

Applied Econometrics
Prof. Tutan Ahmed
Vinod Gupta School of Management
Indian Institute of Technology – Kharagpur

Lecture – 76
Autocorrelation (Contd.)

Hello and welcome back to the lecture on Applied Econometrics.

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The image shows a handwritten derivation on a chalkboard. At the top, it says "DW stat & γ Relationship". Below that, it shows the formula $DW \approx 2(1-\gamma)$. Then, it expands the Durbin-Watson statistic formula: $DW = \frac{\sum_{t=2}^T (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^T \hat{u}_t^2}$. The numerator is expanded as $\sum_{t=2}^T (\hat{u}_t^2 - 2\hat{u}_t\hat{u}_{t-1} + \hat{u}_{t-1}^2)$. The denominator is $\sum_{t=1}^T \hat{u}_t^2$. The final result is $DW \approx 2(1-\gamma)$.

So in this lecture, we are going to talk about autocorrelation. And in that context, we explain Durbin Watson Statistic and how it helps us to identify positive and negative autocorrelation. Now, one thing I really did not explain and I think that important is the relationship between Durbin Watson Statistic and gamma relationship. Now, we kind of simply said that the relationship looks like this, the D W stat is equal to something close to 2 into 1 – gamma.

But I have not really explained it. And I said something like all most equal to. But I did not say why it is almost equal too? So, let me actually explain that. So, this is just a small video too just for that explanation. So, let us again go back to the Durbin Watson Statistic formula and it is t is equal to 2 to T and we know y is 2 and we have $u_t - u_{t-1}$ whole square. Whereas in the denominator, I have something like $t = 1$ to T and I have u_t hat square.

Now, let us just expand the term on the numerator. We can definitely do that. It will be $t = 2$ to T. We have u_t hat square – 2 into u_t hat, u_{t-1} hat + u_{t-1} hat square. And in the denominator, since I have u_t hat for all of this, u_t hat square for all of the terms, I can

actually write down $t = 1$ to T u_t hat square. Here also, I can write $T = 1$ to T u_t hat square and oh sorry, I forgot to give this summation sign here.

And in all these cases, of course in the numerator $t = 2$ to T and in the denominator I am going to have $t = 1$ to T u_t hat whole square. So, now something to look at here in numerator and denominator, the terms are pretty close for, let us say this, term. Only thing that is different is $t = 1$ to T and in the numerator is $t = 2$ to T . Now, if we have sufficiently large number of observations, this difference $t = 1$ to T and $t = 2$ to T will not really matter.

But, if I have a very small number of observation, well then in that case, it might matter. Now, let us say they are almost pretty close to, you know, they are pretty same and I can write this value for this term is equal to 1. If they are pretty close. Similarly for this one also u_{t-1} and u_t , if I take the same logic here, I can pretty sort of approximate it to 1. Whereas for the same, the term in the middle, I will have this u_t and u_{t-1} .

And in the denominator I have some sort of like the steps, basically the variance of u_t . Now, it actually will convert into covariants, the numerator will convert into some sort of covariants and the whole term will look like the correlation coefficient which is essentially the gamma here. So, I can actually write down this equation, simplify as $1 - 2\gamma - 1$. So, there that is why I write approximately equal.

They are not exactly equal because of the approximations I have done here. This $t = 2$, $t = 1$, for all these terms. So that is why I write it approximately equal.

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$$\begin{aligned} &\approx 1 - 2\gamma - 1 \\ &\approx 2 - 2\gamma \\ \underline{DW} &\approx 2(1 - \gamma) \end{aligned}$$

So this is approximately equal to essentially $2 - 2\gamma$ or essentially $2(1 - \gamma)$. This is essentially what we get from here. So that is what essentially our Durbin Watson Statistic. And that is a relationship that we have used when we talked about Durbin Watson Statistic and the correlation coefficient for you know, whenever we try to understand positive or negative autocorrelation.

Thank you. So that is the small teaser video on the Durbin Watson Statistic and gamma relationship. Thank you.