

Manufacturing Systems Management
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Lecture – 14
Algorithm considering sequence of visit of machines

In this lecture we continue the discussion on cell formation using sequence of parts.


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	1	2	3	4	5	6	7	8
1	1			2	1	1		
2		1				4	1	1
3	3			3		2		
4	4				3	3		
5		2	2		2		2	3
6	2	3	1	1			3	2

1, 2, 3 --- 1, 4, 6

4, 5, 6 --- 2, 3, 5, 7, 8

10 moves



So, in the previous lecture we looked at this matrix where we have 6 rows for 6 machines and 8 parts and the incidence data is given in the form of the sequence of visit of the machines.

For example part 1 visits machines 1 3 4 and 6, but in the order machine 1 first followed by machine 6 then by then to machine 3 and then to machine 4. So, the 1 2 3 4 would tell us the order in which each part visits the machine. We also saw the difference between an intercell move particularly when we consider the sequence data versus intercell move when we consider the binary data.

For example, if for part 1 there is an intercell move say on machine 6 if there is an intercell move and machine 6 belongs to some other cell then there are actually 2 moves because from 1 it has to go to 6 and then it has to come back. So, it will be treated as 2 moves whereas, when we used 0 1 data only 1 1 element will be outside of the block and

therefore, we would count it as 1 intercept. So, the intercell move computation becomes different when we consider sequence data. So, we also saw some aspects of a mathematical formulation to solve this problem in the previous lecture.

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C_{ij} be the number of parts that visit machine j immediately after visiting machine i or visiting machine i immediately after visiting machine j . The C_{ij} values can be computed from a given machine component incidence matrix.

Let $X_{ij} = 1$ if machines i and j belong to the same machine group.

Here X_{ij} are defined for values $i = 1, \dots, M-1$ and $j = i+1, \dots, M$.


$$\text{Minimize } \sum_{i=1}^{M-1} \sum_{j=i+1}^M C_{ij} (1 - X_{ij})$$

$$\sum_{i=1}^{k-1} X_{ik} + \sum_{j=k-1}^M X_{kj} \leq N - 1 \quad \forall k = 1, \dots, M$$

$$X_{ij} + X_{jk} - X_{jk} \leq 1$$

$$X_{ij} - X_{jk} + X_{jk} \leq 1$$

$$-X_{ij} + X_{jk} + X_{jk} \leq 1 \quad \forall i = 1, \dots, M-2, j = i+1, \dots, M-1, k = j+1, \dots, M$$

$$X_{ij} = 0, 1$$


So, we define C_{ij} to be the number of parts that visit machine j immediately after visiting machine i or visiting machine i immediately after visiting machine j , X_{ij} will be equal to 1 if i and j belong to the same group machines i and j belong to the same group.


So, in this notation both i and j represent machines. So, C_{ij} is the number of parts that visit both more importantly visit i immediately after visiting j or visit j immediately after visiting machine i .

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The binary IP formulation has 15 variables and 63 constraints.

The optimal solution is given by $X_{13} = X_{14} = X_{34} = X_{25} = X_{26} = X_{56} = 1$
with objective function value = **11**.

	1	4	5	6	2	3	7	8
1	1	2	1	1				
3	3	3		2				
4	4		3	3				
2				4	1		1	1
5			2		2	2	2	3
6	2	1			3	1	3	2



If we consider this incidence matrix particularly if we look at similarity between machines 5 and 6 parts 2 3 7 and 8 all of them visit both 5 and 6 now 2 visits 6 after visiting 5.

So, it contributes to the C_{ij} 3 visits 5 after visiting 6. So, it contributes 7 visits machine 6 after visiting immediately after visiting machine 5. So, it contributes and part 8 visits machine 5 immediately after visiting machine 6. So, it contributes to the C_{ij} . Whereas, if we take 1 and 4 machines 1 and 4 we would see that part 1 visits both machines 1 and four, but it does not visit them immediately after visiting the other therefore, it does not contribute to C_{14} . Part 5 also visits both machines 1 and 4, but it does not contribute to C_{14} because it is not visiting immediately same with part 6. So, even though we have 3 parts that are common the C_{14} will be 0 because there is no part that visits 4 immediately after visiting machine 1.

Now, the formulation is to minimize double summation C_{ij} into $1 - X_{ij}$. Now let us find out what we attempt to do by minimizing C_{ij} into $1 - X_{ij}$, now when we have a large C_{ij} it means we have machines i and j where a large number of parts visit i immediately after j or j immediately after i .

So, it is desirable that i and j belong to the same group because if i and j belong to different groups then there will be 2 intercell moves for every C_{ij} calculated. So, it is preferable that it is they belong to the same group now X_{ij} is equal to 1 if they belong to

the same group $1 - X_{ij}$ will become 1 if X_{ij} is 0 and if they belong to different groups.

So, $C_{ij} - X_{ij}$ will be the number of intercell moves that can happen when i and j belong to different groups. So, we want to minimize $C_{ij} - X_{ij}$. So, the objective function essentially tries to minimize the intercell moves or tries to put machines that have large number of parts visiting them immediately after each other into the same cell, that is what it achieves to do here we have not fixed the number of cells the formulation none of the decision variables X_{ij} is the decision variable X_{ij} does not talk about the number of cells that we wish to create. There is no P or any such number which tells given input which tells us the number of cells. Therefore, we need to restrict the cell size. So, that we have a certain minimum number of cells.

For example if the number of machines is 6 and if the cell sizes for then it will have 2 cells; 1 of the ways of creating more than 1 cell is by restricting the number of machines in a cell. Otherwise it will lead us to a trivial solution where all the machines that were there were only 1 group, all the machines will be in that group, all the parts will be in that group and the inter-cell move will be 0. Now we do not want that to happen therefore, we restrict the cell size.

So, the first constraint which is $\sum_{i=1}^{k-1} X_{ik} + \sum_{j=1}^{k+1} X_{kj} \leq N - 1$ will restrict the number of machines in a cell. The other 3 constraints will ensure that if there are 2 machines i and j that belong to the same cell and if j and k belong to the same cell then i and k should belong to the same cell, if i and k and j belong to the same cell j and k belong to different cells then i and k should belong to different cells and so on.

So, those are taken care of by those constraints and it is a binary formulation. So, when we apply this formulation to this data we have not explicitly shown the C_{ij} matrix, C_{ij} is the number of parts that visit machines i and j considerably which means it visits j immediately after i or i immediately after j .

So, we have not shown the computation of the C_{ij} matrix now when we try and solve this for the example that contains 6 machines and 8 parts we have 15 variables and 63 constraints 15 variables comes from X_{ij} , there are 6 machines. So, 6 into 5 by 2 by 2 comes because the variables are defined as $X_{12}, X_{13}, X_{14}, X_{15}, X_{16}, X_{21}$ is not defined

explicitly 2 1 is the same as 1 2. So, 2 3, 2 4, 2 5, 2 6, X 3 4, X 3 5, X 3 6, X 4 5, X 4 6 and X 5 6. So, that gives us 15 variables $N C 2$ or $M C 2$ if there are M machines, 63 constraints come from these sets of constraints that we have here.


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New Similarity Coefficient

Nair and Narendran (1998) introduced the following similarity coefficient between machines i and j:

$$S_{ij} = \frac{A_{ij}}{B_{ij}}$$

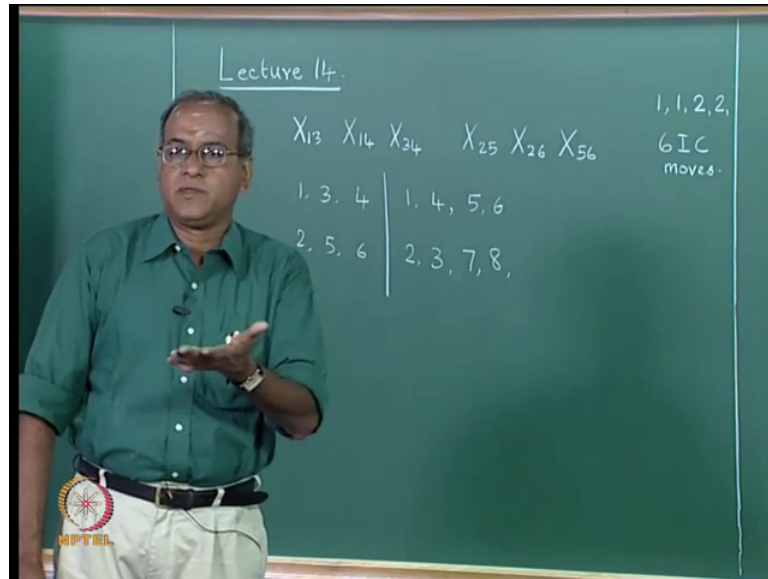
Here A_{ij} represents the contribution of parts visiting both the machines i and j. Part k visiting both i and j contributes 1 if it visits the machine as its first or last machine in the sequence and contributes 2 if it is an intermediate machine. Similarly B_{ij} represents the contribution of parts visiting both either machine i or machine j or both. Part k contributes 1 if it visits the machine as its first or last machine in the sequence and contributes 2 if it is an intermediate machine. The machine groups are formed based on the S_{ij} values.



Now, there are 5 constraints that come from this and the rest of them come from here. So, it is a slightly large formulation in terms of number of constraints particularly because of this these 3 sets of constraints that come this is given from I equal to 1 to M minus 2 j equal to i plus 1 to M minus 1 and k equal to j plus 1 to m for any $i j k$.

So, in this case we will have i equal to 1 to M minus 2 which means i equal to 1 to 4 j will be for the corresponding i plus 1 to M and k will be for the corresponding j plus 1 (Refer Time: 10:44). So, at this gives rise to a large number of constraints. So, there are 15 variables and 63 constraints and when we solve this binary IP we get the solution $X 1 3$ equal to $X 3 4$ $X 1 4$ equal to $X 3 4$ $X 2 5$ equal to $X 2 6$ equal to $X 5$ by 6 equal to 1 with objective function 11.

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Now keeping this solution let us try and look at some of the aspects of this formulation. So, let us first write this solution. So, the solution will be $X_{13} X_{14} X_{34} X_{25} X_{26} X_{56}$ equal to 1. So, the final block diagonal structure is shown here. So, we have 1 3 4 these are the machine groups, now we take part 1 visits machines 1 3 4 and 6. So, it will go to this part 1 1 3 4 and 6. So, it will contribute to only 1 inter cell move because it will finish 1 3 4 in the first group, after that it moves to the second group.

So, 1 inter cell move, part 2 visits to 5 and 6. So, part 2 does not involve any inter cell move, part 3 visits 5 and 6 there in the same cell does not involve an inter cell move, part 4 visits 1 2 and 3 part 4 will go here, but it first visits machine 6 then comes to 1 and 3. So, it starts here 1 inter-cell move to 1 and 3 and back. So, plus 1 inter cell move, part 5 visits 1 4 and 5. So, part 5 will be here, but visits machine 1 first, machine 5, and then machine 4.

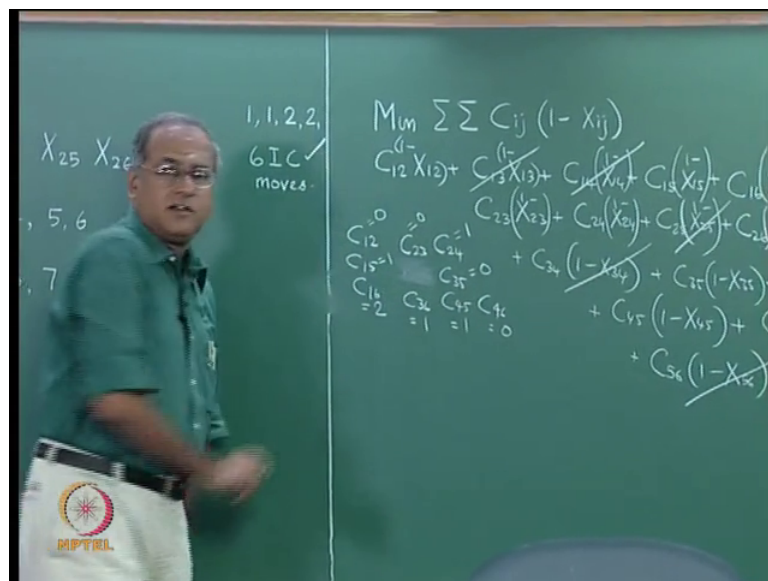
So, machine 1, then machine 5, and then machine 4. So, creates 2 inter-cell most, part 6 visits 1 2 3 4 1 2 3 4. So, creates 2 more inter cell moves, part 7 visits 2 5 and 6. So, part 7 visits here and part 8 also visits here. So, with 6 inter-cell moves it has 6 inter-cell moves, the objective function does not reflect the number of inter-cell moves objective function indirectly minimizes the number of intra cell moves.

The objective function happens to be 11 here. So, we need to explain this the objective function is to minimize.

You look at 2 3 there is only 1 common part which is 6, once again it does not visit immediately after each other. So, this also contributes to 0. So, 2 4 there is 1 common part which is 6 again and part 6 visits to immediately after visiting for therefore, C 2 4 will contribute to 1, C 1 5 we do not have a common part between 1 and 5 there is no common part there is 1 common part between 1 and 5 which is part 5 and it visits 5 immediately after visiting 1 therefore, this contributes to 1.

We look at 3 5 there is no common part between 3 and 5 therefore, this will contribute to 0, look at 1 and 6 part 1 is common that will contribute 1, part 4 is common that will contribute another 1 so 2. So, 1 and 6 will have 2 contributions. So, this will be 2. 3 and 6 part 1 is a contribution. So, it contributes 1 part 4 is common, but does not contribute. So, this contributes 1 4 and 5, 4 and 5 have 1 contribution part 5 is common between 4 and 5 and part 5 visits for immediately after visiting 5.

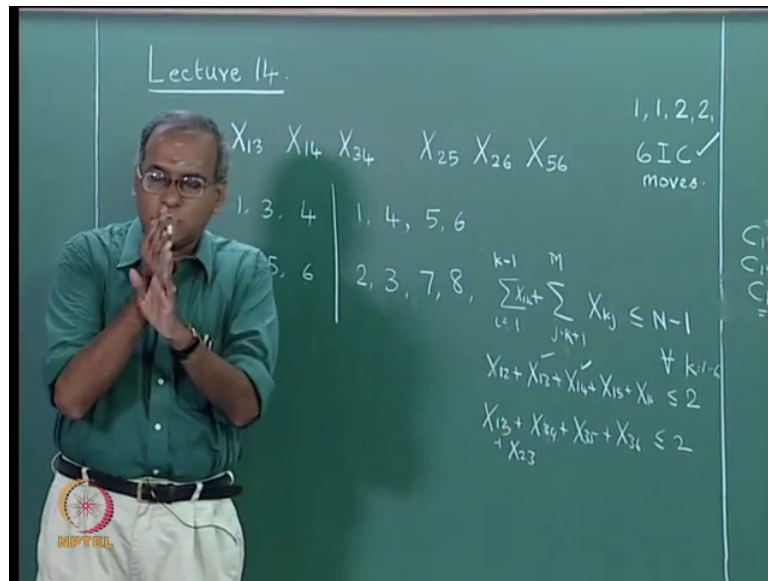
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So, the contribution is 1, 4 and 6 part 1 is common, but does not contribute and there is no other thing. So, it is 0. So, the sum of these contributions is actually 6, 1 plus 1 2 plus 2 plus 4 plus 1 5 plus 1 6 which actually gives us the number of inter cell moves.

So, this formulation essentially tries to minimize the number of inter-cell moves. Let us also try to understand the constraints and see how this solution satisfies all the constraints.

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So, the first constraint is $\sum_{i=1}^{k-1} X_{i,k} + \sum_{j=k+1}^M X_{k,j} \leq N-1$, for every k equal to 1 to M , which means there are 6 machines in this formulation $\sum_{i=1}^{k-1} X_{i,k} + \sum_{j=k+1}^M X_{k,j} \leq N-1$ to M j equal to $k+1$ to M $X_{k,j} \leq N-1$.

So, there are M machines there are 6 machines. So, there will be 6 constraints. So, let us take the first constraint and try to explain what actually happens. So, when k equal to 1 this is for every k equal to 1 to 6, now for k equal to 1 this term does not exist. So, only these terms will exist. So, for k equal to 1 it will become $X_{1,2} + X_{1,3} + X_{1,4} + X_{1,5} + X_{1,6} \leq 2$.

So, this will ensure that if 1 of them if 1 of the $X_{i,j}$ s happens to be taking the value 1. So, in our case $X_{1,3}$ took a value 1, which means 1 and 3 belong to the same group and because I have a restriction of 3 machines only 1 more can be added to this and that can be either $X_{1,2}$ something or $X_{3,4}$ something.

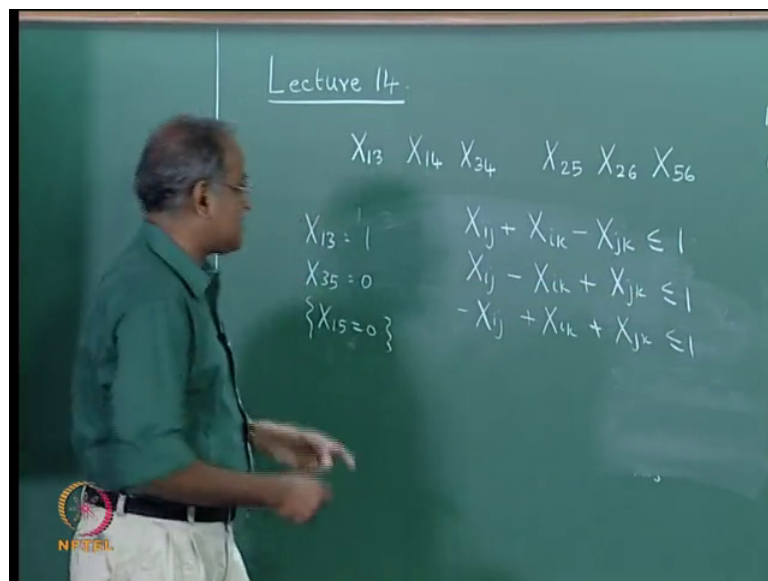
So, in this case 1 3 and then 1 4 so it became less than or equal to 2 it did not allow for example, 1 3 1 4 and 1 5 to come into the solution, which means for machines would have been there. Therefore, this takes care of the fact that there can be a maximum of 3 we may have a situation that only 1 and 3 form a group then the rest of them will take 0 and still this will be satisfied less than equal to 2 will be satisfied.

Similarly, we can explain for example, when k equal to 3, let us write just that equation for k equal to 3. So, i equal to 1 2 k minus 1. So, it will be like X_{13} is X_{12} and k equal to 3 i equal to 1 to k minus 1 will be X_{12} plus X_{34} plus 1 to M . So, when k equal to 3 it will become $4 X_{kj} X_{34}$ plus X_{35} plus X_{36} will be equal to 2.

I think this this should read i equal to 1 2 k i equal to 1 this this becomes i if in k equal to 3 X_{i1} k equal to 3. So, i equal to 1 $\sum_{i=1}^2$. So, there are 2 terms. So, X_{13} plus X_{23} plus X_{34} plus X_{35} plus X_{36} is less than or equal to 2. So, there are there have to be 5 terms in each constraint. Now this will ensure that if 3 has been grouped with another machine which has a lower index than 3, that also comes into the picture plus machines that are grouped with a higher index than 3.

So, if we take this constraint and look at this solution 1 3 is 1 3 4 is 1. So, it satisfies with less than or equal to 2 which again indicates that 1 3 and 4 belong to the same group this also satisfies that. So, similarly we can show that the first constraint ensures that the cell size is restricted to something the next set of constraints we will just show through 1 instance the next set of constraints which are X_{ij} plus X_{ik} minus X_{jk} less than or equal to 1 X_{ij} minus X_{ik} plus X_{jk} less than or equal to 1 minus X_{ij} plus X_{ik} plus X_{jk} less than or equal to 1.

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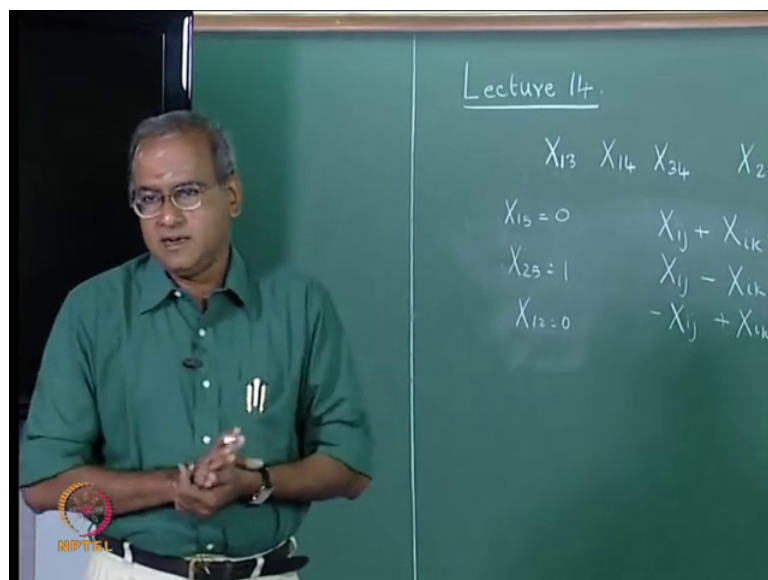


Now, let us try and explain this now we want to show to 2 or 3 things through these constraints. So, if X_{13} is 1 X_{14} is 1, then X_{34} has to be 1. So, X_{13} is 1 X_{14} is 1

this is X_{34} . So, plus plus 1 minus minus 1 is less than or equal to 1, 1 minus 1 plus 1. So, it follows now let us look at a case when this is 1 this is 1 and this is 0. So, this will be 2 less than equal to 1 is not satisfied. So, automatically when these 2 is take 1 this 1 has to take a value 1 because X_{ij} is binary. So, this will force it to 1 and the other constraints will be satisfied. So, if X_{ij} and X_{jk} are 1 then it will force X_{ik} to 1.

Now, let us look at the other case X_{13} is 1 X_{35} is 0 therefore, it should force X_{15} also to 0. So, 1 plus 0 1 plus 0 they should force X_{15} to 0 now from this constraint whether this is 0 or 1 this is satisfied. So, 1 minus 0, 1 minus 0, which is 1 so this will force X_{jk} to 0 otherwise this will become 2. So, this constraint will take care of that.

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So, in a similar manner this will also ensure finally, that if X_{ij} is 0 if X_{15} is 0 X_{25} is 1. So, this will force X_{12} to 0, that will be taken care by the third constraint. So, these constraints put together for every ijk will ensure these conditions that if x_{ij} is 1 and X_{jk} is 1 then X_{ik} is 1 and so on. So, this is how this formulation works.

Now, this formulation is very good in the sense that it provides an optimal solution it is objective function value indicates the indicates the number of inter-cell moves. Now the limitation of this model is that when we considered a matrix with 6 machines and 8 parts, we are 15 variables and 63 constraints. So, the number of constraints grows particularly because the number of variables also increases, but the number of constraints grows because of the 3 constraints for every pair.

So, what happens is when we have a large number of machines the number of constraints becomes very large? So, if it is possible to solve the binary IP and get the optimum solution it is the best thing to happen, but if there are limitations because of problem size or because of solver availability and so on. Then one has to resort to heuristic solutions which are like similarity coefficient based solutions. So, researchers also proposed similarity coefficient based solutions to this problem.


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New Similarity Coefficient

Nair and Narendran (1998) introduced the following similarity coefficient between machines i and j :

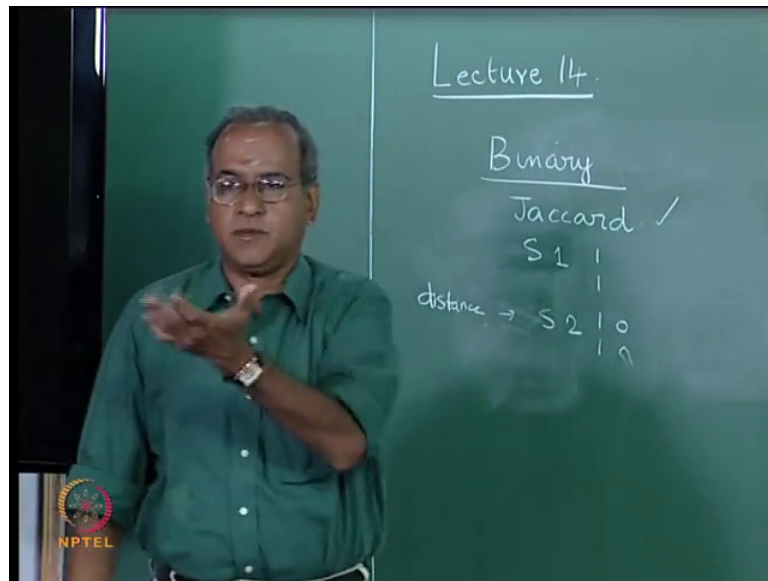
$$S_{ij} = \frac{A_{ij}}{B_{ij}}$$

Here A_{ij} represents the contribution of parts visiting both the machines i and j . Part k visiting both i and j contributes 1 if it visits the machine as its first or last machine in the sequence and contributes 2 if it is an intermediate machine. Similarly B_{ij} represents the contribution of parts visiting both either machine i or machine j or both. Part k contributes 1 if it visits the machine as its first or last machine in the sequence and contributes 2 if it is an intermediate machine. The machine groups are formed based on the S_{ij} values.



We have already seen 3 types of similarity coefficients; we have seen the jacquard similarity coefficient. So, for binary data we have seen the jacquard similarity coefficient.

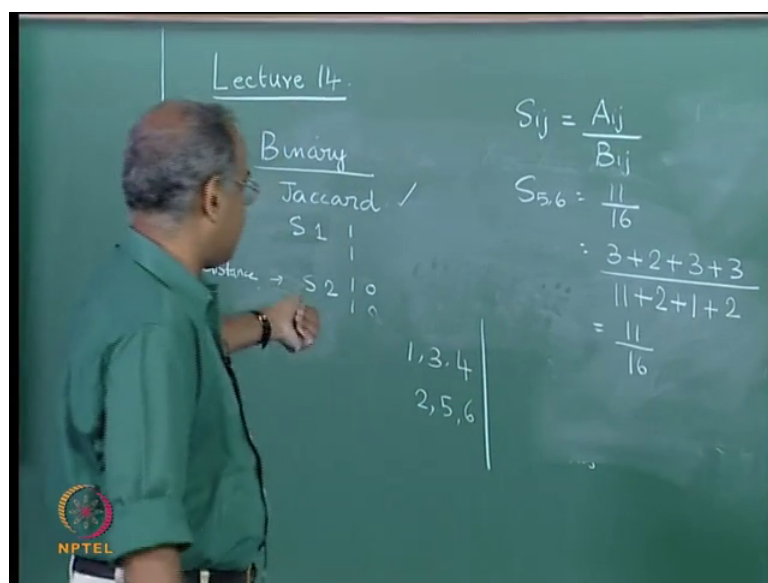
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We have seen a similarity coefficient which uses only the 1 1 pair and then a similarity coefficient that uses both 1 1 pair and 0 0 pair, which you can call as similarity 2 this could be as 1 and this could be (Refer Time: 32:50) and we also saw that this is related to distance.

Now, liar and Narendran introduced a different kind of similarity coefficient between machines where S_{ij} is equal to A_{ij} by B_{ij} .

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Now, the very definition of a A_{ij} by B_{ij} is very close to the jacquard similarity coefficient; jacquard similarity coefficient had a numerator and the denominator the numerator were represented the number of parts that visited both the machines, the denominator represented the number of parts that visited either one of the machines are both. Now the new coefficient also has a numerator and a denominator with a very slight difference in the computation.

So, the new coefficient is A_{ij} by B_{ij} A_{ij} represents the contribution of parts visiting both the machines i and j . So, numerator is similar to the numerator of the jacquard similarity coefficient, the difference is part k visiting both i and j will contribute 1 if it visits the machine as it is first or last machine in the sequence and contributes 2 if it is an intermediate machine.

So, let me read it again part k visiting both i and j will contribute 1 if it visits the machine as it is first or last if it visits one of the machines, as the first or last machine in the sequence and will contribute to if it visits both of them happen to be intermediate machines.


Similarly, B_{ij} represents the contribution of parts visiting either machine i or machine j or both it contributes 1 if it visits the machine as it is first or last machine in the sequence and contributes 2 if it is an intermediate machine.

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Considering machines 1 and 2, we observe that part 6 visits both. A total of 7 parts visit either machine 1 or machine 2 or both. $A_{12} = 2$ because part 6 visits machine 1 as first operation and visits machine 2 as fourth (last) operation. Each contributes 1 to the A_{12} value. The contributions of parts 1, 2, 4, 5, 6, 7, 8 are 1, 1, 2, 1, 2, 1, 1 respectively. Therefore $B_{12} = 9$ and $S_{12} = 2/9$.

Machines i-j	J_{ij}	Machines i-j	J_{ij}	Machines i-j	J_{ij}
1-2	2/9	2-3	3/9	3-5	0
1-3	9/10	2-4	3/6	3-6	5/10
1-4	7/9	2-5	8/12	4-5	3/11
1-5	3/11	2-6	7/12	4-6	3/12
1-6	6/13	3-4	7/8	5-6	11/16

Two machine groups are [1 3 4] and [2 5 6] respectively. There are six inter cell moves.



The machine groups are formed based on the similarity values. So, the similarity coefficients are given here. So, let us try and explain the similarity coefficient now this is the matrix. So, a similarity coefficient between 1 and 2 if we take this take the similarity coefficient between 1 and 2, there is only 1 part which visits both machines 1 and 2 which is part 6. Now it visits 1 and 2 as the last machines because the sequence of machine 1 is 1, sequence on machine 2 is 4 therefore, it visits M 1 as the first machine M 4 as the last machine. So, it should contribute 1. So, let us see what it does.

So, now it contributes 1 this is common. So, it contributes 1 here because it visits as the first machine and contributes another one here because it visits as the last machine. Therefore, it contributes 2 numerator contribution is 2; denominator is if it visits the machine either as either 1 of the machines are both the machines. So, if we take machines 1 and 2 parts 1 2 4 5 6 7 and 8 visit them. So, this will contribute 1 because it is visiting as a first machine, this contributes another 1 this will contribute it is visiting as an intermediate machine. So, this will contribute. So, this contributes 1 this contributes 1 the denominator is 2 by 9. So, the contribution of the denominator is 9. So, let us go back and check 2 from here 3 4 5 7 8 and 9.

So, it contributes 1 because it is the first machine, 1 as the first machine contributes 2 because it visits this as an intermediate 1. So, 1 plus 1 2 plus 2 4 plus 1 5 plus 2 from the numerator 7 1 8 1 9 1 contribution is 1 because it visits m 2 as the first machine. So, the similarity coefficient is 2 by 9.

Let us explain another similarity coefficient say between 5 and 6 let us compute between 5 and 6 the value is 11 by 16, $S_{5,6} = \frac{11}{16}$ or $S_{5,6}$ is equal to 11 by 16, let us show the computation. So, we consider machines 5 and 6 the common parts are parts 2 3 7 and 8. So, part 2 will contribute 2 plus 1 3 because it visits machine 5 as an intermediate machine and machine 6 as the last machine. So, it will contribute 3; part 3 visits machines 5 and 6, but visits only 5 and 6. So, visits 6 as the first machine and 5 as the last machine. So, it will contribute 2, 7 and 8 visit both of them 7 visits machine 5 as an intermediate machine and 6 as a last machine. So, the contribution will be 2 plus 1 3 and part 8 will visit machine 6 as an intermediate machine because of the 2 and machine 5 as the last machine because of the 3 and it contributes another 2 plus 2 3. So, the numerator is 11.

Now, the denominator is the numerator plus the contribution of parts that visit only 1 of them. So, a denominator will be numerator 11 plus contribution from parts that visit either 5 or 6 that happens for part 1, part 4, and part 5. So, part 1 visits only machine 6, but visits as an intermediate machine and therefore, it will contribute 2, part 4 visits machine 6, but visits machine 6 as the first machine and will contribute 1, part 5 visits machine 5, but visits as an intermediate machine and therefore, will contribute another 2. So, numerator is 11 and denominator is 60. So, this way we can compute the similarity coefficient matrix which is now shown in this table.

Now, from the similarity coefficient matrix we will now try and get groups. So, the maximum similarity happens to be for 1 2 3 which is 9 by 10 is about 90 percent 3 2 4 has less than that 2 2 5 is also less 2 2 6 is also less 5 2 6 is also. So, the maximum similarity happens for 1 and 3. So, 1 and 3 will come to the first group. So, 1 and 3 will form the first group the next highest similarity happens for 7 by 9 is about less than 80 percent 3 to 4 is 7 by 8 which is about 90 percent, 2 5 is 75 percent. So, 3 2 4 will be the next 1 with 7 by 8. So, 4 will join 3 the next maximum will be 2 to 5 is about 75 percent 8 by 12 5 to 6 is less than 70 percent to 2 to 6 is also less. So, 2 to 5 is the next highest. So, 2 to 5 will form another group, will form the second group and we will add 6 to it. So, 2 5 6 will come here. Once again the parts can be assigned based on the part assignment rule and we get the same solution that we got for the integer programming formulation also.

So, when we have a large sized matrix 1 can use the similarity coefficient based approach to try and get machine cells and then use the part assignment rule to assign parts and calculate inters-cell moves. Now the similarity coefficient based method works on a simple idea that now we need to explain the idea as to why how it is able to incorporate sequence data, 1 way by which we can show that it incorporates sequence data is by looking at the contributions to the numerator and to the denominator. So, there are parts that contribute to either the numerator or the denominator 2 plus 1 earlier it was contributing only a 1 to a numerator or denominator when we used only jacquards similarity coefficient now we also need to understand how that 2 is going to help us.

So, if we look at this matrix and let us say we look at 3 and 4 which had the max we look at 1 and 3 which had the maximum similarity, if we look at 1 and 3 the ones that have maximum similarity we would get let us compute the similarity for 1 and 3 or we can

even show that let us say 5 and 6 actually belong to the same group. So, 2 5 and 6 let us show these 11 by 16 and how it helps. So, if we take 5 and 6 we realize that the higher value of the numerator came through the contributions of part 2, part 7 and part 8. All these parts contributed at 3 which meant that 5 and 6 are particularly through this they are visited as 2 consecutive machines.

So, when you have 2 consecutive machines their contribution will be large, the only case where 2 consecutive machines will have a contribution of 2 is from part 3 there are only 2 machines it visits both happen to be 5 and 6 it contributes 2 if they are were part which visited 1 5 6 and 2 for example, then that would have contributed 4. So, what happens is when you have consecutive visits even if 1 of the visit is the first or last visit it is going to contribute 3.

So, higher similarity coefficient indicates that there are more parts that actually visit these machines immediately after another. So, the bigger contribution actually comes from the numerator the denominator also helps in its own way by kind of filtering the extent to which the values take and also by ensuring that this ratio is between 0 and 1, because the numerator repeats itself in the denominator the more significant contribution actually comes from the numerator.

Now, this particular contribution where machines are visited continuously or consecutively or 1 after another contributing to a higher similarity coefficient is the same as C_{ij} minimizing C_{ij} into $1 - X_{ij}$ which was the objective function in the integer programming formulation of the problem.

So, indirectly the similarity coefficient captures what the objective function captured in the optimization approach to solving the problem with sequence. For the small sized problem that we have shown here through our example the final solution is the same where 1 3 and 4 came to 1 cell, 2 5 and 6 went to another cell, but for large problem instances where there are several intercell moves it is possible to show that either the optimization approach that we saw now or the heuristic approach that uses similarity coefficients, we are able to get configurations with fewer intercell moves particularly when we count an inter cell move involving an intermediate visit as 2 intercell moves.

Therefore, not only is the inter cell move important the inter cell move for an intermediate machine if it goes to another cell for an intermediate operation and then it

comes back, then it is counted as 2 intercell moves and indirectly it is also seen as a way of losing a little more control. If it were the first or the last operation then you know that if it is a first operation in another cell it is still not very desirable, but you would know that it enters the actual cell after completing an operation elsewhere and 1 is it comes in we know that rest of the things are going to happen in this cell.

So, to that extent it is fine therefore, 1 inter-cell move and to that extent less control. If it finishes all the operations in 1 cell and has to go only for the last operation to another cell. Even then certain amount of scheduling activities can be carried out in this cell such that all the work is done control up to the point up to the penultimate operation and then it goes to another cell and then it leaves, to that extent there is some control even though a little bit of control is lost, but if it has to go for an intermediate operation then it actually affects in 2 ways, because the time at which the other machine is available and is willing to spare certain activities have to be completed here and depending on when it is coming back the rest of the activities will also have to be scheduled.

So, there are 2 physical inter-cell moves that are correspondingly the control is lost in 2 different points in time therefore, it is not very desirable therefore, higher penalty is given and once that is the penalty is minimized, we now get machines which have consultative of operations carried in the same cell, which is the very purpose and spirit of cell formation considering sequence of visits of the parts. The next thing that we will see is what happens when we have when we consider cell load data and how the cell formation problem changes when we consider cell load this we will see in the next lecture.