Applied Econometrics
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Lecture – 74
Autocorrelation (Contd.)

Hello and welcome back to the lecture on Applied Ecommetrics.

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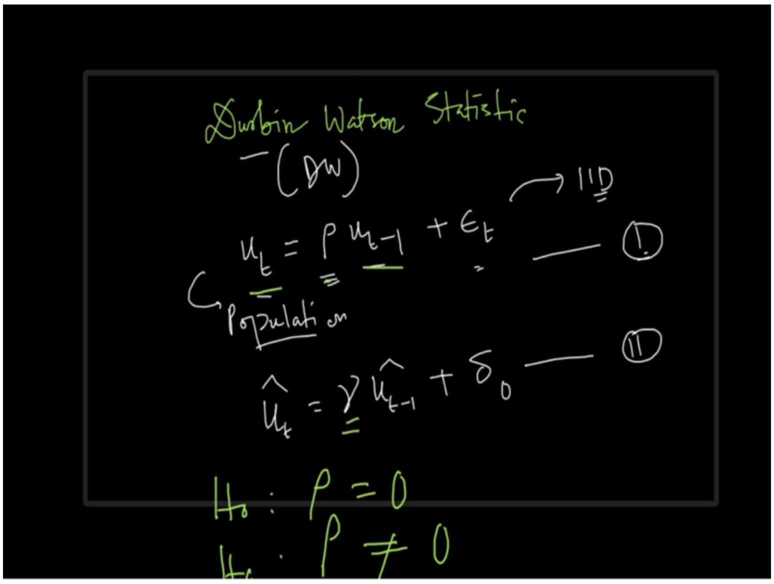

So, in this lecture we are going to talk about autocorrelation and in the previous lecture we have seen what is autocorrelation? What is the consequences of having autocorrelation? So, we really do not want to have a lot of autocorrelation in our model but in order to identify, if the autocorrelation present in the model is significant or not we need to have something quantitative in our hand based on which we can take a decision.

Whether to sort of say that the autocorrelation present in the model is significant or need to do something about it. So, let us say we have to do some tests. And for that there is one test that is Durbin Watson Statistic Test which is you know particularly popular we sometimes in abbreviation we say D W Statistic. So, we use this D W statistic to predict sort of estimate the extent of water correlation present in the model.

And we have some decision making rule based on which we say whether to say that the autocorrelation present in the model is significant or not. So, let us go back to our previous lecture, where we actually wrote down AR 1 process and we sort of create had an equation where we sort of you know estimated the relationship between the error terms of different period. So, for an AR 1 process.

If the error term of tth period is U t and if let us say the autocorrelation coefficient between the error term for different periods rho and the error term in the previous period is U t – 1. So, this is how they are related and let us say there is some innovation term error term. So, it is basically independent it is independent of all the variables present in the model and it is essentially I I D independent and identically distributed with a 0 mean.

So, we can write we can actually express U t and U t 1 U t – in terms of U t – 1 that is a past period error term. So, let us say that is how the errors are related in this process. Now, we know that this is about the population. So, we really cannot get the U t or U t – 1 or even E t because it is in the population. So, we can never estimate that what we can do instead we can have some sample data to estimate something that will you know sort of be a representative of rho.

So, you can sort of estimate U t as U t hat and then we have let us say some gamma and then we write U t – 1 hat which is the previous period error term you know estimated value. And of course when you are estimating this ut or U t – 1 you there is no question of having an innovation term here. Instead what you can have is an intercept term. So, the intercept term let us say I write it as delta 0.

And let us say I name this equation equation 2. Now, if I want to write the null hypothesis for presence of autocorrelation for equation 1 corresponding to equation 1. So, my null hypothesis is going to be let me. So, null hypothesis my H 0 is going to be the rho will have to be 0. So, if there is not a correlation that means rho is 0. So, by this period error term is not dependent on previous period error term.

So, this end of the story there is not a correlation. But, if there is autocorrelation, so, then H a alternative hypothesis is going to be rho not equal to 0. I am not saying, if rho has to be you know greater than 0 or has to be less than 0 as long as rho is not equal to 0 either the this the 2 period error are positively correlated or negatively correlated but there is some correlation. And if there is some correlation.

So, then that means there is autocorrelation. So, that is going to be our null and alternative hypothesis. So, Now, we know that we cannot estimate rho we have to estimate gamma. Now, how do we estimate gamma? Now, before I answer that let me actually write down what is Durbn Watson Statistic in the first place.

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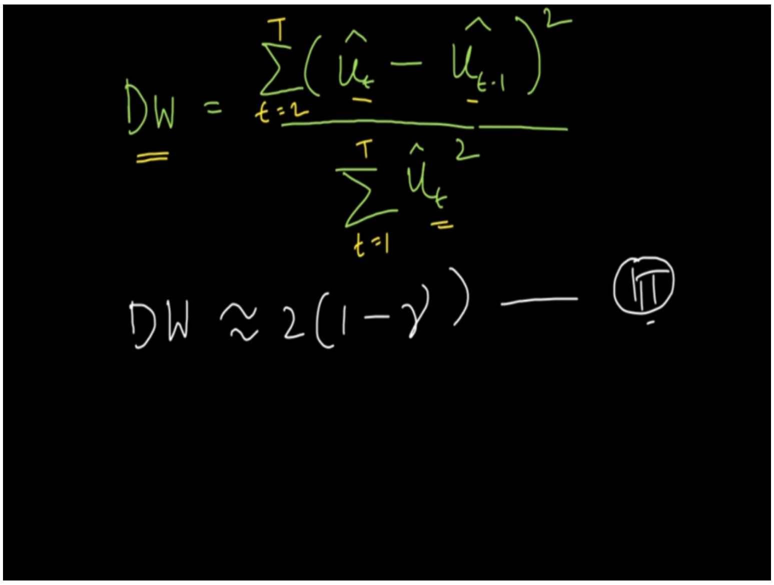

So, I write D W statistic and I am going to write a formula for this and this is equal to let us say U t hat. So, I am estimated I have estimated U t and U t – 1 hat that is the error term corresponding to this period and past period. And then I square them up. I basically take the subtraction between this consecutive terms and I square them up and in the denominator I will have only U t hat and its square.

Now, there is something you need to note when I talk about, when I actually you know sort of indicate the time period. So, here I start t is equal to 2 to T and in the denominator I start from t = 1 to T, why do I do that? Because I am taking this subtraction here. So, essentially, if I take t is equal to 2. So, U t – 1 hat is essentially going to be u 1 hat and whereas U t hat will be u 2 hat.

So, essentially I am basically taking the subtraction for the 2 error terms corresponding to period 1 and period 2. Whereas, since I am taking t here. So, from the very first term or the term corresponding to period one. So, I will start from t =1, t = 1 to T. So, essentially you see that there is one term less here. So, let me actually I will just illustrate that this how do you calculate Durbin Watson Statistic in a moment.

But before that let me write down how Durbin Watson Statistic is related to this gamma here. So, that is the thing that we are going to you know do with the Durbin Watson Statistics. So, it is the relationship is like this Durbin Watson Statistic = 2 into 1 – gamma. I am not going to prove it. So, this is the relationship that we will be using to sort of estimate the value of gamma.

Now, let me illustrate how do we actually calculate Durbin Watson Statistic with an example. So, that will make our life easy.

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			Y = .7 + .41*X			
	Y	X	Fitted Value of Y	Error (Ui)	Error Diff Squared (Ut - Ut-1)2	Error Squared (Ui)2
	20	56.7	23.947	-3.947	-	15.578809
	47	98.7	41.167	5.833	95.6484	34.023889
	34	89.2	37.272	-3.272	82.901025	10.705984
	19	47.8	20.298	-1.298	3.896676	1.684804
	16	30.0	13.0	3	18.472804	9.00000000000001
	10	20.0	8.9	1.1	3.61	1.21
	5	10.0	4.8	0.2	0.810000000000002	0.040000000000000
SUM					205.338905	56.664677
				DW STAT =	3.62375497172604	

Now, let me actually go to one excel sheet I have. So, let us look at the values of y and x. Let us say we have taken from some real experiment or some real valued observations and I have all these values of x 56.7, 98.7, 89.2 and I have corresponding values of dependent variable 20, 47, 34.

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regress y x

      Source |       SS           df       MS      Number of obs   =         7
-------------+----------------------------------   F(1, 5)         =     82.11
       Model |  1177.98096         1  1177.98096   Prob > F        =    0.0003
    Residual |  71.7333282         5  14.3466656   R-squared       =    0.9426
-------------+----------------------------------   Adj R-squared   =    0.9311
       Total |  1249.71429         6  208.285714   Root MSE        =    3.7877

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |   .4144714   .0457405     9.06   0.000     .2968917    .5320511
       _cons |   .7057545   2.711454     0.26   0.805    -6.264261     7.67577
------------------------------------------------------------------------------
```

Now, what I do is I actually get this number on my stata and I actually feed a regression equation. And what I see? I see this is a very nice feed r squared value is pretty good and x value the coefficient of x is going to be 0.41 with a constant term or intercept is equal to 0.7 great. So, I got my regression equation and I again go back to my excel and I have actually written down the regression equation y is equal to 0.7 + 0.41 into x.

Now, this y, the new y I got this is essentially the feeded value of y. I got the first value of y. Now, if I plot these values of actual x into this equation. So, that is 56.7 into the equation so, I will get 23.94 for the period value of y, whereas my actual y is equal to 20. For the next term we, if we plot, if we put x is equal to 98.7. So, the feeded value of y is going to be 41.16, whereas the actual value is 47.

Similarly the next value will get the feeded value of y is 37.2 whereas the actual value is 34. So, this way we get the y values, feeded value of y from the equation whereas the actual y is given. Now, how do we calculate the error? So, we are already familiar how do we calculate the error. So, essentially you just have to subtract these 2 terms. feeded value of y and y. And if we subtract these 2 terms we get this error term.

So, if we subtract 20 – 23.94. So, you will get – 3.94. Similarly for the second term third term for all the different terms we get this error terms. So, the 5.83 we get by subtracting 47 – 41.167 and so forth. So, this way we get all the error terms. Now, if we go back to our D W statistic. For the numerator we have to take the difference between the subsequent error terms. So, that means I have to take a difference between this – 3.947 – 5.833.

So, if I take the difference and then I square them. Now, because both of them are having negative sign, if I square them I am going to get something like 9, it is going to be you know more than 9. So, if I square this up. So, I am get some I am going to get something like 95.64. Similarly, if we take the second consecutive error term so, that is 5.833 and – 3.272. If I take the difference is again they are having the same sign.

So, I add them up, again it is going to be slightly more than 9 and when I square it, so, I am going to get something like 82.9. Next pair I get 3.27 – -1.29. So, this is a opposite sign. So, it is going to be something less than 2, if I basically subtract this and then I square it up so, it is going to be 3.89. So and this way we get the difference basically square of the difference of the error terms. And this way we get all the values here.

Now, we have to according to the Durbin Watson Statistic, I have to sum these things up. So, I sum these things up and I get 205.338. For the denominator I have to basically square the error terms and I have to sum this up. So, I square this up – 3.9 and I square it up I get 15.57, 5.833 square it up 34. something and so on and so forth. I square this error comes up and I sum it up.

So, I get 56.66. Now, according to the again this Durbin Watson Statistic formula, I will have to divide this 2 and if I divide this 2, I get 3.62. So, I am going to explain in the next lecture, how do you interpret this value of Durbin Watson Statistic? What does it mean to have a D W statistic is equal to 3.62? We will explain that. But before we end this one point you need to keep in mind and look at this column for error difference squared and look at the column for error squared.

So, essentially you see that the error difference squared I am having one term less, whereas error squared I am having all the terms. All the terms corresponding to all the rows. Now, this might matter in when we have less number of observations but when you have large number of observations, this will not be a very important factor. So, with this we end the introduction to Durbin Watson Statistic.

In the next lecture we are going to explain D W statistic in the context of positive autocorrelation, negative autocorrelation. We will also talk about the you know we will try to understand the values of D W statistic in different contexts and finally we will talk about the limitations of Durbin Watson Statistics. Thank you.