Applied Econometrics
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Module - 7
Lecture - 55
Multicollinearity (Contd.)

Hello and welcome back to the lecture on multicollinearity. So, in the previous lecture, we explained the concept of multicollinearity. And we said, in case of perfect collinearity, there are some problem, there is an error that occurs, and often that the software cannot run regression; but there are some other software's; they can actually omit some variables and run the regression.

Now, a question may appear that why exactly, like how can I mathematically understand this problem is occurring. So, we have said, in case of, like the different examples where the perfect linear relationship is a problem, because the coefficients are varying simultaneously. Now, if I want to try to explain that in terms of the beta coefficient in a generic term, so, we will see why this problem is occurring.

Let me first write down the expression of a beta coefficient when I have more than 1 explanatory variable, and let us take the case of 2 explanatory variables. So, I will write it down.

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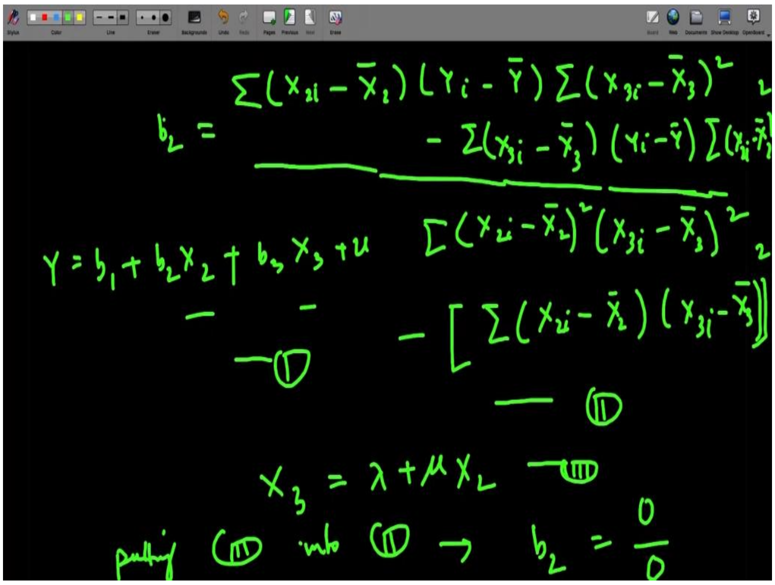

So, beta 2 is going to be in this case summation where; let me actually write down the equation first. So, Y is equal to, so, let us say, the same notation that we are using, beta 1 plus beta 2 X 2 plus beta 3 X 3 and some error term. So, this is my equation. Now, in this equation, if I write it down, the value of beta coefficient in terms of X 2 and X 3; so, it will be like this.

So, basically, you get this value by solving this, the same list square method you have to adopt. And if you do that, X 2i minus X 2 bar into Y i minus Y bar. So, I have not memorised it and I do not recommend you to memorise it, neither you have to actually solve this; you do not have to do it on your own; it is just for the purpose of illustration that if I have something like this; square; and then, I will have just the same way, X 3i minus X 3 bar into; Y bar; into X 2i minus; sorry about this small 4 and set with square.

It is just a symmetric way of writing the first term and the second term as symmetric. Just I have changed 2 and 3. And the denominator is going to be X 2i minus X 2 bar square X 3i minus X 3 bar square minus; it is a bit of a big equation, but anytime you are running, like you are going beyond the simple OLS where you are incorporating more than 1 explanatory variable, it is going to look like a bit messy, but the purpose of writing this equation is not for you to remember this equation, but only for you to understand why the perfect collinearity problem is occurring.

And it is good to have some idea about how it looks like, the final value of beta 2 when I have like more than 1 explanatory variable. See, if I have 3, it will be even more messy. So, X 3i minus X 3 bar. And then, there will be a close bracket, and then there will be a whole square. So, it is a pretty big term. Now, the reason; so, let me actually number the equations. So, let us say this is my equation 1, and this is me equation 2.

Now, for b 2, I got these values. And if I have to get the b 3, I just have to simply change the values of X 2 with X 3. Now, let us have the case where I have my X 2 and X 3 are perfectly linearly related. Let us say this is equal to mu X 2; this is an equation 3. Now, if I substitute this value X 3 with X 2, putting 3 in the equation 2, what I will find? I will find this both term in the numerator is going to be 0, as well as in the denominator, that is going to be 0.

And it is going to be undefined. And this proof, you can actually find from the Christopher Dougherty's textbook. It is a good idea to; in the chapter 3, in the page number around 160 to 170, it talks about the problem of multicollinearity. So, there you can see, the proof is given. So, you can actually take a look why the value of beta 2 becomes undefined here. So, that is basically the proof of why perfect collinearity is a problem for simple OLS, for the OLS regression. With this, we will end this lecture.