Applied Econometrics
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Lecture – 95
Stochastic Regressor

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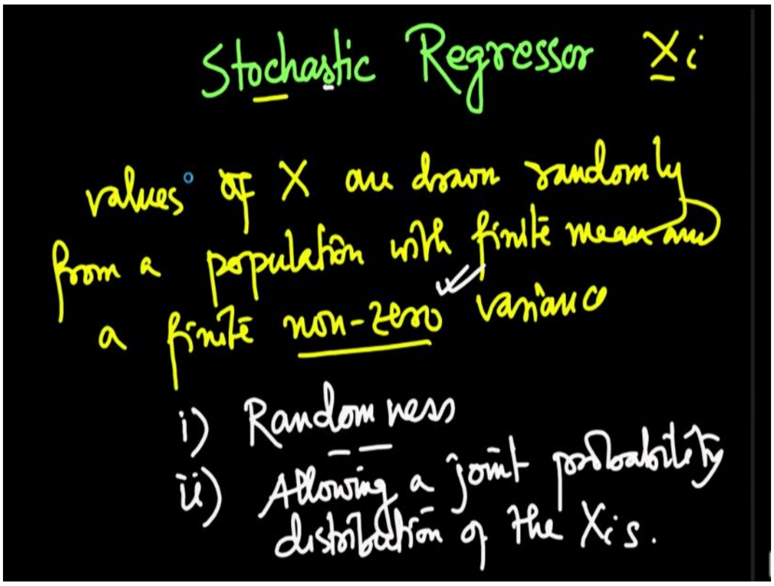

Hello and welcome back to the lecture on applied econometrics. We have been talking about types of regressor. In the previous lecture, we spoke about non stochastic regressor and we said that that is somewhat unreal. Now, in this lecture we are going to talk about stochastic regressor which is more close to reality. But there is a cost because the moment I talk about reality it is going to be more complex.

And we are going to see the problem or the kind of challenges that we might face when you actually take a stochastic regressor instead of a non stochastic regressor. So, to begin with what is the stochastic regressor? Stochastic regressor is a regressor that the values of the regressor is actually values of X are drawn randomly from a population with finite mean and a finite non-zero variance. Now, what do I mean by this, particularly this non-zero variance?

So, as I said the moment I say the regressor is stochastic that is my all regressors X i's that are stochastic, so I essentially allow a sort of randomness into the values of the X. So, I no longer actually determine the values of the X and assign it to the X variables when I run the regressor. Now, here since I am allowing some amount of randomness if I have a non-zero variance, so if I have a zero variance so that would mean there is no variation in the data.

And that would mean that the component of randomness will go away. So, essentially it has to be a non-zero variance so that is the first thing that we need to remember. Now, when I say that they are stochastic, they are random, so then I will actually allow the regressors to actually vary together. So, there could be a joint probability distribution of the regressors which were actually absent in the previous case.

In the previous case, we assumed all X i's are independent to each other and here we are allowing a joint probability distribution of the X i's. Now, that is a really relaxation, but at what cost? So, I will talk about that.

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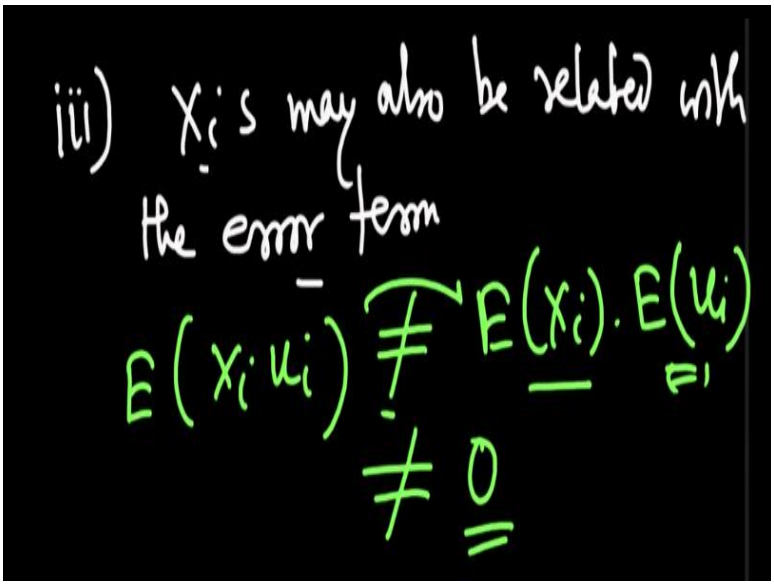

So, the moment actually relax this, what is going to happen is that you can no longer ignore the possibility that the X i's are also or rather may also be related with the error term and that is the problem. So, essentially in the previous case we were having when we talked about the unbiasness of the regression estimator, we were assuming that X i's are required to be independent of the error term.

But the moment I allow the joint probability distributions among the X i variables, I also create the possibility that X i's may be related to the error term. And the moment I allow the X i and the error term to covary, so at that point the bias the condition of base linear unbiased estimator might actually get violated and how is that going to happen let me actually show you. So, essentially what I am saying here is that I cannot say expectation of let us say some X i u i, I cannot say it is equal to expectation of X i and expectation of u i.

I cannot say this or in other words I cannot say this equal to 0 because if it has to be 0, then the expectation of u i has to be 0. And even if the expectation of u i is 0, if this condition does not hold I can never get this value, right. Now as I claim that because of this, because they are not independent, because X i and error term are covarying the bias is going to exist and how the bias is going to exist.

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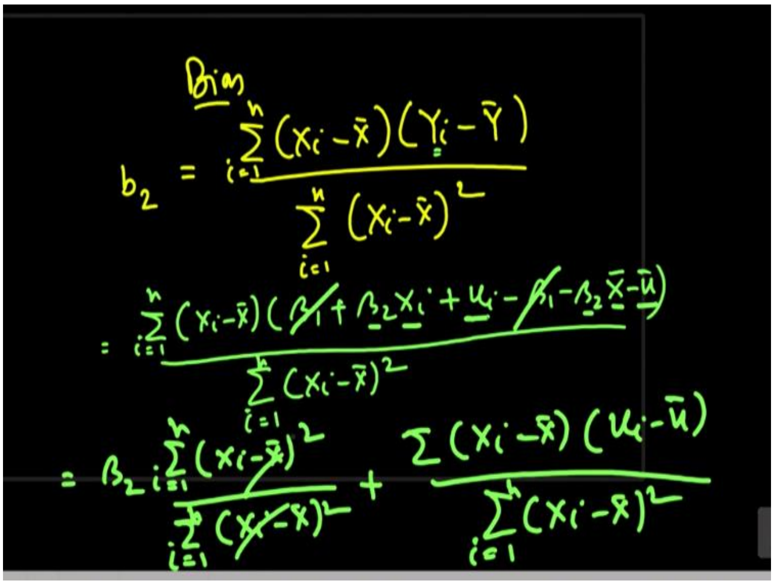

So, let me actually derive the bias term. How the bias is going to come to play? Now, we know by definition that b 2 which is the estimated regression coefficient is equal to summation X i – X bar into Y i – Y bar divided by summation of X i – X bar where i is equal to let us say 1 to n. And this of course also i equal to 1 to n. Now, if I let us say substitute the values of Y i, what I will get is and we have actually done previously, i = 1 to n X i – X bar.

And here I will have let us say beta 1 + beta 2 X 2 or beta 2 X i + u i – beta 1 – beta 2 X bar – u bar. This is something I have done previously and it is of course will be divided by the whole term i = 1 to n X i – X bar whole square; i = 1 to n. Now, we know what is going to happen. So beta 1, beta 1 is cancelled. I will have a beta 2 here, I will have a 2 two here. So, what I am going to do?

I am going to take let us say beta 2 out and then I will have summation i = 1 to n X i – X bar and this also going to be X i – X bar. So X i – X bar whole squared by summation i = 1 to n

X i – X bar whole square. On the other hand, I will have all this u i and u bar present, so it is going to give me summation X i – X bar u i – u bar and that is divided by X i – X bar whole square, i = 1 to n. Now, the first term is going to be 0, this numerator denominator will cancel out. So, I will have beta.

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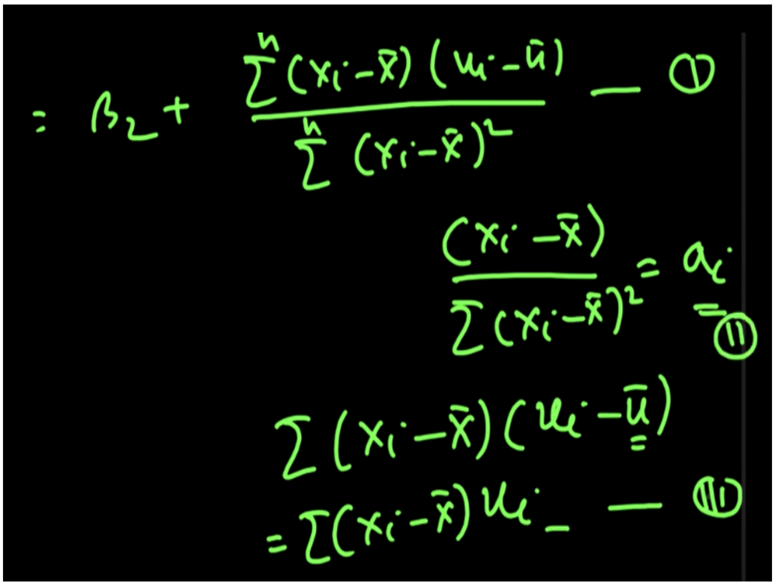

So, essentially I am going to have beta 2 + summation of X i – X bar into u i – u bar divided by summation of X i – X bar whole square over all n. Now, I have actually shown previously that this value could be represented as a i. And if I take X i – X bar u i – u bar over all n so then I will have something like I can actually prove this summation X i – X bar is equal to u i because u bar is actually constant.

So, basically if you multiply, so then essentially you will have u bar to be you can actually take basically sum of all the errors and that will give you a 0. So, essentially you can represent the numerator as this and let me actually write down the equation number 1, 2 and using 1 and 2 I can actually say; let us say this is 2 and this is 3.

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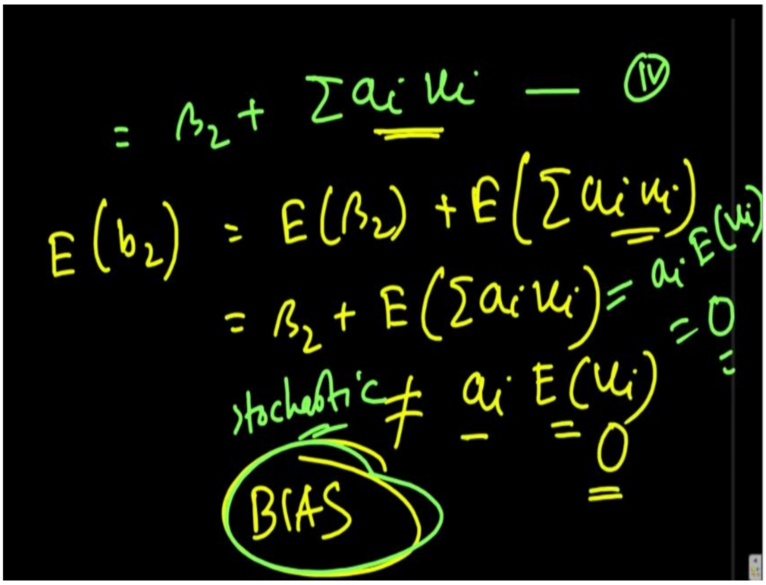

So, using 1, 2, 3 I can actually say this is equal to beta 2 + summation a i u i. This is something that I wanted to have. Now, let us say this is equation 4. Now, from my equation 4, if now go back to the previous condition that we have seen if the X i's are related to the error term, so if X i’s are related to the error term what will happen, let me use a different colour, if they are related.

So, if I take an expectation of beta b 2 here will be expectation of beta 2 + expectation of summation of a i u i. So, this is going to give you a beta 2, but this one is not going to give you a value of 0. Because a i and u i they are correlated, so they are covariant. So as long as they are covariant you cannot write this is equal to let us say a i into expectation of u i or something like that, you cannot write that.

So, if you cannot write it, if you could have written that so you could have gotten the value expectation of u i = 0, but you cannot do that now. And because of that you will have a bias term which is actually a problem. And if you have a non-stochastic regressor, so then you do not have any problem. So, this is for the stochastic regressor. But if it was non stochastic, you could have written this is equal to a i into summation into expectation of u i and that would have been 0.

And then your bias term would have gone from the basically estimator. So, essentially that is a problem. So, the moment you actually have this X i related correlate covariant with the error term, you actually have a bias term and the moment you have a bias term your estimator is no longer going to be a blue estimator, you want a blue estimate. So, it is not going to be a best anymore.

So, that is a problem and we sort of need to actually take care of that and that is why whenever we actually deal with stochastic regressor, we make another assumption that is in addition to the other assumptions that we make for estimating regression.

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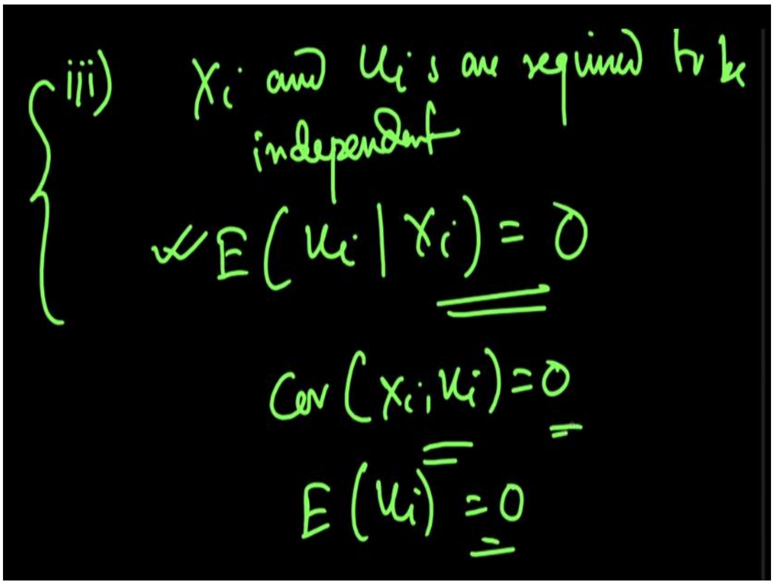

So the assumption that we are going to make is that you need to have X i and u i's are required to be independent. And you write it expected value of u i given all X i = 0, so mean value of the error term given all X i = 0, so that is what you assume. So, if your assumption is correct, if you can actually make this assumption, then your stochastic regressor will behave in a way your non stochastic regressor actually behaves.

And that is why whenever you are dealing with stochastic regressor, you need to make this extra assumption that we have just derived. And when you do that what you will have is you will actually have covariance of X i, u i = 0 or also you can say expectation of u i = 0 so which are basically you need to have for your regression equation. So, with this, we will end the lecture on stochastic regressor.

And in the next lecture, we are actually going to talk about the assumptions that we have been dealing with from for all the past lectures in the second module. So, all of those exercises that you have done will culminate to some assumptions and we will talk about all assumptions in one go. So, we will deal with the stochastic regressor and non-stochastic regressor and their corresponding assumptions in the next couple of lectures. Thank you.