

**Investment Management**  
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**Lecture - 32**  
**Forming Portfolio with ETFs**

Hello there we are talking about exchange traded funds as an investment up class and we know that exchange traded funds are very much similar to mutual funds, but they are listed and traded on stock exchanges. In this session, we will talk about how to form a portfolio using exchange traded fund as an asset.

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**CONCEPTS COVERED**

- Illustrative example of portfolio construction using ETFs
- Application of Portfolio Theory with Solver (Excel Add-in)

The slide features a video feed of Prof. Abhijeet Chandra in the bottom right corner. At the bottom of the slide, there is a navigation bar with various icons and logos, including the IIT Kharagpur logo and the NPTEL logo.

Specifically, we are going to talk about an illustrative example of portfolio construction using exchange traded funds, where the assets will be exchange traded funds. And we will form a

portfolio using our understanding of portfolio theory, but it will be implemented in excel ad-in that is using solver we will construct a portfolio, where the assets will be ETFs.

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**KEYWORDS**

- ETFs
- Portfolio formation
- Correlations and co-variances

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**Portfolio Construction with Exchange Traded Funds (ETFs)**  
**Forming Portfolios with ETFs**

- Data: Monthly closing prices of ten ETFs listed on a stock exchange (Dec. 2017 – Dec. 2022)

Month	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10
Dec-2017	12.34	10.85	3.52	10.20	7.18	2.69	9.78	9.93	268.00	216.38
Jan-2018	12.75	10.85	3.90	11.23	8.52	3.13	9.99	10.25	272.25	216.38
Feb-2018	13.16	12.29	4.19	11.74	9.06	3.43	10.09	10.43	276.25	216.38
Mar-2018	13.38	12.29	4.05	11.04	8.35	3.22	10.11	10.45	273.04	235.88
Apr-2018	13.38	12.03	3.94	11.35	8.10	3.15	10.25	10.59	278.62	233.77
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Oct-2022	18.32	12.56	3.74	13.23	10.02	2.12	10.93	10.96	286.04	259.19
Nov-2022	19.42	12.24	3.55	13.05	9.30	2.19	10.63	10.82	280.68	242.21
Dec-2022	19.67	12.67	3.56	12.47	9.22	2.48	10.44	10.70	279.33	241.94

$R_i = \frac{B_3 - B_2}{B_2}$

To start with, we know that we should have some historical data, we know from our understanding of portfolio theory that we need to have an understanding of historical trends in risk and return for assets that are to be considered for constructing a portfolio or for creating a portfolio of assets. Here, we have the monthly closing prices of 10 exchange traded funds.

Let us name them as ETF1, ETF2, 3, 4 till 10. We have data for 5 years starting from December 2017 till December 2022, where these 10 exchange traded funds that are listed on a stock exchange are selected for including in a portfolio. And these are the data on the prices of these ETF, which implies that this particular number is the NAV or the price of that particular ETF1 as on the last working day of December 2017. And similarly, for January 2018, it is 12.75, further it is 13.16 and so on.

Here these are the missing sales; basically, we have the data in a spreadsheet that we will discuss later. But for now, we have the data set for all these 10 ETFs on a monthly basis, where the prices are available for a 5 year period. Now, once we have the data, the next approach or next step is to calculate the return.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**  
Forming Portfolios with ETFs

- Monthly returns: Change in monthly closing prices

$$R_{i,t} = \frac{(P_{i,t} - P_{i,t-1})}{P_{i,t-1}}$$

Asset i Time t

$t_0 \rightarrow t_1$   
 $100 \rightarrow 110$   
Return =  $\frac{P_{t1} - P_{t0}}{P_{t0}} = \frac{110 - 100}{100} = 10\%$

The slide includes a video inset of a presenter and logos for IIT Bombay and NPTEL at the bottom.

We know that to calculate the return, which is basically returns in terms of changes in monthly closing prices. Here we will use this formula, where we have the return of an asset i.

So, return on asset i at time t, which is basically the monthly time period is based on or is calculated on the basis of price of asset i at time t minus price of asset i minus time, into t at t minus 1 with respect to the price of asset i at time t minus 1. We know this argument, we

have learnt it before, we have discussed in detail, where suppose at time 0, we have 100 rupees of investment and if at time 1, the value of this investment is 110 and we sell it off.

So, we know that return will be calculated as price at time 1 minus price at time 0 divided by price at time 0. So, we can calculate this as 110 minus 100 upon 100. So, this tell us percentage return or percentage change in price. Similar argument is used here, where we will calculate the return on each ETF on a monthly basis.

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The slide is titled "Portfolio Construction with Exchange Traded Funds (ETFs)" and has a subtitle "Forming Portfolios with ETFs". It contains a bullet point: "Monthly returns: Change in monthly closing prices" followed by the formula  $R_{i,t} = \frac{(P_{i,t} - P_{i,t-1})}{P_{i,t-1}}$ . Handwritten in red ink are "Dec 2017" and "Jan 2018" with a dot and "R<sub>i</sub>" next to it. The slide also features a video feed of a presenter in the bottom right corner, a navigation bar at the bottom, and logos for IIT Bombay and NPTEL.

Since we have the data from December 2017, so first return will be calculated in January 2018. So, first return on asset 1 that is ETF1, ETF2 through ETF10 will be calculated in January 2018 and subsequently we will find the return for every month for each of the 10 ETF that we are considering here.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**

Forming Portfolios with ETFs

- Monthly returns: Change in monthly closing prices  $R_{i,t} = \frac{(P_{i,t} - P_{i,t-1})}{P_{i,t-1}}$

$R_p = \sum w_i r_i$

$= (B3 - B2) / B2$

Month	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10
Dec-2017										
Jan-2018	0.033225	0	0.107955	0.10098	0.18663	0.163569	0.021472	0.032226	0.015858	0
Feb-2018	0.032157	0.132719	0.074359	0.045414	0.06338	0.095847	0.01001	0.017561	0.014692	0
Mar-2018	0.016717	0	-0.03341	-0.05963	-0.07837	-0.06122	0.001982	0.001918	-0.01162	0.090119
Apr-2018	0	-0.02116	-0.02716	0.02808	-0.02994	-0.02174	0.013848	0.013397	0.020437	-0.00895
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Oct-2022	0.003836	-0.00397	0.005376	-0.0229	-0.00398	0	-0.00365	0.00828	-0.00908	-0.01759
Nov-2022	0.060044	-0.02548	-0.0508	-0.01361	-0.07186	0.033019	-0.02745	-0.01277	-0.01874	-0.06551
Dec-2022	0.023172	0.035131	0.002817	-0.04444	-0.0086	0.13242	-0.01787	-0.01109	-0.00481	-0.00111

This is the way we can calculate the return. So, this is the snapshot of the returns on 10 ETFs that we have calculated using the price data starting from December 2017. Since we are considering to calculate the return on individual ETFs by using the previous month's data and change from previous month to current month as return.

So, the first return is coming in the month of January. So, here we have the return for ETF1 in the month of January as 0.033225 that is 3.3225 percent. Similarly, next we will calculate the return for individual ETFs accordingly and we will we will have a panel of a data set of returns or rather monthly returns on 10 ETFs for a period of 5 years.

Now, if we are using this example, if we are implementing this example in a spread sheet such as my excel then we know that we can use some sort of function where we can calculate the change in price using simple function where we will have the price data.

So, here if we have this price data and these are the cells that we know we have. So, let us say this is cell number 1, cell number 2, row number 4, row number 5 and so on and this is cell A or column A, column B, column C, column D. We know that the first return on asset ETF1 will be calculated by using the price in B 3 minus B 2 divided by B 2. If we are using this example in excel sheet then we will use this function to find out the percentage change in price which is in the form of return.

So, this will be the a function for excel sheet implementation that we will discuss later. Once we have the return, we know that to form a portfolio we need to have a set of assets which for which we have to calculate the variance covariance matrix because we understand that portfolio return can be weighted average rate of return of individual assets. If you could recall the formula for return of a portfolio its basically weight of individual assets and return of individual assets sum together.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**

Forming Portfolios with ETFs

- Monthly returns: Change in monthly closing prices  $R_{i,t} = \frac{(P_{i,t} - P_{i,t-1})}{P_{i,t-1}}$

$= (B3 - B2) / B2$

Month	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10
Dec-2017										
Jan-2018	0.033225	0	0.107955	0.10098	0.18663	0.163569	0.021472	0.032226	0.015858	0
Feb-2018	0.032157	0.132719	0.074359	0.045414	0.06338	0.095847	0.01001	0.017561	0.014692	0
Mar-2018	0.016717	0	-0.03341	-0.05963	-0.07837	-0.06122	0.001982	0.001918	-0.01162	0.090119
Apr-2018	0	-0.02116	-0.02716	0.02808	-0.02994	-0.02174	0.013848	0.013397	0.020437	-0.00895
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Oct-2022	0.003836	-0.00397	0.005376	-0.0229	-0.00398	0	-0.00365	0.00828	-0.00908	-0.01759
Nov-2022	0.060044	-0.02548	-0.0508	-0.01361	-0.07186	0.033019	-0.02745	-0.01277	-0.01874	-0.06551
Dec-2022	0.023172	0.035131	0.002817	-0.04444	-0.0086	0.13242	-0.01787	-0.01109	-0.00481	-0.00111

$\sigma_p^2 = \sigma_i^2 w_i^2 + \sigma_j^2 w_j^2 + 2\sigma_i w_i \sigma_j w_j \rho_{ij}$

But when it comes to the formula for portfolio risk which is variance of the portfolio, we know that it is based on sigma asset i square weight asset i square sigma asset j square weight for asset j square plus 2 into sigma i weight i sigma j weight j and rho that is correlation between i and j. This is the formula for portfolio risk.

Now, if we have to calculate the portfolio and we need to know which assets are to be included in a portfolio, then we have to have the variance covariance matrix in with us. So, that we can use this variance covariance matrix to construct a an optimal portfolio.



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**Portfolio Construction with Exchange Traded Funds (ETFs)**

Forming Portfolios with ETFs

- Annualized returns and annualized std. dev.: average return and risk

$=\text{AVERAGE}(N3:N62)*12$

	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10
Annualized Return	10.14%	4.19%	1.93%	6.12%	7.61%	1.98%	1.48%	1.72%	0.96%	2.64%
Annualized STD	10.85%	14.90%	18.69%	20.69%	23.11%	27.19%	5.90%	6.73%	5.07%	9.11%

$=\text{STDEV}(N3:N62)*\text{SQRT}(12)$

Now, here we have the return and once we have the return, we can proceed further to calculate the annualized return and standardized or annualized risk. Now, this is required because for any set of data if we do not know the average rate of return, we cannot know the expectations. So, we first calculate the annualized return and annualized standard deviation to get an idea about average return and average risk on each of the 10 exchange traded funds.

So, in a simple way what we did was we found the total return of the return of series of individual assets and annualized it by dividing multiplying it with 12 that is number of months in a year since we have the monthly data. So, we find the average of all the returns for individual assets for individual ETF and annualize it by multiplying it with 12. So, this is the data that we get where we have for ETF the average rate of return is 10.14 percent per year and annualized standard deviation is 10.85 percent.

For ETF 2 we have annualized rate of return as 4.19 percent and annualized standard deviation is 14.90 percent, then we have ETF 3 where we have annualized return as 1.93 percent and annualized standard deviation as 18.69 percent and so on. We know that none of the assets none of the ETFs has generated more than 10.14 percent of return in any case. So, that is the highest return generated by this ETF during this 5 year period on an annual basis and if you look at the risk then we see that this is the highest risk that ETF has carried or ETF data has shown.

Now, with this understanding now since we were talking about implementing this example in an spread sheet or excel we know that we can use the function of average from for all the returns multiplied with 12 and similarly for standard deviation we need to annualize it since standard deviation for all the assets put together on an average and multiplied with square root of 12. And this way we should be able to find out the annualized rate of return for individual ETF in this case ETF1 through ETF10.

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### Portfolio Construction with Exchange Traded Funds (ETFs)

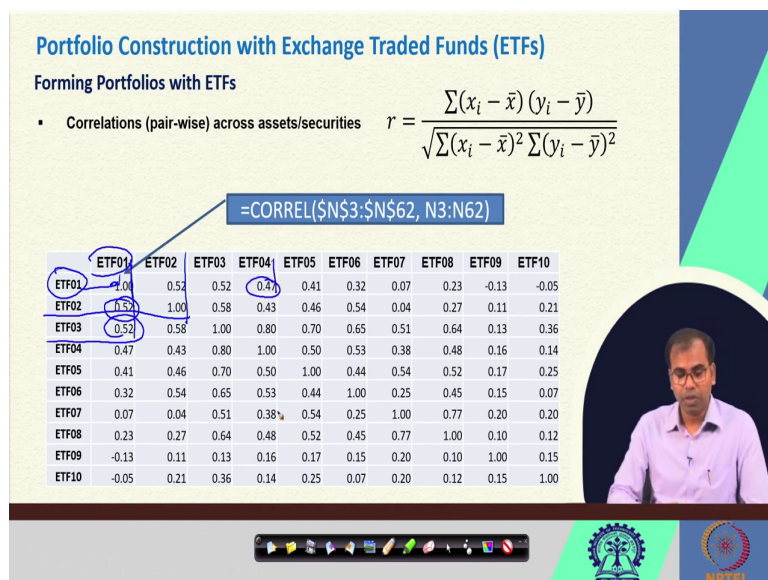
Forming Portfolios with ETFs

- Correlations (pair-wise) across assets/securities

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

=CORREL(\$N\$3:\$N\$62, N3:N62)

	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10
ETF01	1.00	0.52	0.52	0.47	0.41	0.32	0.07	0.23	-0.13	-0.05
ETF02	0.52	1.00	0.58	0.43	0.46	0.54	0.04	0.27	0.11	0.21
ETF03	0.52	0.58	1.00	0.80	0.70	0.65	0.51	0.64	0.13	0.36
ETF04	0.47	0.43	0.80	1.00	0.50	0.53	0.38	0.48	0.16	0.14
ETF05	0.41	0.46	0.70	0.50	1.00	0.44	0.54	0.52	0.17	0.25
ETF06	0.32	0.54	0.65	0.53	0.44	1.00	0.25	0.45	0.15	0.07
ETF07	0.07	0.04	0.51	0.38	0.54	0.25	1.00	0.77	0.20	0.20
ETF08	0.23	0.27	0.64	0.48	0.52	0.45	0.77	1.00	0.10	0.12
ETF09	-0.13	0.11	0.13	0.16	0.17	0.15	0.20	0.10	1.00	0.15
ETF10	-0.05	0.21	0.36	0.14	0.25	0.07	0.20	0.12	0.15	1.00



Now, once we have the annualized rate of return that will give us an idea about average return and average risk to be expected from individual assets individual ETFs. Next comes the pair wise correlation. So, we will calculate the pair wise a correlation or across correlation or across securities across ETFs. We know the correlation formula here for example, if we implement this formula in excel, we know that the correlation can be calculated by using this formula where x is series 1 and y is series 2 or observation 2.

So, if we have 2 data series x and y then we can calculate by implementing this formula to find the correlation or pair wise correlation for the data set that we have. This is the correlation that we can generate. We can see here that for correlation between ETF1 and ETF1 its 1. Similarly, for ETF2 and ETF2 it is 1 and so on. So, if we look at the correlation

between ETF1 and ETF2, then it is 0.52. If it is correlation between ETF1 and ETF3 it is 0.52. If it is correlation between ETF4 and ETF1, then we have 0.47 and so on.

So, in this way we can have the pair wise correlation across correlation across 10 securities that we have. Once we have the correlation the next step that we need to implement is covariance.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**

Forming Portfolios with ETFs

- Co-variances (pair-wise) across assets/securities  $COV_{x,y} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{N - 1}$

*Avg. Return  
Avg. Risk  
Correlation  
Covariance*

`=COVAR($N$3:$N$62, N3:N62)*12`

	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10
ETF01	0.0116	0.0083	0.0103	0.0104	0.0102	0.0092	0.0004	0.0016	-0.0007	-0.0005
ETF02	0.0083	0.0218	0.0158	0.0132	0.0156	0.0216	0.0004	0.0026	0.0008	0.0028
ETF03	0.0103	0.0158	0.0343	0.0303	0.0297	0.0326	0.0055	0.0080	0.0012	0.0061
ETF04	0.0104	0.0132	0.0303	0.0421	0.0234	0.0291	0.0046	0.0066	0.0016	0.0026
ETF05	0.0102	0.0156	0.0297	0.0234	0.0525	0.0273	0.0072	0.0079	0.0020	0.0052
ETF06	0.0092	0.0216	0.0326	0.0291	0.0273	0.0727	0.0040	0.0081	0.0020	0.0018
ETF07	0.0004	0.0004	0.0055	0.0046	0.0072	0.0040	0.0034	0.0030	0.0006	0.0010
ETF08	0.0016	0.0026	0.0080	0.0066	0.0079	0.0081	0.0030	0.0045	0.0003	0.0007
ETF09	-0.0007	0.0008	0.0012	0.0016	0.0020	0.0020	0.0006	0.0003	0.0025	0.0007
ETF10	-0.0005	0.0028	0.0061	0.0026	0.0052	0.0018	0.0010	0.0007	0.0007	0.0082

Now, covariance formula is pretty straightforward. Here we know that if we have 2 data series where x and y are 2 data series then we can calculate the covariance between x and y by implementing this particular formula. And here we have the covariance of all the 10 securities or 10 ETFs calculated as follows.

If we are implementing this particular example in an spread sheet then we know that we can use the function covar for these data points that is N3 through N62 which is basically the return for asset ETF1 for the entire period. And similarly, we can use this function for calculating the covariance between ETF1 and ETF1, similarly ETF1 and ETF2, ETF1 and ETF3 and so. Now, we have average rate of average return, we have average risk, we have correlation and we have covariance.

Now, with all this can be implement in Excel for example, the portfolio theory or the approach through which we can find how much of investment, how much of money of total pool or total fund should be invested in ETF1 and ETF2 and 3 and 4 throughout 10. So, that we can either have the maximum return as an investor or we can have the minimum risk. Remember as an investor our dream our goal is to achieve maximum return for the minimum risk, but both typically is not achieved together.

So, we can achieve, we can obtain a portfolio, we can construct a portfolio of assets which will give us maximum return for a given level of risk or minimum risk for an expected level of return. From our understanding of portfolio theory, we know that if we are sure about what return to expect then we can using portfolio theory we can figure out what will be the minimum risk that I need to assume if I want to achieve that rate of return.

Similarly, if I know about how much risk I can bear, what is the maximum risk that I can tolerate then using portfolio theory we can figure out what will be the maximum return that I need to ask for. So, here we have all the ingredients average return, average risk correlation, covariance.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**

Forming Portfolios with ETFs

- Finding the weight of individual securities to be included in the portfolio through Solver (add-in)
- First: activate 'Solver Add-in' in the Excel program, if not done already:
  - File > Options > Add-in > Go...

**Taskbar:** Windows taskbar with various application icons.

**Logos:** IITM logo and NIFTM logo.

And now we will proceed further for implementing this example in a portfolio theory. Now, the task of a portfolio theory is twofold. One to quantify the relationship between risk and return which means to tell us how much extra return should be asked for every unit of risk or every unit of additional risk that we are assuming. So, if I move from 1 unit of risk to 2 unit of risk what should be my expectation in terms of extra return?

If we remember in portfolio theory we have learned that if this is the risk that I have I can carry and we know that the portfolio theory suggest about something like this where we have this kind of relationship this is a frontier of risky asset, this is risk free asset and this particular curve. So, this is the curve that tells us about the frontier of risky asset and this basically is risk free asset and when we combine risk free asset with risky asset this is the

frontier that we obtain and if we move along this frontier this curve, we get a portfolio that will have exposure to both risky assets and risk free asset.

So, if we know that as an investor I have this much of risk tolerance or this much this is the risk that I need to I can bear then this portfolio theory tells us that this is the maximum return that I can earn. Similarly, if we know that this is the risk this is the maximum risk that I can bear then this is the maximum return that I have to expect if we combine both risky assets and risk free asset and if we do not combine risk free asset then this is the return that I have to accept if this is if this is my level of risk tolerance.

So, this is the first task of portfolio theory. Second task of a portfolio theory is tell us in what proportion funds should be invested or should be allocated across different assets available for investment such that investor is able to achieve either return maximization objective or risk minimization objective. So, second task of a portfolio theory is to find the weight of individual securities that are going to be included as part of the portfolio.

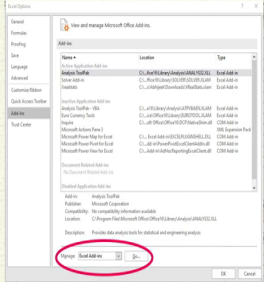
Now, here we are going to use this this data to find the weight of individual ETFs that are included in the portfolio through solver that is an add in. So, if you have not gone through it before then I would request you to see if you can activate solver add in beforehand and in the excel program and then you can proceed further.

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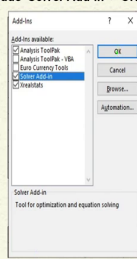
### Portfolio Construction with Exchange Traded Funds (ETFs)

#### Forming Portfolios with ETFs

- Finding the weight of individual securities to be included in the portfolio through Solver (add-in)
- First: activate 'Solver Add-in' in the Excel program, if not done already:
  - File > Options > Add-in > Go...



Then, Include 'Solver Add-in' > OK



So, to add this solver add in in the excel program we go to file, then option, then add in and go and here we have this this type of window where we have excel add in and we go with this excel add in that gives us a solver add in along with other options. So, if we use this solver add in and subsequently, we can just click on ok and figure out that solver add in is added here. Now, with this we are ready to repeat this exercise or implement this example of finding weights of individual ETFs that are to be included in the portfolio.






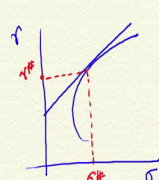
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**Portfolio Construction with Exchange Traded Funds (ETFs)**

Forming Portfolios with ETFs

- Recall the  $n$ -asset Markowitz problem: 
$$\text{minimize } \frac{1}{2} \sum_{i,j=1}^n w_i w_j \sigma_{ij}$$

subject to: 
$$\sum_{i=1}^n w_i \bar{r}_i = \bar{r}, \quad \text{and} \quad \sum_{i=1}^n w_i = 1,$$



Now, let us recall the  $n$  asset Markowitz portfolio theory or Markowitz problem. Remember typically our investor wants to minimize the risk. So, when we try to minimize the risk basically its minimizing the total risk of the portfolio which comprises of asset  $i$  and  $j$ .

So, we use this objective function of minimization of total risk subject to certain constraints and these constraints are in whatever proportion we us allocate the asset across allocate the budget allocate the funds across different assets it should give us a desired rate of return.

So, we want to minimize the risk for a given level of return. Remember that example the scenario where we have efficient frontier and we want to minimize the risk for a given level of return. Suppose an investor says that this is the return that I want to earn. So, what will be the minimum risk that I need to bear?

So, this task is done by portfolio theory. So, it asks the optimizer to minimize the risk for the portfolio subject to the return constraint that is return should be the return on the portfolio should be the weighted average rate of return for individual securities and 100 percent weight 100 percent of the funds should be allocated to different assets as part of the portfolio. So, the sum total of all the individual weights should be equal to 1.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**  
Forming Portfolios with ETFs

- Recall the  $n$ -asset Markowitz problem: 
$$\text{minimize } \frac{1}{2} \sum_{i,j=1}^n w_i w_j \sigma_{ij}$$

subject to: 
$$\sum_{i=1}^n w_i \bar{r}_i = \bar{r}, \quad \text{and} \quad \sum_{i=1}^n w_i = 1,$$

- Here, let's allow the portfolio weight vector  $w$  to take on any real value, but sometimes we'd restrict the portfolio weights, by adding more constraints to the above objective function.
- If we demand  $w_i \geq 0, i = 1, 2, \dots, n$ , then we do not allow short-selling.
- If we demand  $w_i \in [l_i, u_i], i = 1, 2, \dots, n$ , then we require the portfolio weight  $w_i$  to lie between the lower and upper boundary  $l_i$  and  $u_i$ , respectively..

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Now, with this here we know that we if we allow the portfolio weight vector  $W$  to take any real value, but sometimes we would restrict the portfolio weights by adding more constraint to the above objective function and if we demand weight of individual securities being greater than equal to 0 where individual securities could be 1, 2, 3, n that we do not then then we know that we are not allowing short selling which means we are putting the constraint of not having non-negative weight, negative weights.

So, essentially, we mean to say that we do not want to have any negative weights rather all weight should be 0 or greater than 0. Similarly, if we demand another function then we require the portfolio weight  $w$  to lie between lower and upper boundary  $l$  and  $u$  respectively. So, yeah if we have this these conditions, we can put these constraint and subsequently we can find the value of  $w$  for irrespective assets.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**  
 Forming Portfolios with ETFs

- Recall the  $n$ -asset Markowitz problem: 
$$\text{minimize } \frac{1}{2} \sum_{i,j=1}^n w_i w_j \sigma_{ij}$$

subject to: 
$$\sum_{i=1}^n w_i \bar{r}_i = \bar{r}, \quad \text{and} \quad \sum_{i=1}^n w_i = 1,$$

- Here,  $\bar{r}$  is the desired level of expected rate of return (on the portfolio).
- The factor  $\frac{1}{2}$  is a scaling factor in order to get nicer/simpler formula.

Here  $r$  that is the expected rate of return it desired or expected rate of return on the portfolio and factor  $1/2$  is scaling factor in order to get nicer simpler formula.

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**Portfolio Construction with Exchange Traded Funds (ETFs)**  
Forming Portfolios with ETFs

- Finding the weight of individual securities to be included in the portfolio through Solver (add-in)

Target Return	StdDev	Return	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10	Total Weight
1%	0.04731	1%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	100%

*Annualized Rate of Return  $\sum w_i \times r_i$*

So, we know already about this Markowitz portfolio theory and using this argument we are trying to find the weight of individual securities that we want to include in the portfolio and here this is the way we calculate the weight. So, what we do here is in this particular table we start with 10 ETFs ETF 1 through ETF 10 and to begin with we have one equally weighted portfolio where 10 percent each is invested or allocated to the 10 ETFs that we have which mean total weight should be 100.

This particular column indicates about the target return of whatever percentage of return that we expect from the investment in portfolio and this return should be basically a function of average rate of return or just recalling from previous calculation annualized rate of return of individual ETFs multiplied with these percentage or these weights of individual securities.

So, remember the weight  $w_i$  into  $r_i$  where  $r_i$  is annualized rate of return and weight here for the time being is given as 10 percent each, but we will try to optimize it we will try to change this and this will be sum together to find this rate of return and this is the standard deviation that we want to reduce we want to minimize.

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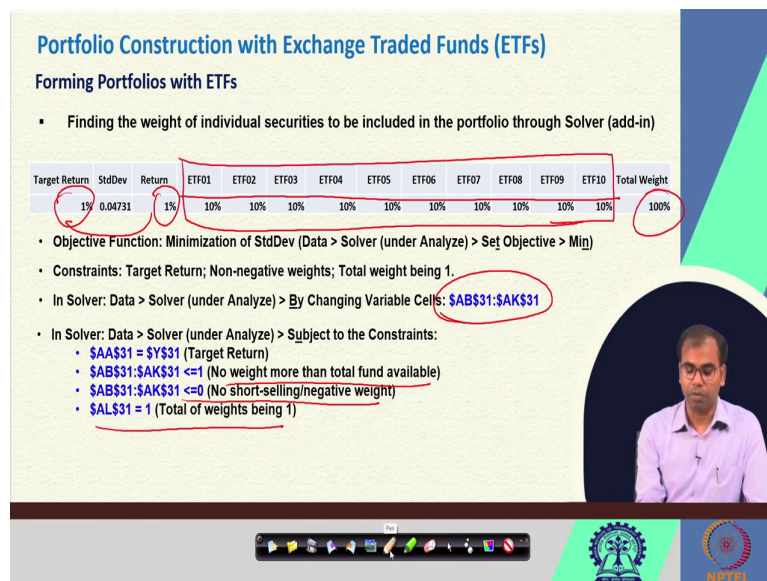
### Portfolio Construction with Exchange Traded Funds (ETFs)

#### Forming Portfolios with ETFs

- Finding the weight of individual securities to be included in the portfolio through Solver (add-in)

Target Return	StdDev	Return	ETF01	ETF02	ETF03	ETF04	ETF05	ETF06	ETF07	ETF08	ETF09	ETF10	Total Weight
1%	0.04731	1%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	100%

- Objective Function: Minimization of StdDev (Data > Solver (under Analyze) > Set Objective > Min)
- Constraints: Target Return; Non-negative weights; Total weight being 1.
- In Solver: Data > Solver (under Analyze) > By Changing Variable Cells:  $\$A\$31:\$AK\$31$
- In Solver: Data > Solver (under Analyze) > Subject to the Constraints:
  - $\$A\$31 = \$Y\$31$  (Target Return)
  - $\$B\$31:\$AK\$31 \leq 1$  (No weight more than total fund available)
  - $\$B\$31:\$AK\$31 \geq 0$  (No short-selling/negative weight)
  - $\$AL\$31 = 1$  (Total of weights being 1)



So, what we do here is we put the objective function of minimization of standard deviation that is if we are implementing this example in excel sheet, we use the data data option data tab to go to solver under analyze then we have an option set objective and we have the option to minimize that minimization of objective. And then we put the constraint the constraints are target return that is whatever return we are want to achieve non-negative weights and total weights or sum total of individual weight should be 1.

So, in solver we put the data tab go to solver under analyze then by changing variables sales of a particular set of sales which is basically nothing, but the weight assigned to individual ETFs. So, to start with we will have a by default weight of 10 percent each, but we want to change this by using this solver.

Now, we also have to put constraint and the constraint will be in solver we go to data tab solver under analyze function then subject to constraint where we will put the constraint such that the target return should be return that we are here.

So, this particular return should be equal to this particular return that we have decided. So, second constraint is about the weight being not more than 1 which implies that we cannot invest more than 100 percent of the money. Third constraint is about non-negative weights which is basically we cannot allow short selling and finally, the last constraint is about total sum total of individual weight should be 1 or 100 percent of the funds available.

With these constraints and objective function, we will go ahead with implementing this in solver where we have all the constraint and objective function written.

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### Portfolio Construction with Exchange Traded Funds (ETFs)

#### Forming Portfolios with ETFs

- Finding the weight of individual securities to be included in the portfolio through Solver (add-in)
- Constraints: Target Return; Non-negative weights; Total weight being 1.
- In Solver: Data > Solver (under Analyze) > By Changing Variable Cells: **\$AB\$31:\$AK\$31**

**Min: \$Z\$31**

**The Constraints:**

- \$AA\$31 = \$Y\$31
- \$AB\$31:\$AK\$31 <= 1
- \$AB\$31:\$AK\$31 <= 0
- \$AL\$31 = 1

So, here we have the objective function which is the minimization of standard deviation then we have by changing the sales particularly changing the weights and then we have the constraint that are in input given here and with this we solve this and the for further results we will discuss in next session.

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**CONCLUSIONS**

- ETFs are taken as a set of assets that can be combined to form a portfolio to achieve minimum risk (for an expected return) or maximum return (for a given level of risk);
- Five year monthly data on ten securities are used for demonstrating the portfolio formation using spreadsheet example.
- Under certain constraints, we aim to solve for the objective function using the Solver (as a used case here).

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So, so far in in this session we know that ETFs are one of the emerging asset classes and using ETF data as a set of assets we that can be combined to form a portfolio we are going to implement.

This as an a portfolio optimization exercise using excel solver where we have the five year monthly data on ten ETFs that are used for demonstrating this portfolio formation using objective function constraint and we try to find the optimal weight that can be assigned to individual ETFs to minimize the risk of the portfolio. With this I end up this session.



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**REFERENCES**

- [Introduction to Exchange Traded Funds \(ETFs\) - CDSL](#)
- <https://etf.nipponindiaim.com/#etfmodelportfolio>
- <https://www.nseindia.com/market-data/exchange-traded-funds-etf>

The slide features a light green background with a dark blue and green geometric design on the right side. A presenter's video feed is visible in the bottom right corner, and a software toolbar is located at the bottom center. Logos for IIT Bombay and NIPTE are visible in the bottom right corner.

Thank you very much.