here I will show the same example through another slide. The same example we have seen here through this the initial machine component incidence matrix is shown there and the cells are shown here and the 4 intercell moves are shown here. Now we have to try and minimize the 4 intercell moves. Now let us look at it by after we create cells using an algorithm, we assume that this would have been created by using either rank order clustering or a similarity coefficient based method or even a p median based method and so on.

So, first one would say when I say alternate machines, it will say can I do there is one operation part 5 on machine 5 which is here, part 5 on machine 5 this is an intercell move well the question is can I have this operation done by this. It sounds a little simple and perhaps a little redundant to say at the moment, but then we also have to understand that the machine component incidence matrix itself is created based on current practice based on which machine part 5 visits to carry out this operation. It may happen that this operation could be done on machine 3 itself, but current practice its visiting machine 5 therefore, the incidence data captured a 1 for p 5 m 5 and did not capture for p 5 and m 3 if such a thing is possible, then we should try and put this 1 here so that we can save an intercell move.

Sometimes as machines get added during expansion, newer machines come into the factory. When newer machines come into the factory they are little faster and they have higher capability and because they are new and fast immediately some operations done on older machines get shifted to the newer machines. Sometimes this is increases the workload on the new machine, but it also takes away some work load from the old machines. So, an older machine which is already there might be capable of doing it. So, 1 needs to check this sometimes even at slightly higher processing times and at that point we understand that if machine 3 can do this operation higher processing time, it is still all right to do it within the cell rather than send it outside to another side. So, the first aspect of alternate machines is taken care of this way second aspect is called alternate process plan for example, if I take p 5 and if p 5 is right now doing m 5 m 1 and m 4.

If there is an alternate plan process plan by which it can do in m 5, m 6 and m 2 if it could then we could choose the other process plan and p 5 will come into this group and it will visit m 6, m 5, m 2. So, this cell will have 5 parts the other cell will have 3 parts. Now the alternate process plan again has to be taken a relook based on 2 considerations one is the existing plan which again would be a common practice, what is currently being followed would be captured the other is to look at can we make some very minor changes in the design which will result in change in process plant and have it to another cell where it minimizes the inter cell move. So, that is another way of doing third one is machine duplication.

Now, if we look at this solution very clearly we understand that if we duplicate machine number 6 into the second cell, we can eliminate 2 inter cell moves. So, first cell will have 2 5 6 second cell will have 1 3 4 6. So, components are parts 1 and 4 will visit the other 6 that will come here. So, this will have 3 machines, this will have 4 machines and there will be 2 inter cell moves which will be on part 5 machine 5 and part 6 machine 2. So, 2 inter cell moves can be cut down if we duplicate machine 6. In fact, this is the same idea that we used in production flow analysis, when we looked at duplicating machine 6 and 8 in that example. Part subcontracting would be to entirely take out that part from the matrix. If we really have to do parts subcontracting then we have to take out assuming that we duplicate 6, we have to take out both 5 and 6 which will go completely away and will not be made in the cells the entire part is out sourced.

In such a case the part families will be 2 8 7 3 for the first cell and 1 and 4 for the second cell the machines will be 2 5 6 for the first cell 1 3 4 and 6 for the second cell and entirely 5 and 6 part 5 and part 6 will completely go out of the matrix. Operation subcontracting would mean that we simply take these 2 operations and subcontract only these operations outside. In such a case after a duplication the this machine cell will have 6 5 and 2 2 7 8 3 this will have machines 1 3 4 and 6 which is duplicated the parts will be 1 4 5 and 6, but these 2 part 5 machine 5 and part 6 machine 2 will be subcontracted operation subcontracting and there will be no inter cell move.

Now, in terms of cost part subcontracting will be costlier than operation subcontracting. Part subcontracting can sometimes result in less utilization of available machines also because when we do part subcontracting of p 5 and p 6; obviously, certain loads on all the 3 machines comes down because this part of the matrix is removed. So, some load comes down. So, there is a cost element which is higher than operation subcontracting there is a load reduction.

So, utilization comes down. There could be issues related to quality because 3 operations are done outside outside of the plant for part 5 and 4 operations are done outside of the plant for part 6 and costly. Operation subcontracting does not have all these disadvantages, it would even be less costly than doing part subcontracting, but the real issue is to get someone who can do only that operation for a less cost; might actually be easier to get some subcontractor who will do the entire part and give it because the earning to the subcontractor will be high.

So, the organization has to judiciously look at do we do part subcontracting or operation subcontracting. Now we will look at some algorithms that handle all these. Particularly algorithms that handle the 3 issues, the 3 issues of alternate process plans the issues of machine duplication and the issues of part subcontracting. Now let us look at alternate process plans as the first one that we will study and how we look at optimization to do the alternate process plan case.

Now, if we have machine components the incidence matrix with.

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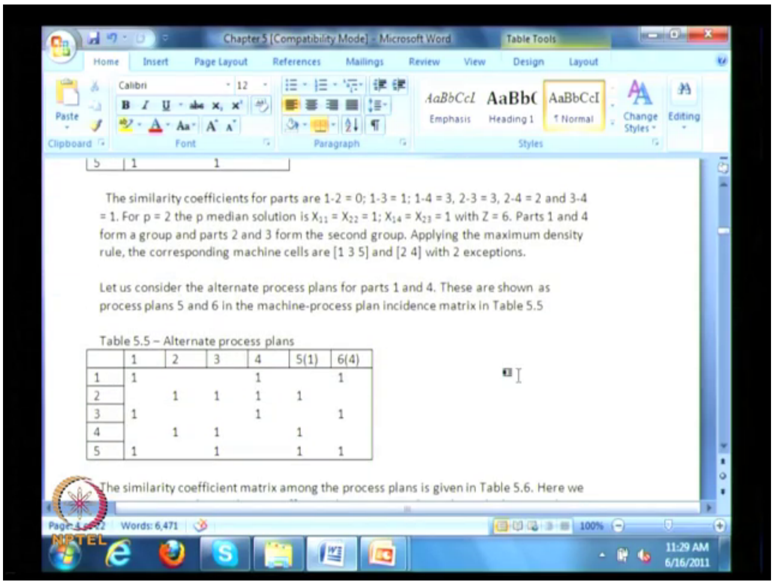

	1	2	3	4	5(1)	6(4)
1	1			1		1
2		1	1	1	1	
3	1			1		1
4		1	1		1	
5	1		1		1	1

Now, let us look at this machine components incidence matrix, which is shown here now this has 5 machines. 1 2 3 4 5 machines and it makes 4 parts 1 2 3 4 parts.

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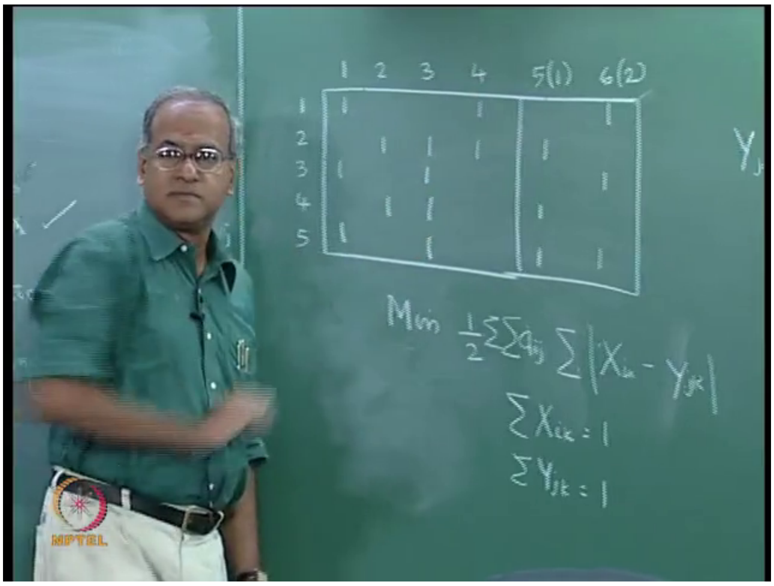

So, incidence matrix is 1 1, 1 plus we assume that 5 is an alternate process plan for 1 and 6 is an alternate process plan for 2. So, 5 has 2 4 5 and 6 has 1 3 5 suppose we have this. Now both the p median formulation as well as the (Refer Time: 48:47) formulation can be modified to consider alternate process plans. Now let us just modify the (Refer Time: 49:00) formulation for this. So, the (Refer Time: 49:01) formulation will minimize inter cell moves half into sigma a i j into X i k minus X j k this will be the objective function where machine I X i k equal to 1 if machine I goes to cell k and Y j k equal to 1, if part j goes to cell k in the earlier formulation that we have seen.

Now, we will replace Y j k equal to 1 if part far if process plan j goes to cell k. Now there are 6 process plans including the 2 alternate process plans, then we will have this condition that sigma X i k equal to 1, because each machine goes to 1 cell sigma Y j k equal to 1 each process plan will go to 1 cell plus we now know that it is enough to choose this or enough to choose this we will not choose both because part 1 can be done by using this process plan or this process.

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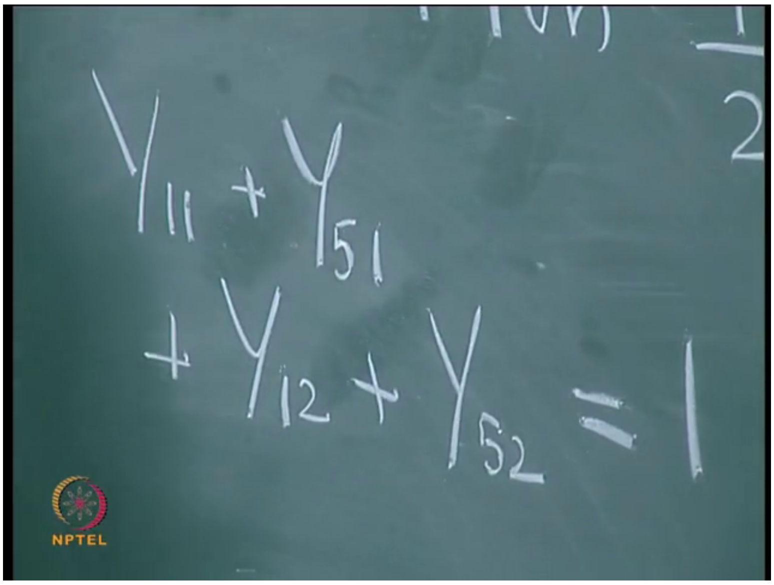

So, we will now, say if there are 2 groups, Y 1 1 plus Y 5 1 plus Y 1 2 plus Y 5 2 is equal to 1, which means either 1 is chosen or 5 is chosen and whatever is chosen will either go to cell 1 or cell 2 depending on where it optimizes. In a similar manner we can write Y 2 1 plus Y 6 1 plus Y 2 2 plus Y 6 2 equal to 1 will ensure that only 1 out of them we will take 1 the others we will take 0 which means only 1 out of the 2 possible alternate process plans will be chosen and that process plan will either be allocated to cell 1 or allocated to cell 2.

So, the problem of alternate process plans can be formulated and brought into an optimization problem by simply adding these 2 constraints. In a similar manner the machine duplication problem can also be done we will see those aspects in the next lecture.