Investment Management
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Lecture - 17
Capital Asset Pricing Model

Hi there. So, in last session we talked about the concept of portfolio. Once we understand the concept of portfolio, can we derive the understanding of portfolio theory for the asset pricing model? This session focuses on understanding the capital asset pricing model which is one of the most commonly used asset pricing model in finance literature and practice.

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Basically, the issues that we are going to discuss in this session is the concept of portfolio, particularly portfolio of risky assets and how it changes with an introduction of risk free asset

and subsequently can we take this understanding to explain or to derive the Capital Asset Pricing Model or CAPM.

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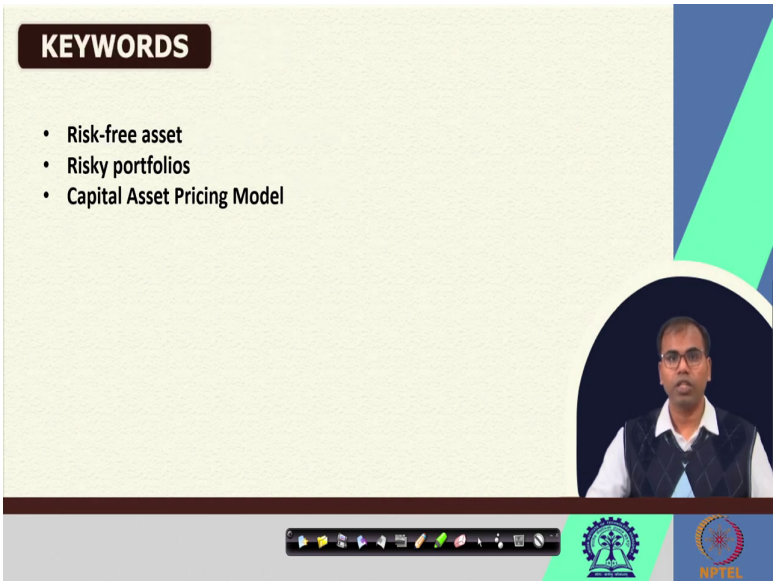

Let us try to understand the idea of asset pricing first and then we will try to see how asset pricing is derived or dependent on the understanding of portfolio theory and availability of risk free asset in that matter.

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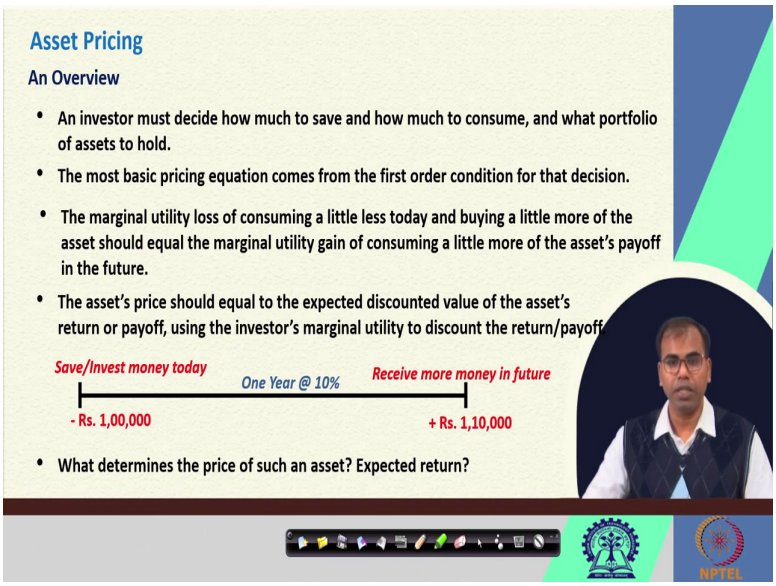

So, when we talk about asset pricing, we know that an investor has to decide from his income how much to save and how much to consume. Suppose a person has received an income or a salary of 100 rupees for a particular period he or she has to decide whether to consume 100 rupees altogether or consume may be part of it and remaining can be shaved or invested in some financial assets. That is the whole idea of investment.

Now, when we talk about investing the amount or the part of money, the part of income that the investor has saved, we have to address the question what portfolio the investor should hold, what portfolio of assets investors should hold. So, the first question comes how much to consume and how much to save and if at all something is saved then what portfolio of assets to hold.

The most basic principle or most basic pricing model comes from the first order condition for that decision which means, an investor has to decide how much to save depending on how much he or she is going to get if that saved money is invested in some financial assets.

So, we can say that the marginal utility loss of consuming a little less today and using that money to buy a little more of the assets should equal the marginal utility gain of consuming a little more of the assets pay off in the future. Which means, if I save little more today and hold this saving in the form of some investment, it should be generating some pay off in future which should be the marginal utility loss that I am going to face today because of consuming less.

Because I am taking a decision to save more today essentially, I am consuming less. If I have 100 rupees and I can consume 100 rupees or anything less than that then the less I consume the more I am sacrificing in terms of utility and if that loss in utility that the loss in marginal utility of consuming less today that will be used for saving.

And subsequently investing in financial asset should be equal to the marginal utility gain that I expect by consuming a little more of the income or the payoff that I am expecting from the investment in certain assets. So, when we talk about asset pricing the assets price should be equal to the expected discounted value of the asset returns or pay off where we use the discount discounting process through the investor's marginal utility to discount the expected payoff.

We have already discussed in more detail when we talked about calculating the present value where we know that if an investor is investing some amount of money in some financial asset and that asset has promised certain pay off in the form of dividend or coupon or interest or any other cash flow.

We need to bring those expected pay off in the form of dividends or interest or coupons or any other form of cash flows to the present time by discounting it with some discounting rate. And that discounting rate is basically determined keeping in mind the investor's marginal

utility or in some sense opportunity cost or in some way the weighted average cost of capital as well.

So, if you try to understand this if an investor has saved 100,000 rupees today this 100,000 rupees can be invested for one period let us say 1 year and the investor expects to earn 10 percent of return. So, at the end of 1 year the investor will receive a money is equal to 1,10,000 or 1,10,000 rupees and this 1,10,000 will be brought back to the present time to see if it is in present terms value more than 100,000 that is being invested.

So, what essentially determines the price of an asset where investor is investing or an asset that investor intends to hold in the form of a portfolio? Is it the expected return or is it anything else? If it is expected return, how do we derive that expected rate of return that is addressed by the capital asset pricing model?

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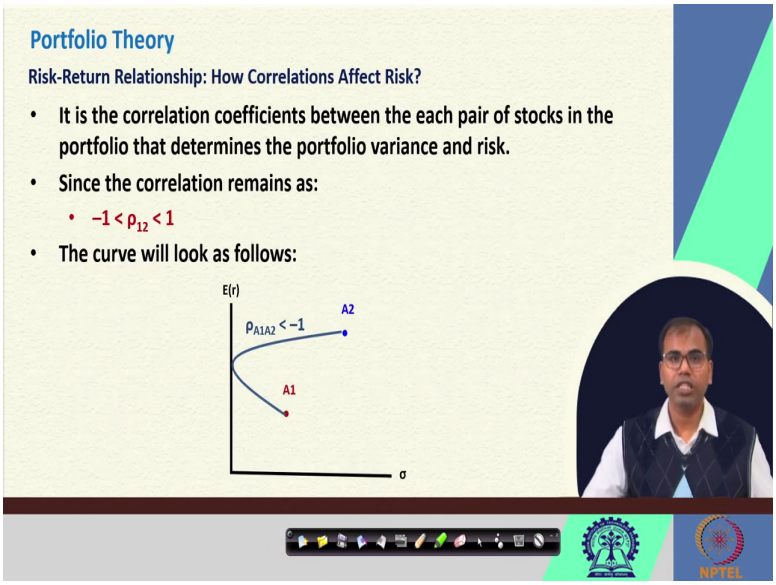

So, let us begin with the same the basic understanding that we had derived in the previous session where we know that the return on a portfolio of asset is calculated using the weights invested or proportion of money invested in individual assets and the return coming out of individual investment combined together in the form of weighted average rate of return and risk depends on not only the individual risk associated with the assets and the proportion of money that are invested in individual assets, but also the interrelationship between different assets of the portfolio.

Now, this interrelationship typically is indicated in the terms of correlation coefficient and we know that the return on assets could be correlated with each other from to the extent of minus 1 to plus 1 and depending on the correlation the shape of the curve indicated in this graph will look like and accordingly our portfolio of assets will be placed.

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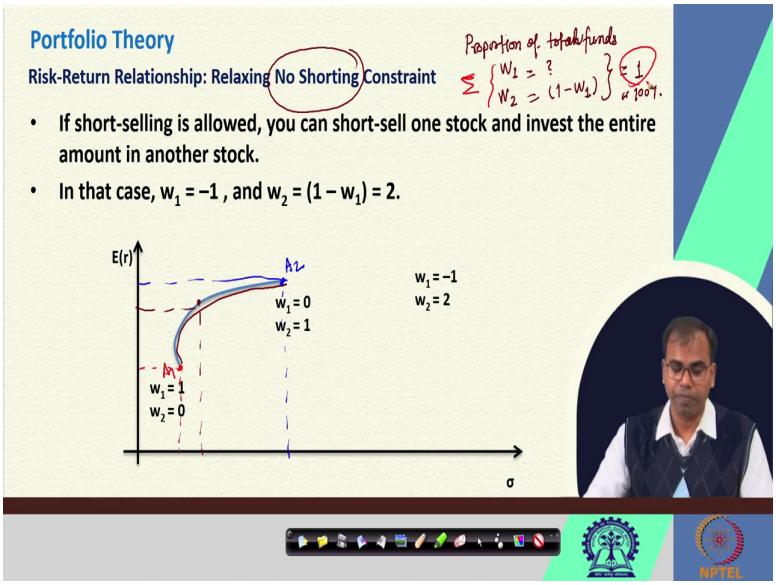

Now, this is a scenario which is very simple, but earlier we have assumed that there are two assets an investor has to invest in those two assets only and we have also seen that the when we talk about proportion of investment in those asset proportion of total fund or total money that investor has for investment is given by weight.

Let us say if this is a two asset case then weight 1 and weight 2 to be something and for weight 2 it is 1 minus weight 1 which means, the sum total of these two assets should be equal to 1 or the total money has to be invested in two assets. Now, here the basic assumption is no sorting which means, no short selling essentially and it restricts the investor to move along this line only this graph only to play with the position of the portfolio.

Which means, if I invest 50 percent of my money in each of the asset I can be placed somewhere here probably where my risk and return will be driven and if I invest 100 percent of my money in asset A 1 then my portfolio will be here and if I invest 100 percent of my money in portfolio A asset A 2, then my portfolio will be lying somewhere here.

This is the basic assumption when short selling is not allowed. So, let us lets quickly understand what short selling implies here. So, short selling implies that you can sell an asset without actually holding it with the assumption that in future the price of such an asset will fall you will buy and settle the deal and in that process, you will probably make a money because you have sold it at a higher price and bought it at a lower price.

So, when we talk about short selling relaxing the short selling constraint or the basic idea of having weight some of these two weights to be one we are essentially relaxing this condition of some of all the weights being one where some of weights can be in some of the weights let us say weight 1, weight 2, weight 3 in case of multiple assets.

So, one of the weights could be non positive non positive as well. So, here in this case if we look at this scenario, we are assuming that the situation is such that the weight is negative and this way we can probably extend our argument and see the position of the portfolio subsequently.

So, let us move forward with understanding that short selling can be constrained and once the short selling is relaxed the no short selling constraint is relaxed. Then we know that the portfolio can move beyond this point and similarly it can move below this point as well and as we allow short selling.

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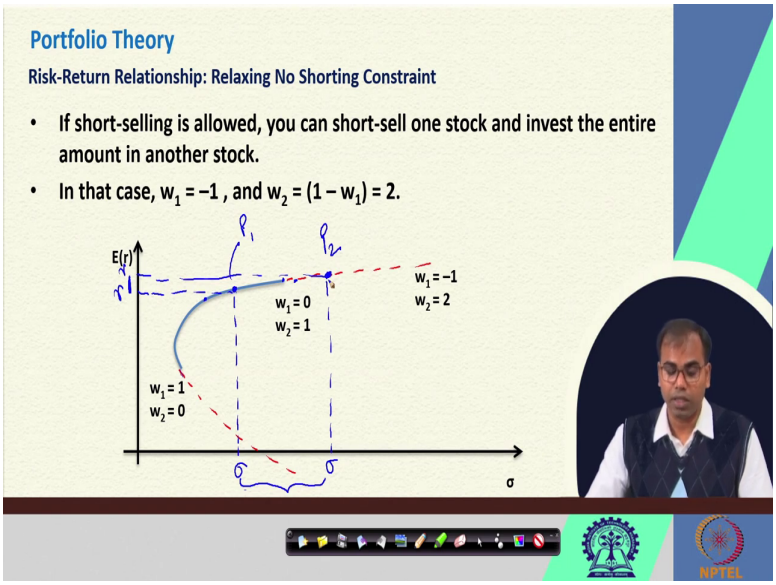

We know that depending on the risk and return investor can invest here or here or here anywhere in this entire curve. In fact, an investor will have a choice of investing somewhere here also depending on the preference for risk and return. As we understand if investor is investing here, she has to carry this much of sigma or this much of risk and she can expect this much of return.

Similarly, if an investor chooses to invest here in a short selling portfolio, then risk will be somewhere here and return will be somewhere here. Now, we understand from the basic intuition that the marginal risk that investor is carrying by investing in this portfolio.

Let us call it let us say portfolio 1 and this portfolio which is a short selling portfolio P 2 the marginal risk that investor is carrying is much higher than the marginal return that the investor is expecting to gain by investing from instead of portfolio 1 by investing in portfolio 2, P 2. Now, with this assumption of no short selling relaxation, we can try to understand how this essentially translate into a more sophisticated form of portfolio.

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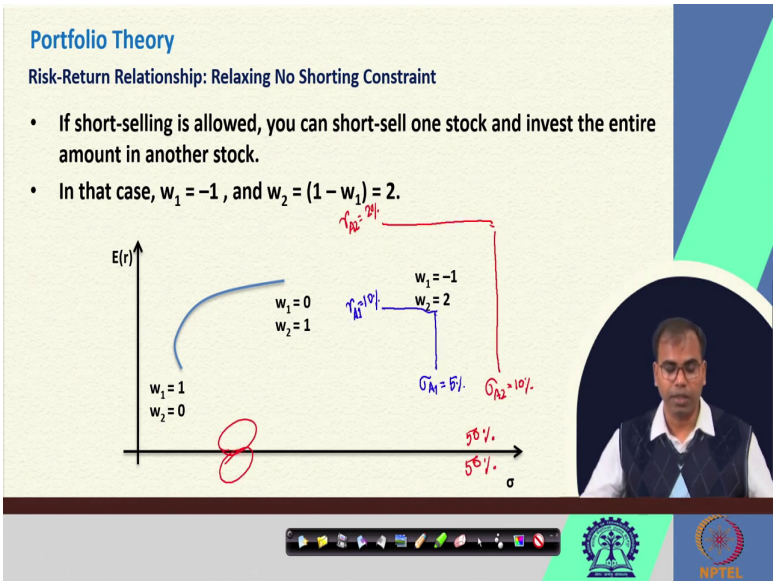

And we can see this in the form of capital asset pricing model for further processing.

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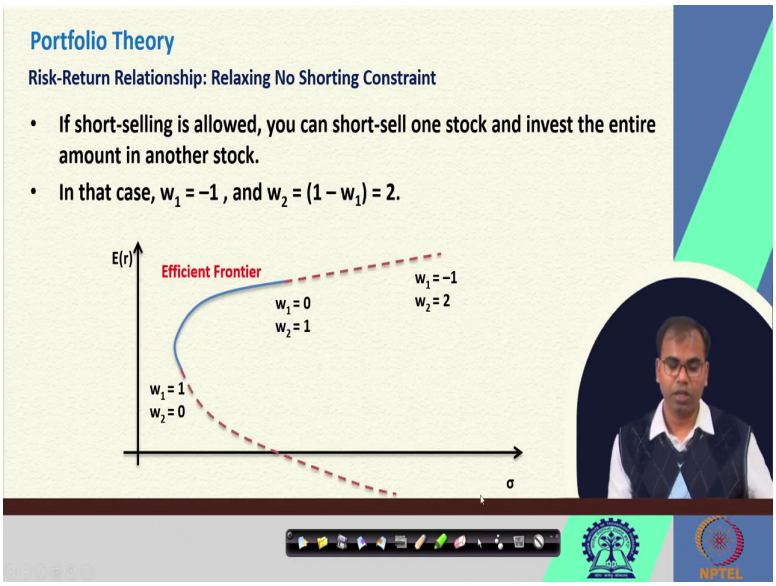

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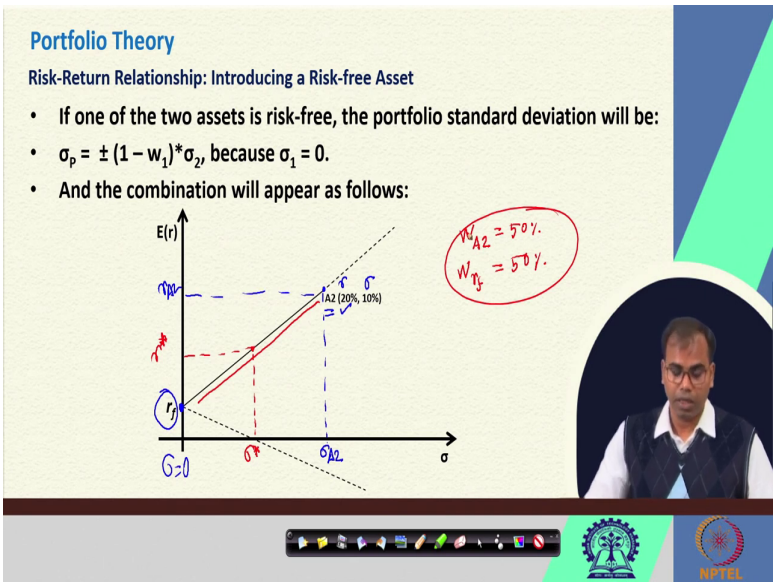

Now, let us take a look at the situation where an investor has an option to invest in a risk free asset as well. Now, as we understand that there are certain financial assets in the market where investor does not have to carry any risk which means, the return is guaranteed there is no uncertainties about the return.

And one of the risk free asset or one of the examples of risk free asset is treasury bills or government bonds most preferably because they provide you assured return and there is no risk associated with that which means there will be expected rate of return indicated in the form of rf, but there is no sigma.

So, the asset that I am talking about is placed here where there is no sigma or sigma is going to be 0 for this particular asset and return is going to be something that an investor can expect. Now, if an investor chooses to invest entire money in a risk free asset then the

portfolio will be somewhere here, but if investor chooses to invest in the combination of asset A 2 which has 20 percent of return and 10 percent of sigma. So, investor has a choice to choose choice to invest in asset sigma A 2 and this is basically (Refer Time: 15:03) A 2.

Now, if investor chooses a combination of asset A 2 and rf then we know that investor's portfolio will be moving a cross across this line. So, if an investor puts 50 percent of her money in A 2 and remaining 50 percent in let us say risk free asset. So, it will be indicated as weight A 2 as 50 percent and weight RF as 50 percent and the portfolio will be somewhere on the middle of this curve and the risk will be indicated by a sigma star here and return will be here.

So, we know that this will be the placement of the portfolio if investor chooses to invest 50 percent of money in each of the two assets. So, this will be the scenario where some part of investment is made into risky assets and some part of the investment is made into risk free asset.

Now, again this scenario is also two asset keys. Imagine that you have one asset as risk free asset, but another asset as not a single asset, but rather a portfolio of assets and to be particular portfolio of risky assets. Now, if you try to understand this argument before that we need to see how does it look like when we have multiple assets multiple risky assets in a portfolio.

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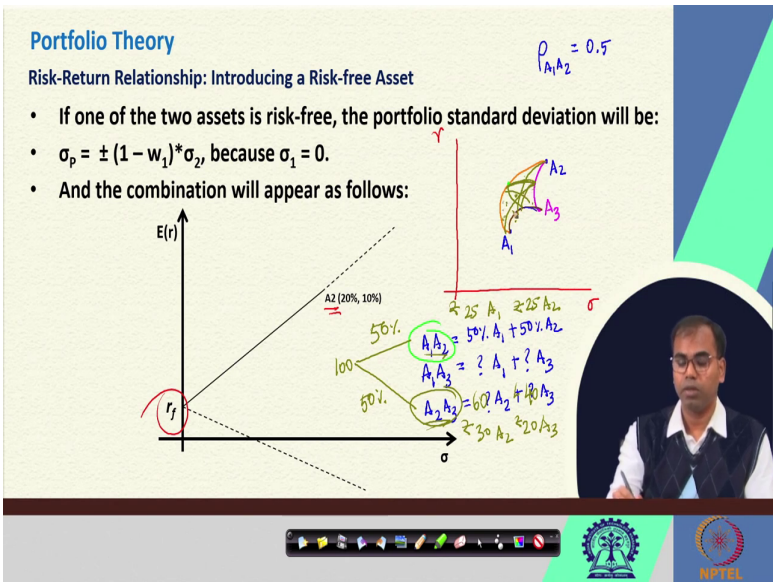

So, we will go a little one step back and see that we have risk return plane here risk return graph where we have two assets let us say asset A 1 and asset A 2. We have seen that depending on the correlation the shape of the curve might look like. So, let us assume that the correlation is moderate 0.5. So, the shape of the curve between asset A 1 and A 2 might look like something like this.

Now, let us hope that there is another asset A 3 introduced in the market. So, when we combine investment in asset A 2 and A 3 to form another portfolio, assuming that they also have moderate correlation the portfolio will be looking something like this and if we believe that we can combine A 1 and A 3 also.

So, there will be some portfolio lying on this curve as well which means, I have a choice to move along a portfolio of A 1 and A 2. So, if I hold a portfolio of A 1 and A 2 where I may

invest let us say 50 percent of my money in A 1 and 50 percent of my money in A 2. So, my portfolio will be my placed along with this orange curve. If I invest in A 1 and A 3 in the same proportion certain proportion of money in A 1 and certain proportion of money in A 3 then my portfolio will be lying around this brown curve.

And if I invest in A 2 and A 3 portfolio in some proportion where some money goes into A 2 and some remaining money goes into A 3 then my portfolio will be lying across the pink line. If we derive our understanding basis of two fund theorem and the concept of two fund theorem where it is argued that if you have two or more assets you can create many more assets in the portfolio by forming them into a different portfolios.

So, here suppose an investor is investing not only in A 1, A 2, but also investing in a portfolio of A 1 and A 3. So, suppose I have a certain amount of money invested in portfolio A 1 and A 2. So, my portfolio will be somewhere here and some part of money is also invested in let us say A 2 and A 3. So, I have let us say 100 rupees, what 50 goes into A1, A 2 portfolio where I have 25 rupees into A 1 and 25 rupees in A 2.

But at the same time, I am also investing remaining 50 percent into a portfolio of A 2 and A 3 where let us say if it is 60 and 40. So, 30 rupees goes into A 2 and 20 rupees goes into A 3. So, in total I will have probably 55 rupees in A 1, but in two different portfolios A1, A 2 and A 2, A 3 and remaining in A 2 and A 3 and this way I have exposure to all three assets available in the market. So, my portfolio would be lying somewhere between the combination of A 1 and A 2 portfolio and A 2 and A 3 portfolio. So, somewhere here on this curve.

Similarly, I can have combination of A 2, A 3 and A 1, A 3. I can have combination of A 1, A 2 and A 1, A 3. I can have a combination of A 2, A 3 and A 1 and similarly I can have combination of A 2 and A 1, A 3 and so, on. So, basically, we can believe that there will be multiple points within this region where I can make my investment depending on preference for exposure to different assets available in the market.

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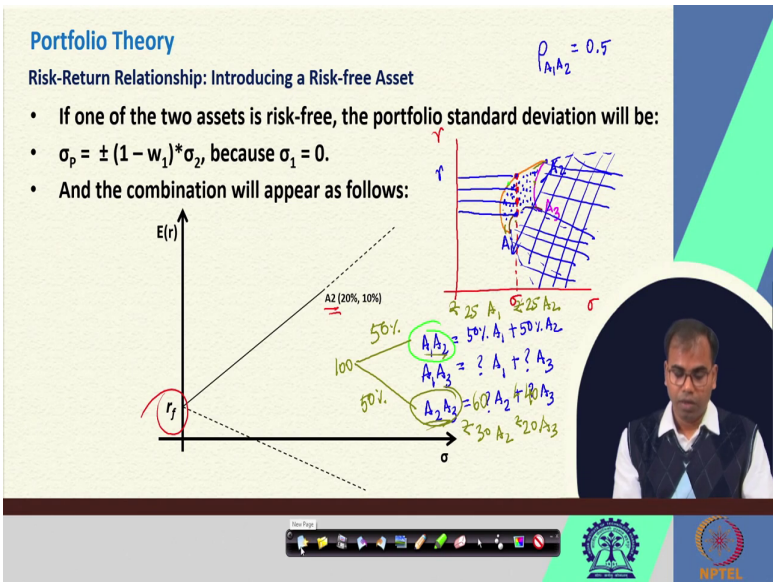

So, if we generalize this argument, we can say that any point within this region can be a possible combination of assets in some proportion, some sort of unique proportions of A, A 1, A 2 and A 3 or to say so, all the assets available in the market.

Now, if we extend this argument a little further and relax the condition of short selling, we know that this can be extended further if we short sell, if we are allowed to short sell and then in that case there will be all these points as possible combination of asset A1, A 2, A 3 or for that matter all the assets available in the market.

However, at a rational investor I would not want to invest in any asset that is available here let us believe that if there is an asset available here or rather there is a portfolio available here

and at the same time there is a portfolio available here as well. So, given my rational investor assumption I can invest anywhere along this red line.

I have a choice to invest in a portfolio here, I have a choice to invest in a portfolio here maybe here or here all these portfolio will give me same risk, but mind it these portfolio are giving different return and as a same rational investor I would want to go for the highest return which means for a given level of risk an investor would want to have highest risk a possible return.

No investor would like to invest in a lower return portfolio if he can get higher return portfolio for the same level of risk and that is where the portfolio theory explains about the risk return relationship. It implies that given the level of return, given a particular level of return an investor would want to invest in the portfolio which provides highest return and subsequently if a portfolio investor has an expectation of return, we can determine what is the minimum level of risk that investor needs to carry.

With this assumption we can move further and combine all the scenarios together.

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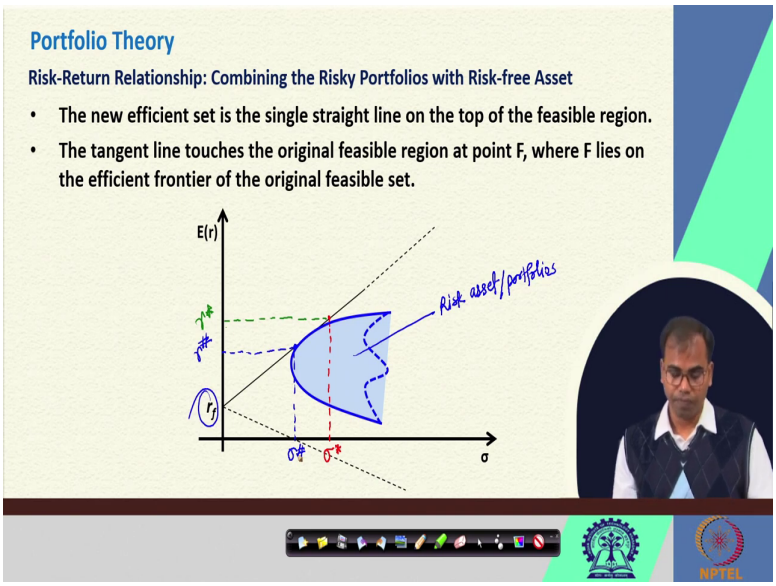

So, all the scenarios means, we can combine the situation where risk free assets are risk free asset is available and risk free asset is combined with risky portfolios and that will provide us a new efficient frontier, new efficient set of combination of assets available in the market and these assets comprise of both risky as well as risky assets and this way we get a new efficient frontier where the investor has a choice to invest money.

So, this particular graph this particular reason is where we have all the combination of assets basically risky asset or risky asset portfolios. And this is risk free asset and when we combine these two we get some sort of portfolio or the choices of portfolio where we have some exposure to risk free asset and some exposure to risky assets and depending on our expectation of risk and return we can decide where the investor would be better off in terms of the preference for risk and return.

For example, if an investor has a tendency to or expectation of carrying let us say this much return sigma as strict mark. So, we know that investor has this much of risk bearing capacity. So, the investor has a choice to invest anywhere along this red line, but going by the same rational investor, rational expectation assumptions we know that the investor has the possibility to earn highest return to this level.

Similarly, if we are given by an investor that I want to let us say some investor says I want to earn this much of return. So, we know that this is the point where the portfolio has to r hash this is the expected return of the investor. So, we can tell the investor that this is the minimum risk that the investor has to bear.

And this is the way the combination of risky portfolios and risk free asset help us in determining the combination of asset in certain proportion and this forms the basis of the portfolio theory and subsequently the capital asset pricing model.

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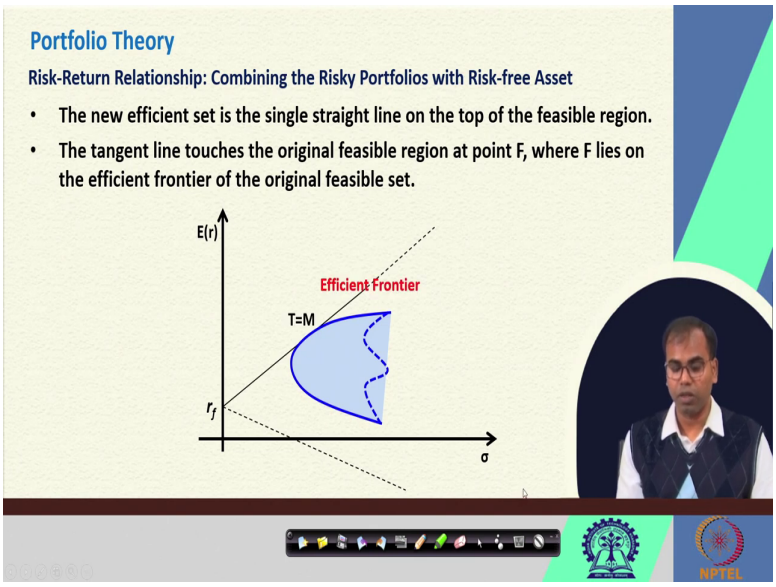

Let us take a look at how this helps us in understanding the capital asset pricing model.

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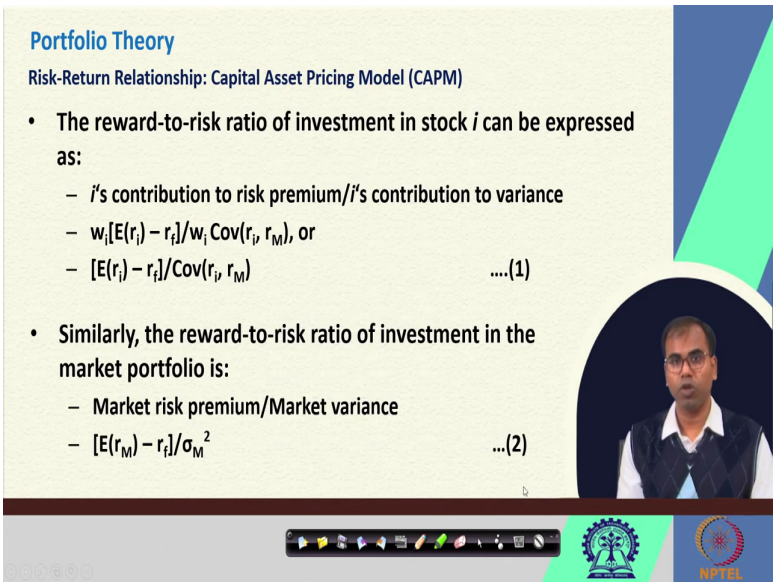

So, to understand this let us start with the simple assumption that if there are multiple assets available in the market then the marginal or incremental reward to risk ratio of investment in stock 1 is going to be same for all the assets all the stocks available in the market.

So, if you start with let us say any single asset let us say stock i. So, the marginal reward to risk ratio can be expressed as the expected rate of return from investment in asset i minus risk free rate that is over and above risk free rate how much return a particular asset i is generating with respect to the risk that it is carrying.

So, this is basically the reward to risk ratio. This particular expression indicates the reward to risk ratio for investment in stock i and if there are multiple assets in the market reward to risk ratio of investment in those assets or incremental reward to risk ratio of investment in those

assets will be given as same; because no investor can be able to arbitrarily over or over and above return than other investor in the market on an average.

Similarly, if there is a market portfolio then a market portfolio reward to risk ratio of investment will be given by this expression where market risk premium divided by market variance will be the expression for indicating the reward to risk ratio of investment in the market portfolio.

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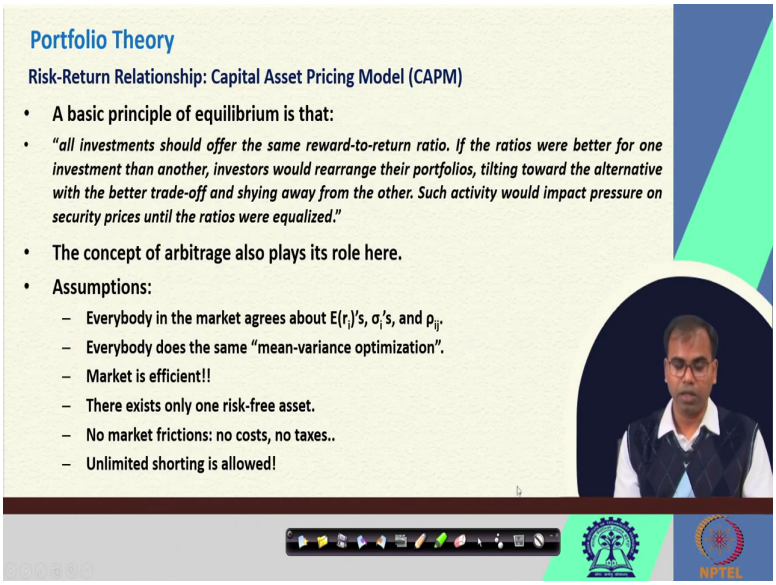

Now, if we try to see the assumption that whatever investment whatever investment an individual is carrying, let us say any other investor is carrying a different investment and that investment has higher risk than the first investor. Then whatever reward that second investor is getting should be equal for all investors available in the market with respect to all the assets with different combination of risk and return.

Because all investments should offer same reward to risk ratio if the ratios who are better for one investment than another investor would; obviously, rearrange the portfolio tilting towards the situation or the combination of assets with the better trade off or better payoff in terms of reward to risk ratio. And sowing away from those with worse reward to risk ratio and such activity would impact pressure on security prices until the ratios were equalized.

Because we know that there is no arbitrage assumption in the financial market where an investor cannot outperform others in general the basic assumption here is everybody in the market agrees about all the possible expected rate of return from individual assets. Sigma's basically the risk associated with assets and the correlation between the pair of assets everybody also is able to do the same mean variance optimization because that is how they get the portfolio choices or efficient frontier.

It is also assumed that markets are efficient and there exist only one risk free asset as we have seen in the previous case. For the sake of simplicity, we also assume that there is no market frictions which means, there is no transaction cost no taxes and at the same time unlimited short selling unlimited sorting is allowed. With these assumption if we simplify the argument of reward to risk ratio for individual assets being same as the reward to risk ratio for the market we can write the expression as following.

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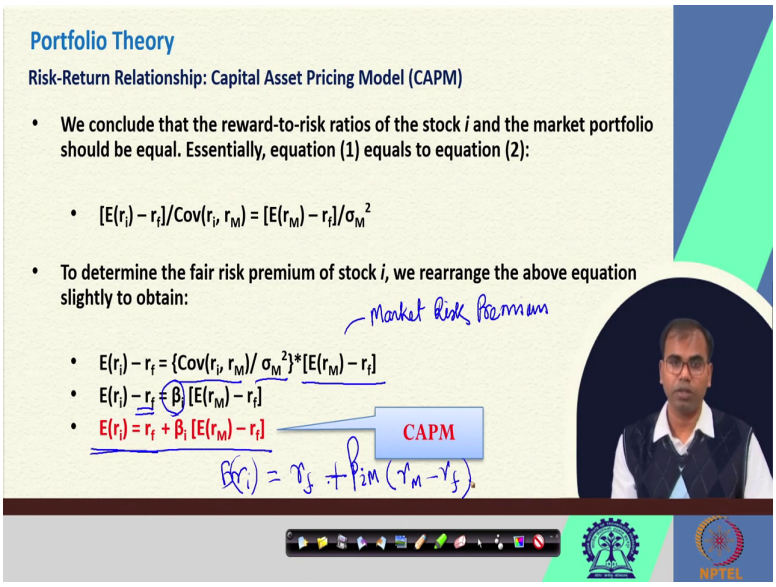

So, here we have reward to risk ratio for individual assets is equal to reward to risk ratio for the market and if we rearrange this expression, we can simply see here to determine the fair risk premium of stock i. We rearrange the above equation such that we bring all the factors which are related to individual asset on one side and then bring other factors on the other side.

So, this way we can express this simple expression into expected rate of return on asset i and over and above the risk free rate which is E r minus E r i minus r f which is expected rate of return on asset i minus risk free rate of return is equal to the covariance of asset with respect to the market. And variance of the market which is also denoted as the beta and the risk free rate of return the return on the market asset over and above the risk free rate of return also known as the market risk premium.

With this we rearrange this formula to arrive at this expression where we have expected rate of return on asset i is equal to r f that is risk free rate of return plus beta i m which is the covariance variance relationship of the asset and the market asset into r m minus r f. So, that is how we arrive at the capital asset pricing model and we use this capital asset pricing model to determine the expected rate of return on any asset given their riskiness.

Here the riskiness is with respect to the market asset and that is why we call it market model or single factor model.

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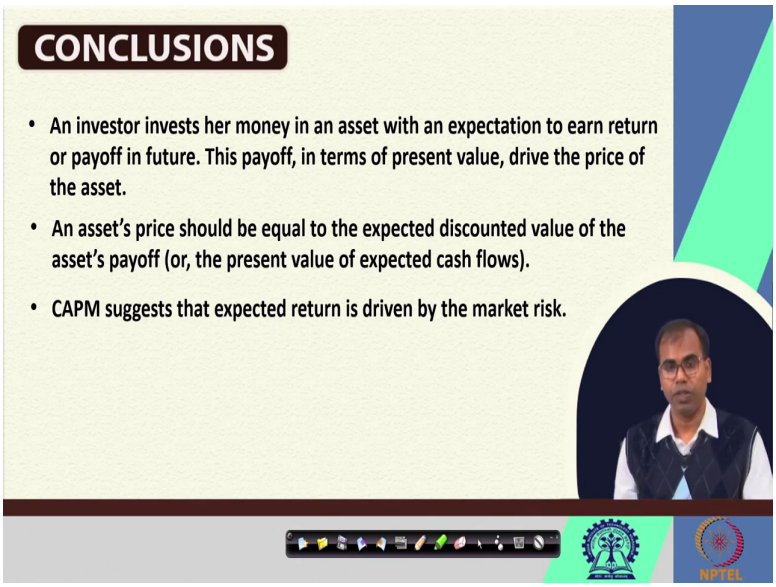

With this I think we can conclude this session and suggest that we have we have learned in this session and previously that investor would want to know what will be the payoff or the present value of the payoff that the investor is going to get in future by investing today.

Because investing today implies that there will be some sort of reduction in consumption today with the hope that the investor would get more to consume in future. And that is why we can say that an asset's price should be equal to the expected discounted value of asset's payoff or return that is the present value of expected cash flow coming in the future.

And these relationships is indicated in the form of capital asset pricing model which suggests that expected return is a factor driven is basically driven by market risk factor or the risk sensitivity of asset with respect to the market portfolio or market asset. With this I end this session.

Thank you very much.