Commodity Derivatives and Risk Management
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Lecture 15
Pricing of Futures

Welcome to the 15th lecture on Commodity Derivatives and Risk Management. And today we are going to discuss the Pricing of Futures Contract. In fact, if you recall the last couple of sessions, we have many a times discussed whether turmeric farmers have entered long or short futures position or squared up the contract at different price points. Now the question arises of how these traders arrive at what price they are going to enter long or short futures contract. Are they free to decide any price? The answer to this question is no, they are not free to bid any price. For example, if a trader who is interested in taking a long futures position would be very happy to give a price which is very low.

Similarly, a short futures position holder would be interested in giving a price bid which will be very high, but the moment this happens there may not be any counter party who is willing to take position at such high or low prices. Hence, both buyers buy order and sale order has to be realistically priced. Now the question arises does there exist a model which will help these traders to arrive at the price and that model is known as your cost of carry model. Now let us understand more about this cost of carry model in today's session.

Now before we go into the discussion related to cost of carry, it is very important to understand the distinction between an investment commodity and a consumption commodity. And all of us know an asset is an investment asset when somebody is investing in this asset just to generate certain financial returns. For example, equity, mutual fund, bonds, debenture, currencies etcetera are investment assets. People hold these assets just to generate financial returns. Now coming to commodities, commodities can be investment commodities and commodities can be consumption commodities.

So, what do we mean by investment commodities? Please note that a commodity will be an investment commodity when those commodities are primarily held for investment purposes. For example, gold and silver, all of us we know retail individuals, household, banks, financial institution, hedge funds, mutual funds etcetera invest in gold and silver because they want to generate some financial return out of it. And it is very important to understand that people who invest in gold and silver or entities who invest in gold and silver, have nothing to do with gold or silver for their day-to-day business. They are not

jewelry makers, they are not you know silver filigree companies, they are not wholesalers or traders of gold or silver. However, they invest in gold and silver just because they want to make money out of this investment.

As compared to this investment commodity investments we also have commodities which are known as consumption commodities. So, consumption commodities are those commodities which are primarily held for normal business activities. For example, wheat, aluminum, coffee, sugar, cotton, natural rubber or for that matter most of the commodities are held by traders for day-to-day business. For example, if ITC is buying a significant amount of wheat they are doing so, because they want to make you know atta and other kinds of biscuits, pasta etcetera to run their day-to-day business. So, whenever any trader is buying or selling commodities for their day-to-day business activities those kinds of commodities are known as your consumption commodity.

In this context it is very important to understand that the thin line between a consumption commodity and investment commodity is increasingly getting blurred. Of late many banks, financial institutions, hedge funds have started investing money in you know commodities which were earlier categorized as consumption commodities. For example, crude oil, copper, lead etcetera is attracting large inflow of investment capital, and these commodities are emerging as a new asset class. And when you know a lot of money is being invested in these commodities this is giving rise to a concept called financialization of commodities. And financialization of commodities has become a very debated topic, because all of us know when a lot of money comes to a particular asset the volatility associated with that particular asset increases.

And whenever there is high volatility in a commodity that Hence financialization of commodity is attracting significant debate and those who are you know more interested about what is the positive and negative aspect of financialization of commodity is available in the web link given here. If you are more interested in understanding different aspect of financial commodity I financialization of commodity, I would urge all of you to go through this link in detail and have a general or more ideas related to the financialization of commodity. Now with this let us move to today's agenda which is pricing of futures contract using the cost of carry model. Before we go to the cost of carry model, I would like to draw your attention to this slide, this detail given in the table shows the actual commodity futures trading. This data, I have taken from MCX and NCDEX websites. As you can see the trading date this is the 19th of May 2023 data this is the silver is the underlying the contract maturity is on June, August and November and open high low close price.

And an open price is the price at which the first long or short futures trading has matched. And close price is the daily settlement price, trading volume and open interest we have already discussed in detail and polled spot price is the spot price polled by the exchange

on that given date. Similarly, like silver this is the data related to coriander the underlying coriander and trading date is 19th May 2023 contract maturity is 90th June and 20th July. Similarly, open high low close trading volume open interest and polled spot price data is given. And here I would like to draw your attention that the in the case of silver the polled spot price is much less compared to your close price.

However, in the case of your coriander, the polled spot price is higher compared to your ah you know close price for both June contract maturity as well as July contract maturity. And we have discussed the concept of contango and backwardation in the previous lecture. With this let us move to today's agenda point or today's focus on pricing futures contract using cost of carry model. Now what does this cost of carry model say? The model assumes that the price of a futures contract should be equal to the price of the underlying asset in the spot market plus the cost of carrying the asset from the spot date to the maturity period of the contract. Now let us understand from this picture let us say a trader who needs turmeric 3 months later.

Now he has the option of entering a long futures contract today and taking delivery of the ah turmeric after 3 months or he has the option of buying turmeric in the spot market and storing it or carrying the turmeric for the for the 3 months. So, whatever cost this gentleman will incur by buying the turmeric and storing or carrying it for 3 months he should be happy to quote a future price today. So, the dilemma he is facing is that he wants to enter into a long futures contract, but at what price. So, the cost of carry model is going to help him decide what is going to be the price he should enter the long futures contract. Now coming back to the cost of carry model can have a different you know different variations.

So, here we have given I have given 3 variations to the cost of carry model. So, the first model is when the underlying asset does not provide any return or income. And the second model is the underlying asset provides a known income or dividend within the contract maturity period. And the third you know third variation of the cost of carry model is your underlying asset provides a known yield. Please note that these 3 variations are applicable to the futures contract when the underlying are financial assets.

Now what does $F_{0,t}$ stands for today that is on spot date, the spot price is S_o what should be my future price as on today for a contract maturing on capital T Day. So, it would be nothing but S_0 and this is my spot price and e^{rt} is my carrying cost. Similarly, when the underlying asset is going to be giving some dividend then the formula is changed to subtract present value of the dividend from the spot price and multiplying this difference with the carrying cost of e^{rt} that is going to be the cost of carry model. And in the third case when the dividend is not dividend or known income is not in terms of absolute number, but in terms of percentage we adjust the r, we subtract the r from r the q, q is our

continuously compounded known yield. So, $S_0 \times e^{(R-q)\,T}$ is going to be the model for finding out the future price. And please note that the carrying cost or interest rate is expressed here in the continuous compounding rate. Now let us come back go back to understand a little more on what do we mean by continuous compounding rate though it is a very basic concept, but I thought that it is appropriate that I must cover the concept of continuous compounding now. Let us understand what do we mean by carrying cost or interest rate as a continuous compounding rate? Please note that when we go and make a bank fixed deposit, all of us know the bank will be giving us an interest rate and suppose we want to make a bank fixed deposit we go to the online portal of the bank and the bank is saying that if you want to invest for 1 year you are going to get let us say 7 percent continuous compounding you are going to get 7 percent compounded annually. And the same bank may say if you want to invest for let us say ah if you want to ah get interest in with more compounding frequency you made the bank may say that you are going to get 6.7 per cent with the you know semi-annual compounding.

So, with this kind of a discrete interest rate. So, when what bank is providing us is the discrete interest rate. So, we will call it an R_m which is our annual interest rate and with the annual interest rate we have different compounding frequencies and R_c is our continuous compounding interest rate. So, our objective is to find out what is the R_c, please note that in real life we never get to see the continuous compounding rate what we get to see is your annual interest rate with different compounding frequency from that detail, we find out what is going to be the continuous compounding rate because we require this continuous compounding rate for the pricing of our futures contract. Now here let us take an example suppose one makes a bank fixed deposit at 7 percent interest per annum with quarterly compounding of 4 with quarterly compounding.

So, that makes m equal to 4 and the question is what the equivalent continuous compounding rate is and as you can see, we have plugged in this formula this detail and it is leading to 6.939 percent. And this table shows the nominal interest rate or annual interest rate remains same 7 percent, but as the compounding frequency increases the effective interest rate is increasing and simultaneously, we can calculate the equivalent continuous compounding rate. So, what is the interpretation of 6.939 percent here? The interpretation is if somebody is investing money at 7 percent with compounding frequency as 4 whatever he or she is going to get at the end of the time will be same as if the bank is giving interest at 6.939 percent with continuous compounding. Let us take this example to understand this aspect more. So, if one invests 500 rupees a year with an annual compounding of 7 percent per annum with quarterly compounding frequency or 6.939 continuous compounding rate, how much money the investors should have after a year? So, as you can see the P_t, that is the price which is going to be prevailing or investment value after the time period which is going to be $500 *(1+ 7\%/4)^{4*1}$ is going to

give us 535.93 and when we are using the equivalent continuous compounding rate, we are also getting exactly the same amount.

So, in real life nobody is going to provide us the interest rate or carrying cost in continuous compounding rate we will be able to we have to convert that you know discrete compounding or discrete interest rate into the continuous compounding rate because that is what we require to calculate the future price. With this let us take numerical example of cost of carry model for financial assets. This explanation I have given 5 minutes before but let us take some numerical examples to understand what the cost of carry should be based future price. So, we have 3 you know underlying assets one is the single stock future which is your underlying asset does not provide any return or income during the life of the asset and when do we mean by life here the contract time is 6 month or 0.5-year continuous compounding rate is 6.639 per cent and today's spot price is let us say 1750. So, if the spot price is 1750, we will be using this first formula to arrive at what is going to be the future price and please note that the S_0 is 1750, t is our 0.5 and r is your 6.639 percent. So, going by that we are finding out $F_{0,t}$ is to be 1809.

Similarly, the second type of cost of carry model when the underlying asset has a known dividend. Normally individual stocks or shares have some dividend component at a certain point in time. So, let us say the spot price is 1750 and this share has already the company has already declared a dividend of 8 rupees per share, which is going to be received after 25 days and using the second formula S 0 minus I, which is the present value of the dividend. Please note that I is not 8 rupees. Here dividend is 8 rupees, I is going to be the present value of the dividend.

So, the present value of the 8 rupees must be calculated and this must be subtracted from the S_0 to arrive at the $F_{0,t}$. Now, why are we subtracting the present value of the dividend? The logic is if somebody is investing in the stock, he or she is going to get a dividend, but if somebody is entering into the long futures contract that party is not going to get a dividend. Hence, we must adjust or subtract the present value of the dividend to arrive at the cost of carry based futures price. Similarly, the third kind of you know when the asset underlying asset provides a known yield or known return, we do some modifications to the same formula which is $S_0 * e^{(R-q)t.}$ Normally, this formula is applicable for index futures and let us say the index spot value is 19,780 continuous compounding yield dividend yield is 1.49 percent. Now, how do we get this continuous compounded formula? Please note that the exchanges on a regular basis publish what is the dividend yield associated with a particular index. This table which is given below this shows on 18 May 2023 for the index nifty 50 what is the dividend yield as you can see the dividend yield is 1.5 percent. And this 1.5 percent must be converted into continuous compounded yield which is coming to your 1.49 percent and using the third formula will be able to find out what is going to be the future price for the you know for index having a 6-month time to maturity. So, that is coming to your 20,296. Of course, these numbers

have been rounded up to you know whole number and here in this context the value of E is your this is known as your Euler's number which is 2.71828. So, when we are using this formula $S_0 * e^{rt}$, the e will take the value of 2.71828. With this let us go to find out what is the cost of carry model associated with commodities as underlying asset. This discussion is related to the financial futures, but commodity futures cost of carry model will be slightly different. And this slide shows the cost of carry model for commodity underlying. And please note that the commodity underlying when somebody is investing in the spot market, he or she has to incur an additional cost which is your storage insurance and other costs. So, along with the $S_{0,}$ we will be finding out what is the present value of the storage cost and any other you know cost the buyer of the underlying asset will be incurring when he or she is buying the underlying asset that path has to be added into it.

So, the formula for cost of carry formula for commodity asset changes to $(S_0 + U)\ e^{Rt.}$ And many a times please note that the storage insurance and other cost can be expressed as a percentage to the underlying commodity price. If storage cost is expressed as a percentage, we modify the formula into $(S_{0)}\ e^{(R + u)\ t}$. Now, the question is where we get the details related to the you know storage cost that these two tables as you can see this table data, I have taken from NCDEX clearing corporation India Limited website (NCCL). And this table shows the warehouse charges for the LME that is the London metal exchange. So, exchanges or a warehouse associated with the ah exchanges regularly inform what is going to be the storage and warehouse ah charges. So, those factors have to be taken into consideration when we are pricing out when we are pricing the ah futures contract for commodity a commodity asset. So, as you can see from here please note that $F_{0,\ t}$ will be always higher than S_0 because S_0 is positive, u is positive of course, r is positive, interest rate is positive, t is greater than 0. So, going by this formula so, $F_{0,\ t}$ should be higher than your S_0 and this leads to the concept of contango market as we know we have discussed when the future price is more than the spot price, or a distant future price is more than a near month future price we call that market is a contango market. Hence as you can see contango market seems to be a normal market.

Now, with this with this let us go to a real-life example related to a cost of carry model for commodity assets. So, here as you can see the spot price of an underlying commodity is 6240 the storage cost is INR 3 rupees per day the continuous compounded interest rate is 10 percent per annum and in addition to the storage cost the come in the buyer also has to pay a quality assessment cost of 50 rupees which is payable at the end of the storage period. So, this particular block shows the calculation related to the cost of carry model associated with this particular underlying and here the formula the right-hand block shows the formula related to the cost of carry model for commodity assets. With this we will come to end of today's session and to summarize that the futures prices of any underlying asset is decided or governed by the cost of carry model and the cost of carry

model for financial assets are different than the cost of carry model for commodity assets. Commodity assets have a storage cost and quality assessment cost while financial assets will have a dividend or ah dividend yield component to be incorporated into the cost of carry model. Now, the next question is in reality does the cost of carry model holds true or it is just a theoretical model, and it does not hold true. So, this aspect we will be discussing in the next session. Thank you all of you and I am looking forward to interacting with all of you in the next session.