Investment Management
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Lecture - 18
The CAPM and Index Models

Hello there, we are discussing the course Investment Management and in this module, we are talking about investment management through portfolio theory. And earlier we have discussed about capital asset pricing model. In this session we will talk about the Capital Asset Pricing Model and other Index Model that are based on the same philosophy.

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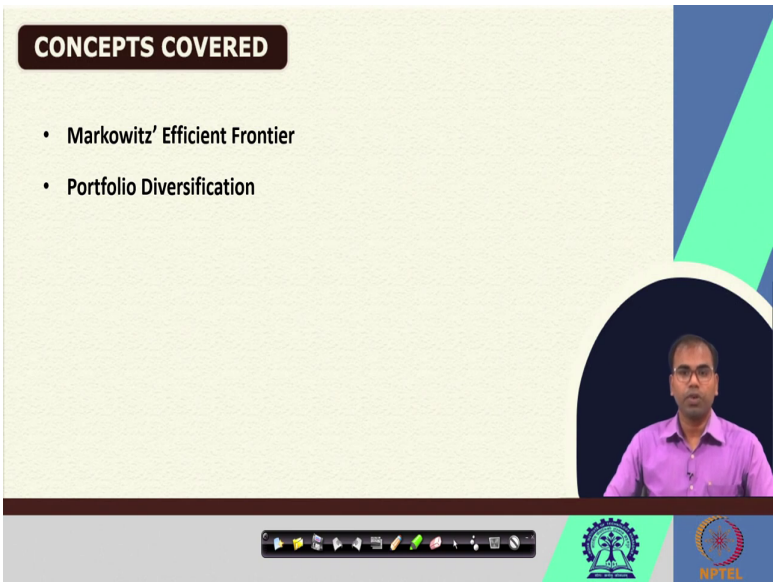

Basically, we are going to talk about in in detail Markowitz efficient frontier and portfolio diversification as a tool for investment management. This session will take us in detail about

how the risk return combination can be used to draw Markowitz efficient frontier and how an investor can use the efficient frontier for deciding whether to invest in a particular asset or not based on for preference for risk and return.

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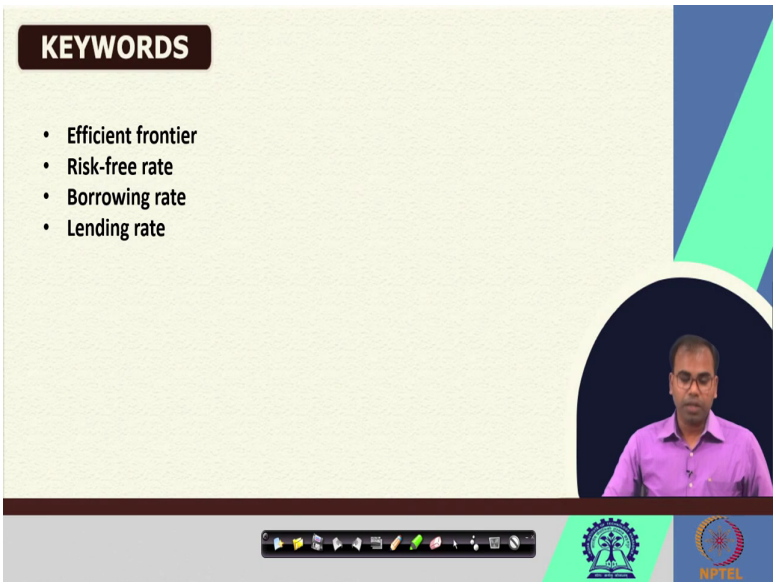

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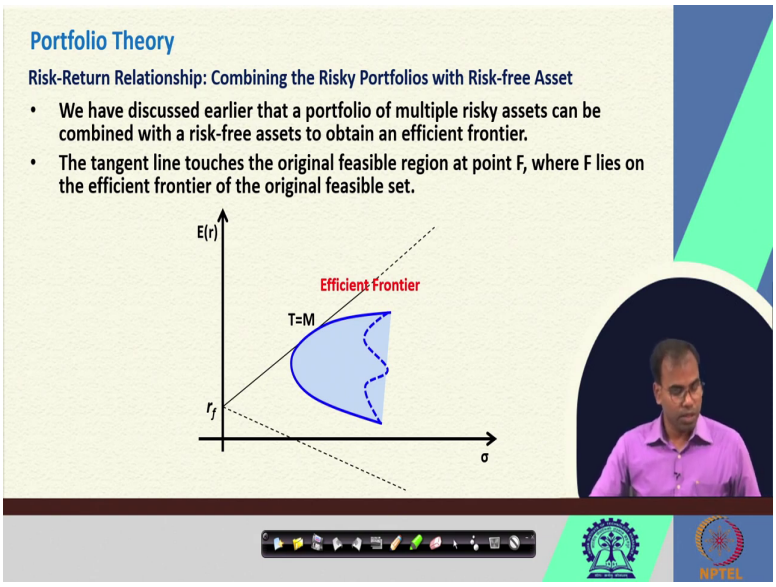

So, basically when we talk about Markowitz efficient frontier earlier we have seen that typically an investor faces a choice problem where there are different assets both risky and risk-free available in the market. And investor has to pick a combination of risky assets and may wish to have some exposure to risk free asset in order to find an efficient or optimal asset allocation.

What happens in typical capital asset pricing model is we have a combination of risky assets and these risky assets in the form of portfolios are combined with some proportion of risk free asset. And then we find a new efficient frontier where we have a tangent or a market portfolio that is essentially the best possible combination of risky and risk free asset for an investor.

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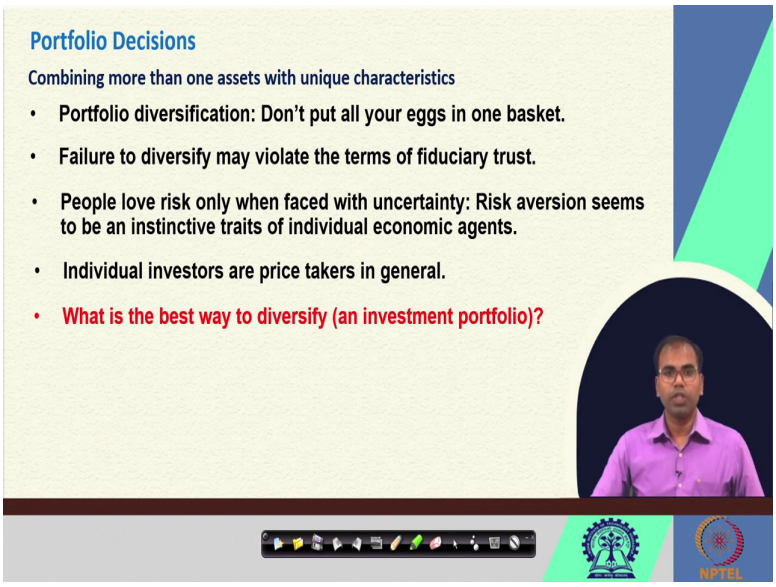

Now, drawing or understanding from this if you try to relate the concept that we have discussed previously we know that efficient frontier is used for portfolio diversification. Basically, the underlying philosophy is we should not have too much exposure in similar asset in terms of risk and return.

We need to diversify our portfolio. So, that in case of unexpected turns in of event in the market our portfolio is not affected extremely in one direction either too much loss or too much risk being carried out in terms of portfolio risk. If we fail to diversify our portfolio it essentially does not go down well for any investor and people typically prefer to have risk only if they are compensated with substantially above return compared to the case where they are previously holding a portfolio with less risk.

Essentially risk aversion is the basic choice an investor makes particularly in context of individual investors. Now, when we use these assumptions and try to understand the portfolio theory or capital asset pricing model or for that matter as basic tool as efficient frontier, we need to understand it from the scratch and we know that an investor is always looking out for a combination of assets with unique risk return a combo for diversifying an investment portfolio. Where do we begin with when we try to diversify our portfolio?

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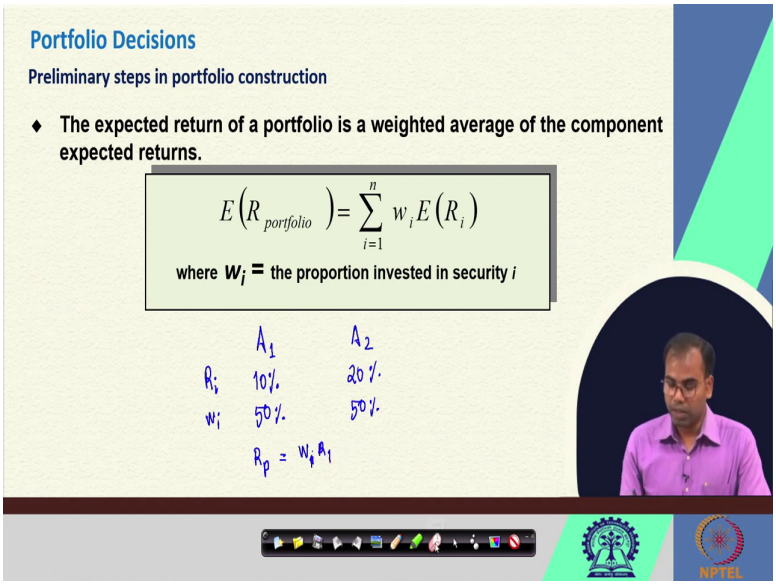

Typically the initial steps in order to diversify portfolio or investment portfolio is to understand the expected rate return for a portfolio which is basically a combination of individual returns along with the proportion of funds or money that is invested in those individual assets.

We know we have discussed earlier that the calculation of expected return on a portfolio is based on individual return given by R i and weight associated with each of the assets or the proportion of funds or proportion of money that is invested in each of the security in the portfolio.

We have earlier seen how we can calculate return on portfolio. So, we we we saw in previous example where if we have two asset; asset 1 and asset 2 and we know that these two asset have 10 percent return and 20 percent return respectively. If we try to identify these assets and invest let us say 50 percent of money in asset A and 50 percent of money in B then we know that return on portfolio is going to be calculated in terms of weight in asset 1, return in asset 1.

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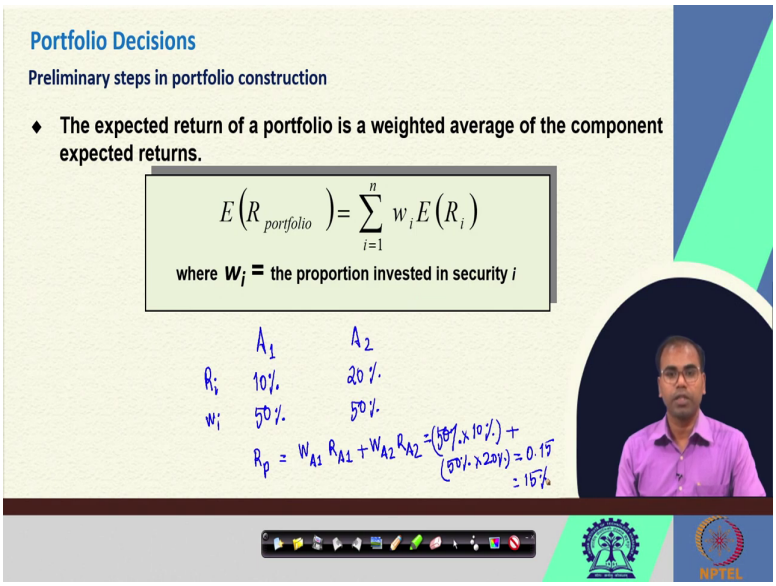

Essentially, we are trying to calculate the weighted average rate of return, which means weight in asset 1 into return in asset 1 plus weight in asset 2 into return in asset 2. If we try to calculate it will become 10 percent, 50 percent of weight in asset A 1 into 10 percent of return plus 50 percent of weight in asset 2 with 20 percent of return and this gives us 0.15 which is essentially 15 percent of return.

We have seen this example earlier and we know that in this way we can calculate a portfolio return with as many assets included in that portfolio. If we know the assets given are known for the investor. For example, if we have more than two assets, we will keep on adding weight of that asset respectively along with the return of that individual assets.

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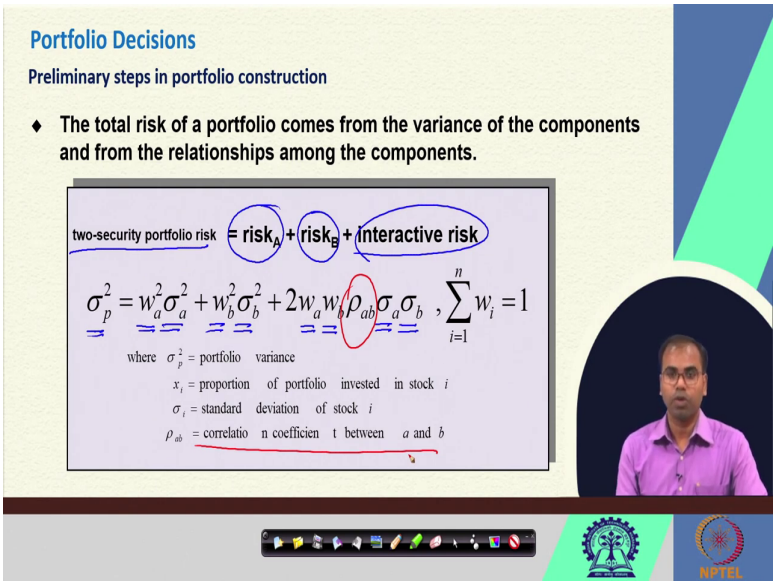

When it comes to calculation of risk for a portfolio the calculation is slightly different because total risk of a portfolio comes from the variance of the components and from the

relationship among individual components. Effectively it implies that the if we have a portfolio of two asset let us say asset A and asset B or asset A 1 and asset A 2 we know that the risk of two security or two asset portfolio will be calculated as individual risk of first asset plus individual risk of second asset and interactive risk.

Earlier we have seen in detail how we can arrive at this formula where we have variance of portfolio sigma square p is equal to weight in first asset squared sigma of first asset squared weight in second square as second asset squared and sigma of second asset as squared and 2 into weight into first asset weight into second asset sigma of first asset and sigma of second asset along with the interactive risk which is basically correlation.

Again, this correlation coefficient determines the total risk of the portfolio and whenever we come, we want to calculate the portfolio risk where it is portfolio of two asset then we can calculate portfolio risk in this way.

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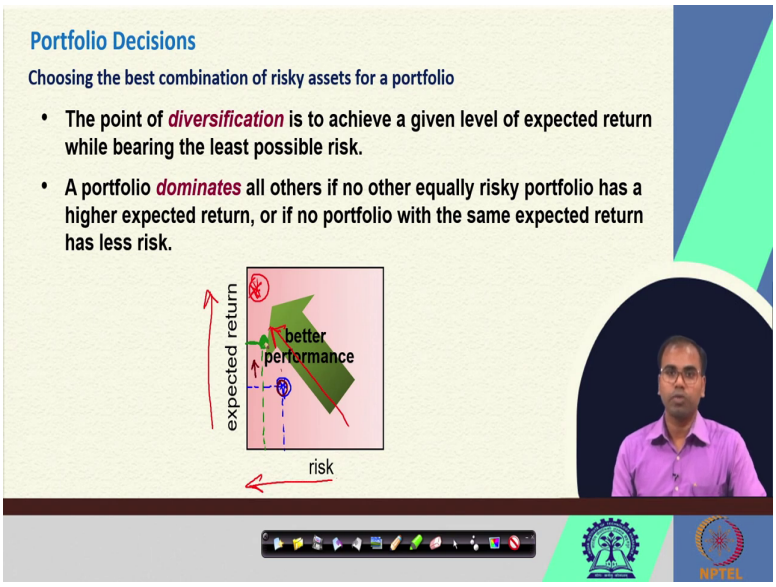

Now, investor's job is to find the best combination of risky assets in order to construct a portfolio. Now, when it comes to construct the best portfolio essentially what investor wants is to reduce the risk for a desired level of return or to maximize the return for a given level of risk.

Earlier we have discussed that if we know the risk tolerance or expected risk of an investor we can determine how much maximum return the investor can generate by combining the assets available in the market. Or we can if we know that investor expects a certain level of return from investment, we can decide how much risk the investor needs to carry in order to generate that much return from the investment in the market.

Now, the whole idea is diversification. So, diversification is to is carried out to achieve a given level of expected return while bearing the least possible risk. Any investor would want

to carry out the diversification exercise in order to minimize the risk for a given level of return.

Although the investor's dream might be to minimize the risk for the maximum level of return, but that maximum level of return cannot always be fulfilled, but for a given level of return the risk can be minimized using this diversification approach.

Now, if we have a portfolio where or a combination of assets where we have expected return and risk on to plane to access, we know that for any investor the best opportunity or the best set of opportunity will be somewhere if we can find any asset around this point where the expected return will be highest possible and risk will be lowest possible. So, as the investor wants to increase the return and at the same time reduce the risk then investor has to move in this direction in order to find a combination of assets.

However, there might not be sufficient number of assets with such properties such that the return will be higher; however, risk will be lowest. So, typically a portfolio dominates all other portfolios if no other quality risky portfolio has higher expected return or if no portfolio with the same expected return has the has any less risk.

Which means if there is a portfolio that is placed here which provides this much of risk and this much of return and there is no other portfolio for this level of risk with generate any higher return then we can say that this portfolio dominates because this is the combination of assets where the investor gets the maximum return for a given level of risk.

Or if the investor has some sort of expectation in terms of return and it knows that if there is an asset a portfolio which is placed here and the investor has this much of return that this portfolio will have to carry this level of risk. So, this this portfolio will dominate others. This is the concept of diversification where portfolio dominates if no other risky portfolio has a higher expected return or no other portfolio with the same expected rate of return has any less risk.

With this concept we will move further and try to see how this combination of risk and return in in the form of portfolio of risky assets behave and also when there is a risk free asset introduced how the behavior of efficient frontier changes and so on.

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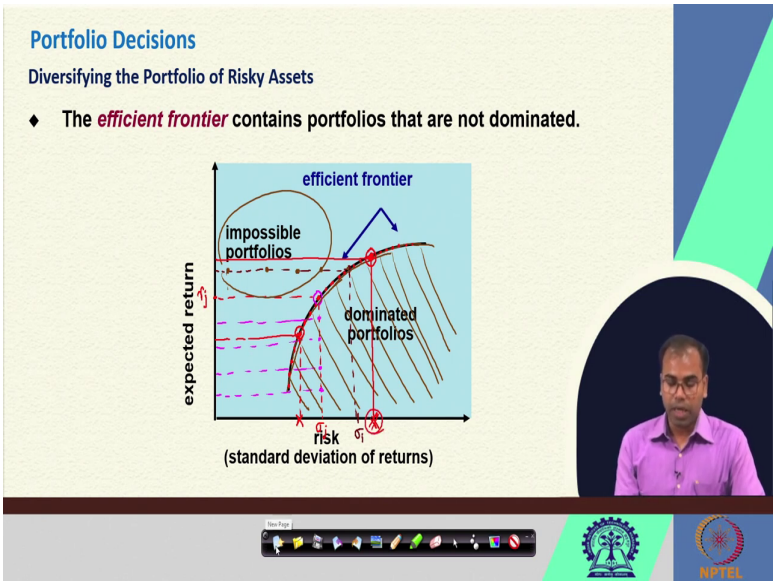

This idea if we move further in the in diversifying the portfolio of risky assets, we know that there must be an efficient frontier that contains portfolios that are not dominated. Which means there are no other better combinations. And this is the way we essentially project or visualize a portfolio with efficient frontier and efficient frontier is the combination of assets where they get investor get to choose best combination of risk and return.

So, just to reiterate if we know that an investor desires this much level of risk then this is the best point to which the investor's portfolio can be placed, which means the investor has the potential to generate a return to this extent. If investor would want to have this amount of this

level of return this is the expected return of the investor J and the investor wants to know what is the minimum level of risk that investor can generate then this is the level of risk that investor has to carry.

Now, we know that for an any investor it is possible that investor can carry this much risk and have a portfolio somewhere here, can have a portfolio here, can have a portfolio here as well and here as well along this entire line the investor can have portfolio, but this portfolio will be the best portfolio because it generates highest return for the given level of risk.

In a similar fashion if you look at the investor I where investor has a choice to generate this much return and it can carry this much risk, but possibly there is no such asset available here, there is no such asset available here either or here or here for that matter. So, this particular reason could be considered as a combination of risk return where there are impossible portfolios or non-existence portfolio.

So, portfolios are available in this region and the portfolios lying along with the efficient frontier would be considered the portfolio that are plausible for any investor to invest in and they are the best combination of risk and return where the investor can invest and generate the highest level of risk and carry the minimum level of risk and generate the highest level of return.

We know that in in such cases where the portfolio could be lying along this line and these portfolios present the best combination of risk and return for any investor where the investor can choose any of the portfolio and generate the highest expected return for a given level of risk. So, if an investor choose to invest in this portfolio she has to take this much of risk and she can expect this much of return.

So, this is the highest return for this level of risk. Similarly, an investor can choose to invest in this portfolio and she can expect to generate this much of expected return for this level of risk. There is no other portfolio which will generate higher return than this portfolio for a given level of risk. So, this is the idea of efficient frontier.

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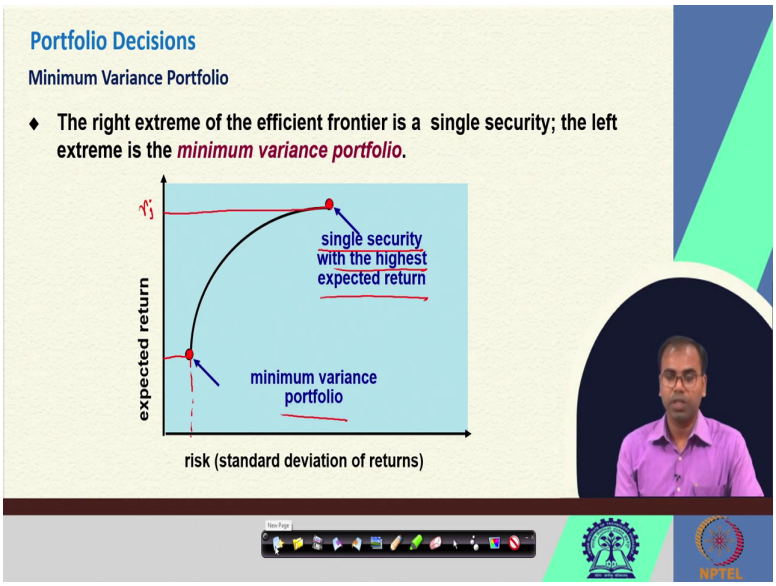

With this understanding of efficient frontier, we can move on further to understand the minimum and maximum variance portfolio.

So, let us try to understand first the minimum variance portfolio. Now, if we look at this graph here we see that there are portfolios along both extremes of the curve. So, right extreme of the efficient frontier is the single security and left extreme is the minimum variance portfolio. So, we know that this particular asset this particular security is the single security here which might have the highest expected return if we extrapolate this from here.

So, this will be the highest return generating security and this is the security which carries the lowest risk. So, this is minimum variance portfolio and this is the single security with highest

expected return. Now, let us try to connect this concept of minimum variance portfolio and try to understand the maximum variance portfolio.

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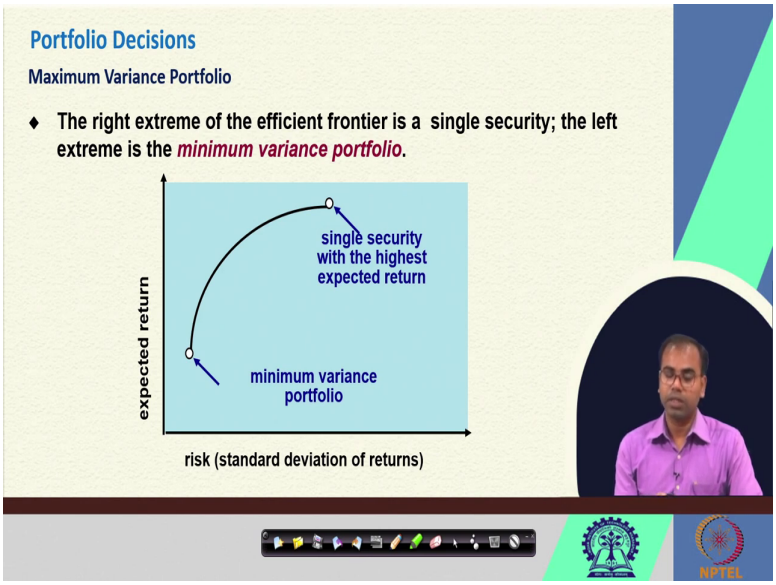

Now, when it comes to maximum variance portfolio we know that the right extreme of the efficient frontier is single security portfolio, single security and highest expected return.

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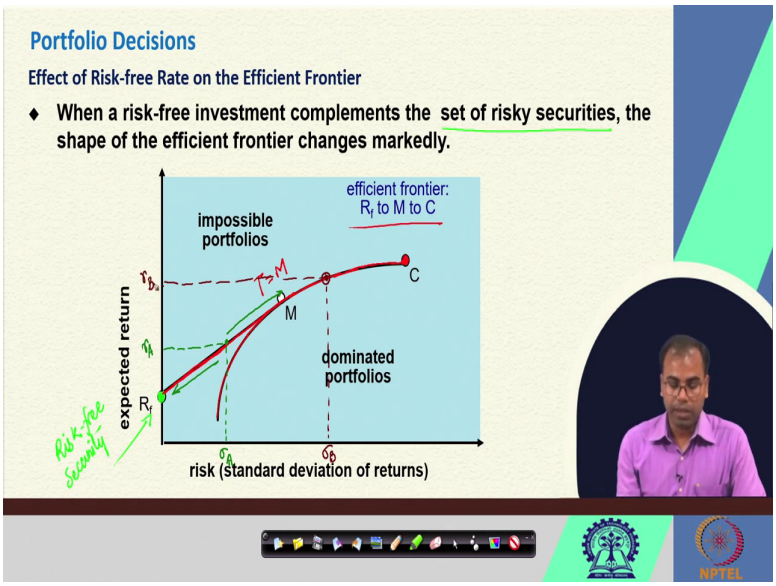

And similarly, we have minimum variance portfolio and we want to see how this maximum variance portfolio could effectively translate into a into as an investment or a portfolio that essentially does not provide an appropriate investment opportunity for the investor.

So, when a risk free investment complements the set of risky securities as we understood earlier in terms of introducing a risk free asset where there is no risk associated at all. If we try to understand this so this portfolio this security is the risk risky risk free security and this implies that the security presents some amount of return, but carries zero risk or no risk at all.

So, if we introduce the risk free asset in the picture in the scenario we know that earlier we have a set of risky securities. Now, we have a combination of risky and risk free security as well. So, when we try to combine the two types of securities together, we know that this

portfolio curve connecting the risk free security with the risky asset will be tangent to a point and this portfolio is essentially the tangent portfolio also known as market portfolio.

And here the efficient frontier will be starting from R f which is basically the risk free asset and it will move on to be tangent to the risky portfolio which is basically this curve and once it be tangent then investor would have a choice to move along with this curve to go to the extent of point C which is essentially the new efficient frontier.

So, if an investor wants to invest in this portfolio which has some sort of exposure of exposure to risky assets and risk free asset then investor can invest here which means there will be some proportion of investment in risk free asset and remaining proportion of investment in risky portfolio. And this way the investor would have this level of risk and can generate or can expect a return to this extent.

Now, if we change the scenario here there would be a situation where an investor would want to invest at this point with some exposure to risk free asset and most exposure to risky portfolio the investor can increase the return from r A to r B and this way he has to carry some additional risk as well.

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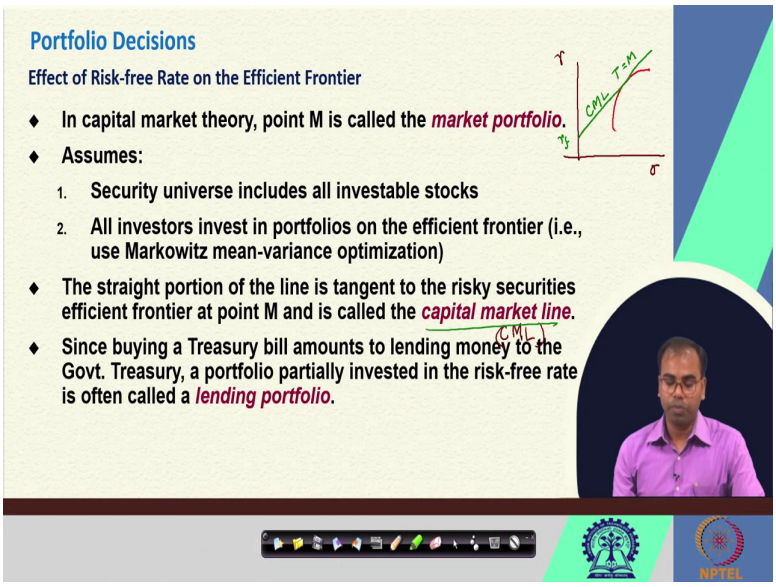

When we have this risk free rate in the scenario we know that in a typical capital market theory this point T or point M is also called the market portfolio which is basically the best combination of risky and risk free assets and essentially it has all investors agreeing that this is the best combination of risky and risk free assets and this presents the most optimal allocation of assets towards risky assets.

And it there are certain underlying assumption here which says all the securities in the universe which is basically the securities available for investors to invest includes all stocks or all assets that are investable which basically means that investor can invest in any or all of the stocks in some or the other proportion. All investors are assumed to be investing in portfolios on the efficient frontier not below that which is basically the dominated stocks and this is done by using the Markowitz mean-variance portfolio optimization.

Another assumption is the line connecting R f to M is the line tangent to the risky securities and efficient frontier at point M and this is this particular line is called the capital market line or also CML which we earlier know in terms of risk and return where we have a risk a portfolio of risk risky assets and then we introduce a risk free asset and we know that this will be tangent or the market portfolio.

So, this particular line connecting the risk free asset and risky portfolio is considered called as capital market line. If we see here, we know that this risk free asset is nothing, but some sort of government securities or treasury bill or any other similar asset and since buying in such securities or such asset like treasury bill amounts to lending money to the government treasury.

Because when you invest in government bond you are investing in government bond by and effectively you are lending your money to the government and that is why a portfolio partially invested in the risk free rate is also called the lending portfolio in this context.

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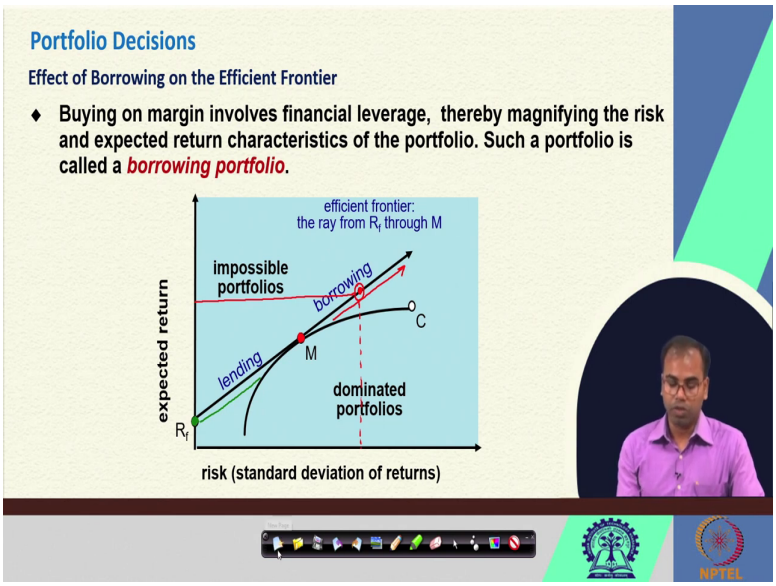

When we see this borrowing concept or the idea of borrowing money or the portfolio which is created by using the funds borrowed from some other entity. Then typically we see that investors buy in assets particularly stocks on margin and it involves financial leverage, thereby it magnifies the risk and expected return characteristics of the portfolio. And that is why it is also called as borrowing portfolio where investor borrows money from other entities and use that borrowed fund for investing in the securities.

And we see thus this kind of portfolios on a curve like this where we have a typical setup where we have this efficient frontier and this efficient frontier is effectively moving from R f going to the market portfolio. And then it moves along with the borrowing zone where investor have borrowed money or leverage their portfolio by borrowing money and this is called borrowing portfolio.

So, it can be easily understood when you try to connect this borrowing portfolio or any portfolio on this borrowing line if you assume to have a portfolio here you know that you are carrying heavy risk and at the same time you are also expecting much higher return compared to the other portfolios on the efficient frontier. So, the whole idea of leveraging the portfolio or creating a borrowing portfolio is to have leverage effect on the return, but at the same time the risk is also increased.

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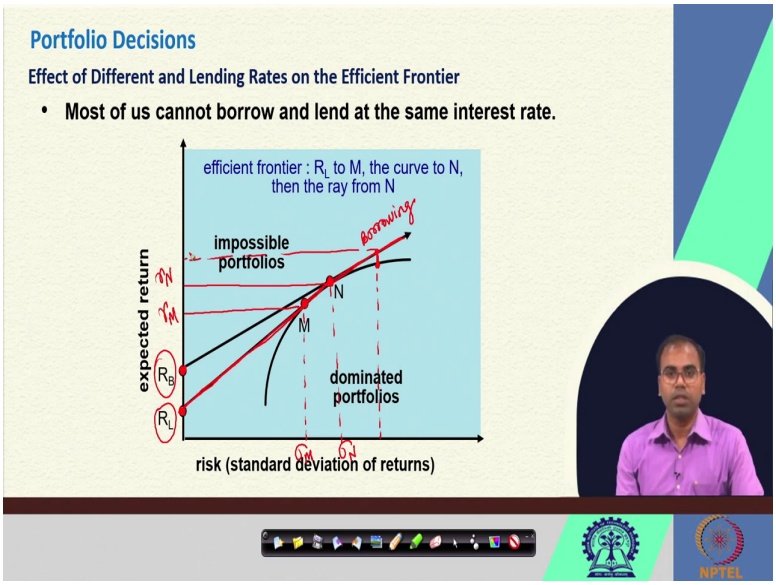

When we believe that the lending and borrowing rates are different then the scenario can change as well if we try to visualize the scenario where lending and borrowing rates are different; which means if an investor can lend money at certain rate, but if he needs to borrow money then he has to pay a different interest rate then we can see that scenario would be different.

Here we see that lending and borrowing rates are different and in this case there will be two possible portfolios where there will be tangent line and these two portfolio can be of can be offering different combination of risk and return. We can see here this portfolio M carries this much risk; however, provides this much of return and another portfolio that is based on borrowing rate is providing this carrying this much of risk and providing this much of return.

So, we see that this efficient frontier now the new efficient frontier would be from R L to M and the curve to N and then the ray from N to other terrain where you borrow money and then invest that money in the portfolio. So, you can see if this kind of possibilities are there where a person can lend at this rate.

However, the person can borrow at this rate and this money can be invested in this stock then investor has to move along this line where the portfolio would be lying somewhere here where he can generate this much extra return compared to a typical portfolio in risky asset because of the leverage effect. But we need to keep in mind that with additional return comes the additional risk and that the investor need to be mindful of.

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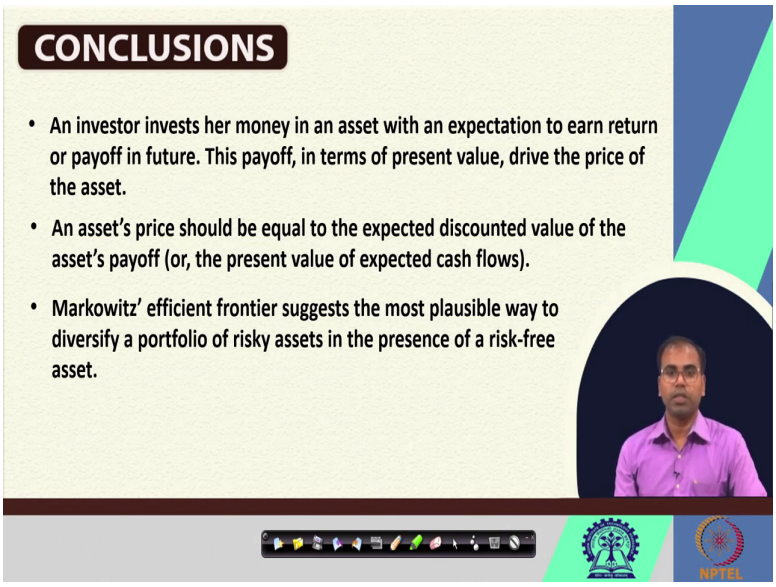

So, to conclude this session we started with the idea of having an investor looking out for diversification strategy where the combination of risk and return can be designed in unique way such that an investor can minimize the risk for a maximum possible return or the given level of return. And the investor can also try to reduce the risk for given level of return or increase the return for a given level of risk.

But we should also understand that Markowitz efficient frontier suggest the most plausible way to diversify a portfolio of risky asset, in the presence of risk free asset as we have seen and there might be a different scenario when borrowing and lending rates are different for an investor. And there will be no similar situation where investor can move from one scenario to another scenario and diversify the portfolio for better return or minimum risk. With this I conclude this session.

Thank you very much.