Investment Management
Prof. Abhijeet Chandra
Vinod Gupta School of Management
Indian Institute of Technology, Kharagpur

Lecture - 20
Consumption-Based Asset Pricing

Hello there. So, we are discussing asset pricing model in the course Investment Management. And in this session, we are going to talk about Consumption Based Asset Pricing Model.

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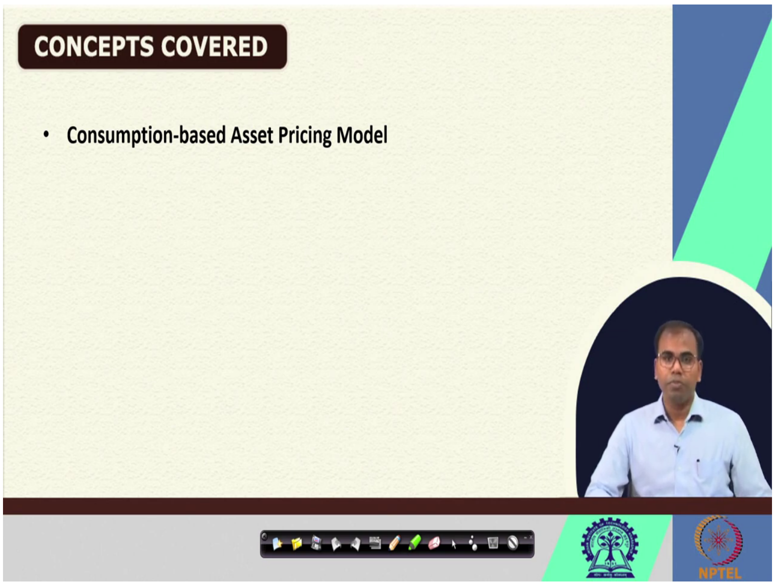

So far, we have discussed about the asset pricing framework in general, where we try to understand what should be the expected rate of return that an investor should expect when it comes to investing in a particular asset.

And in a very generic asset pricing model, we have learnt that the expected rate of return from any investment depends on not only the return that investor can get without holding any additional risk. But also, the return premium that the market asset in general provides, given certain level of riskiness in the asset that investor desires to hold.

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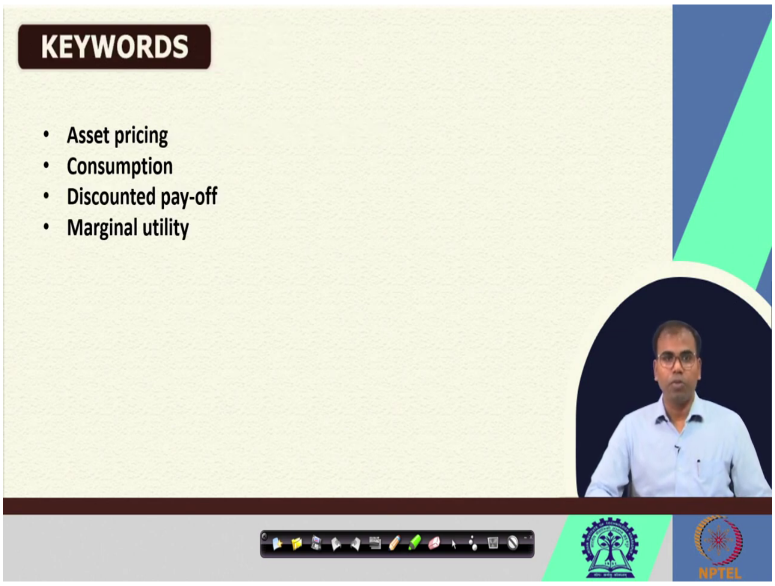

In consumption based asset pricing model, we will try to understand the implication of payoffs that is expected in future, and how this payoff can be used to determine the value of investment or the type of portfolio an investor should hold.

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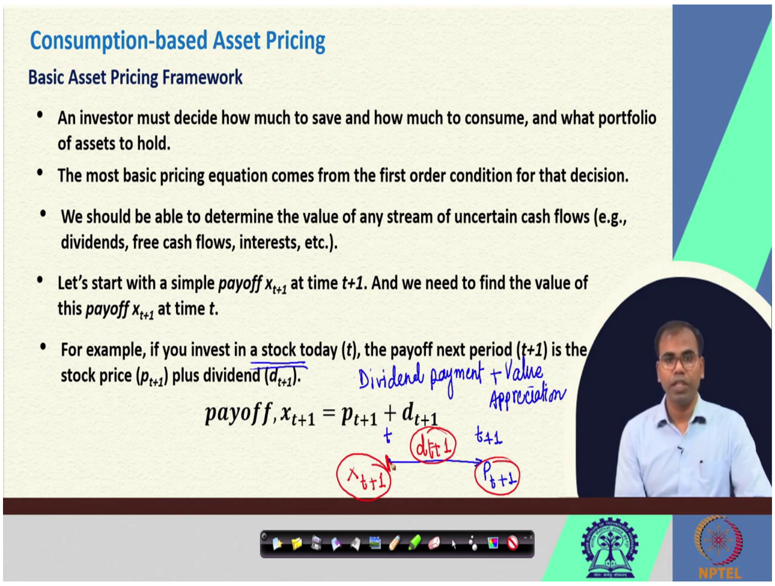

So, effectively when we are talking about basic asset pricing framework, we want to understand for an investor how much money, how much funds should be saved and of that saving what portfolio of asset the investor can hold. So, essentially, if we assume that an investor has a cash flow or a revenue stream of x value, what part of this x should be saved, and what part should be consumed, and out of this saving what portfolio of investment portfolio of asset the investor should hold.

In a typical framework, the most basic asset pricing a equation comes from the first order condition for that decision, where the investor has to decide how much to consume out of the total revenue or total income and where to save or invest the remaining stream of cash flows.

If we understand this first order condition, then we should be able to determine the value of any stream of uncertain cash flows. For example, in case of stock investment, it is dividends.

In case of investment, in any project it is free cash flow for the firm or the equity holders. In case of fixed income securities such as bonds or debentures, it is a coupon or interest.

So, when we try to understand the value of any stream of cash flows that is coming in future, we can start with a simple payoff, let us say x t plus 1 at time t plus 1. So, t plus 1 is the time in future where we are standing at time t at the moment. And we need to find the value of this payoff x plus x t plus 1 at time t, which is basically the present time.

So, we need to understand the value of future cash flows that is expected at time t plus 1 at time t which is basically the value of future cash flows at present time. For example, if we invest in a stock or a bond or any asset today at time t, the payoff next period which is time t plus 1 is the stock price plus dividend.

We have already discussed in detail how this payoff will look like, if we invest in any financial asset today. We expect a payoff that is coming from future cash flow or future time in the form of the increase in price and dividend. We already know that in case of stock, typically, the payoff is in the form of interim dividend payment, which the investor is expecting to receive and any value appreciation in the price. So, value appreciation which can happen because of any other factor.

So, basically, when it comes to stock investment, if an investor is holding a stock today or deciding to invest in stock at time t and the price of the stock at time t plus 1 can be anything. So, this will be the price at t plus 1.

And in the meantime, the investor can also receive some sort of dividend at time t plus 1, and this these two cash flows combined together will determine the payoff at x t plus 1 which need to be brought in today's time at time t to understand the value of that asset. Given that, the asset is expected to generate a dividend as well as value appreciation in future.

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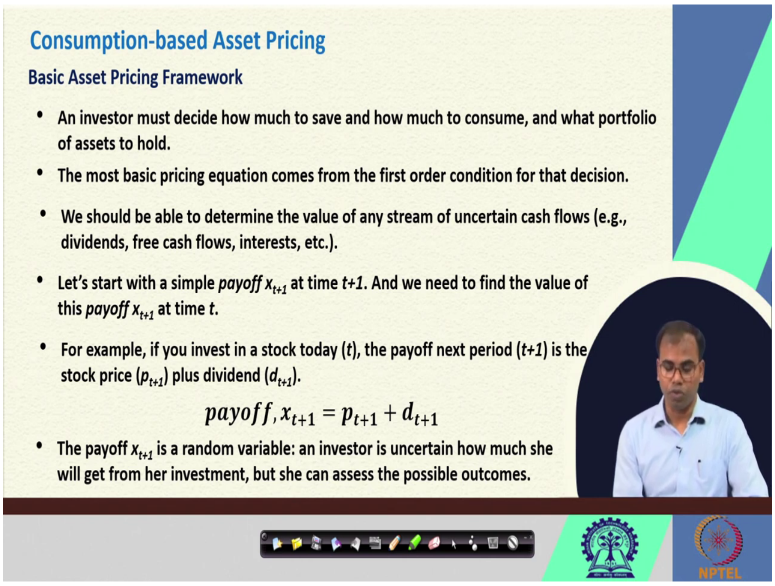

So, here we need to be careful because x t plus 1 that is the payoff is a random variable and any investor is not certain how much this random variable would be in future which is effectively how much the investor is going to get from her investment. But she can assess the possible outcomes because we understand that when whenever we need to have some investment today, we know that the time t and time t plus 1 that is future time, the price of the asset can either go up or it can go down.

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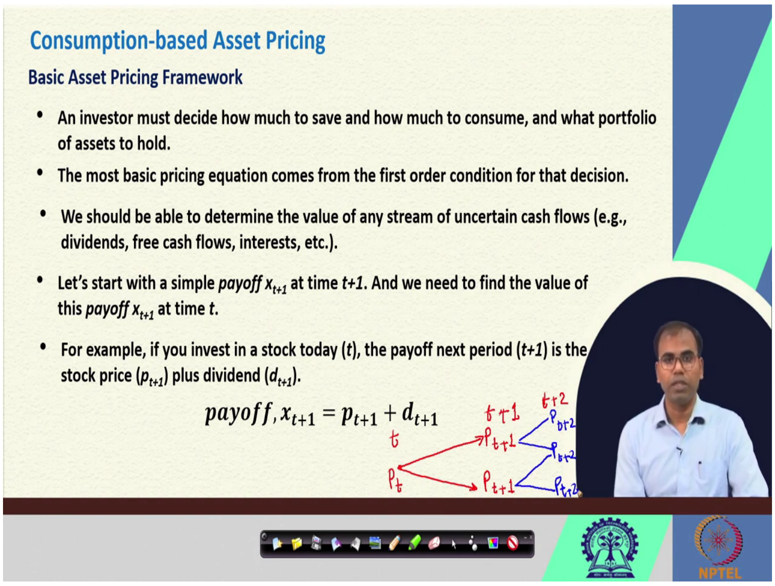

So, this will be price t plus 1, scenario 1. This would be price t plus 1, scenario 2, which means if at time t if price is p t at time t plus 1, the price will be price p t plus 1 either upward or p t minus t plus 1 or downward. So, either in case of stock, the price can either go up or go down. In some cases, it can remain stable also, which is still another scenario.

And similarly, if we move further in time, so let us say time t plus 2. So, at time t plus 2 if price is at time t plus 1 here then it can either go up where it will be price t plus 2 or it can go downward from this point at time t plus 2. And similarly, from this point, it can either go up or it can either go down.

So, in this case, we know the possible scenarios or likely scenarios of price. And based on the experiences or historical trend, we can assign the probabilities or expectations of these scenarios of price in future. And then accordingly, we will try to calculate the value at any point of time by discounting those cash flows or those prices that are expected.

So, effectively, here that payoff x t plus 1 is a random variable and investor is uncertain how much she is going to get from her investment, but she is in a position to assess the possible outcomes.

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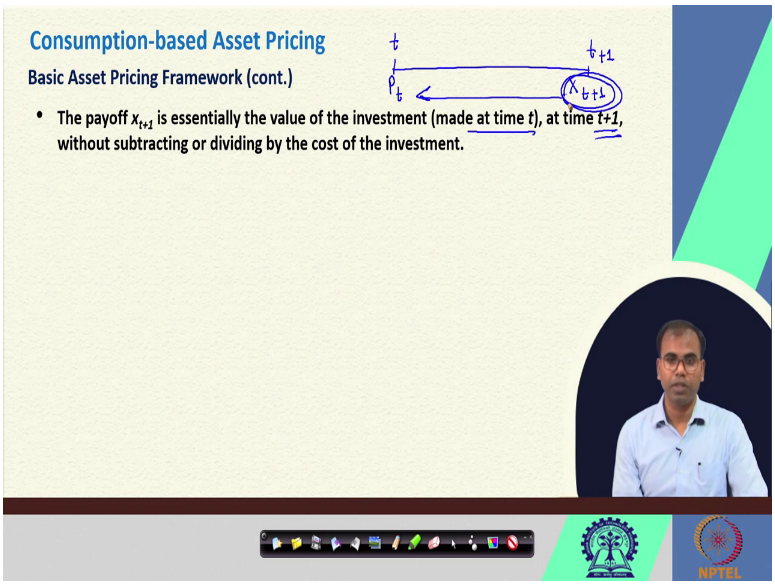

Now, with this background if we know that this payoff x t plus 1 is essentially the value of the investment made at time t at time t plus 1 which is the argument here. Because if it is time t, and this is time t plus 1, we know that we have to make an investment today and whatever payoff that we are going to get is basically the payoff that we are going to get at time t plus 1. But this x t plus 1 is the value at time t.

So, you make an investment at time t and this payoff will be available only at time t plus 1. And we are not deducting or dividing the cost of investment here, which indicates that this is the absolute payoff that investor is expected to receive at time t plus 1, if the investor decides to make an investment of p t t at time t.

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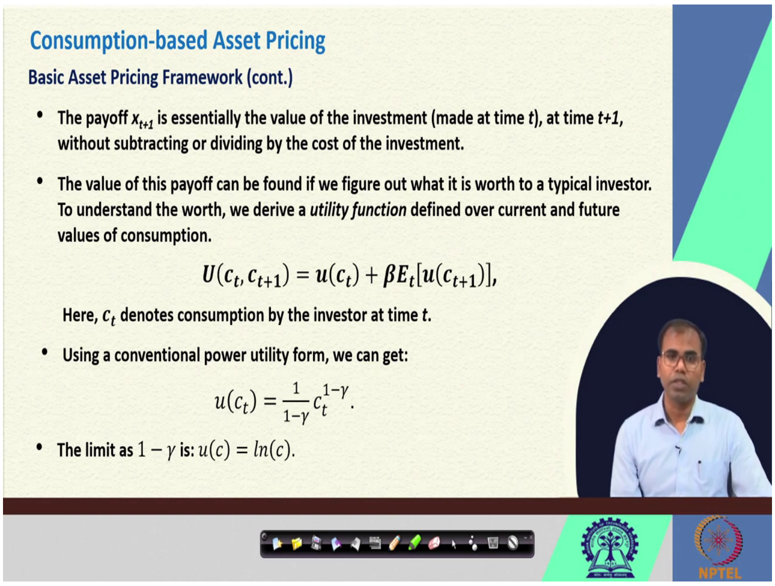

With this assumption, if we move further, we know that the value of this payoff can be calculated if we figure out what is the worth of this particular investment for a typical investor. And to understand the worth of this cash flow or this payoff, we have to derive a utility function defined over current and future value of consumption.

So, effectively, what we are trying to say here is the utility function for this consumption for this particular investor for a typical investor is defined such a way that you want to know this c t. That is consumption at time t by the investor and the utility that is derived from the payoff or the consumption in future, and their expectation combined together in terms of utility of consumption at time t with respect to consumption at time t plus 1.

What it means in simple language is if an investor decides to invest certain amount of money today, what he or she is going to get in future will determine her consumption in future. So, it is basically the trade-off between the choice to consume today versus the expectation or likelihood of having more funds, more resources, more utility to consume in future.

With this, if we use a conventional power utility form, we can get from this equation what is referred to as this function here where, utility of consumption at time t which is basically the resources or the fund or the cash flow or the money that is available for consumption today. The utility of that money will be given by 1 by 1 minus gamma c that is consumption power 1 minus here.

If you can see this particular function, and just to highlight this, we know that we have to see the limit which is given as u c, that is utility of consumption at any given point of time will be natural log of consumption a value or the resources that is available for consumption for this investor.

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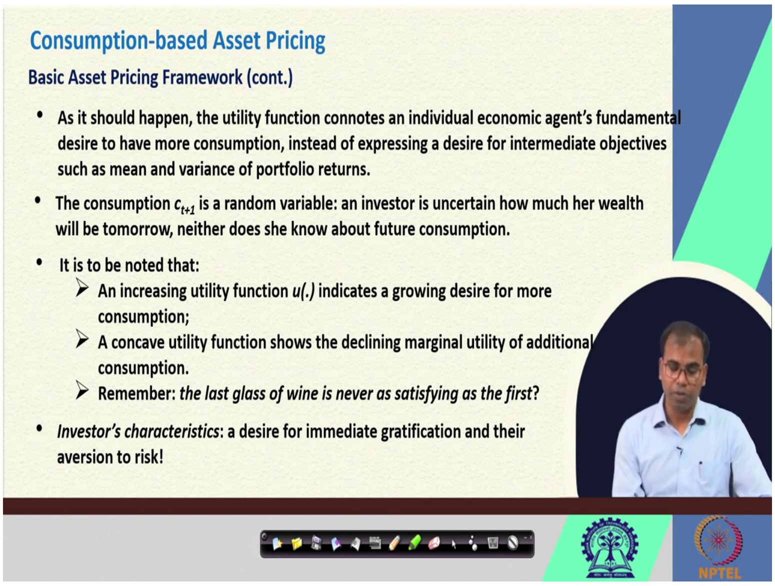

With this basic utility function, if we move further and try to understand the implication of this limit function, we know that the utility function comes to explain the individual economic agent's fundamental desire to have more resources, more funds, more value to consume, instead of expressing a desire for an immediate objective such as mean and variance of portfolio return.

So, investor would look at this investment decision or this choice between consume today consume more today versus have more savings today. So that consumption will be more in future versus the choice to decide about the mean and variance of portfolio returns if the investment is made today.

So, this consumption that is denoted by c t plus 1 which is basically the consumption that is available in future point that is time t plus 1 is again a random variable. And here again, we know that an investor is uncertain, how much her wealth will be tomorrow and she also does not know about the resources or funds available for future consumption in future time.

Now, here you must be careful, and note that a typical increasing utility function indicates a growing desire for more consumption that happens with most of a typical investors. As a basic typical economic agent, everyone wants to have more funds, more resources for more for consumption.

A concave utility function however, shows the declining marginal utility of additional consumption. And we know the basic concept that the last apple that we eat never gives as much satisfaction as the first. So, we should be taking care of this declining marginal utility of additional consumption.

However, we already know that there is in a growing desire of investor for more consumption. And based on that investor's characteristic is defined where a desire for immediate gratification and their choice or their aversion for risk decide their behavior to convey save more today versus consume more and save less in future.

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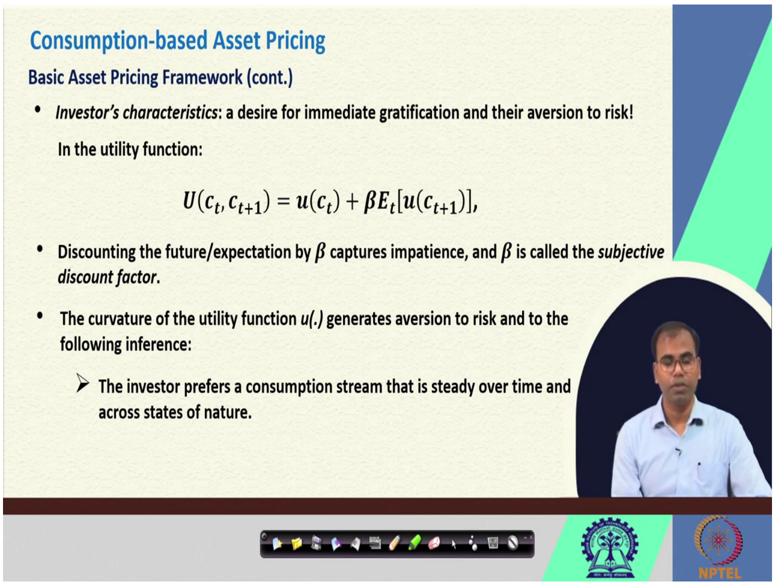

With this objective function decided, where the consume investor or consumer of financial services or financial investment is characterized by a desire for immediate gratification and their aversion for risk. If we draw this in the utility function which we have defined earlier, where utility function is indicating about the expectation of future consumption and the utility that is derived from the consuming more today or consuming certain amount of resources today.

We, if we discount the future or expectation by beta, which captures impatience of the investor, and this is also known as subjective factor in the literature. With this concave curvature of this utility function generates aversion to risk and we have the following inferences based on this utility function. One, the investor prefers a consumption stream that is steady over time and across states of nature.

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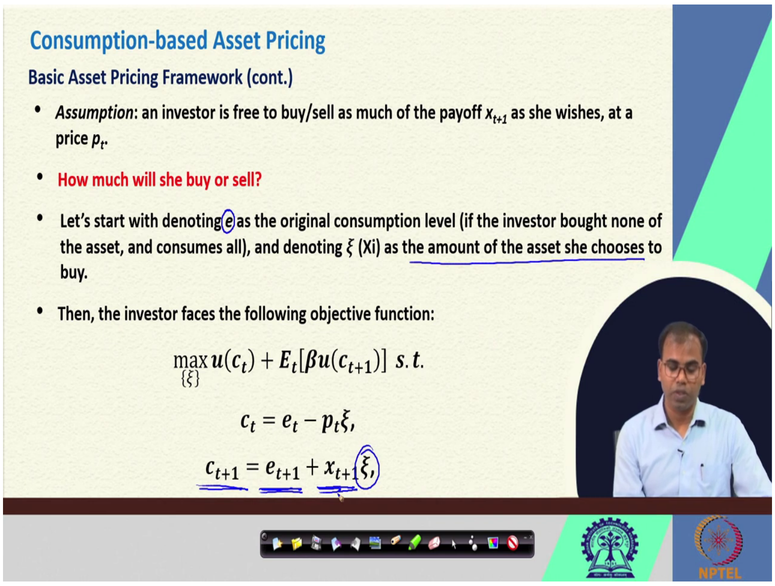

And we also know that under the assumption of investor's choice or freedom to buy or sell as much of the payoff as she wishes at any point of time in price t that is at price of the asset at time t. With this assumption, we have to find a solution or we have to recommend the investor how much she would want to buy or sell of an any asset.

Given these consumption, the utility function associated with consumption, if you start with denoting e, as the original consumption level where investor bought none of the assets and consume all the resources that she has. Let us say an investor has an income of 100 rupees and investor decides to consume all 100 rupees, then the e denotes the original consumption level and chi denotes the amount of the asset she chooses to buy.

With this basic assumption, if we a rewrite the investor objective function, the investor faces the choice in the following term where the investor has to maximize the utility of the consumption in present point of time. And the expected utility of the consumption that she is expecting in future with respect to or basically which is basic which is multiplied with the stochastic discount factor or subjective discount factor related to the consumption that she is expecting in future.

So, what it implies is, the future expectation has to be discounted back to present time using this subjective discount factor that is beta. So, this objective function should be defined in such a way such that we have c t as a function of e that is the original consumption at time t minus p t which is the price at time t, price of the asset at time t and chi which is basically the amount of asset that she chooses to buy.

With this constraint and one more constraint saying that the consumption at time t plus 1 that is denoted by c t plus 1 is basically nothing but the consumption level at time t plus 1 and this payoff that she is expecting and the amount of asset that she chooses to buy.

So, if she chooses to buy 100 rupees worth asset, this will be interacting with the payoff that the investor is expecting out of this decision. If we have this kind of relationship between the consumption that is expected or desired by the investor today, that is c t, and the consumption that investor desires to have in future. And we maximize this objective function within these two constraints.

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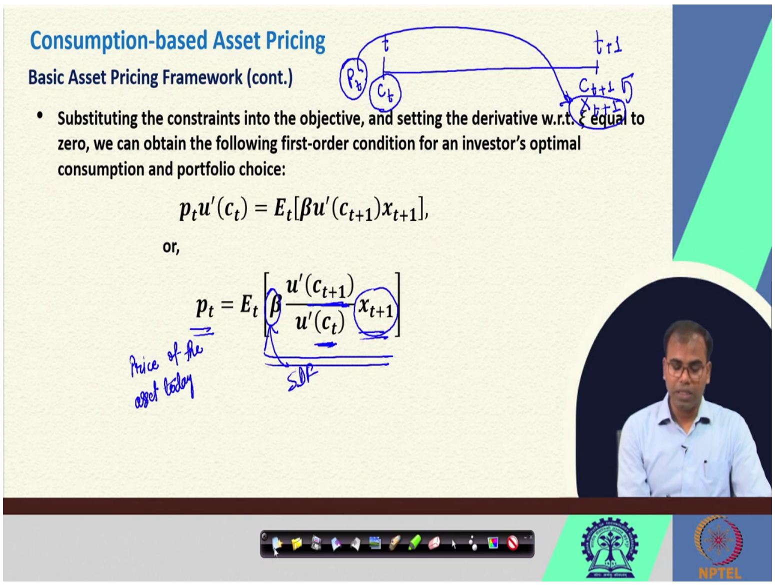

We know that if we process it further and substitute the constraint into the objective function and setting the derivatives with respect to chi equal to 0, we can obtain the following first order condition for an investor's optimal consumption and portfolio choice.

And remember, the consumption choice or the portfolio choice for an investor is how much of any asset the investor should buy or sell with the expected consumption or expected utility derived from the consumption. So, if we use this previous objective function with two constraint and substitute these factors into these two constraint into the objective function, and set the derivative with respect to chi equal to 0.

We can obtain this objective function such a way that we have this value of price and utility of consumption that the investor is expecting today as given as expectation of sub consumption that is coming in future or that is expected in future. And the payoff that is expected in future along with the subjective utility factor or adjusted with subjective utility factor and their expectations should be equal to the price and utility that investor is desiring in present time point of time.

This can also be rewritten as the price of the asset at time t should be expectation of the subjective discount factor that is denoted in terms of beta. And the utility that is coming because of the expectation expected consumption in t plus 1 time that is in future, and the expected utility of current time, and the multiplying it with the payoff that the investor expects at time t plus 1.

So, effectively what we are trying to show here is, if this is the timeline where we have t and t plus 1, investor has a choice to consume c t today or and c t plus 1 in time future time. Investor also has a payoff x t plus 1. And investor has to decide how much of the asset need to be bought or sold at p t that is price t. So, this objective function of utility based objective function essentially shows that the investor has a choice to buy or sell any asset at price t at p t that is the price of the asset today.

And this essentially is given by the expectation of the function basically the this price of the asset today at which the investor can buy and sell any amount of asset is given by the function of or an integrated function of the utility that the investor is deciding desiring today. The utility that the investor expects to have in future, and the payoff that investor is expecting if the investment choice is means today that is x t plus 1.

So, if this investment is done at P t today, this will result in this payoff that is x t plus 1. And if the investor chooses to have c t today which means this will be the total revenue or total stream of cash flows minus c t that is consumption today that will be invested at price t to in today’s term. And this will lead to the payoff of x t plus 1, and this x t plus 1 will give the investor consumption of c t plus 1 within this function given this subjective discount factor.

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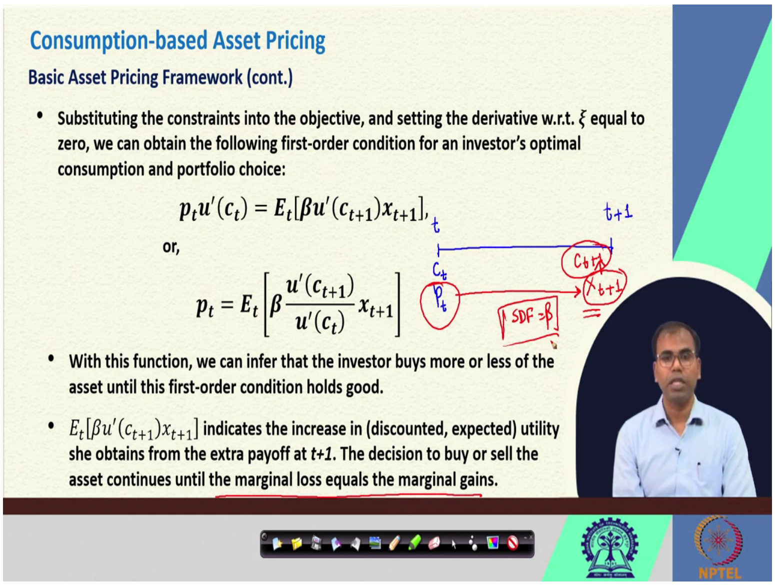

Now, with this, we can use with this function we can infer that the investor buys more or less of the asset at price t, p t, until this first order condition holds good and accordingly the investment choice is made. Here we need to be understanding that this factor expectation of subjective discount factor and utility of future cash flows earlier indicates the increase in discounted or expected utility that investor obtains from the extra payoff at t plus 1.

And the decision to buy or sell the asset continues, which means the investor will continue to buy or sell asset at price t, p t until the marginal loss equals the marginal gains. So, basically as highlighted earlier if investor is, if investor is having a decision at time t and he is expecting some pay off at time t plus 1 and suppose this investor has to consume some value that is c t and expect that the remaining will be invested at price p t.

This p t will effectively generate a future cash flow which is basically p t invested at time t will which will generate a future cash flow of x t plus 1. And this x t plus 1 will give the sources to the investor for consumption in terms of c t plus 1 given this stochastic subjective discount factor of beta.

So, until the marginal loss equals the marginal gain for the investor, the investor will continue to buy more and more of this asset or if she has taken an opposite position. Then, it will, she will continue to sell more and more of asset at price t in order to maximize the value of payoff at t plus 1 that is x t plus 1, to generate higher resources for consumption in terms of c t plus 1 given this subjective discount factor of B.

This particular framework essentially gives us the first order condition for consumption based asset pricing model.

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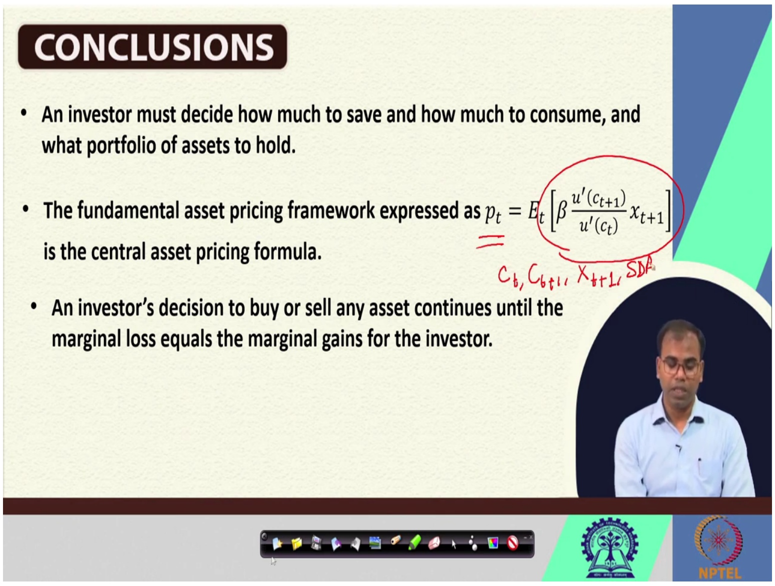

And this can be used for deciding for any investor about the level of investment that investor should made in an asset given these factors such as the expected payoff in terms of x t plus 1 subjective discount factor in terms of beta and the expected consumption in terms of c.

So, investor can use this model, this framework for deciding how much to save and how much to consume. And once the saving decision is taken care of, then the investor can decide what portfolio of assets should be held. The fundamental asset pricing framework such as this is expressed as, we have seen earlier price t is determined by the expectation of these 3 factor combined that is consumption today, consumption in future, a payoff in future, and subjective discount factor that is beta. And investor's decision to buy or sell any asset at price t, p t continues until the marginal loss equals the marginal gain for the investor.

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With this, I conclude this session.

Thank you very much.