Commodity Derivatives and Risk Management
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Week-04
Lecture 17
Convenience Yield, Minimum Variance Hedge Ratio

Welcome to the 17th lecture on Commodity Derivatives and Risk Management. And today we are going to discuss various aspects related to the convenience yield, minimum variance, hedge ratio and hedge effectiveness. If you recall in the previous session, we had discussed arbitrage opportunities for financial as well as commodity underlying. And arbitrageurs are the third categories of investors in addition to the hedges and speculators. And arbitrage arbitrageurs take positions in both spot market as well as futures market to make risk less profit. And even though they take a position in the spot market, they are not commodity producers, consumers, or value chain partners.

They see an arbitrage opportunity and they undertake cash and carry or reverse cash and carry arbitrage to get some financial benefit out of it. And they are a significant value to the commodity derivatives market. And as you know that the arbitrage brings the actual futures price to the theoretical or model-based price. And the arbitrageur ensures that the cost of carry model holds true for pricing forward or futures contract.

And invariably the cost of carry model holds true for financial assets having underlying of equity, index, foreign currency etcetera. It holds true for investment commodities which are gold and silver. It also holds true for consumption commodities, but consumption commodities which do not have any supply constraint. The moment consumption commodities have supply constraint, the cost of carry model does not hold true. And this aspect we had discussed in the previous session that is when the underlying commodities are in short supply, it may so happen that the spot price will be much higher than the future price giving rise to a significant arbitrage opportunity.

However, nobody will be able to take that arbitrage opportunity because the underlying commodity is in short supply. Whoever is holding the underlying commodity, they are holding the commodity because it is giving them some benefit, and that benefit is nothing but your convenience yield. Let us take an example of how we find out what is the convenience yield associated with a you know consumption commodity which is facing a supply constraint. And how do we define convenience yield? Convenience yield arises from the benefit of holding the asset and such benefits are not available to the holder of forward or futures contract. So, somebody who is holding the underlying spot commodity, they are benefiting by holding that commodity.

And whoever is entering into a long or short futures contract, they do not have that benefit. And low availability of the underlying commodities associated with the higher convenience yield or vice versa. Now how do we calculate the convenience yield? Convenience yield is the rate which equates the cost of carry model price to the actual future price. So, as you can see, we do a slight variation to the cost of carry model to find out the convenience yield which is your y here. Please note that the future price for the underlying commodity is calculated as $S_0 * e^{(R+U)\,t.}$

To factor in the convenience yield, we are subtracting y, and that y is nothing but your convenience yield. Let us take an example what do we mean by convenience yield or what is the quantity of convenience yield. Let us revisit our example of coriander. The spot price is 6,688, future price is 6,540. We see that the future price is significantly less than the spot price.

However, nobody is able to arbitrage away this benefit because thus coriander may be in short supply. And in this contract the maturity period is 31 days, and the applicable continuous compounding interest rate is 6.693 percent per annum, and we have to find out the convenience yield. And as you can see, this particular calculation shows the convenience yield calculation, and this is self-explanatory and as you can see the convenience yield is 24.5 percent. So, this indicates that you know people who are holding or companies who are holding the coriander they are benefiting significantly by holding the coriander and they do not want to lend coriander to somebody else. So, that somebody else will be able to make arbitrage benefit. And this convenience yield is positive in case of a backwardation market. Please recall a market is a backwardation market when the spot price is much higher than the future price. Contango market is a normal market where the future price is higher than the spot price.

And when commodities are in short supply commodities you know specifically Agri commodity just before the harvest the quantum of Agri commodities available in the market goes down significantly at that point in time you will have the spot price much higher than the future price and that is that gives rise to a positive convenience yield. And in this case as we can see the convenience yield is coming to 24.15 percent. With this we will come to the end of discussion related to the cost of carry model and the applicability of cost of carry model for pricing futures contracts related to commodity assets both you know investment commodities as well as consumption commodities. Now, let us understand little bit on commodity hedging policies of companies.

And here I have taken two examples of two companies Glencore and Rio Tinto. These are big multinational companies; they are into production of various metal and mineral commodities. For example, Glencore produces a wide range of metal and mineral such as copper, cobalt, zinc, nickel, and ferroalloys as well as this particular company also markets aluminum or alumina and iron ore from third parties. So, that is Glencore's main

business. Similarly, the other company Rio Tinto, this company undertakes mining exploration and processing of various mineral resources including aluminum, diamond, uranium, copper, gold, and iron ore.

And the next two blocks clearly indicate the stated commodity hedging policy of both these companies. I would like to read out this information because this is very critical to what we are going to discuss in the future ah lecture sessions future one or two lecture sessions. Please note that Glencore mentions you know all this information I have taken from their annual report 2022 annual report. And this annual report mentioned that Glencore is exposed to price movement for the inventory it holds and the product it produces. And Glencore manages a significant portion of this exposure through futures and options transaction in worldwide commodity exchanges or in over-the-counter market.

So, this detail very clearly indicates that Glencore manages its ah commodity price risk by entering into futures and option contract at different commodity exchanges as well as enters into bilateral over the counter ah transactions. And interestingly Glencore also mentions that commodity price risk management activities are considered as an integral part of Glencore's physical commodity ah marketing activities. So, this sentence also clearly indicates the importance given by Glencore with respect to commodity price risk management. In contrast to Glencore, let us read out what is the stated commodity hedging policy of Rio Tinto and Rio Tinto annual report mentions our broad commodity base means our exposure to commodity prices is diversified. Our normal policy is to sell our products at prevailing market prices, we do not generally consider that using derivatives to fix commodity prices would provide a long-term benefit to our shareholders.

So, we have two comparable companies operating in a similar industry. However, their commodity hedging policy varies significantly whether one is good over the other that we will be discussing in a later part of the ah lecture sessions whether hedging is beneficial for a company or not that we will be discussing in one or two sessions going forward. But today what I want to what I want to highlight here is that companies have stated hedging policy depending upon their understanding, depending upon their risk perception, depending upon their shareholder expectation companies formulate their commodity hedging policy. Now with this let us come to the next important aspect which is the hedge ratio. Now let us say a company which is exposed to commodity price risk and is interested in hedging the commodity price risk.

Now the question is how many futures contracts this particular company will be buying or selling to hedge the exposure in the underlying spot market. So, hedge ratio is the ratio which is measured by the size of futures contract divided by the size of the spot position. Many companies go for one-to-one hedge or naive hedge. So, what do we mean by

one-to-one hedge that is hedge ratio of one that indicates hedge ratio of one indicates that the size of futures contract is same as your size of the spot position. For example, let us say a coriander farmer wants to hedge the price risk of 5 quintals of coriander and has will take probably 2 or 3 short futures contracts as the contract lot is for 2 quintals.

So, it will be treated as a one-to-one hedge, that is it is exposed to price risk for 5 quintal and it will be entering into a futures contract which is equivalent to 5 quintal. In addition to the one-to-one hedge there is another concept called minimum variance hedge ratio. In that case the hedge ratio is not set to one company identify what is going to be the minimum variance hedge ratio and please note that the word minimum is used with respect to the hedgers' risk at the portfolio level. So, this minimum hedge minimum variance hedge ratio is calculated in such a manner that the hedgers' risk is minimized at a portfolio level. So, combining both spot and hedge unit of futures.

In the previous example please remember hedge is one, but in that minimum variance hedge ratio we will find out what is going to be the hedge. Now, let us go to the you know discussion related to how a company or how a hedger will identify how many units of futures contract he or she should take keeping in mind the spot underlying position risk it wants to mitigate. This particular slide shows the calculation related to the minimum variance hedge ratio and as you can see in the first block let us say a farmer who wants to hedge coriander risk for one quintal. We have taken for a simplistic calculation for one quintal we will you know the same thing can be extended to any quantity of underlying. So, the farmer wants to hedge the coriander for one quintal.

So, the farmer is now a long asset, and it will be able to mitigate that price risk by entering short futures and the farmer will not take one unit of short futures let us say the farmer will be taking h unit of short futures. Now, the farmer has a portfolio one unit in long asset and h unit of short futures. Now let us say on day one the spot price of coriander is 6000 rupees, the future price is 6300 rupees, on day 2 spot price increase to 20 rupees and the future price increase by let us say 15 rupees. So, what is going to be the benefit to the farmer because of the spot? So, this is going to be represented by delta S or change in the spot price which is your 20 rupees. How much benefit the farmer is going to get because of the short futures position? Please note that when the spot when the future price increases because he has taken a short future position, he will be incurring a loss or paying a mark to market margin.

Hence his benefit is going to be h (-15). Similarly, for different days for different price movement of both spot and future price the benefit of the farmer from the spot market and benefit or loss of the farmer from the futures market is mentioned here. And from this series of data, we will be able to find out the standard deviation of delta S and from this series data we will be able to find out the standard deviation with respect to the futures contract. Now coming back to the minimum variance hedge ratio calculation

related to h related to the h. So, the portfolio the farmer has one unit of long asset, and the farmer has h unit of short asset and change in the portfolio value on a daily basis will be governed by delta S - h delta F.

And portfolio variance which would be represented as sigma square portfolio and please note that the sigma square portfolio will be combination of the sigma square variance of the spot variance of the delta spot plus h square variance of the delta future minus 2 into h into covariance of delta S and delta F. And this covariance is nothing, but the correlation between delta S and delta F multiplied by sigma delta S and sigma delta F. So, this particular formula now changes to sigma square delta S plus h square sigma square delta F minus 2 into h into correlation between delta S delta F and sigma delta S and sigma delta F. Now for what value of h the sigma square portfolio it will be minimum, and we know that when you take a first order derivative and equate the same with 0, we will be able to find out the lowest value of the portfolio for a value of h. So, when you take the first order derivative and equate with the 0 and we will be able to find out the h which is the correlation between the delta S and delta F into the standard deviation of delta S divided by standard deviation of delta F.

So, in case of our naive hedge ratio we were taking h as 1 in case of a minimum variance hedge ratio we will be finding out the minimum variance hedge ratio by first finding out the correlation between change in spot price and change in future price multiplied by the standard deviation of change in spot price divided by the standard deviation of future price. And as you can see from this diagram if we plot h and the portfolio variance, we will be choosing that combination or that value of h for which the portfolio variance will be the lowest. Now let us take some numerical examples to calculate the h or the minimum variance hedge ratio. This table in the left block shows about 25 days data point this is the future price this is the spot price we find out the delta future we find out the delta spot we find out the standard deviation of delta futures we find out the standard deviation related to delta spot. And we also found out the correlation which is 0.58 and by taking into consideration all these factors what we find out the minimum variance hedge ratio minimum variance hedge ratio is 0.35. So, for every unit of long assets the farmer should enter 0.35 units of futures contract. Now let us check whether really this minimum variance hedge ratio happens at a h value of 0.35 or not. So, the right-side block shows us the value of the portfolio when the farmer is 1 unit of long asset and h unit of short futures. And from here we can find out the portfolio variances to portfolio variance of 2191 occurs at a h value of 0.35. The moment we increase the h the portfolio is also portfolio variance also goes up we have reduced the h also various combination and portfolio variance also has gone up. In fact, the excel file related to this calculation will be available for all of you who would be part of this NPTEL course.

Hence companies enter into companies explicitly mention what is their hedging policy whether they would like to hedge the commodity price risk or not that is explicitly

mentioned. And some companies may go for 1 is to 1 hedge ratio or naive hedge ratio some companies may identify what is going to be the minimum variance hedge ratio and accordingly enter into the number of futures contracts. So, let me explain the minimum variance hedge ratio in you know the interpretation of minimum variance hedge ratio.

So, we found out h to be 0.35. So, the number of futures contracts somebody will be entering into, or the company will be entering into will be 0.35 into number of spot position. Let us say the coriander farmer has 630 quintals to hedge. Now, 1 future contract has 1 quintal as underline. So, number of futures this party will be entering into the number of futures contract this farmer will be entering into would be governed by 0.35 into 600 quintal which is equal to your 220.5 quintal which is equivalent to your either 220 contracts or 221 contracts. Now, the next question is why the standard deviation of the spot price, and the future price will be different. Please recall we calculated the minimum variance hedge ratio of 0.35 and we also check that the standard deviation of future price is you know 95.79 and standard deviation of spot price is 57.66. Now, the interesting part is that both spot and future price are related to the same underline. If it is so, why will the future price variation in future price be different than the spot price. The logic is future prices have a daily price limit unlike spot prices.

Spot markets do not have a daily price limit. Because they are exchange traded, the exchanges impose a daily price limit. Spot market operates 7 days a week while futures market operates only 5 days a week and that is 7 to 8 hours per day. And all of us know as the contract approaches maturity hedgers who are not interested in taking delivery or giving delivery and speculator also close their futures position making the futures market more volatile. As you can see you know from the open interest calculation, once a contract starts open interest builds up very slowly it picks up and as the contract comes to a maturity open interest goes down very quickly. And going down of a reduction in open interest leads to greater variability with respect to the futures contract.

Hence futures contract will normally have a higher variability in the contract which matures immediately and those futures contract will exhibit a higher amount of variability. In this context there exists a hypothesis which is known as Samuelson hypothesis. This was given in the year 1965 and this hypothesis says that the future price volatility increases as the contract approaches its expiration. So, these are some reasons why the futures price volatility will be different than the spot price volatility. Now, coming to the interesting aspect can h be negative the answer is yes.

Please note that the standard deviation in this formula standard deviation of delta spot and standard deviation of delta futures can never be negative, but what can be negative? Negative can be the correlation. So, h will be negative only when the correlation is negative. Normally spot and future price mark futures contracts move in the same direction. So, a negative hedge normally does not happen, but if the correlation is

negative this could be because the person who is finding out the correlation may have considered a very short period, or the future price is not really reflecting the spot price that is the far more serious concern. If spot price is moving in one direction and future price is moving in another direction that is not a good situation.

That means, spot and futures market as not related to each other. In normal cases spot and future price will be moving together and will have a positive correlation. Now, the question is, can hedge be greater than a, the answer is yes, is hedge can be greater than a 1 only if you have a this r is positive and as well as delta is is delta sorry sigma delta s is much higher as compared to sigma delta f. So, with this we will come to an end of today's discussion with respect to the minimum variance hedge ratio. As I have mentioned, companies explicitly mentioned what is going to be their hedging policy whether they will hedge commodity risk or not hedge the commodity risk.

And if a company decides to hedge the risk company has the option to go for naive hedge vis-a-vis a minimum variance hedge. So, minimum variance hedge ratio takes into consideration the portfolio combination of spot and future price and from that portfolio we arrive what is going to be the optimal hedge ratio where such that the combination of spot and futures gives the lowest risk portfolio. With this we will come to an end of this session, and I look forward to interacting with all of you in the next session.