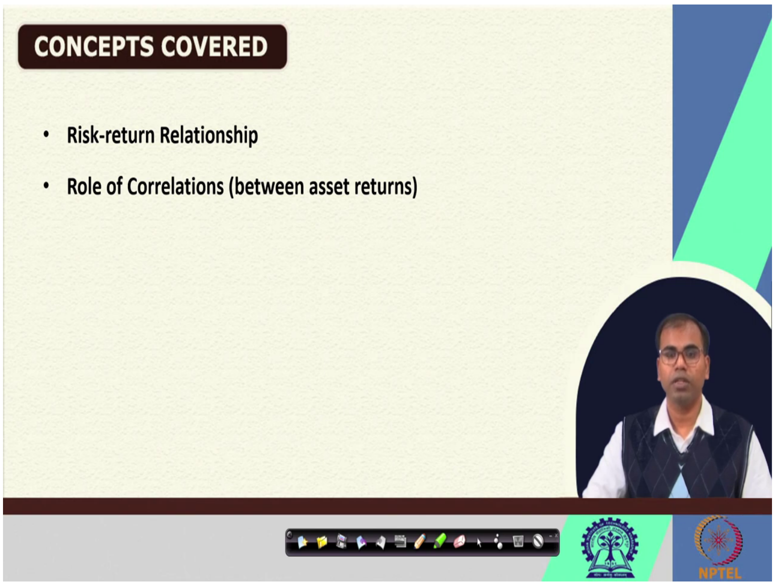
Investment Management
Prof. Abhijeet Chandra
Vinod Gupta School of Management
Indian Institute of Technology, Kharagpur

Lecture - 16
Portfolio Theory - Primer

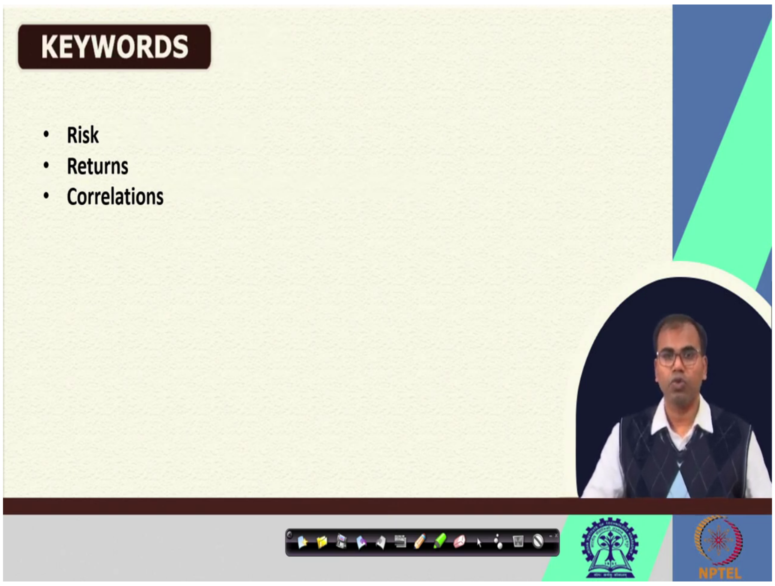
Hello there. Welcome back to the course Investment Management and we are learning about different financial assets, their valuation and how we can combine them for making investment portfolio. In this session, we will learn about Portfolio Theory.

(Refer Slide Time: 00:44)



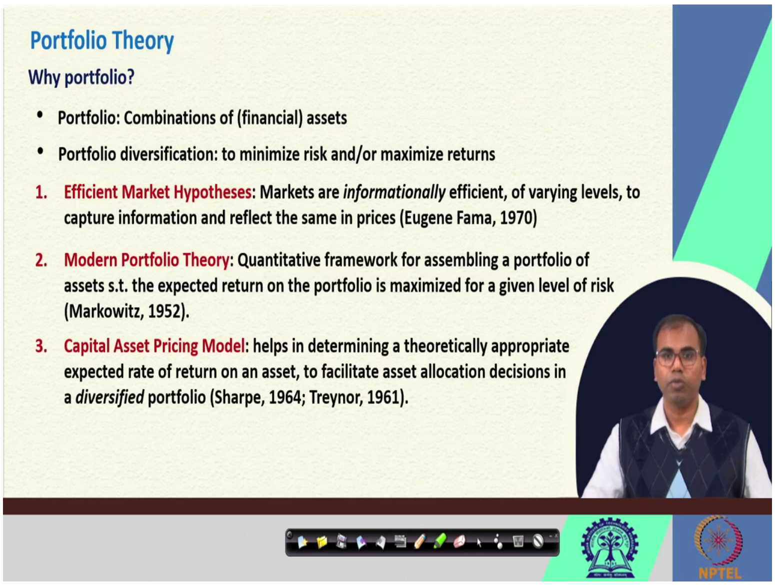
Basically, we will try to discuss about the prerequisite for understanding a portfolio or rather portfolio theory through understanding risk return relationship and how correlations between the returns on assets affect the relationship between risk and return.

(Refer Slide Time: 01:05)



So, the ideas that we are going to talk about today will form the basis of portfolio theory in larger context.

(Refer Slide Time: 01:11)



And portfolio management eventually; as we understand from our previous discussions, portfolio is basically a combination of financial assets in this case here, where we can have assets such as equity bonds or any other fixed income security and other financial assets.

As an investor, when we combine these financial assets together, we basically form a portfolio, which means if I have certain amount of rupees for investment in financial assets and I invest part of my funds in asset A and remaining part in asset B, basically I am holding a portfolio of asset A and asset B in certain proportion.

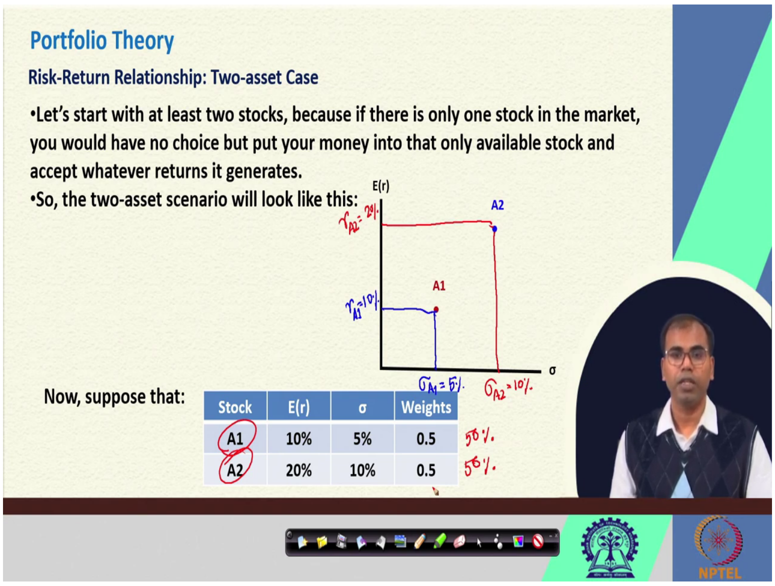
Earlier, we had discussed how we can calculate the return on individual financial assets, where we learn that return on financial asset is basically the change in prices and any intermittent income. For example, if I invest in a stock of a company on first day of the year for let us say 100 rupees and on the last day of the year, I sold it for 110 rupees, then the change in price is from 100 to 110, which means 10 rupee as return.

But if the company has given any dividend in the meantime during the year and let us assume that the dividend income is 5 rupees. So, my total return will be 10 rupees of price change and 5 rupees of dividend income. And thus, my total return 15 rupees on an investment of 100 rupees will be translated into a 15 percent of return on my investment.

Similarly, we also learnt about calculating return and risk for a portfolio, where return of a portfolio is weighted average rate of return on individual assets in the portfolio and risk in the portfolio is basically the summation of individual risk with respect to their weights and the correlation.

We know that there are three major theories namely efficient market hypothesis, modern portfolio theory and capital asset pricing model that basically play behind the scene when we talk about portfolio risk and return and subsequent applications.

(Refer Slide Time: 04:05)



Let us begin with the simple idea of a portfolio of two assets. Suppose as an investor, I invest in two assets, the case that we have seen earlier where an investor is investing in asset A1 and asset A2. If we start with two asset case, let us begin with the argument that these two assets have different return and corresponding risk.

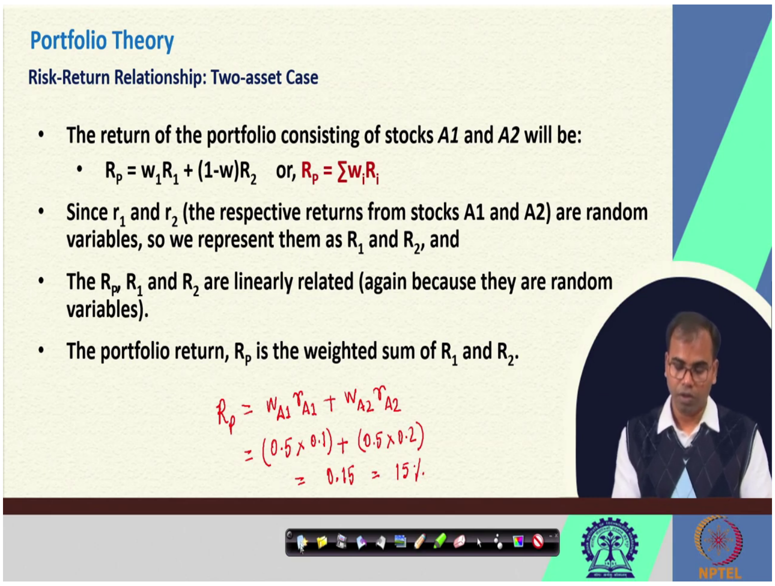
For example, if the two assets are placed or visualized on a risk return graph, it will look like this where we have risk and return shown as following, where we see that there are two assets A1 and A2 and their corresponding risk and return and also the weight in which or the proportion in which the investor has invested in these two assets.

So, if we look at this particular graph, we know that this is an asset A1 where the return and risk is here. So, r A1 is 10 percent and sigma A1 is 5 percent and we have another asset that we can see here as r A2 as 20 percent and sigma A2 as 10 percent. And if an investor is investing 50 percent of the total investable fund into A1 and remaining 50 percent of the total investable fund in A2, then the portfolio should be consisting of these two assets A1 and A2 in certain way.

Now, when we talk about visualizing a portfolio, we have to understand how these two assets are related. When it comes to investing in a single asset, the relationship of that single asset with other assets might not be important, but when we combine two or more assets together, then the relationship of the assets in the portfolio becomes critical. And that is why when we try to calculate the risk of an individual portfolio comprising of two or more assets.

We discussed earlier that the correlation of returns on different assets in the portfolio will be an important factor that drives the risk of the portfolio. Given this case of two portfolio where two asset portfolio where asset A1 has 10 percent of return and 5 percent of risk and asset A2 has 20 percent of return and 10 percent of risk. And the corresponding weights are the proportion of money that are invested in these two assets A1 and A2 are 50 percent each.

(Refer Slide Time: 07:31)

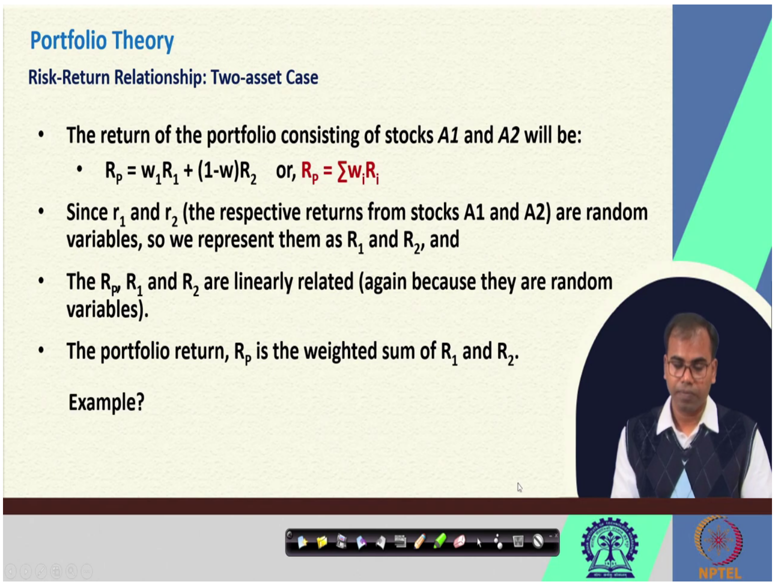


Now, if we try to understand the return calculation as we discussed earlier, the return on the portfolio will be the weighted average rate of return of individual assets because these returns are random numbers and when we combine them, they are linearly related. So, return on a portfolio is basically linearly dependent on return on individual assets and the weight in which the asset has been invested in those the fund has been invested in those assets in the portfolio.

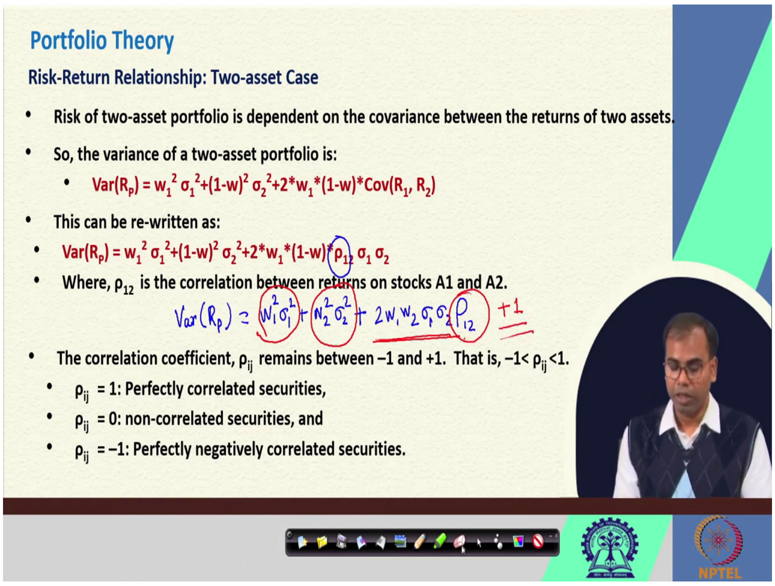
Now, if you try to calculate the return as highlighted earlier, return on the portfolio of these two asset will be weight into asset A1 and return on asset A1 plus weight in asset A2 and into return an asset A2. So, as we have seen it is 0.5 into return was 10 percent and 0.5 that is 50 percent of the investible fund goes into asset A2 into 20 percent of return.

So, this gives us 0.15 which is basically translating into 15 percent of return on this portfolio of asset A1 and A2 in where investor has invested 50 percent of the money in A1 and 50 percent of the money in A2.

(Refer Slide Time: 09:18)



(Refer Slide Time: 09:20)



However, when it comes to calculation of risk, we know that the risk calculation is not so straightforward where the risk of individual asset combined with risk of another asset in the portfolio will drive the total risk of the portfolio. Such that the correlation between two asset returns will become more critical and that is how this risk of the portfolio will be driven by the interrelationship of the assets or what we know as covariance between the returns of two asset as discussed earlier.

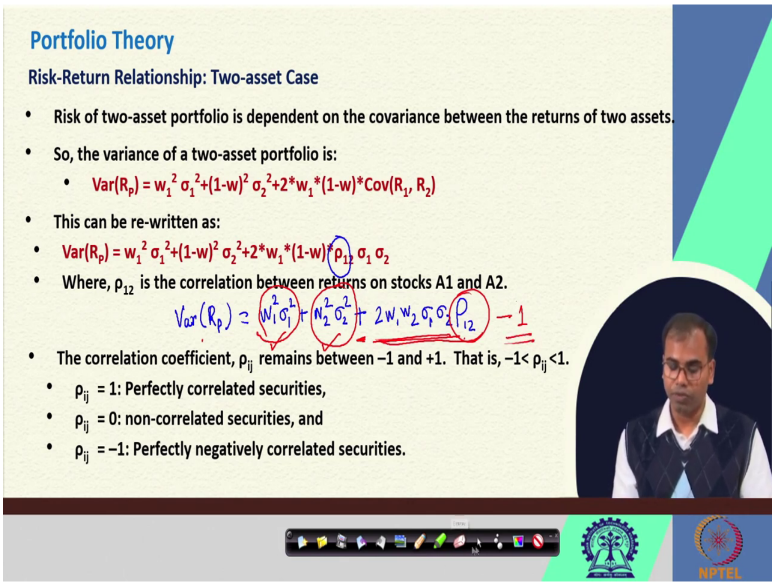
This part will become more important and as we understand the correlation between two series here the two asset returns may range from minus 1 to plus 1 and if the correlation between asset A1 and asset A2 returns is plus 1, we can say that these are perfectly positively correlated financial securities. And if the correlation coefficient between asset A1 and A2 is minus 1 then we will claim that these two assets are perfectly negatively correlated securities.

And if the correlation coefficient is 0 then we can say that these two securities are not correlated at all. Most of the time in real world you will find that financial securities or assets are correlated to certain extent there is hardly any pair of asset that might not be correlated where you will find zero correlation. Most of the time the correlation between two or more assets is always going to be either positively correlated or negatively correlated.

So, let us try to understand how this correlation affects the risk of the portfolio because earlier we have seen that the correlation of individual correlation between individual asset is going to drive the total risk of the portfolio. As we understand if we rewrite variance of portfolio. So, if we have weight of asset A square risk of asset A1 square plus weight of asset 2 square risk of asset 2 square plus 2 into weight 1, weight 2 risk of asset A1 and risk of asset A2 and correlation.

Now, if this correlation coefficient is perfect perfectly positively correlated, we can see mathematically that the total risk of the portfolio is the risk coming from asset 1 plus the risk coming from asset 2 and the risk that is coming from the interaction of the two asset or the correlation driven by the correlation positive correlation of the two asset.

(Refer Slide Time: 12:47)

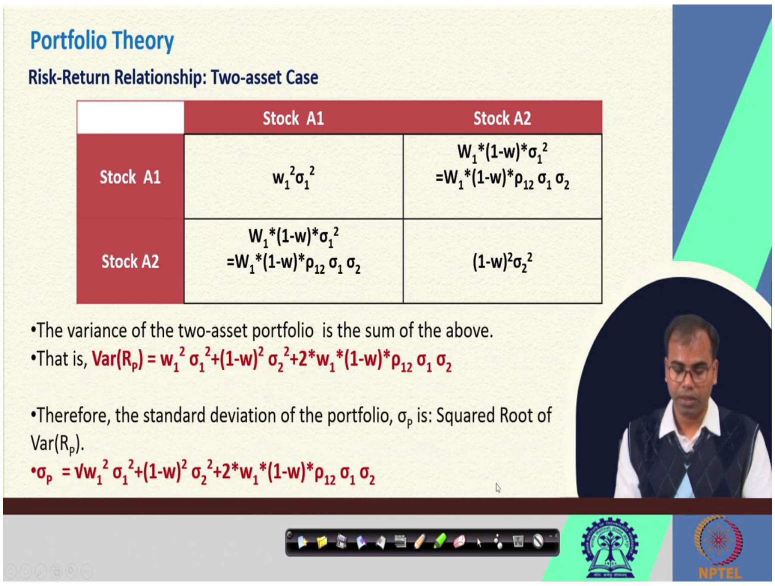


But if the case is opposite where the correlation is minus 1 then we know that the total risk of the portfolio will be the sum of risk coming from first asset, risk come plus risk coming from second asset minus the risk driven by the correlation of the two asset. So, in this way we can reduce the total variance or total risk of the portfolio if we have negatively correlated assets in the portfolio.

So, let us try to understand it in a visual form where we can see three scenarios with perfectly positively correlated assets, perfectly negatively correlated assets and assets where the correlation ranges between plus 1 and minus 1. As I said earlier there will hardly be any financial securities that are not correlated at all, which means there will be no financial assets with zero correlation.

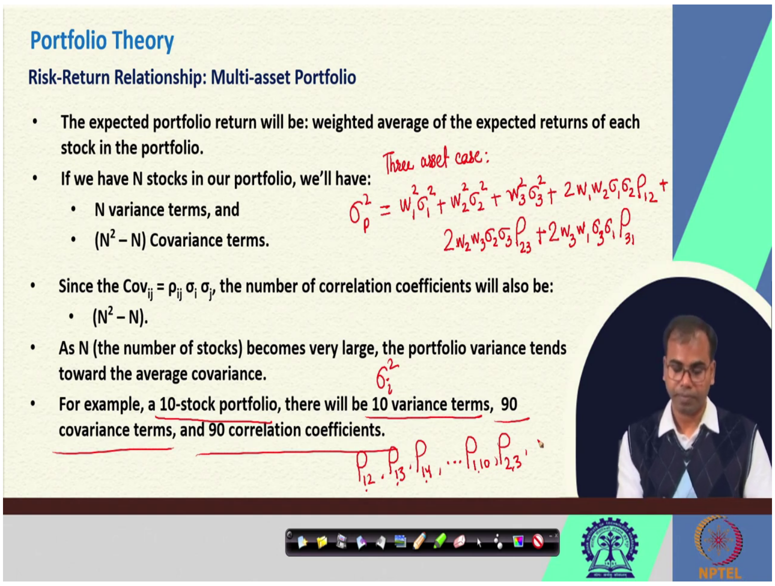
So, we are assuming three cases one with perfectly positive correlation that is plus 1 and another is correlation of minus 1 that is perfectly negatively correlated assets and any pair of assets with correlation ranging between plus 1 and minus 1. So, let us say on an average 0.5 or so.

(Refer Slide Time: 14:08)



So, the first situation that we are going to discuss is the perfectly positively correlated assets.

(Refer Slide Time: 14:12)



Before that if we extrapolate our argument of two asset portfolio to more than two asset portfolio where multi asset portfolio we should realize that the expected portfolio return will be weighted average value of expected return of each of the individual assets in the portfolio. But when we talk about risk as we discussed earlier as well where the risk of the portfolio variance of portfolio will be.

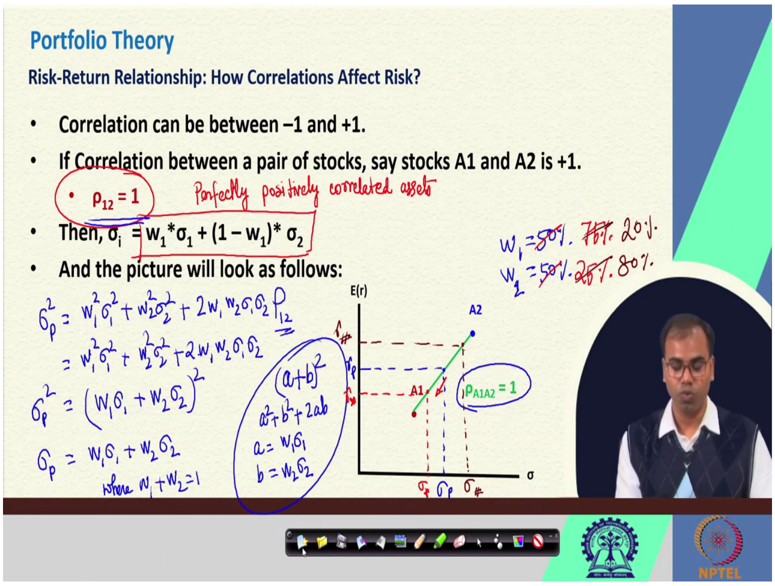
Let us say three asset case, weight of asset 1, sigma of asset 1, weight of asset 2, sigma of asset 2, weight of asset 3, sigma of asset 3 plus 2 into weight of asset 1, weight of asset 2, sigma of asset 1, sigma of asset 2 and correlation of 12, correlation between asset 1 and asset 2 plus 2 into weight 2, weight 3, risk of asset 2, risk of asset 3 and correlation of asset 2 and 3.

And similarly weight 3, weight 1, sigma 3, sigma 1 and correlation between 3 and 1. So, we see here that when it was two asset case we had one correlation coefficient between asset 1 and 2, when we have three asset scenario we have three correlation coefficient. And similarly, as we increase the number of assets in the market we will have correlation coefficients increasing accordingly and that will drive the total risk of the portfolio as well.

For example, if we have 10 stock portfolio there will be 10 variance terms right every for every unique stock there will be sigma or variance terms, 90 covariance terms and 90 correlation coefficient because from ranging from correlation coefficient between 12 correlation coefficient between 13, correlation coefficient between 14, tail correlation coefficient between 1, 10.

And then again, we will start with correlation coefficient between 2 and 3, and 2 and 4, 2 and 5 and so on. So, we will have 90 correlation coefficient and that is why when we handle multi-asset portfolio or a portfolio of large number of assets, the risk proportionately increases because of the presence of covariance and correlation coefficient between the pair of pairs of assets.

(Refer Slide Time: 17:34)



So, let us start with the first scenario where we have correlation coefficient as plus 1 which means if we have 2 assets, asset A1 and A2 and these 2 assets are perfectly positively correlated. Let us take 30 seconds to understand what it means in real terms. So, if we have 2 assets A1 and A2 and they are perfectly positively correlated, the row coefficient is going to be plus 1.

What it means is if there are 2 assets A1 and A2 and the price of asset A1 moves up by 1 unit and it is perfectly positively correlated with asset A2, then price of asset A2 will also move up by 1 unit. That is what it essentially means when we talk about perfectly positively correlated stock.

If the correlation coefficient is less than 1, then we can say that if price of asset A1 increases by 1 unit and the correlation coefficient between A1 and A2 is 0.5, then we can say that the price change in asset A2 will be up to the extent of 0.5. So, here the scenario is the correlation coefficient between asset A1 and asset A2 is plus 1.

So, when we translate this into the risk total risk of the portfolio, then we can say that the total risk is going to be indicated by this expression. Now, if we just take a minute to understand, we know that the portfolio variance is given by weight 1 square sigma 1 square plus weight 2 square sigma 2 square plus 2 into weight 1, weight 2 sigma 1, sigma 2 and correlation coefficient between 1 and 2.

And assume here that correlation coefficient between 1 and 2 is 1. So, effectively it will be weight 1 square sigma 1 square, weight 2 square sigma 2 square plus 2 into weight 1, weight 2 sigma 1 sigma 2 because the correlation coefficient is plus 1. We know that this reminds us of basic algebraic expression where we can say that a, if we refer to this basic algebraic expression as a plus b to the power whole square is a square plus b square plus 2ab.

And if a is indicated by weight a sigma a and b is indicated by weight 2 sigma 2, then this essentially translate into what we can say is weight 1 sigma 1 plus weight 2 sigma 2 square. So, standard deviation of the portfolio will be weight 1 sigma 1 plus weight 2 sigma 2 where weight 1 and weight 2 should be equal to 1. So, we can say when we have the correlation coefficient between two assets to be perfectly positive which is plus 1, then the risk of the portfolio will be linear, right. Risk of individual assets combined together.

So, we can see if we on a two dimensional graph where we have asset A1 and asset 2 indicated like indicated here. So, we can visualize the portfolio the portfolio to be standing on this curve indicated in green. So, we can see that we can see here that if there are two assets A1 and A2 and the correlation coefficient between the two assets is plus 1, then my portfolio will be moving across this green line.

Which means if we have weight to be 50 percent each in asset A1 and A2, then my portfolio will be lying somewhere in the middle of this line, which means my return will be here and risk will be here. So, my portfolio will be lying exactly in the middle of this green line because 50 percent of my asset is invested in A and 50 percent of my asset is invested in B. So, the return on the portfolio will be determined on and so, will be the risk.

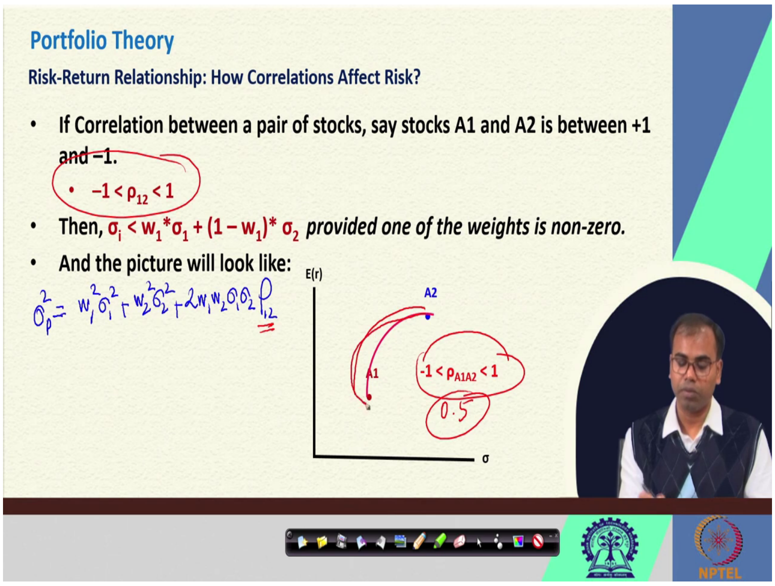
If we assume that the weight is changed, which means an investor invests more of the money in asset A. So, let us say 75 percent of the money goes into asset A and only 25 percent of the money goes into B. If that happens, then my portfolio will be moving closer to A because more of my money is invested in A and less of my money is invested in B. So, my portfolio will be probably lying somewhere here, where this will be the value of risk and this will be the value of return.

And if it is the case of otherwise, where we have let us say a portfolio where more money is invested in B, asset A2 and less money is invested in asset A1, then let us say 20 percent in asset A1 and 80 percent in asset A2, then my portfolio will be lying somewhere closer to B A2, where the risk will be determined here and risk return will be given accordingly, alright.

So, this is the way we visualize a two asset portfolio on a risk-return plane. Given that we have the correlation coefficient between asset A1 and A2 to be plus 1. Now, once we understand this argument of correlation coefficient being a factor that drives the risk of the portfolio, we can just change the correlation coefficient from plus 1 to minus 1 or anything else and try to see how this curve indicated by green line here will look like.

So, let us take a look and try to understand what it means for the risk of the portfolio if the correlation coefficient changes from plus 1 to any other value.

(Refer Slide Time: 25:30)

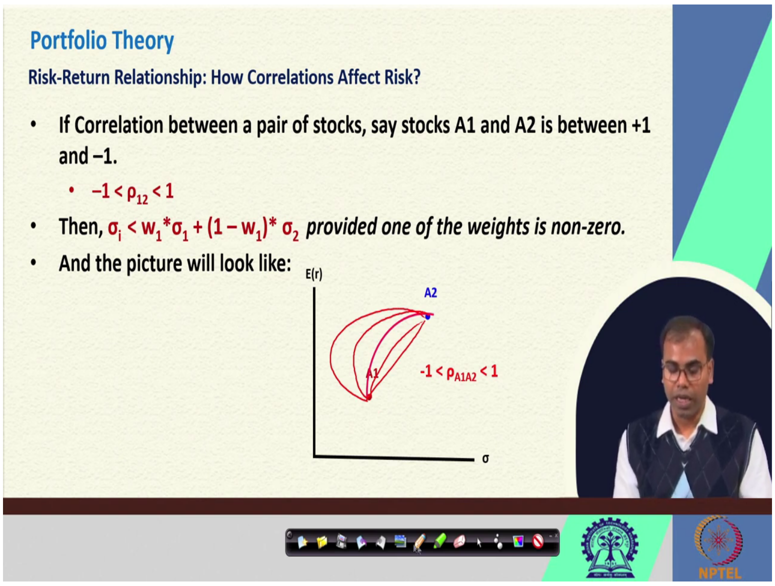


So, here is the scenario, where we have a correlation coefficient between the two asset A1 and A2 as any where between A plus 1 and minus 1. Now, if it is between plus 1 and minus 1 and we go by the same argument, where we had the formula for portfolio risk to be given as sigma of portfolio as or variance of portfolio given as weight 1, sigma 1, weight 2, sigma 2 and 2 into weight 1, weight 2, sigma 1, sigma 2 and correlation coefficient between asset 1 and 2.

Now, if this correlation coefficient is anywhere between plus 1 and minus 1, we know that the curve that will be coming out to be in this case will be indicated as following. So, this will be the curve looking like. So, here the basic assumption is the shape of the curve will be moving according to the value of this correlation coefficient ranging between minus 1 and plus 1.

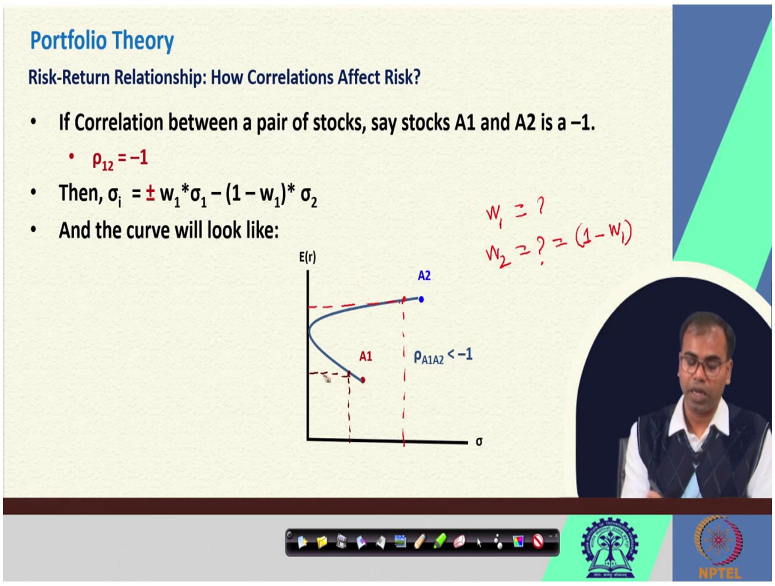
So, let us assume that the correlation coefficient here is 0.5 which is between minus 1 and plus 1. So, in this case the curve might look like this. As we change the value of this correlation coefficient, the shape of the curve might change.

(Refer Slide Time: 27:23)



It can be more it can be more steep, it can move according to different according to the value of correlation coefficient between return on asset A1 and A2 and subsequently the shape of the curve will also change. Because eventually the sigma of the portfolio will be less than the sum of the total risk arising from investing in asset A1 and asset A2.

(Refer Slide Time: 27:56)

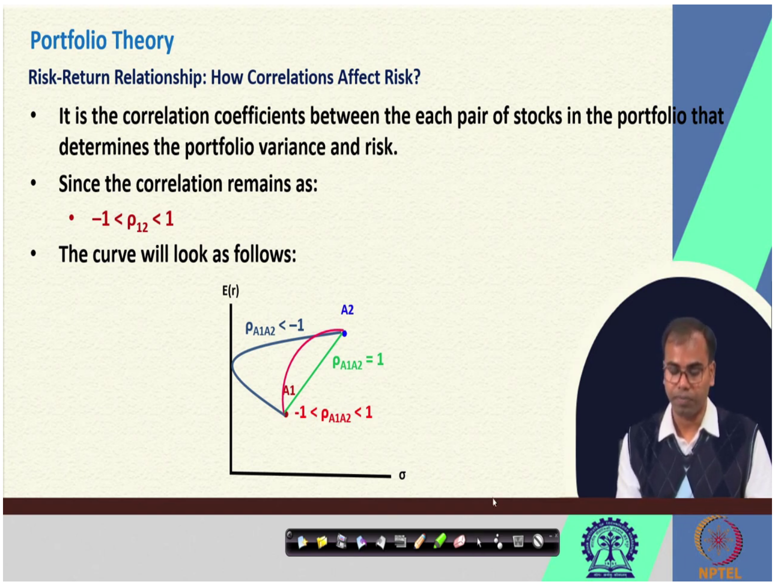


Now, if we extrapolate this argument for a scenario where we have perfectly negatively correlated assets assume that we have perfectly negatively correlated assets which is asset A1 and A2 return on asset A1 and A2 are negatively correlated. So, the value of correlation is minus 1.

And the shape of the curve should look like this where it will not it might touch the axis, but it will not cross the axis and that that is how the value of the correlation is affecting the shape of the portfolio curve. And just highlighting the example that we were discussing earlier depending on how much money is invested in A1 and A2 which is basically the value of w 1 and w 2 will be determining the shape of the position of the portfolio on this curve.

So, if we have more investment in asset A2, we can be more closer to A2, the portfolio will be lying more closer to A2 and accordingly the risk will be determined. And if we have a scenario where we have more investment in A2, A1 then my portfolio will be lying somewhere closer to A1 and accordingly the risk and return will be determined.

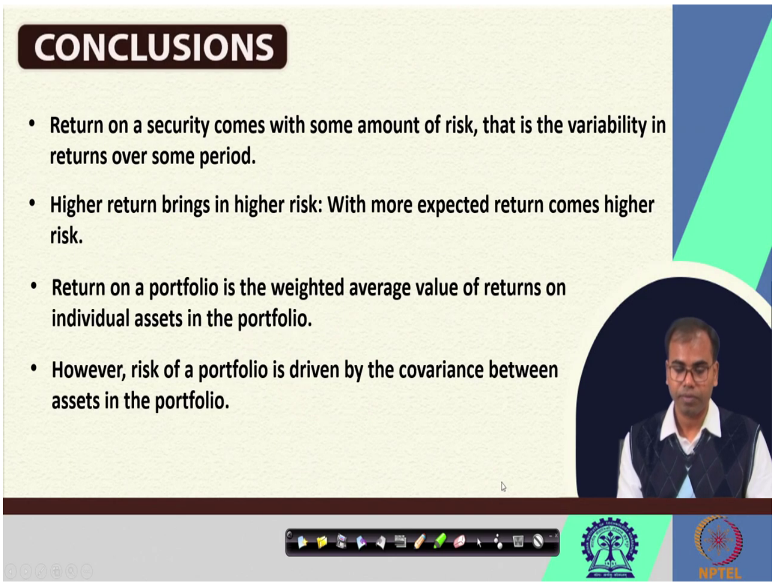
(Refer Slide Time: 29:27)



With this argument if we try to see the 3 scenarios of correlation between minus 1 plus 1 and anywhere between minus 1 and plus 1, then the 3 scenarios will look something like this where we have correlation coefficient ranging from minus 1 to plus 1 and anywhere in between.

And if we have such a scenario or such a pair of stocks or such a pair of assets, we can see where our portfolio would be lying depending on how much money is invested in A asset A1 and how much money of the remaining money is invested in asset A2. However, this is the case of two asset portfolio where two assets are A1 and A2 and we know the correlation between them if we have more than two assets the scenario will definitely change accordingly.

(Refer Slide Time: 30:23)



So, to conclude this session I would again reiterate that return on a security, return on a financial assets can be calculated in terms of the price change as in and any interim income. However, when we talk about return on a portfolio of assets, it is basically the weighted average sum of individual assets which means the return on individual assets and the proportion of money that is invested in the asset in the portfolio will be determining the total return on the portfolio.

In on the other side when we talk about risk of individual assets denoted by standard deviation in general will be combined not only in the proportion of the money that is invested, but also the correlation between them correlation between the return on assets and that is how the return on risk of portfolio is driven by individual risk as well as the interrelationship between the assets. With this I end up this session.

Thank you very much.