Applied Econometrics
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Lecture – 75
Autocorrelation (Contd.)

Hello and welcome back to the lecture on Applied Econometrics.

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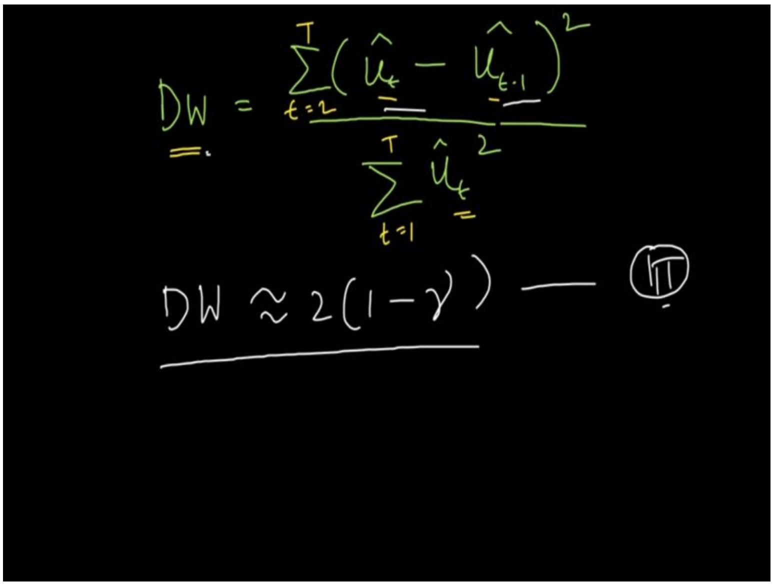

So, in this lecture we are going to talk about auto correlation and in the previous lecture we introduced the concept of Durbin Watson Statistic. So, this is essentially a test to detect, if there is any significant autocorrelation in our model. In this lecture we are going to see the value how can, we get the values of Durbin Watson Statistic and gamma for positive autocorrelation, negative autocorrelation.

We will also talk about the limitations of Durbin Watson Statistic we will see that we cannot apply this statistic everywhere. So, let us begin with the Durbin Watson Statistic for positive auto correlation. So, before we begin just let us recall that we actually introduced this formula where Durbin Watson Statistic and gamma are related. Now, using this formula we will see what happens for positive autocorrelation and for negative autocorrelation.

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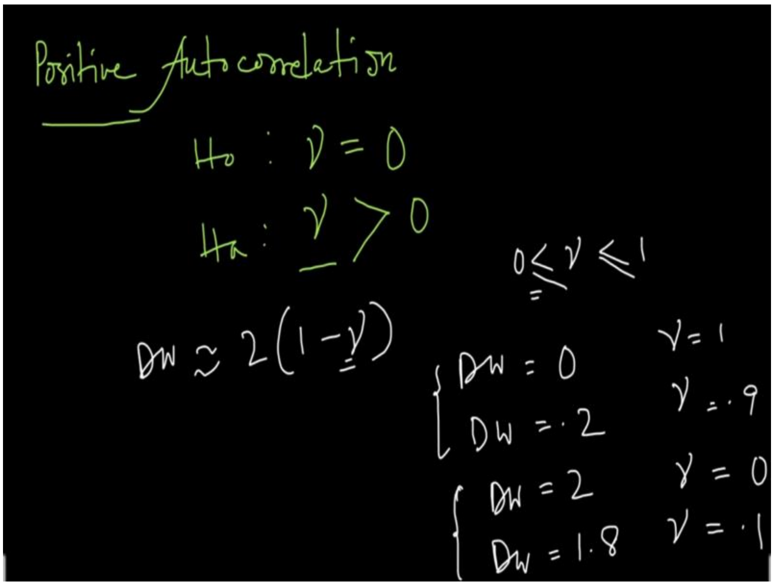

So, let us first start with the positive autocorrelation. Now, positive autocorrelation is something that is pretty common in economics problem. So, let us write down positive autocorrelation, for positive autocorrelation what would be our null hypothesis? Null hypothesis H 0 is my gamma = 0. So, there is no correlation between the errors for tth period between tth period and t – 1 period.

We remember we have already introduced this equation here and whereas, if the alternative hypothesis is going to be gamma greater than 0. So, for positive autocorrelation gamma needs to be positive. So, with this in mind let me actually write down the equation that I have already written previously. So, D W is equal to is close to 2 into 1 – gamma. So, now, we will fit some values here.

So, we know this is a correlation coefficient. Its value can vary between 0 to 1 for a positive autocorrelation. And if I fit the values here, what I will see here? If gamma is equal to 1, I will see my D W value is equal to so, 2 into 1 – 0 = 2 into 1 – 1 = 0. Whereas, if gamma is equal to let us say 0.9 which still signifies a quite strong positive auto correlation. So, my D W value is going to be so, 1 – 0.9 is 0.1, 2 into 0.1 is 0.2.

So that means my D W is going to have a smaller value. Now, let us put gamma value on the extreme opposite. So, if gamma = 0, I will have 2 into 1 – gamma which is D W is equal to 2. Whereas, if I put gamma is equal to let us say 0.1 which is very small. So, then I will have my D W statistic is equal to 2 into 1 – 0.1, 1.8. So, these are the different values we get. Now, we can see that if the gamma, if there is significant positive autocorrelation.

So, gamma is 1 or close to 1. So, my D W value is pretty low. Whereas, if my gamma is, so, if there is no significant positive auto correlation. So, what is basically gamma is close to 0. So, then my D W value is going to be 2 or close to 2 which is pretty high. Now, for all these cases actually Durbin e Watson Statistic they also provided us some tools to actually determine whether I can call the positive autocorrelation to be a significant positive autocorrelation.

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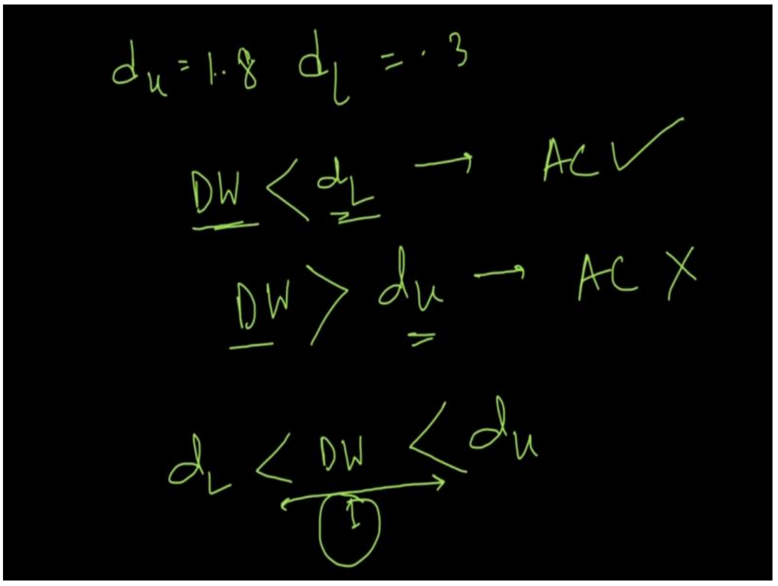

So, what they provide is the d u or the Durbin Watson Statistic upper limit or upper critical and d L Durbin Watson Statistic lower critical. Now, for positive autocorrelation, if we can see, if D W is pretty pretty low 0 or 0.2. So, then or you know around that. So, if in other words you can say that if D W is less than some lower critical. So, let us say that the D W here is let us say 0.3 here.

So, if my Durbin Watson Statistic is less than 0.3 then only I can say the positive auto correlation is significant. So, it makes sense, it is close to 0. So, I have a lower critical 0.3 and my D W is less than that. So, it is 0 to 0.3. So, then I say that well there is a positive autocorrelation. Similarly, if there is a let us say upper critical and let us say its value is maybe it has to be 0 to 2.

So, let us say it is 1.8. So, if the upper critical is 1.8 and if my D W is let us say greater than d u. So that is pretty high. So, we have seen, if D W is pretty high. So, then perhaps my gamma is very very low. So, then that means there is no positive autocorrelation. So, then I will say the autocorrelation is actually there is no autocorrelation. So, these are pretty understandable when the values are at the extreme, the D W values are at the extreme.

So, we can using this boundary conditions we can actually sort of decide whether I would call D W to be a positive or significant positive articulation or no autocorrelation but, if my D W value is in between. So, let us say D W is greater than the lower, there not some lower and smaller than d u. So, basically let us say D W something like in our case let us say D W is 1. Now, for this if D W has to be 1.

So, in that case my gamma has to be 0.5. So, in that case we cannot say very concretely that there is a positive autocorrelation. So, what we do usually? If the D W value falls in between, we say that we cannot you know conclude concretely whether there is a positive autocorrelation or not. So that is the kind of a grey area.

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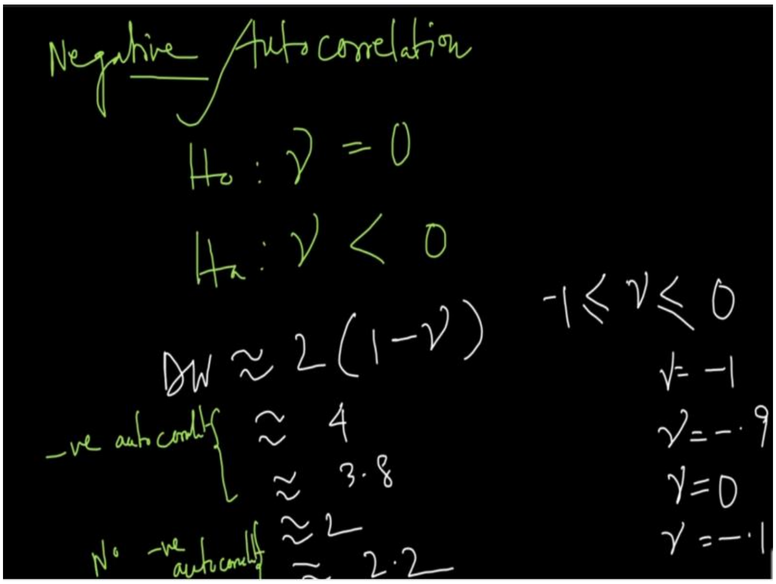

So, now, let us talk about the negative articulation part. In the same fashion will write negative autocorrelation again we will use the same formulation where we will have our null hypothesis and for null hypothesis like the positive autocorrelation the gamma has to be equal to 0. So, there should not be any correlation between u t and u t – 1. So, the gamma should be 0. Whereas alternative hypothesis gamma has to be less than 0 this time because we are talking about the negative autocorrelation.

So, correlation coefficient has to be negative. So, it has to be less than 0. Now, again we write down the formula for Durbin Watson statistic and gamma relationship. So, D W is equal to 2 into 1 – gamma. Now, gamma here. Now, let us say again let us fit some values. Let us say gamma is equal to. Now, in this case the value of gamma will vary between it is gamma is going to be greater than – 1 and gamma is going to be less than 0.

So, let us first put gamma is equal to – 1. So, if gamma is equal to – 1 that means a perfect negative autocorrelation. So that would mean the Durbin Watson statistic would be 2 into 1 – – 1 is equal to 2 into 2 4 it is close to 4 or, if my gamma is equal to let us say – of 0.9. So, similarly it will be 2 into 1 – -0.9. So, it is going to be 3.8. So, gamma value is going to D W value is going to be pretty high.

What, if my gamma is actually 0? On the other extreme, if my gamma is equal to 0, my D W value is going to be is equal to 2. Just like what you have seen for positive autocorrelation. And if my let us say it is pretty low, let us say gamma is equal to – of 0.1. So, my D W stat value is going to be 2 into 1 minus of 0.1. So, 2 into 1.1 is equal to 2.2. So, what we see here? We see here that, if my D W stat is actually close to 4 or very high.

So, then I will say that there is negative autocorrelation. Whereas, if it is close to 2 we say that there is no autocorrelation, no negative autocorrelation. So, we will again use the same sort of decision rule to sort of decide whether there is negative autocorrelation or not.

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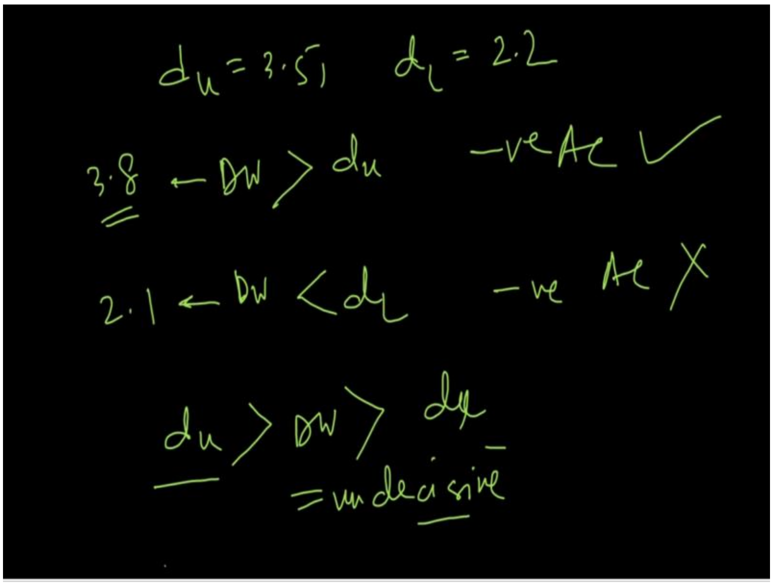

So, similar to the positive autocorrelation here also we have the d u and d L which is you will get it and this is created by Durbin Watson based on the number of explanatory variable as well as sample size. So, let us say in this case our d upper let us say is 3.5. Now, if my, if my calculated D W, if my D W is greater than the upper. So, let us say D W is something like 3.8 maybe.

So, what I will say? I will say, if this is the case then there is a significant negative autocorrelation. So, negative autocorrelation is present. On the other hand, if my D W is lower than the d lower, the lower critical, if D W is lower than d lower critical. So, let us say this value is in this case let us say it is maybe 2.2. So that is the case, if my D W is let us say 2.1. So, then I will say negative autocorrelation is not present.

So, we can just match what we are saying here. So, if it is lower than the d lower. So, we can actually say that this value is pretty close to this. And similarly, if my D W is let us say lower than d u and higher than D W is lower than d u and higher than d L. So, then we say that it is undecisive. We really do not know what is whether there is a significant negative autocorrelation or not.

Now, one case, one point that we need to be careful here I mean I have done it just for our understanding. But actually the d u and d L, they are calculated for negative autocorrelation. We have to see it slightly differently. So, here what we do is I will just explain it the formula is something like this.

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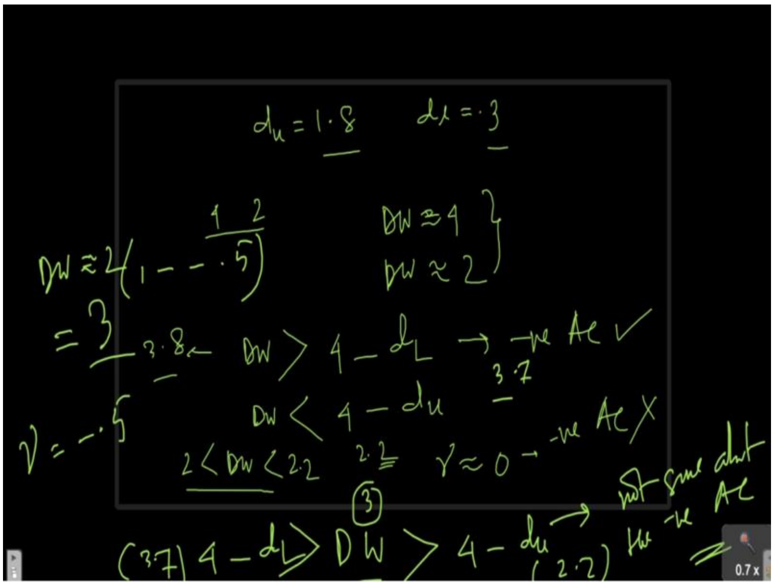

Now, let us use the same d u and d L. Let us say the for negative autocorrelation let us say we have the same d u and d L which is now, for negative autocorrelation let us say we use the same d u and d L, du is 1.8 and d L is let us say 0.3. It is calculated based on the sample size and the number of explanatory variables. So, my model does not know whether it is a case of positive attribution or negative altercation.

So, d u and d L is going to be calculated in the same manner. Now, remember that in case of negative autocorrelation, we have the value ranging between 2 to 4. So, if it is between 2 to 4 and my d u and d L is 1.8 and 0.3, we cannot really make any sense. So that is why there is a little change made in case of negative autocorrelation and that is we say that if and we already know the highest for negative autocorrelation.

The D W stat has a highest value of 4 and D W stat for negative autocorrelation D W has the lowest value of 2. So, it ranges between 4 and 2. Now, for negative autocorrelation how we create the decision rule is this. So, if my D W stat is greater, if it is greater than 4 – d L, if it is greater than 4 – d L. Now, d L is a small quantity, let us say 0.3 and if my D W is greater than 4 – 0.3 that is 3.7 and if my D W is something like let us say 3.8.

So, then I say there is significant negative autocorrelation. Whereas, if my D W on the other hand, if I see, if my D W is lower than 4 – d upper. So, what will happen 4 – d upper means? It is going to give me a value of 2.2. So, if my D W is less than 2.2. So that means D W has to be in the range of 2 to 2.2. It is given. I know that for negative autocorrelation my D W value can range between 2 to 4.

So, if it has to be less than 2.2, it has to range between 2 to 2.2. So, in that case because I know that if my D W is close to 2, my gamma is actually close to 0. So, because of this I will say that there is no negative autocorrelation. Whereas, if my D W stat actually falls in between. So, then just like in the case of positive autocorrelation, I will have some grey area. I will not be able to say anything conclusively.

So, I will write it as. So, if my D W is less than 4 – d L. So that means, if my D W is less than in our case, it was I think we calculated 3.7. If my D W is less than 3.7 and if my D W is greater than 4 – d u which is in our example 2.2 and 3.7. So, let us say I have my D W which is let us say, I do not know 3. I have my D W which is 3. So that I will get from the formula, I can get D W is equal to 3, if it is 2 into 1 minus let us say – of 0.5.

So, basically 1 + 0.5 so that will give me 3. So, if my rho is equal to – of 0.5. I have my D W is equal to 3. So, if D W is equal to 3, I will not be able to be very certain about the presence of negative autocorrelation. So, of course the D W and d L will vary. So, it might be a little though you know the range the D W can take can vary but that is essentially the idea. So, in this case we will say we are undecisive.

We are not sure about the negative autocorrelation. So that is basically how we understand and interpret the D W statistic and how we use D W statistic to understand, if there is any positive autocorrelation or negative autocorrelation.

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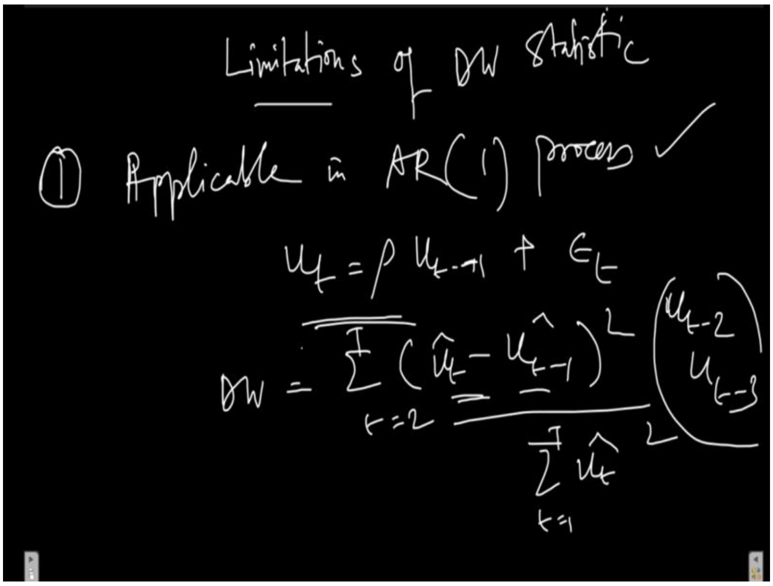

Now, the last part of the lecture is essentially the limitations of D W statistic. So, I will jot down some of the limitations and I will explain, why the limitations? Where from the limitations are coming from? Limitations of D W statistic. Let me explain the limitations. Limitation number 1 is that the D W statistics is applicable in AR 1 process and that was quite obvious and I will explain why this is obvious.

It is applicable in A R 1 process. Remember when you wrote the equation u t is equal to rho u t – 1 + some E t. So, this is essentially representation of AR 1 process and when we took in we express the D W stat in terms of this formula, u t hat and u t – 1 hat square and in the denominator I had u t hat square and I had this t = 2 t and t = 1 to T. So, I took only these 2 values u t and u t – 1.

I did not take u t u t – 2 or I did not take u t – 3 which would essentially mean my process would have been a AR 2 or AR 3 process. So, because of these essentially the Durbin Watson Statistic is applicable only for AR 1 process that you need to remember.

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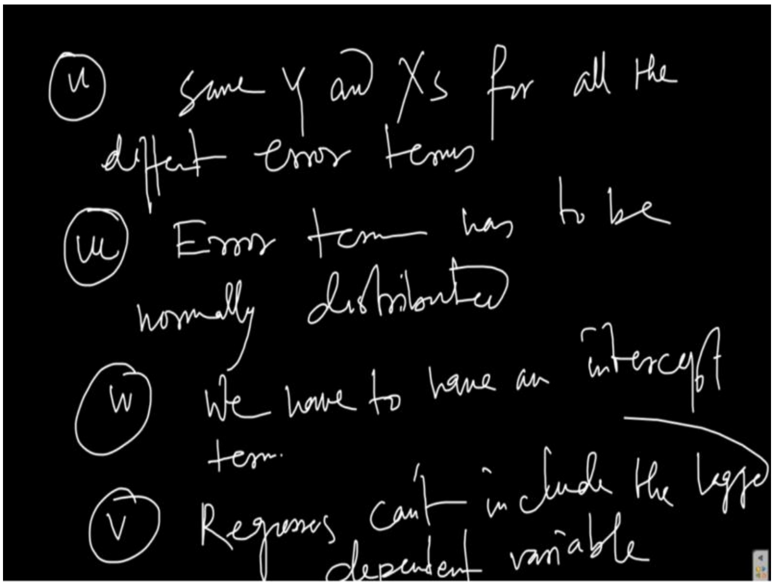

So, this is one limitation. The limitation number 2 is that and that is kind of quite intuitive is that the for different observations the different u t’s we observe. So, all the regression equations should have the same y and x s for all for the different error terms let us say. So, all the u t's that we get they should come from the same process that is quite obvious that is quite intuitive. So that is the second limitation.

The third limit so, basically you cannot really change your models when you are estimating the error term. Now, the third one is that we have to have the error term normally distributed. So that is error term has to be normally distributed. The 4th limitation of it is this there has to be an interceptor. May be you remember when we wrote down this equation, we had this interceptor.

So, there has to be an interceptor when you are estimating D W statistic. And why is that? We have to have an interceptor. Now, why we have to have an interceptor? Because, if we do not have an interceptor, the error term will basically assume whatever is left. So, then that is not what we want. So, we want the interceptor to be there. Now, the fifth criteria, the fifth restriction is that the dependent variable the lag dependent variable cannot be in the regression equation.

So, regressors cannot include the lagged dependent variable. So that means it cannot have y t – 1, y t – 2 and so on. And why is that? Because we; are trying to understand the relationship between u t and u t – 1. Now, if I you know introduce a y t and y t – 1, the whole process will be somewhat distorted. So, y t will actually in y t – 1 actually influence the equation the relationship between u t and u t – 1. So, we do not want any of this to happen.

So, essentially we have to be careful that we do not have within the x variables the regressors we do not want to have y t – 1 or any of the other y t's. So, these are the limitations that we have to keep in mind when you are applying Durbin Watson Statistic. It is a easy to use and quite popular statistic but we should be careful about this limitations. So, with this will end the lecture here. In the next lecture I will talk about another process of identifying autocorrelation. Thank you.