Applied Econometrics
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Lecture – 73
Autocorrelation (Contd.)

Hello and welcome back to the lecture on applied econometrics. So, in this lecture, we are going to talk about autocorrelation.

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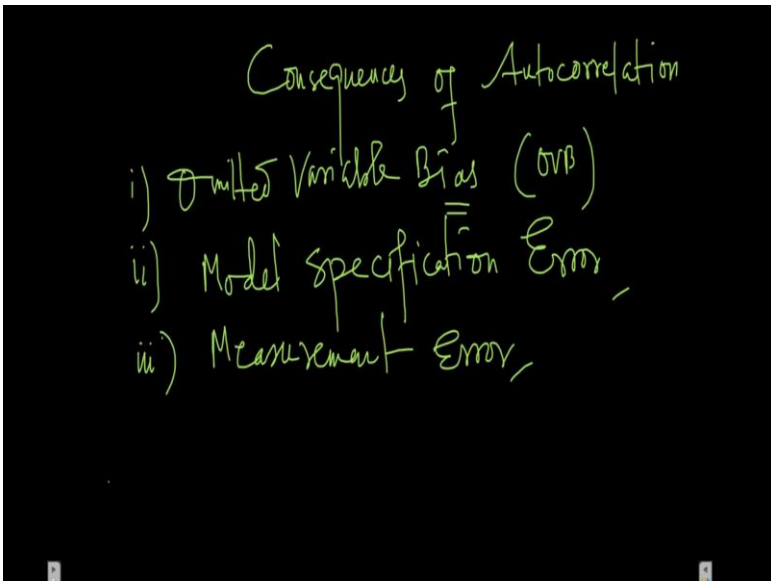

And in this lecture, we are going to talk about the consequences of autocorrelation. What happens if autocorrelation is present in my integration? So, in the previous lecture, we spoke about why autocorrelation is present? And we also spoke about the mathematical expression of autocorrelation. So, let me again recap. So, why autocorrelation is present? One reason we explicitly mentioned is omitted variable if there is any omitted variable.

So, what happens is that because of the omitted variable which is actually influencing my model. What happens in different periods? The omitted variable will show a pattern and that is something captured in the error term. S, we call it omitted variable bias. Going forward, we are going to explain omitted variable bias in details. So that is one reason we have already explained. The other reason could be model specification error.

So, what we mean by model specification error? We briefly may say, you have a model that X and Y are plotted like a straight line. Whereas in reality, X and Y could have a U shaped. It happens is that when you do not plot it, when you do not get the actual shapes? So, what happens the U shape or the other kind of shapes which are not captured, they are actually captured by the error term.

And then you can see a pattern in the error term and U t and U t - 1, you can see there are some correlations. So, modular specification error could be another reason. The third reason is measurement error. So, what is measurement error? So, let us say you do not have some data to actually represent the variable. So, what happens? You use some proxy or something and then you basically leave out a part of it.

Now, the part which you have left out that is really, you know that is really important and because, it is influencing your model, it is being captured in the error term and then you will see this pattern in the error term. So, these are essentially the reasons we usually see why autocorrelation is present omitted variable bias often an important cause but other causes are also they may also cause autocorrelation.

What happens if you autocorrelation? So, you know, why should we care so much? If let us say there are some error terms which are correlated but why do we care so much? And I will explain that and I will show how that actually matters or how actually that does not matter.

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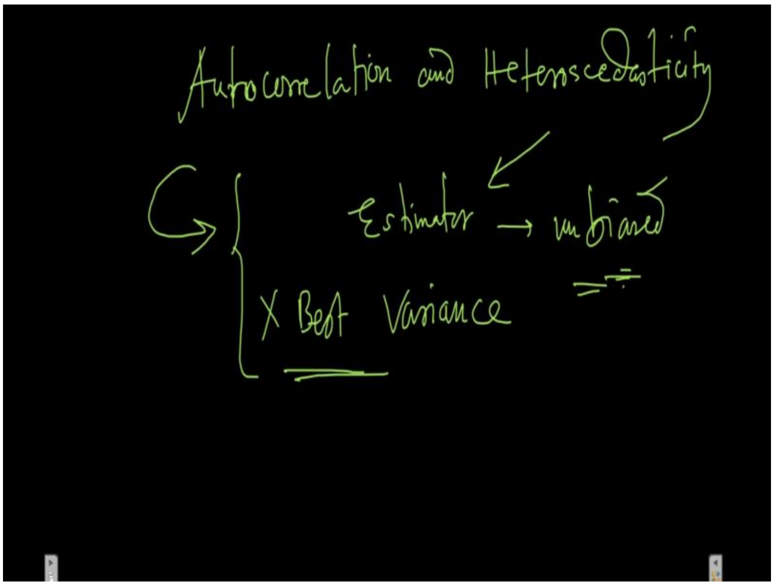

So, one thing we need to remember is autocorrelation and heteroscedasticity. Let me again write it down. Autocorrelation and heteroscedasticity are pretty close, they pretty close and like in heteroscedasticity, we have seen that because of the presence of heteroscedasticity, the estimator is unbiased which is a good thing. It actually fulfills one criteria for blue estimated best linear unbiased.

So, you get the unbiased part. But, it is not the best because, the variance is not the minimum variance because you can always get an estimator which has a lower variance than the estimators you are estimating when your model has heteroscedasticity. The same is true for autocorrelation as well. So, the estimator is unbiased but the variance estimated because the variance is actually high.

So, you can always find another estimator which you can have a lower variance or the minimum variance. And there is a problem with; if you have autocorrelation. So, the variance part could be understandable. So, whenever you have autocorrelation, the variance is going to be higher inflated. But, how do I actually prove the unbiased part? So that is what is the main topic of this lecture, I am going to actually explain why despite having autocorrelation in your model.

Your estimator could be unbiased. So, let me actually show you that. So, let us first start with this simple formula. Let us actually I use a new page here.

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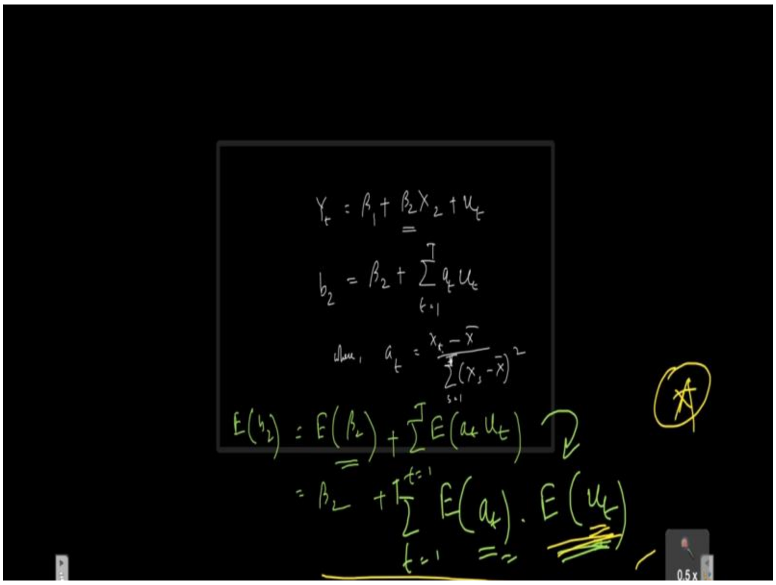

So, let us actually write down an equation we have been using this. So, Y t = beta or beta 1 let us say, beta 1, beta 2 X 2, I guess we are writing and then write U t. Now, my concern is about beta 2. I want to actually you know estimate beta 2. So, I always get a b 2. This is my estimate of beta 2. And I know this mathematical expression I am familiar with this is equal to beta 2, I am not going to explain it again.

So, this is how, it looks like is equal to t equal to let us say, we write it a t U t, where we use the same. Now, here a t where let us a t = X t - X bar by let us say s = 1 to t, X s - X bar whole square. So, note that I am writing here, X t and here I am writing X s, where x = sorry x = 1 to capitality. So, I am just using different notation t and s. And nothing this X t has a specific value and I am just using all this different X s here.

So, now with this I will get now; let me take expectation on both the sides. So, what I will get let me actually use this page itself. So, if let me use a different colour, if I take expectation on both sides, so, I will have expectation of beta 2, b 2 = expectation of beta 2 + t = 1 to t expectation of a t U t. I can write that. Now, symbol expectation rule. Now, if I know this is a constant term, expectation of beta 2, because beta is constant and this is equal to beta 2.

So, I do not have any problem with this. But, with this one, I am going to write this as let me further reduce the size. So, I am going to write t = 1 to t here I will write expectation of a t into expectation of U t. And what I when I can do that? Only when a t and U t are independent. Now, how I can write that.

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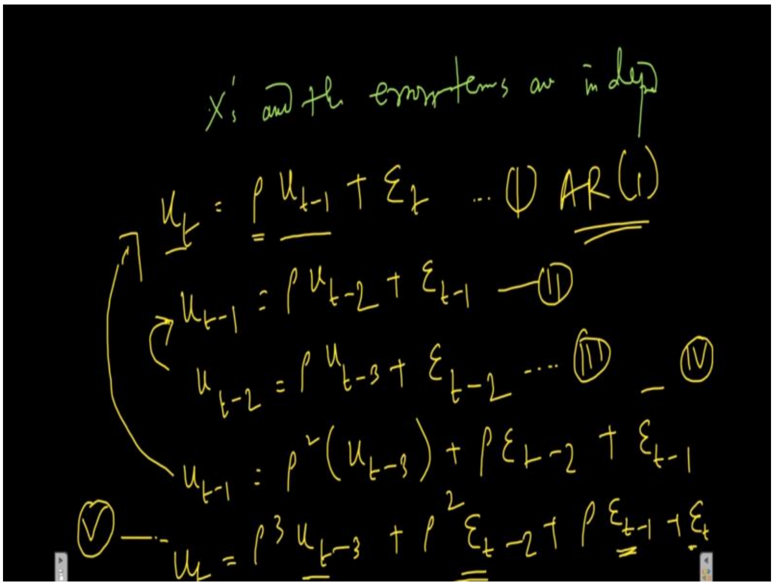

So, you can write that because there is a Gauss Markov assumption that exists and the error terms are independent. Something for my estimated to be glue, I my Gauss Markov assumption states that these 2 terms are needed to be independent because a t is nothing but the X terms and U t is nothing but the error term. So, they need to be independent. And so, using that I actually you know wrote this equation in the following manner.

Now, till now, I have not spoken anything about autocorrelation. Now, I am actually talking about the U t the error term. So, let us actually talk about the U t now. So, what will be the form of U t and if I have autocorrelation what is going to happen? So, let us see, if we have autocorrelation let us say it is an AR 1 process. So, for an AR 1 process, you already know, it is going to be U t is equal to U t rho U t -1 into some innovation or error term which is IID let us say E t.

So, this is how we write an AR 1 process. It could be AR 2 process, 3 process, whatever it is but for simplicity's sake, I am just doing an AR 1 process. Now, I can write the same U t here in the form of this equation here. Now, let us look at U t - 1, U t - 1 could be written as rho U t - 2 + E t - 1. Whatever U t - 2 is equal to rho U t - 3 + E t - 2. So, with this equations, you can clearly see where I am going. So, let us say this is equation 1.

This is a question 2, this is equation 3. And if I substitute 1, basically 3 into 2 and 2 into 1, what I will get is, essentially, I will get, let us say, I substituted here. So, I will get U t - 1 = rho square U t - 3. And then I will have my E t – 2. So, I will have rho E t - 2 + E t - 1. So, if I have substituted this into this 3 into 2. Now, what if I substitute 2? So, now I got like this equation 4. And now, I substitute 4 into 1, let us say I substitute 4 into 1.

So, what I will get is that? I will get U t is equal to now, I get this rho here. So, I multiply the whole thing here. So, rho cube U t - 3 rho square; this is going to be E t - 2, rho, E t - 1 + E t. So, now, I can see the let us say this is equation 5. Now, I can see, I can express U t in terms of some remote error term, U t and then I have all the E terms which is essentially a series.

So, I have E t, E t -1, E t - 2. And as I sort of, you know, further express U t in terms of E t. So, I will get more and more E t.

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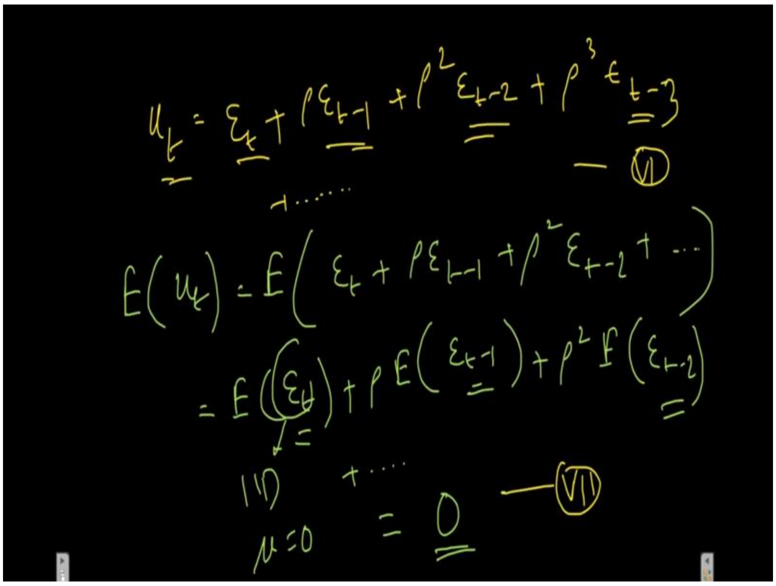

So, what will happen is essentially I will get something like a E t + rho E t - 1, + rho squared E t - 2, + row cube E t - 3 and so on, it will continue like this so on. Now, if I have this, let us say this, the U term that I will have. So, U t - n, or something that is going to be very, very far away from U t and that let us say hat is the first time on this 0, let us say.

Now, if I have this, I have an infinite sort of the express U t in terms of all these different E ts. Now, I actually come back to the expectation part which I wrote here, expectation of U t. Now, if I want to express the whole equation in terms of the expectation. So, I will get expectation of U t = expectation of the whole thing here. E t rho E t - 1 rho square E t - 2 and so forth.

Now, we know this by from very expectation rule that you have learned previously. It is going to be expectation of E t + rho expectation of E t - 1 + rho square expectation of E t - 2 and so forth. IID, Independent identically distributed with the mean is equal to 0. So, if mean is equal to 0 and they are IID. So, what I will get is essentially all are 0.

So, essentially that will give me a value expectation of U t is equal to 0. So that is a very, very important derivation here. And here, I can name some this equations like the number the equation. So, is question number 6, equation number 7, let us say. Now, if I substitute equation number 7 to this, I did not really mark it. So, let us this equation star.

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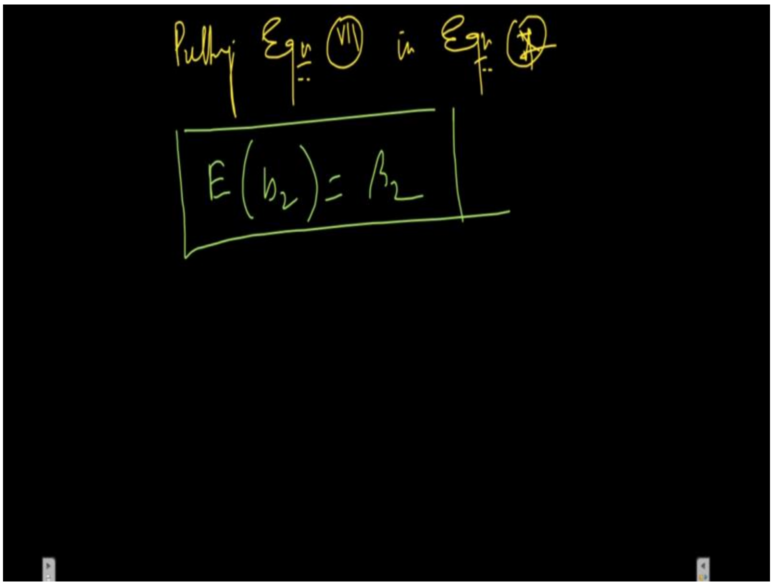

So, if I put equation 7 in putting, let us use a different page, VII in equation star let us say what I get is, this term becomes 0. So, this term becomes 0. So, essentially the result I get is expectation of b 2 is nothing but beta 2. So, then the b 2, I can claim that b 2 is an unbiased estimator of beta 2. So that is the proof I wanted to do here.

So, essentially, if the autocorrelation is present, what I can claim is that my estimate is unbiased but, it is no longer the minimum variance estimator. There are better estimators available where the variance is lower and that is the most efficient estimator. So, with this, we will end the lecture here.

And in the next lecture, we are actually going to see some of the techniques like Durbin Watson technique to actually, you know detect autocorrelation. Thank you.