Applied Econometrics

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Lecture – 77
Autocorrelation (Contd.)

Hello and welcome back to the lecture on Applied Econometrics.

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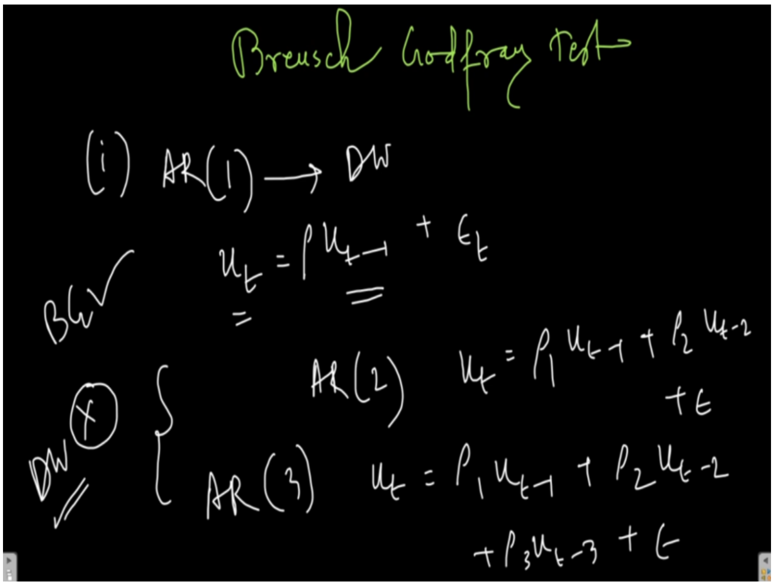

So, in this lecture we are going to talk about autocorrelation and in the previous lectures we introduced Durbin Watson Statistic as a test of autocorrelation. Now, in this lecture we are going to talk about another test that is Breusch Godfrey Test which we use to sort of get rid of certain limitations that we have seen in case of Durbin Watson Statistic. So, if you remember we actually outlined some of the limitations that you can see in Durbin Watson Statistic.

And one of the most important limitation for Durbin Watson Statistic is that it is only applicable for AR 1 processor. What we mean by AR1 process is that the error term of t th period so, we write the equation is like that u t = rho u t – 1 E t. So, the error term of the t th period u t is only related to the error term of the previous period t – 1. But it is not related to any other previous error terms. So, which is basically that would be indicated by higher order AR processes.

So, let us say AR 2 process would be represented, if I write down the error term. So, it will look like u t is equal to let us say rho 1 u t – 1 + rho 2 u t – 2 + the innovation term. And similarly, if I keep on writing here 3 process in the same manner I will write u t = rho 1 u t – 1 rho 2 u t – 2 rho 3 u t – 3 and the innovation term. So, this is how we will write but these are the processes for which Durbin Watson Statistic cannot be used and that is a problem.

So, for this kind of situations when we have higher order AR processes we can actually use the B G test, Breusch Godfrey Test. So, in this cases Breusch Godfrey Test is applicable.

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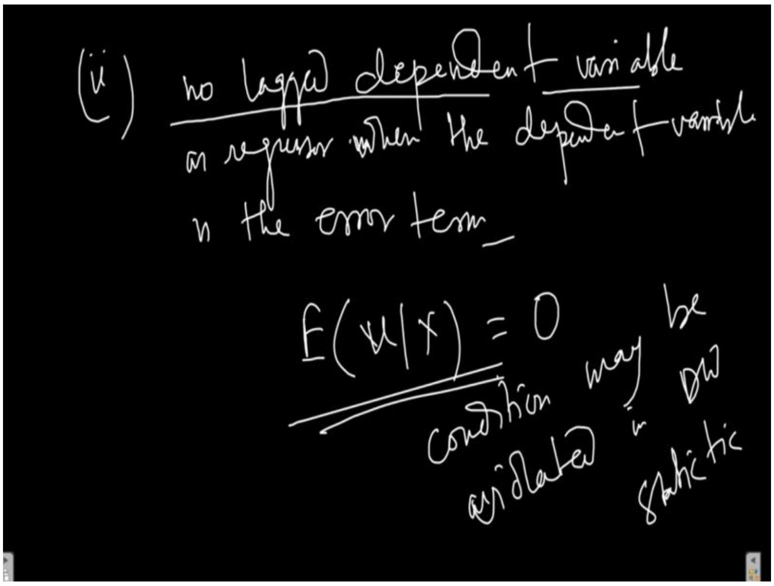

The second limitation that we have seen in case of Durbin Watson Statistic is that we cannot introduce the lag dependent variable, no lag dependent variable, lag dependent variable as regressor when you are regressing when the dependent variable is the error term. So, we have seen previously that we actually identify a regression equation and from where we actually do the prediction of the value of rho 1 or rho in this case for Durbin Watson Statistic.

So, in that equation we cannot have the lag dependent variable, no lag dependent variable as regressor when the dependent variable variable is the error term. I just explain it is the error term. Now, there is a problem with this. The problem is, if I cannot introduce the lag dependent variable or any of the independent variables of the original equation into the equation of the error term.

So, what I will have is that I may actually end up violating the zero conditional mean of error term. So, zero conditional mean of error term we have seen sorry this is written the error term. This conditional mean of error term is written as this. So, this condition may get violated. And I will explain how this is happening and why by the inclusion of lag dependent variable this condition actually you know the condition the basically the violation may be avoided.

So, in this condition may be violated and this is one of the condition for Gauss Markov Assumptions. So, we need to sort of maintain we sort of uphold this condition when we are doing the regression of the error term. May be avoided condition will be violated. If violated in D W statistic and that happens because we cannot introduce a lag dependent variable in the regression equation. Now, let me explain what I am trying to say here.

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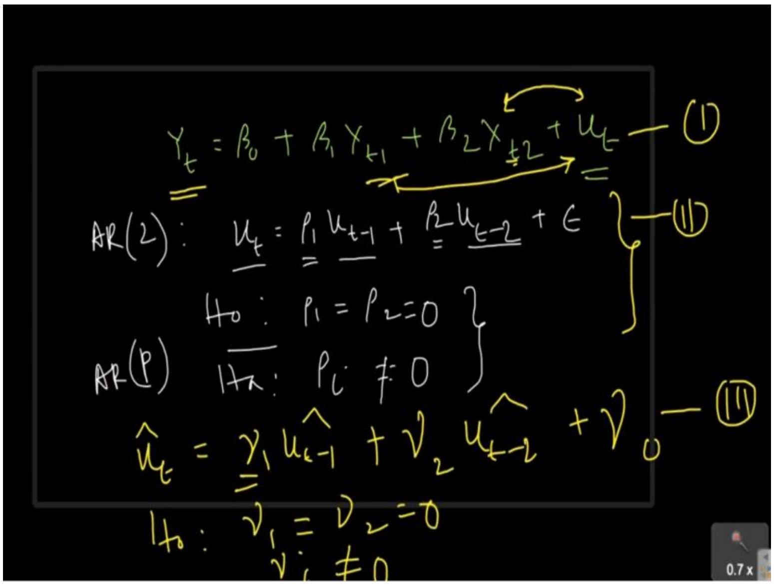

Now, let us say we have the original regression equation is something like this. Let us say my y t is equal to some beta 0 + some beta 1 x t 1 beta 2 x t 2 and sum t 2 and sum u t. Now, u t here is the error term and the problem that we are trying to deal here is that we are assuming that u t has some autocorrelation structure and we have seen let us say in this case we are talking about a AR 2 process.

Let us say, I am assuming that u t has a AR 2 autocorrelation structure. So, u t would be written as u t = rho 1 u t – 1 + rho 2 u t – 2 plus some innovation term. Now, what would be the hypothesis test for this? So, my H 0 would be all the rhos are 0 because my in this model autocorrelation is 0. So that if that is the case. So, then all the rhos needed to be equal to 0. Whereas the alternative hypothesis goes like this is that rho i.

So, it could be like for AR p process. I can have like 5 rhos for AR p process. So, rho i not equal to 0. So, any of the rhos, if they are not 0. So, if there are some sort of autocorrelation is present. So, I will say that well autocorrelation is present. So, the null hypothesis of no autocorrelation is no longer true. So, this is the null hypothesis but we have seen previously that we cannot really do it for the population.

So, we actually take some samples. And what we do there? We actually take let me use a different or I can actually reduce the size here. So, what I do there let me use a different colour, I actually estimate the u t and I run the regression. So, let us say I have some gamma 1 u t – 1 hat + gamma 2 u t – 2 hat and + some gamma 0. So that is basically we have seen previously how we get this. Now, here the null hypothesis is going to be in the same manner.

The null hypothesis is going to be my gamma 1 = gamma 2 = 0 and alternative hypothesis is going to be gamma i is not equal to 0. Essentially the gamma from gamma you are sort of getting an estimate of rho. Because you cannot get this for the population. So, you are actually estimating gamma and from gamma you are actually getting an estimate of rho. So that is the idea for a higher order AR process on the error term.

And how do you actually have your null and alternative hypothesis for autocorrelation? So, now, I will come down to the second point where I can introduce a lag dependent variable. Now, let us go back to our main equation here. So, let us say our u t is related to some of the x t’s. Now, the error term is related to some of the independent variables. So, what happens here? So, there is some endogenous problem.

So, error term is and the x variables so, they are supposed to be independent u t is supposed to be independent of all the x t's and that is where the Gauss Markov Assumption of zero conditional mean of error is so important.

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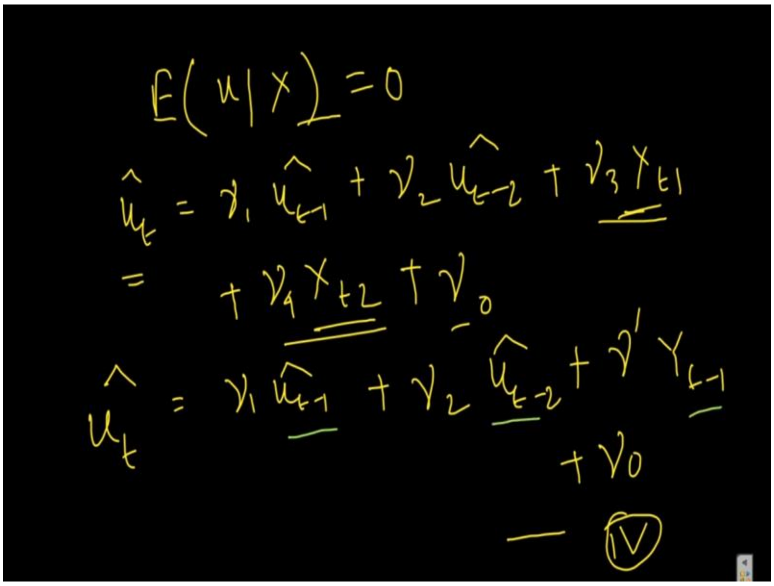

But, if it is not independent of x. So, then u is going to be biased and then the expectation term of this is not going to be equal to 0. So that is not basically that is essentially violating the O L S regression condition. So, we need to sort of ensure that and how do I ensure that? So, if I have let us say, if I have in my regression equation, if again I write down this equation, let us say this equation I could have actually numbered this equation.

So, let me actually number this equation. So, let us say this is my equation 1, this is the main equation I have written and then I have written down this equation 2 and then I have written down equation 3. And here I have let us say I will write down equation 3. But I will now, think about this relationship. I will now, think about the fact that u t and x t is somewhere related. So, this condition is no longer correct.

So, I have to sort of you know do some error correction to take care of this bias problem. And so, to do that I just rewrite the equation first. So, let us say I write down u t = rho 1 u t – 1 + sorry gamma 2 u t – 2 hat + I already have a gamma 0. So, I will write it at the end of it and let us say this x t's are related. So, let us say I have a gamma 3 x t 1, the variable name was x t1 and then I have a gamma 4 x t 2 and then I have a gamma 0 like I had as an intercept term.

So now, why I am actually including these independent variables in my regression equation is because since they are related with error terms. So, it is important that I sort of introduce them in the regression equation. So that I can make them free of this influences. So that is the idea why I introduce this x t terms in the regression equation when I am regressing, when I am actually, when my u t is my dependent variable.

So, when I do that essentially to make things simple I can actually replace this x t 1, x t 2 with that lag dependent variable with the y t variable the dependent variable. Because the x t's are actually determining y t. So, if I include y t. So that would essentially take care of the bias problem that we have. So, essentially we can write this equation as gamma 1 u t – 1 hat + gamma 2 u t – 2 hat + some let us say gamma prime.

Let us say y t – 1 I have taken lag dependent variable of previous period. So, we can take other periods as well. So, this is my regression equation. So, my regression equation when I do this correction so, my regression equation is going to be this. So, now, this is the final equation I got for the Breusch Godfrey Test. Now, I will be doing this regression. Now, how do I know? Now, I have to sort of decide some decision rules based on which I can say the autocorrelation is present or not.

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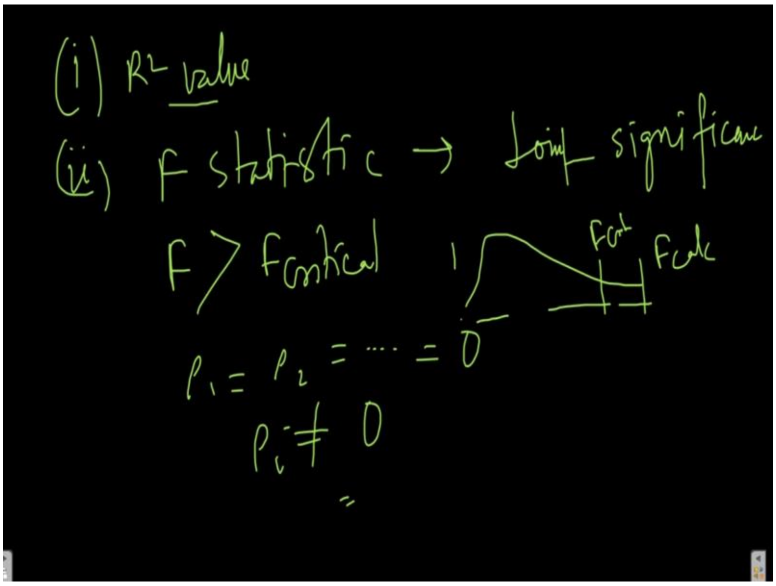

So, the first test is that a very simple one, we basically see the R square value in this regression equation, if the R square value is high, we say that the model is good. So, the variables the independent variables are actually determining the dependent variable and the independent variables are essentially the previous period error term or the lag dependent variables. So, then that would mean the autocorrelation is present in the modular.

Now, the second problem or second way to identify is that basically to check the F statistic. So, we know the F statistic is actually helpful to determine the joint significance of a model. So, what do you mean by joint significance? So, essentially, if all the variables are actually making sense, if the model is making sense. So, how do we do that? So, we have seen it in case of wireless process.

So, in wireless process what we have seen is that if let us say, if statistic that you calculate is higher than F critical. So, then we say that the model is significant. So, essentially, if we have let us say, F critical is here and then you have your F calculated is here. So, then you say that your model is actually having significance. So, your there is autocorrelation structure present in the model.

So, this is how we use F statistic we are already familiar with this. And what is the null hypothesis here? The null hypothesis rho 1 = row 2 and whatever rho other rho you have is equal to 0 and alternative hypothesis is any of the rho is not equal to 0. Now, so, we do that. So, there is another way of actually identifying or actually you know taking a decision based on the Breusch Godfrey Test.

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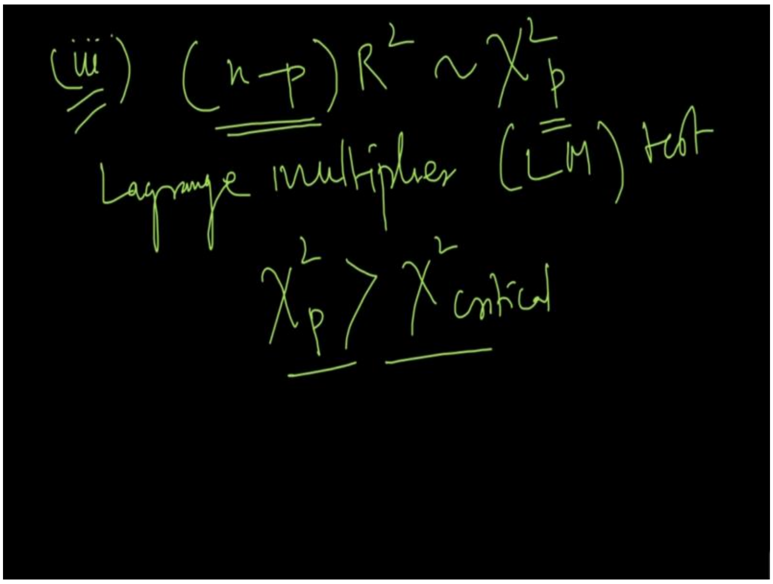

And that is basically we have n – p, where n is the number of independent variables and p is the order of the AR process into the R square. The R square that you obtain that actually follows a chi square distribution with p degrees of freedom. So, this is called Lagrange Multiplier Test. The sample size has to be large, Lagrange Multiplier Test or L M Test sometimes it is called LM test.

So, the idea is that if the chi square value. So, chi square value essentially is this and if the chi square value that you calculate is greater than chi square critical for p degrees of freedom. So, then I will say the model is significant that is there is autocorrelation in my model. So, one point that we need to note is that the test 2 or the decision rule 2 and decision rule 3 at this there are the test 3.

They are essentially the same or similar because the chi square distribution and F distribution are similar. So, then the results that we get you by you know using this 2 or 3 would essentially give us similar kind of result. So that is how we sort of use the Breusch Godfrey Test to identify the presence of autocorrelation in my model. So far we have discussed the theory part and in the next lecture we are actually going to do some hands-on to show autocorrelation.

We will use data, we will use R, using R and will also show you Durbin Watson Statistic, Breusch Godfrey Test and so forth. So, with this we will end this lecture here. Thank you.