Manufacturing Systems Management

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Lecture - 11
Optimization based algorithms

In this lecture we look at some Optimization based methods for cell formation. So far we have seen few algorithms to create pat families and machine cells. Starting with the production flow analysis, we also looked at rank order clustering, we looked at similarity based approach or a hierarchical based clustering based approach and then we followed it up with a non-hierarchical clustering based algorithm. All of them provide good solutions to the cell formation problem, but we address another issue of is there scope for optimization or how does an optimum solution to this problem look like, how does an objective function look like for this problem and so on. So, we will be looking at a couple of optimization based algorithm.

Let us revisit the grouping problem that we have already seen more than once.

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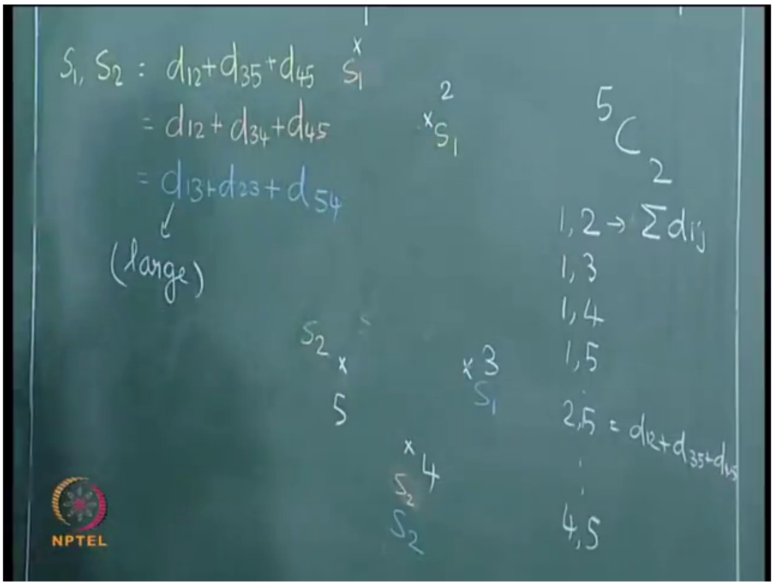

So, let us go back and look at how we group let us say we have 5 points which we call as 1 2 3 4 and 5. Now let us go back to the non-hierarchical clustering algorithm where we said that we want to have 2 groups and we fix 8 points, now let us say that we want 2 groups, we have already seen that 1 of the ways of getting 2 groups is to have 2 same points and we have already seen that it is good to have the seed points as the points themselves and we have seen that the seed points should be far away from each other.

Now, for this simple example we know that if say this S 1 and S 2, we could do to get the best group. In fact, even if we had tried this as S 1 and this as S 2, we would still have got the same groups at the end of the exercise. We also know that if we have tried this as S 1 and this as S 2 we will not have got a very good solution, because if we have tried this as S 1. Let us look at the yellow solution where this is S 1 and this is S 2.

So, if we had S 1 S 2 by which I mean 2 is seed S 1 and 5 is seed S 2, if I group with this 1 would have gone to this let me remove this for a moment. So, 1 would have gone to 2 2 would have gone to itself, 5 would have gone to itself, 3 and 4 would have gone to 5. So, the groups would have been 1 2 3 4 5, but distance would have been d 1 2 plus d 3 5 plus d 4 5 this would have been the distance. Because d 2 2 is 0 and d 5 5 is 0 if we had chosen this 1 or before that, if we had chosen this as S 1 this as S 2 now for this solution the distance will be d 1 2 which is a same as d 2 1 d 1 1 is 0 plus d 3 4 plus d 4 5. If we had chosen this then the distance would be d 1 3 plus d 2 3 plus d 5 4, because 3 3 would be 0 and 4 4 would be 0 5 would have gone to this these 2 would have gone to 3.

Obviously this is a bad solution and by judging either from the points themselves or by understanding the value this is not a good solution, because this value is large compared to either this or this, now even though both of these would give the same groups 1 2 and 3 4 5 can happen that 1 of them is slightly smaller than the other. Therefore, 1 of the ways of solving this problem optimally is if there are 5 points in this case 2 seeds can be chosen out of 5 points in 5 C 2 ways and for each par or seats starting 1, 2 1, 3 1, 4 1, 5 upto 4 5 5 C 2 is 10. So, 10 such solutions 1 can do the clustering get the groups and more importantly get the total distance which I call as sigma d i j total distance.

For example, for the combination 2 and 5 for 2 and 5 somewhere this would be the yellow 1 d 1 2 plus d 3 5 plus d 4 5 compute all of these and then choose the 1 which has minimum distance of these that would certainly give us the best group. For example, there could be many which would have given us this there could be many pairs of seeds that could have given us the solution 1 2 3 4 5 1 3 would have given that, 1 4 would have given that, 1 5 would have given that, 2 3 would have given that, 2 4 and 2 5 actually 6 out of these 10 pairs would have given us the same groups 1 2 and 3 4 5, but 1 out of the 6 or more than 1 out of the 6 would have the smallest distance.

So, certainly the 1 with the smallest distance will give us the best solution. So, if we are looking at an optimization problem, the criterion to choose would be minimum distance would be to minimize the distance. So, the optimization problem will look like choose 2 points and cluster around them a very rudimentary algorithm would be to choose all possible pairs clustered around them find out the distance as an objective function or a measure of performance of the cluster and choose that pair of seeds, which has minimum distance that pair of seeds and the associated groups that has minimum distance.

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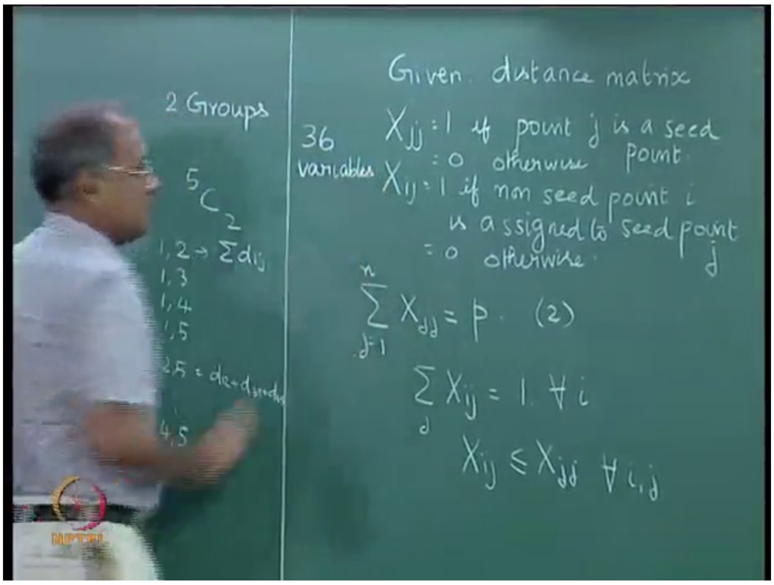

Now, that can be formulated as an optimization problem with an objective of minimizing the distance. So, given a distance matrix given a distance matrix get points to out of these 5 and take it now there are several ways of formulating this problem 1 of which is a well-known formulation.

So, we will do this formulation and then we will go back and explain and name the formulation. So, the formulation will be like this now we could say that we want to seeds. So, we could either say that let if point j is a seed we could say y j equal to 1 if point j is a seed maybe instead of saying y j equal to 1 we now say X j j equal to 1 if point j is a seed point. We could have defined it as y z, but we simply define it as X j j equal to 1 if point j is a seed point. We could have defined it has y j, but we simply defined it has X j j equal to 1 if point j is a seed point, equal to 0 otherwise and there are 5 points there are going to be 2 seed points. So, there are going to be 3 non seed points these 3 non seed points are going to be assigned to seed points and these 3 are the ones that are going to create the distance that we have here, because assigning the seed point to itself gives a distance of 0.

So, X i j equal to 1 if non seed point i is assigned seed point j. So, a non-seed point I have to be assigned to a seed point j. So, we have written the decision variables equal to 0 otherwise here I can write equal to 0 otherwise, again equal to 0 otherwise. So, what are my constraints sigma X j j equal to p j equal to 1 to n summation. Now here m is the number of points that I have. So, there are 5 points 1 2 3 4 and 5. So, I want 2 out of the 5 points to be seed points. So, X 1 1 plus, X 2 2 plus, X 3 3 plus, X 4 4 plus, X 5 5 equal to 2, where p is the number of groups that I want so sigma X j j equal to p now at the end of the day we are going to form groups. So, each point out of these n points should get assigned to 1 group the seed will get assigned to itself the non-seed will go to 1 of the seeds.

But every point will have to be allocated to a group. So, that is given by a constraint sigma X i j equal to 1 summed over j for every i every point I has to be assigned to a group or to a seed. So, that j represents a seed as well as the group i is always a non-seed. So, each I will go including j including itself will be assigned to a group. The last 1 is that if for example, 1 and 4 are seeds then 2 has to be either assigned to 1 or assigned to 4 it cannot be assigned to 3 it cannot be assigned to a non-seed. So, this constraint will take care of that X i j less than or equal to X j j for every i j. Now let me explain all of these again.

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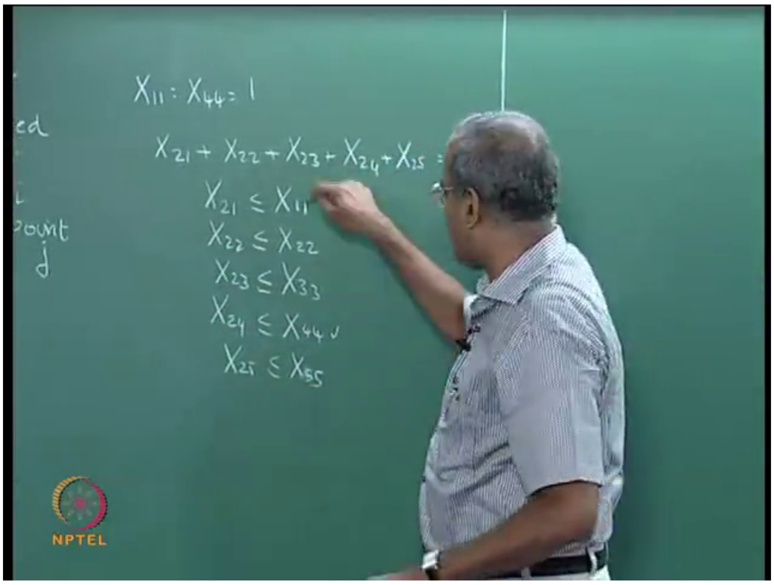

If we consider a solution say with 1 and 4 as seeds then for this solution X 1 1 equal to X 4 4 equal to 1 they are the seeds.

So, sigma X j j equal to p is taken care we need 2 groups. So, 1 1 and 4 4 is equal to p. So, it will not create a third seed it creates only 2 seeds, this is 1 of the solutions. Now 2 3 and 5 are the non-seed points 2 will get attached to 1 or 2 will get attached to either 1 or 4. Now what is this constraint this constraint is X 2 1 plus X 2 2 plus X 2 3 plus X 2 4 plus X 2 5 equal to 1 for a non-seed 0.2. So, 2 will get attached to either 1 or 2 or 3 or 4 or 5 that is what this will ensure, but then we know that 2 is a non-seed point and for this solution 1 and 4 are seed points.

So, we can have either X 2 1 equal to 1 or X 2 4 equal to 1, we cannot have for this solution these 3, but then we cannot write this formulation specific to 1 set of choice of seeds we have to make it generic. So, we also write X i j less than or equal to X j j, which means you have X 2 1 less than or equal to X 1 1, X 2 2 less than or equal to X 2 2, X 2 3 less than or X i j less than or equal to X j j X 3 3 X 2 4 less than or equal to X 4 4 X 2 5 less than or equal to X 4 X 5 5.

These are the 5 constraints corresponding to this for non-seed point number 2. Now, out of these X j j's for any feasible solution only 2 of them are going to take value 1 in this example 1 and 4 will take 1. So, this will be there these 2 will be 0 these 3 will be 0 therefore, 2 cannot be assigned to 2, 2 cannot be assigned to 3, 2 cannot be assigned to 5, 2 can be assigned to either 1 or 2 4 and since we are going to minimize some distance it will go to the nearer 1 between 1 and 4. So, it will go to 1. So, this is the combination of all 3 of them, now the objective function will be to minimize. So, the objective function is to minimize minimize I equal to 1 to n j equal to 1.

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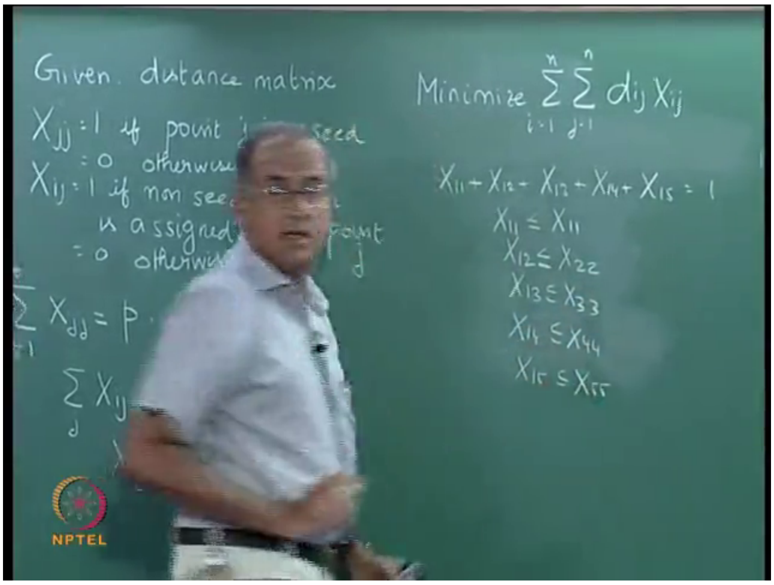

So, now if we have 5 points here n points we also observe that by now we have n square decision variables I equal to 1 to n j equal to 1 to n n square decision variables out of which only n out of the n square decision variables will take value 1 because if a particular point is a seed then that X j j is 1 correct if a particular point is a seed that X j j is one. So, it will be assigned to itself with distance 0 we explained it for the non-seed for the seed it will actually do not need and explanation because if you take the in the same example if you take seed 0.1 if you take seed 0.1 X 1 1 is a seed.

Now for that you will have X 1 1 plus X 1 2 plus X 1 3 plus X 1 4 plus X 1 5 equal to 1 will be this constraint this set will be X i j less than equal to X j j less than equal to X 1 1 X 1 1 less than or equal to X 1 2 X 1 1 less than or equal to X 1 3 X i j less than or equal to X j j X 1 2 less than equal to X 2 2, X 1 3 less than or equal to X 3 3, X 1 4 less than or equal to X 4 4, X 1 5 less than or equal to X 5 5.

If 1 is a seed straight away this is going to be 1. So, this is taken care all of them will be 0 and this is taken care 1 is less than equal to 1 and all of these will be 0, because this is a non-seed non seed non seed this is a seed, but X 1 4 will be less than equal to X 4. In this case it will become 0 because X 1 1 is also a seed. So, this will automatically take value seed. So, whether it is a seed or a non-seed we can explain this formulation. Now there are n square decision variables. So, there will be n square terms here, but out of the n square X i j’s only n X i j's will take value 1. The seeds will be assigned to themselves. So, if there are 5 points 2 of them are assigned to themselves, the remaining 3 non seeds will go to only 1 seed.

So, there will be 3 1 each assignment. So, there will be 5 assignments, there will be p seeds and n minus p non seeds, there will be n assignments n minus p plus p n assignments even though there are n square terms and out of these n assignments which will contribute to the objective function.

The seed being assigned to itself will contribute a 0 therefore, only the non seeds will contribute a distance that is why we got d 1 2 plus d 3 5 plus d 4 5 there are 5 points 2 of them are seeds 3 non seeds are contributing 3 distances and the 1 with minimum d a i j X i j based on our earlier argument will give us the optimum solution to this problem.

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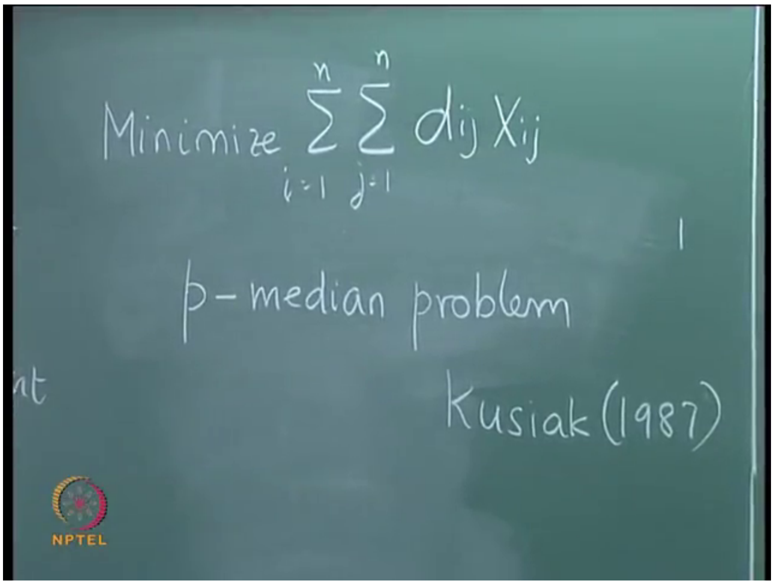

So, this problem is called the p median problem this is well known in the operations research literature it is sometimes called as a k median problem, depending on whether you define this as p or k and this problem comes from location allocation problems from location theory or location allocation theory, it is 1 of the important formulations in location problems, but the p median formulation was applied to cell formation by Kusiak in 1987. And now we will see how the p median problem can be now used to solve the cell formation.

So, far we have seen how the p median problem can be used to get groups, we have already seen how through similar examples we have seen how we get groups using hierarchical clustering as well as non-hierarchical clustering. So, all we need to do is to construct a distance matrix which we know how to do. So, if we are solving a p median problem for the rows we construct a distance matrix for the rows and then solve a p median problem because all of them are fairly straightforward. So, let us look at an example to see how we apply the p median algorithm to do this.

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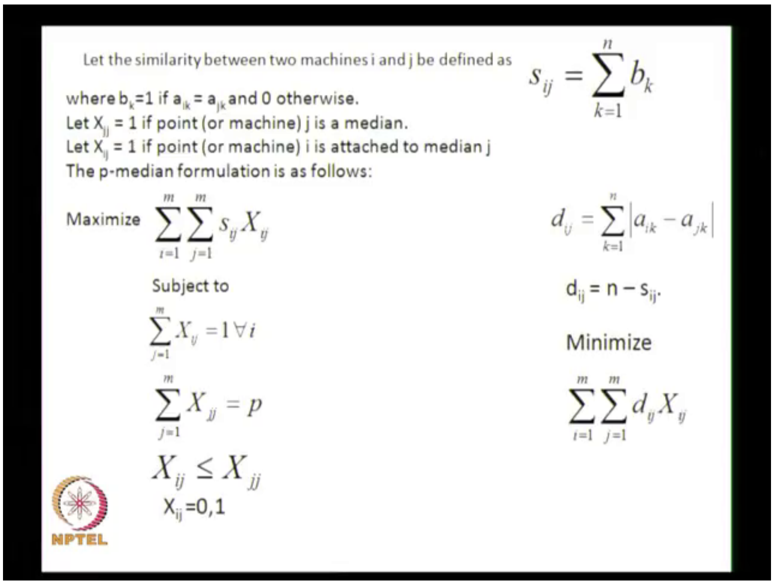

So, whatever we have seen is shown in this presentation. So, the p median the constraints are the same X j j equal to p sigma X i j equal to 1 X i j less than equal to X j j and X i j is our binary in this slide we have shown the objective function to be a maximization where it maximizes similarity. Here we have shown the familiar formula for distance or dissimilarity which is the same as n minus similarity and we could solve a p median problem that minimizes distance.

So, either we could solve a p median problem that minimizes distance or we could solve with a median problem that maximizes similarity.

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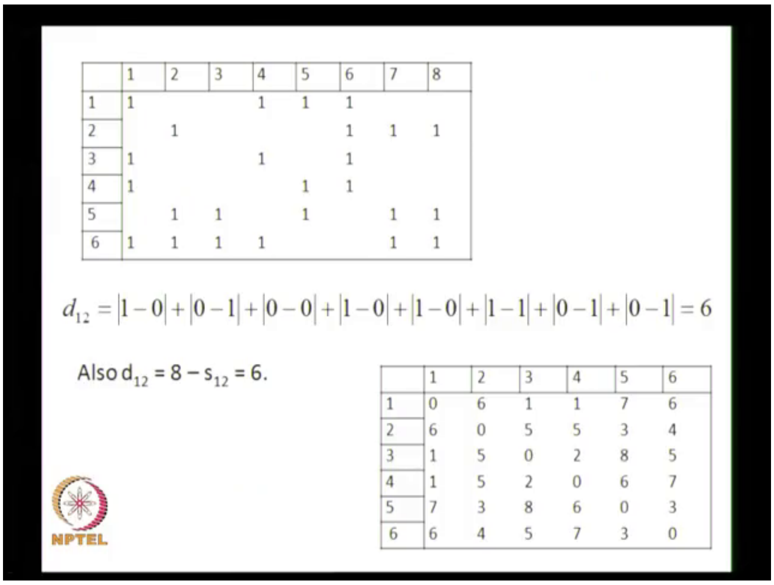

	1	2	3	4	5	6	7	8
1	1			1	1	1		
2		1				1	1	1
3	1			1		1		
4	1				1	1		
5		1	1		1		1	1
6	1	1	1	1			1	1

$$d_{12} = |1-0| + |0-1| + |0-0| + |1-0| + |1-0| + |1-1| + |0-1| + |0-1| = 6$$

Also $d_{12} = 8 - s_{12} = 6$.

	1	2	3	4	5	6
1	0	6	1	1	7	6
2	6	0	5	5	3	4
3	1	5	0	2	8	5
4	1	5	2	0	6	7
5	7	3	8	6	0	3
6	6	4	5	7	3	0

So, we go back to explain the p median problem using the same incidence matrix which is a 6 by 8 that we have been seeing for the earlier examples and we now show the same distance matrix that we used in the non-hierarchical clustering algorithm and we have already shown that the distance between 1 and 2 rows 1 and 2 is 6. So, if we want to apply the p median algorithm to form machine groups we now compute a distance matrix for the machines and solve a corresponding p median problem.

So, if we look at this as our machine component incidence matrix which is a 6 by 8 matrix, 6 rows and 8 columns, the distance matrix amongst the machines will be a 6 into 6 matrix which is shown here.

So, the p median will now have 36 variables 36 X i j's.

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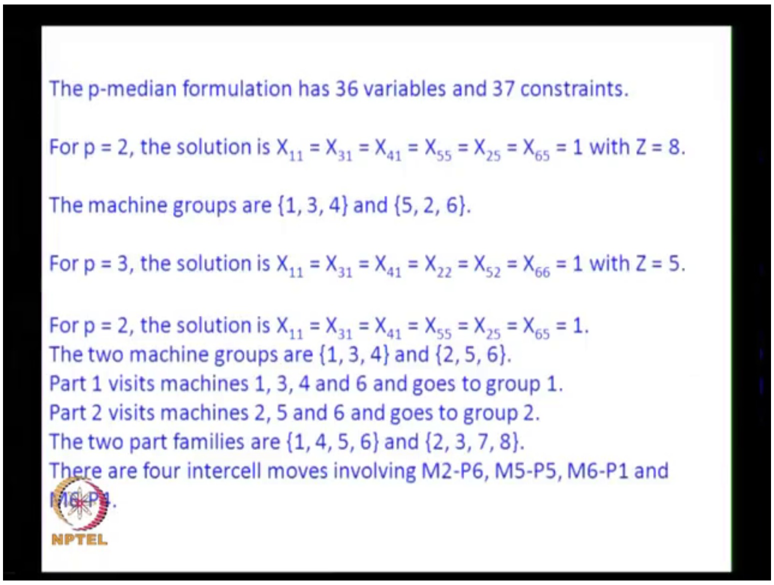

So, the p median problem will have 36 variables here. So, 36 variables X j j equal to p is 1 constraint sigma X i j equal to 1 will give us 6 constraints X i j less than or equal to X j j in principle has 36 constraints, but 1 of which is X j j itself. So, you can remove those 6 you there is no need to write a constraint which says X 2 2 less than equal to X 2 2. So, it is generally written for I naught equal to j. Therefore, there will be 30 constraints. So, there will be 37 constraints, 36 variables binary variables and we can solve this for p equal to 2 or p equal to 3 or whatever p that we wish to solve.

So, if we solve this as a binary integer programming problem that minimizes d i j X i j for p equal to 2 the solution is shown here.

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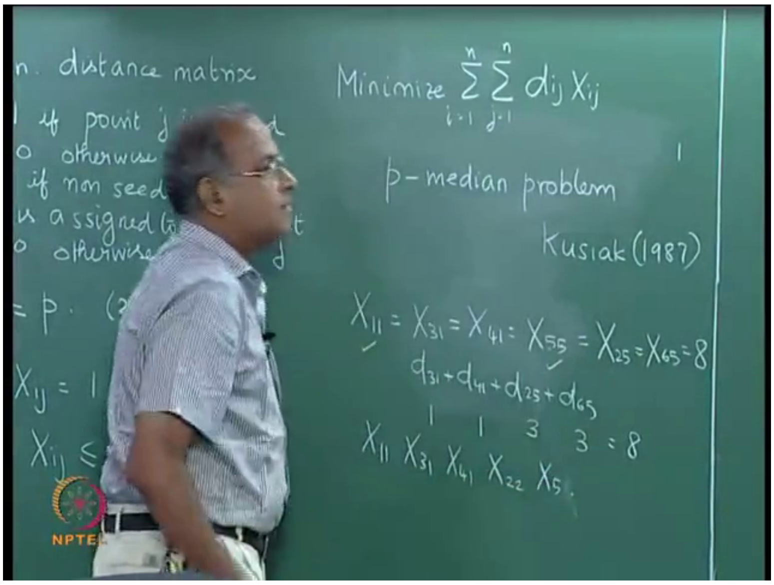

The solution is X 1 1 equal to X 3 1 equal to X 4 1 equal to X 5 5 equal to X 2 5 equal to X 6 5 with distance equal to 8 is shown as the solution.

Now, from this solution we can go back, but please note that this solution is for the machine component incidence matrix it is not for these set of points. So, from this solution we understand that X j j equal to 1 happens here as well as here which means points 1 and 5 are the seed points for the given machine component incidence matrix. So, 1 and 5 are the seed points.

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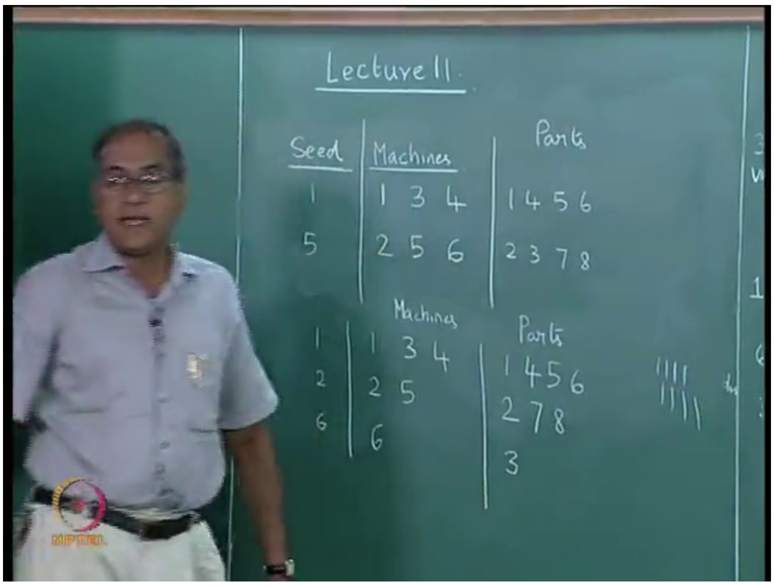

So, seed 1 and 5 machines 1 goes to 1 2 goes to 5 you can see that 1 1 3 1 4 1 5 5 2 5 machine 2 goes to seed 5 or 0.2 goes to seed 5. So, 2 goes to this 3 goes to 1 X 3 1 equal to 1 3 goes to 1 4 goes to 1 4 goes to 1 5 goes to 5 5 goes to 5 and 6 goes to 5 6 goes to 5.

Now, what is the value of the objective function objective function value will be this will contribute a 0 this will contribute a 0. So, this will be d 3 1 plus d 4 1 plus d 2 5 plus d 6 5 will be the objective function. So, let us go back to the matrix now you realize that d 3 1 is the same as d 1 3 1 d 1 4 is also 1 d 2 5 is 3 and d 6 5 or 5 6 is also 3 with objective function equal to 8.

So, if we solve a p median problem with 2 groups we get a solution like this 1 3 4 and 2 5 6, now we can apply our part assignment rule to try and get the parts and when we do that we will get the same previous solution which is which assigns 1 4 5 6 and so we will get 1 4 5 6 2 3 7 8 with 4 inter cell moves.

Now, if we solve a p median problem with 3 groups now the solution is given there. So, X 1 1 X 3 1 X 4 1 X 2 2 X 5 5 or X 5 2 and X 6 6 equal to 1 with z equal to 5 now for this for 3 groups the seeds are 1 2 and 6 1 2 and 6 3 2 1 so 1 2 1 2 to 2 3 to 1 4 to 1 5 to 2 and 6 to 6. So, once again we can apply the part assignment algorithm to see what happens part 1 goes to 1 3 4 and 6. So, part 1 this is parts, parts, parts. So, part 1 goes to 1 3 4 and 6. So, part 1 here 1 inter cell move part 2 visits to 5 and 6, part 2 will come here 1 inter cell move, part 3 visits 5 and 6. So, part 3 will go here or it can go here there is a tie. So, it will go to the smallest cell. So, part 3 will come here.

There is 1 inter cell move part 4 will go to 1 3 and 6. So, part 4 will go here 1 3 and 6. So, 1 inter cell move part 5 will go to 1 4 and 5. So, 1 4 and 5 part 5 1 more inter cell move, part 6 is 1 2 3 4 part 6 will be here 1 more inter cell move, part 7 is 2 5 6. So, part 7 1 enter cell move, part 8 is also 1 inter cell move. So, will be 8 inter cell moves for this solution.

As the number of groups increases it the generally the number of inter cell moves will also increase. So, we may use this solution, but then we understand that if we look at this we very quickly understand that this is better than this there is more inter cell moves and so on. So, this is how the p median algorithm can be used to solve this problem.

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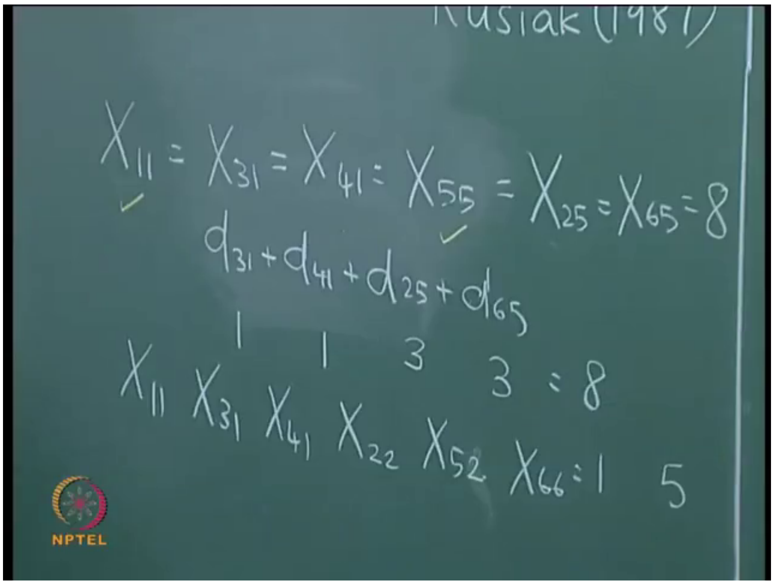

Now, the same p median algorithm can be used to solve the part families problem we showed how it is used to solve the machine cell formation problem by considering a distance matrix for machines. Similarly we can compute a distance matrix for the parts or components now that will be an 8 by 8 matrix and the p median formulation will have 8 into 864 decision variables all of them are binary. The number of constraints will be 1 plus 8 plus 56 which would give us 65 constraints for the part families problem. Now depending on the value of p we can find out number of part groups and which part belongs to which part group. And then we could go and assign the part groups to the machine groups themselves, but we consistently use our part assignment rule once the machine groups are formed we can use our part assignment rule to try and get the best assignment of part families.

Usually in a machine component incidence matrix the number of rows will be less than the number of columns, which reflects the practical situation that the number of machines that are taken for cell formation will be fewer than the number of parts that are available. So, a p median problem for machines will be a smaller sized optimization problem compared to p median problem for parts. So, it is advantages to solve the p median problem for machines, depending on the number of groups that we want depending on p we solve it and then use the part assignment rule to assign or allocate parts to known machine groups. So, this way we can solve the cell formation problem as an optimization problem, now there are a couple of issues with respect to the p median problem the first 1 is that the number of groups is not known a priori.

So, while p median can solve it very well for a given p p this particular formulation of p median does not treat p as a variable, it treats p as a known given number and solves the problem for a known instance of p. Subsequently there were formulations in the literature where p was also taken as a variable and people try to solve it by treating p also as a variable. Now we are not going to look at those algorithms in this lecture series, but 1 of the limitations of the p median algorithm the way we have applied it to cell formation is that it expects us to give the value of p. If we do not know the value of p or if we want to find out the value of p, then we have to solve for p equal to 2 3 4 up to n minus 1 and then choose the best.

Now, even for a known p the problem is a binary integer programming problem with 0 1 variables and the problem is a hard problem as the problem size increases it will be difficult to provide a good solution within reasonable computational time. Another way of looking at the p median is actually p median tries to find out the optimal selection of seeds and once the seeds are selected assigning the non-seed points to seed points is a simple algorithm. So, the whole focus of the p median is in trying to find out the optimal combination or the optimal set of seeds. So, if there are n points and if we want p groups or p seeds then they can be chosen in n C p ways. So, in the worst case p median looks at n cp ways of choosing the seed points.

Now, n cp gets large as n is large and p is p tends to n by 2. So, once again p median problem is a difficult optimization problem to solve. The other issue to be looked at when we consider p median for cell formation is given a p let us assume that we can solve the problem optimally, let us say we can solve the given instance optimally. Now for a given p it provides an optimal solution to a grouping problem where the n points or n machines are grouped based on the objective of minimizing the distance. Now the other question is now is minimizing distance it is certainly related to minimizing inter cell moves because minimizing distance maximizes similarity and maximizing similarity in a way reduces inter cell moves, but maximizing similarity reduces inter cell moves when parts are assigned to machines.

Now, machines are assigned to machines which is what p median bus by looking at it as a clustering problem, now is the objective of minimizing total distance or does the objective of minimizing total distance adequately represent the important objective of minimizing inter cell moves in cell formation is another question, which p median will not answer because as a model p median is created to minimize total distance. Now p median also solves the cell formation problem in 2 stages, when I say cell formation problem I mean forming the machine cells as well as forming the part families. So, either p medium solves it in 2 stages which is solving the machine machine group formation problem separately, solving a part family problem separately and then mixing them together by assigning the correct part family to the correct group or the approach that we have followed by solving the machine formation machine cell formation problem separately and applying the part assignment rule to assign parts.

So next step in optimization is can we overcome these 2 and look at an optimization algorithm which explicitly considers minimizing inter cell moves and is capable of forming machine cells and part families concurrently and not sequentially. So, that leads to the next formulation that we are going to see under the optimization approaches.

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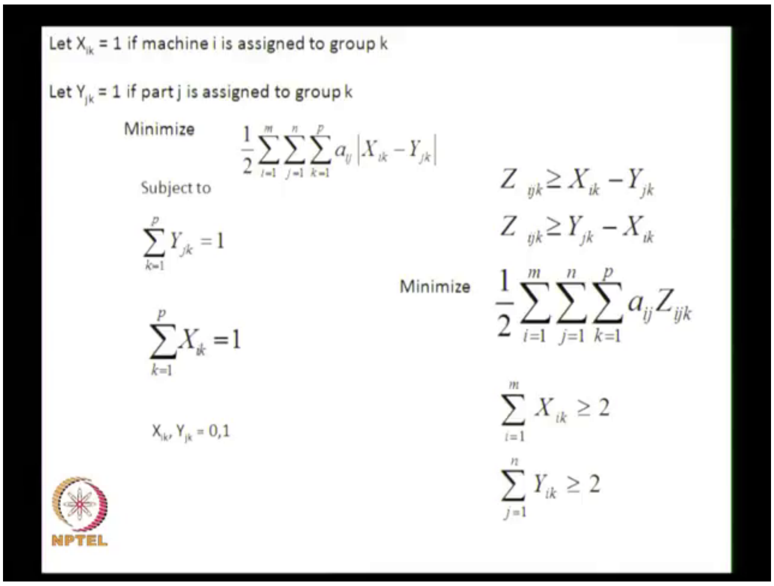

Now let us define decision variables X i X i k equal to 1.

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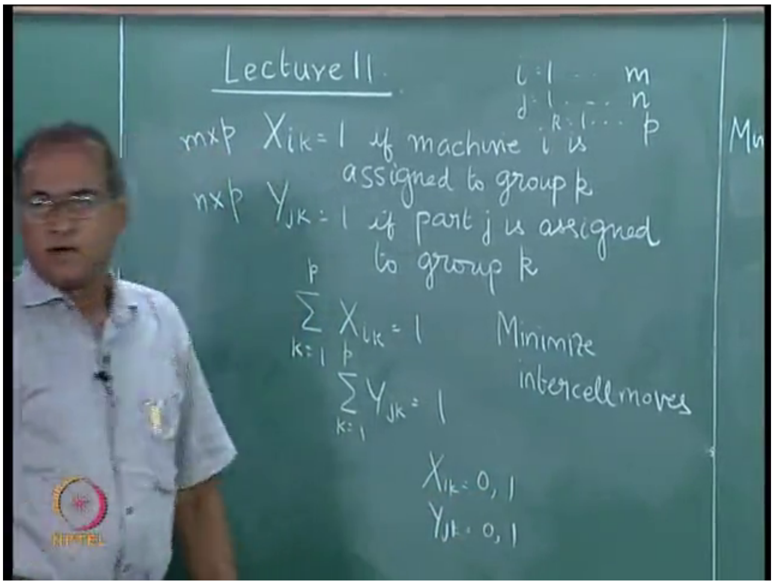

If machine i is assigned to group k, let Y j k equal to 1 if part j is assign to group k. So, now, we have i is the number of machines we say i equal to 1 to m there are m machines j equal to 1 to n there are n parts k equal to 1 2 p there are p groups.

Now, this is these are the decision variables. So, if there are m machines n parts and p groups this will be m into p this will be n into p because each part goes to only 1 group. Now each machine has to be assigned to only 1 group. So, sigma X I k equal to 1 k equal to 1 to p each machine will be assigned to only 1 group, if there are 2 groups machine number 1 can go to either group 1 or group 2. So, this will be a constraint X 1 1 plus X 1 2 equal to 1 and both are binaries similarly y j k equal to 1 k equal to 1 to p for each part j of course, X i k and y j k are binaries now what is the objective function.

Objective function is to minimize inter cell moves is to minimize inter cell moves now when do we get an inter cell move, if we go back to this when we go back to a solution 1 3 4 2 5 6.

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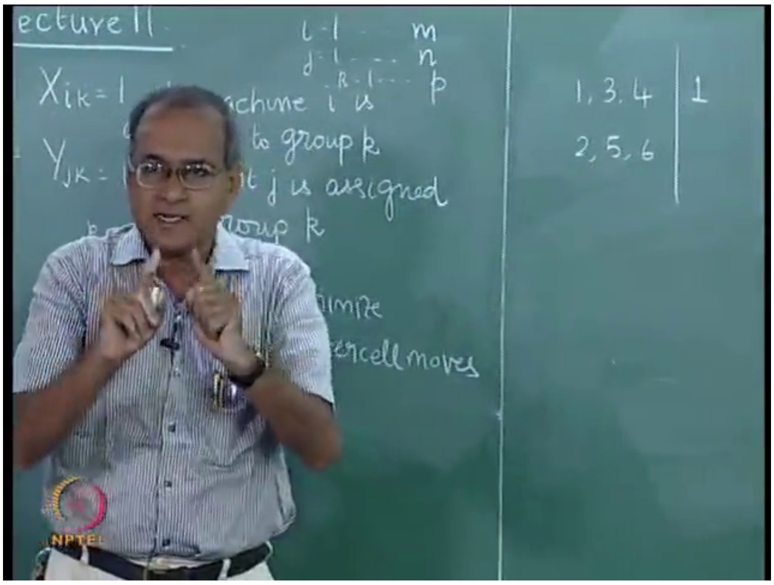

If we go back to a solution like this and if we take part number 1 part number 1 visits machines 1 3 4 and 6 1 3 4 and 6. So, part number 1 goes here now there is an inter cell move what is the inter cell move between part 1 and machine things. So, there is an interesting move if there is a visit between part j and machine i and part j and machine i belong to different groups if part j and machine i belong to the same group and if there is a visit then there is no inter cell move.

So, inter cell move comes when part j requires machine i and parts j and machine i belong to different groups.

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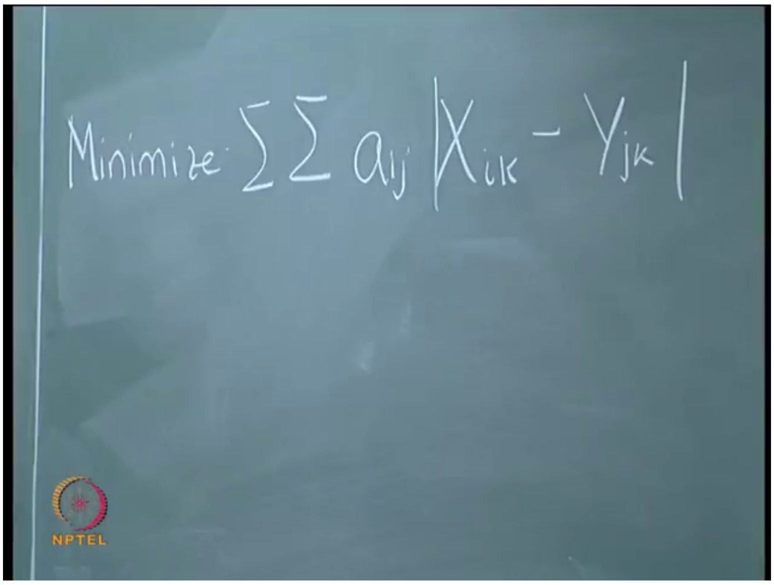

So, minimizing inter cell move is like looking at an objective function like this which is to minimize X i k minus Y j k absolute value a i j summation. So, if a particular a i j is 1 if a particular a i j is 1 only then inter cell move can happen may not happen if a particular part does not visit a particular machine there is no question of an inter cell move. So, a i j should be 1 for an inter cell move to happen and machine i and part j should belong to different groups. So, when machine i and part j belong to the same group then X i k and Y j k will be 1 both will be 1 and the difference will be 0, if they belong to different groups when this will be 0 and this will be 1 or this will be 1 and this will be 0. So, the absolute value term is a difference between a 0 and 1 it is either a 1 minus 0 or 0 minus 1 and the absolute value will give us 1.

So, that 1 when multiplied by the incidence 1 will give us an inter cell move of 1 for example, in this particular instance that we just saw when we wrote the machine groups 1 3 4 and 2 5 6 when we wrote 1 3 4.

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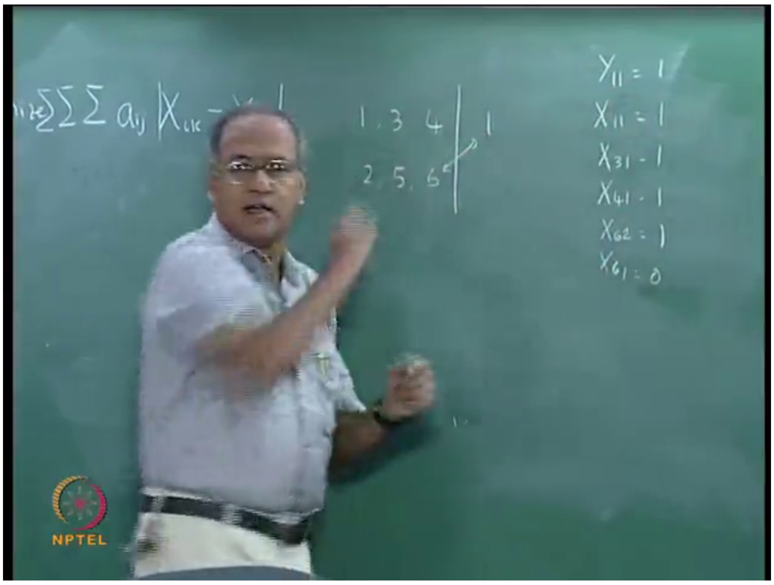

And 2 5 6 and when we assigned part number 1 here and said there is an interesting move happening the corresponding solution would be Y 1 1 is equal to 1 X 1 1 is also 1 X part 1 requires 1 3 4 and 6. So, X 1 1 is 1 X 3 1 is 1 X 4 1 is 1 X 6 2 is 1 in this solution machine 6 goes to cell 2. So, if we go back to this objective function you now see that there is an incidence between 1 and 1, but X 1 1 is 1 y 1 1 is 1. So, 0 no inter cell move there is a visit between part 1 and machine 3.

Y 1 1 is 1 X 3 1 is 1 no inter cell move, there is a visit for part 1 and machine 4 1 1. So, absolute value subtracts no inter cell move, but there is a visit for part 1 on machine 6. So, there is a 1 here there is a 2 here which means X 6 1 is 0. So, there will be a contribution. So, it will add 1 to the inter cell move. So, inter cell moves are calculated this way, the only difference between what I have written here and what you see on the slide is a half now that happens because it is counted twice because of the absolute value term it is counted twice.

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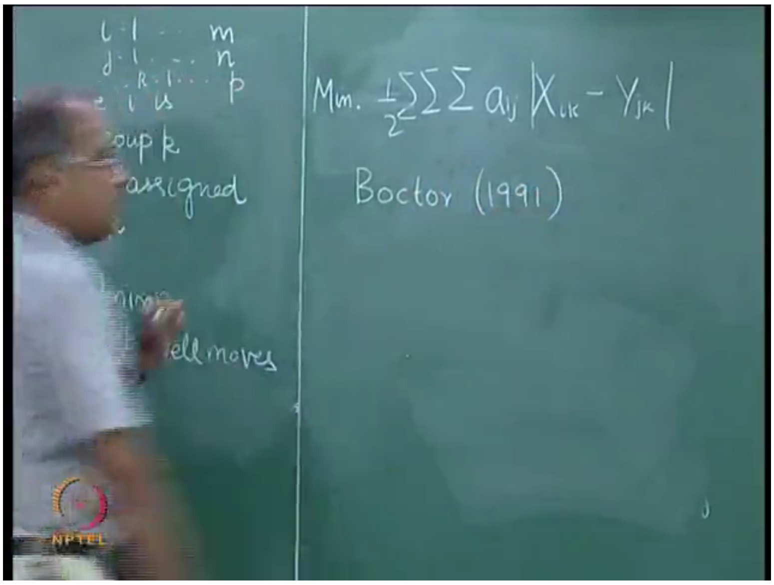

Therefore it is good to write minimize half in to minimize half into this we also know that multiplication by every term multiplying it by the same number does not make a difference, but we still write the objective function as minimize half into this summation. So, this is the objective function that minimizes inter cell moves this make sure that each machine and each part goes to only 1 cell and these are the binary variables.

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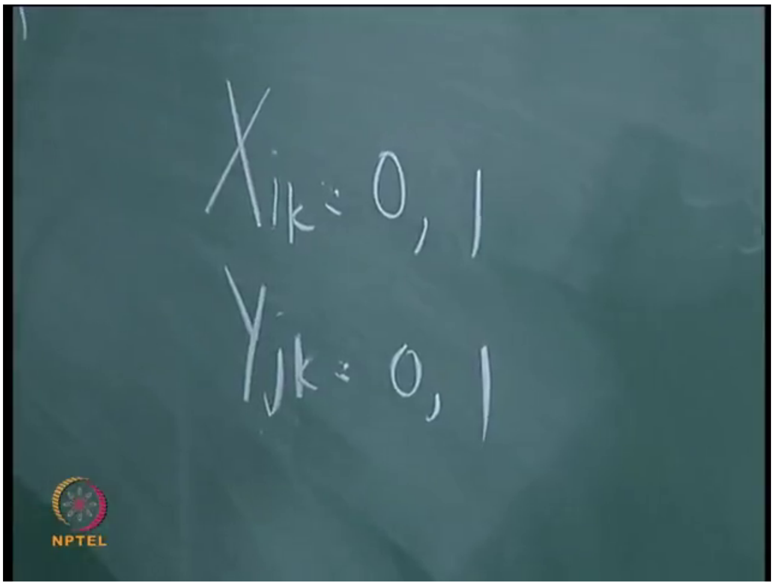

So, X i k equal to 0 1 Y j k equal to 0 1 so this formulation will minimize inter cell moves for a given number of groups. Now this formulation is needs to be further looked at this formulation is from Boctor 1991. This is a binary variable formulation, but something needs to be done because an absolute value term cannot go directly into a formulation. So, we have to make it linear.

And we will see how we do that and how we solve the cell formation problem by linearizing it; we will see that in the next lecture.