

Applied Econometrics
Prof. Tutan Ahmed
Vinod Gupta School of Management
Indian Institute of Technology-Kharagpur

Lecture - 33
Decision Rules

Hello, and welcome back to the lecture on Applied Econometrics. We are talking about hypothesis testing. And in this lecture, we are going to introduce the decision rules that we need to have to actually do the hypothesis testing. Now the idea of doing hypothesis testing, actually we will introduce some concepts, basically some tools to actually do the hypothesis testing.

So before we actually get into the rules, let me actually talk about the tools and concepts that we are going to use when we do the hypothesis testing.

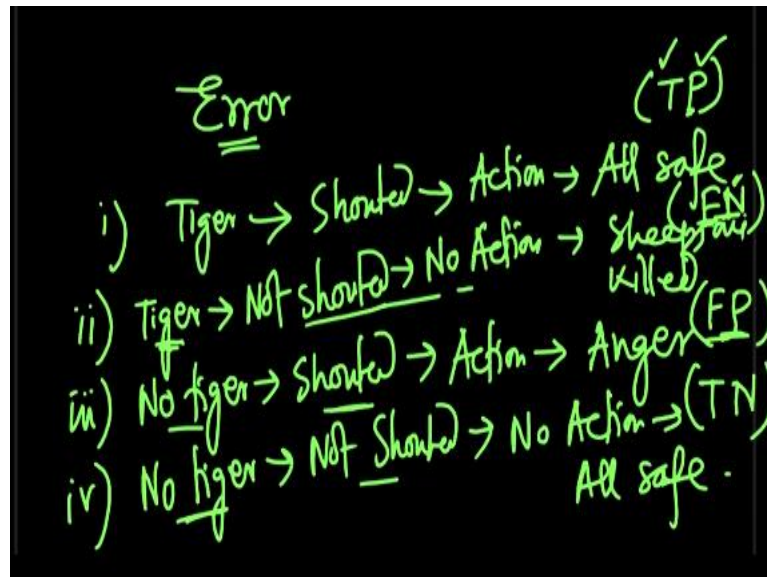
(Refer Slide Time: 01:00)



So the tools or concepts are like, let us say, we talk about level of significance. We talk about confidence level. We will talk about confidence interval, and we will talk about P-value. We will use something called power. It is a very important concept when we do any statistical design for survey and other things. And of course, the assumption, assumption about the distribution is also very important.

So essentially, we need to familiarize ourselves with these different concepts to sort of come to the decision rule. But before I go there, let me actually explain something called the error.

(Refer Slide Time: 02:15)



We know that any statistical estimation is, you know there will be error because we are always approximating problem, many observations right? Now to understand error in the context of hypothesis testing, we need to sort of, you know see what are the different types of error that might, you know that we may actually come across. And we will often hear terms like type one error, type two error, false negative, false positive, true negative, true positive, and so forth.

So we need to familiarize ourselves with these kind of concepts. Now before I actually try to explain that, I think it is always a good idea to start with a story, because stories are something that you always remember, right. And when you remember the stories, it is easier for us to connect to the concepts. So let me tell you a fable, right. So where a shepherd actually took his sheep to the field for grazing.

And let us say, there is a tiger who might attack the sheep, right? And the boy, the shepherd boy, you know he can, you know there are many possibilities, how the interaction may happen. So one possibility is that the tiger may actually attack the sheep, because the boy is actually, you know grazing around the forest and the tiger may just come out and you know because it wants to hunt, and the boy may actually shout.

And when the boy will shout what will happen, the villagers will come, and the villagers will actually make sure the tiger goes away and the boy and the sheeps are safe, right? On the other hand, it can happen that so let me actually write it down. So what happened? So let us say the, let us say there was a tiger or a wolf, and the boy shouted. And when the boy shouted, so there was an action, the villagers came.

I am writing action. And the repercussion is all safe, because the villagers came and actually they took an action so everybody is safe now. The second possibility, let us say the tiger came, okay. And what will happen? Let us say the boy may actually decide not to shout. I mean, he might be just paralyzed in fear and whatever, he might just run away whatever. So he did not shout. Not shouted, he did not shout.

And when he did not shout, what will happen? There will be no action, because the villagers did not know that the tiger has attacked the sheeps, right? So there will be no action. So what will happen? The sheeps are killed, sheeps are killed, right? Now this is the second possibility. What is the third possibility? Let us say there was no tiger, no tiger.

Now our shepherd boy, he decided to have a prank and he actually ended up shouting, right? He ended up shouting that there are tigers. And when he shouted, what will happen? The villagers will come because the boy shouted, everybody wants to save him. So there will be an action. People will actually come to save him.

And when they will come to save him and when they will see that it is just a prank so they will just get angry, and they will be you know they will not like the boy and there will, there will be you know there will be trust issue, they will not come next time, right? So there are so many possibilities. So let us say anger. So villagers get angry, okay. Now let me come to the fourth possibility.

And as you may guess by now the fourth possibility is that there is no tiger. And the boy did not shout, not shouted. Quite naturally, there was no action. And the outcome is there is no change. Everybody is you know as it is, right? All safe. Let me write down there is no problem at all. Now let us, why did I give you this example.

And it will actually, you know it will actually help us to sort of connect the ideas of different kind of errors that may happen when we, you know deal with this data and do a hypothesis testing. So let me actually explain that. So in the first case, the tiger came. So the fact happened, and the boy shouted, the action happened. And so everybody is safe. So we call it we call the first case as, or I can actually use the previous one.

This is called true positive, right? So that case the phenomena happened. And the action was taken. So it is a case of true positive. So in the second case, we see that the tiger came, but the boy did not shout, and there is no action, and because of that the sheeps are killed. Now what happened here is that the action, there was no action, no action is taken. So it is negative. And it is falsely negative because the tiger came.

And despite the appearance of the tiger, the boy decided not to shout. So we call it a false negative case, a false negative. It is a negative, the action was not taken. But the negative is false, right? It is a wrong thing to do. Like it is a false thing. So that is why we call it a false negative. Whereas in this case, in this case, in this case, we see that there was no tiger. There is no tiger and yet the boy shouted, so action is taken.

So it is positive action is positive. But there was no tiger. Again, it is a false premise, right? The premise here is that he shouted, so the ideal situation should have been, there was tiger, but there was no tiger. So the premise is false. So we call it false positive, okay? So just try to keep these two things in mind, which is false positive, and which is false negative. So always we look at the action.

The action here is negative. Then we see whether the premise is correct. The premise is actually here false because the tiger was there. So actually he should, you know he should have shouted, right? So that is why it is a false negative. Whereas this one is, the action is positive, but the premise is false. There was no you know the tiger should have been there, right? So in both the cases, the premise is false.

Whereas in the final case, there was no shout, so the action is negative. And there was no tiger, the premise is correct. So there was no tiger. So that is why the boy did not

shout. So we call it true negative, okay. So this is something we need to sort of conceptualize the concepts of what is a true positive, what is a false negative, what is a false positive and what is the true negative, okay?

Similarly, likewise, I hope you got this idea. So likewise, I will just give you another example. Because this is extremely critical for us to conceptualize this four terms. And we will see, we will connect these four terms with type one and type two error this going forward. So let us say the example we have used previously, let us say COVID test okay, COVID test.

(Refer Slide Time: 09:23)

The diagram is handwritten on a black background with green text. It is divided into two main sections: 'Covid test' and 'Test Result'.

Covid test

Actual

T

NT

NT

T

Test Result

P → TP

P → FP

N → TN

N → FN

And let us say you actually let **let** me draw it differently. Let us say this is the actual thing, whether you actually had COVID or not. And let us say this is a test result okay, test result. Let us say you actually have COVID, and the test result is let us say positive, okay. So when if you have you actually have COVID then and the test result is positive, so the premise is true.

You can have test result positive, but you may actually not have COVID, right? So the actual is not true, but you still get a positive result. You can get a negative result whereas you actually do not have COVID. So not true and negative, right. And finally, you can have a negative result whereas you can actually have COVID, right? So there are four possibilities, just now we will try to see what I just written down here.

So what happened here? I have in the first case, I had COVID, and I am tested positive, so it is true positive, okay. In the second case, what happened? I actually did not have COVID, but I am tested positive. So basically, the premise is again, again the premise is false, right? So this is a false positive case, okay. So I do not have COVID, but I am tested positive.

In the third case, I am negative, I am negative. And I did not have COVID. So the test is test is negative. But and the premise is also true that I did not have COVID. And that is why I am tested negative. So this is true negative. And the last case is that I have my result negative. But the premise is that I actually have it. So the negative is actually false. So this is called false negative, okay.

So this is what you have to understand how we sort of interpret these true and false positive and true and false negative. Let me give you another example.

(Refer Slide Time: 11:26)



Actual	Judgement	
Thief	Jail	→ T P.
Thief	No Jail	→ F N.
Not thief	Jail	→ F P.
Not thief	No Jail	→ T N.

And this is also a pretty common example that we use, and this is an example of thief, okay. Now let us say someone is in a judicial in let us say, there is a, you know in the court, the judge is giving his judgment if someone is a thief or not, right. So let us say a person is actually a thief. Actual, let me use a different color. Actual and here is the judgment, okay. So the action is that the person is thief, okay.

And the judge says that well, he is thief, so he should go to jail, right? Let us say he is thief. And either the judge either there is not enough evidence, the judge is linear, and

there was some mistake in the judgment and he says that no jail, the thief can actually be freed, okay. The other possibilities is that the convict is not a thief, the convict is not a thief. And the judge has actually wrongly put him in jail, okay.

The fourth case, not thief, but the judge and judge has also freed him. So no jail, okay. Now try to again, like the previous examples, this example also we will try to think what is happening here. So if the person is thief, and he is jailed, so the action is that he is put in jail, so the action is positive, so something is done. And the premise is correct. So it is a true positive case, okay?

Second, the person is, there is no action is taken, okay? But the person was actually a thief, okay, the person was actually a thief. So what we will do is that the action is negative, but the premise is false right? Here the person is not thief, but he is put in jail. So action is positive, but it is falsely positive, right? So it is a false positive. And finally, the person is not in jail. So it is a negative and the premise is he is not a thief.

So this is correct. The action is taken on the true premise so it is a true negative. So I with this three example we should be having enough clarity to understand these four terms true positive, false negative, false positive true negative. So I have you know the orders here are different the way I have given the examples but I am sure you will be able to understand how we derive this idea of what is true positive, what is true negative, what is false negative, what is a false positive, okay. Now we have seen the examples.

(Refer Slide Time: 14:33)

Type I error
Type II error.

	T	F
P	3	3
N	6	2

Now going forward, we will connect these ideas with type I, type I error and type II error, okay. Let me explain what is a type I error and what is a type II error. Now let me put basically some numbers here. Let us say I have a grid here. I have a grid here. And in this grid I have some, let us say, it is positive, negative. And the premise is could either be true or could be false.

I will use some other color here, I can use yellow and let us say these are the numbers 3, 3, 6, 2 okay. Now I have to basically identify. So basically we have already seen what is a true positive, what is a true negative, what is a you know false positive, what is a false negative. Now from here, I have to let me if I ask you, which are the cases representing the, representing the false positive?

So the false positive is here, the false positive here, okay. So essentially that would mean that someone wrongly let us say for the COVID example, someone wrongly diagnosed as COVID positive, okay. And if I ask the, what are the examples of false negative, this is the number, false negative. So basically someone is diagnosed as negative, but that is false, right? So he actually has COVID.

So essentially if you see, this two are problematic, these two are problematic. I do not want someone to be diagnosed wrongly, right?

(Refer Slide Time: 16:37)

Covid test	
Actual	Test Result
T	→ TP → treated
NT	→ FP → wrongly treated
NT	→ TN → free
T	→ FN → should have been treated

So if I go back to my COVID example, so what will happen if my test results if I actually have positive and if, sorry if I actually have COVID, if it is true and the test result is positive. So then I will get treated, right? The action is I will get treated, so that is good. I have COVID and I am tested positive, so I am getting treated. Now if I am tested positive, but I actually do not have COVID. So what will happen?

I will have wrong treatment, right? So I will be admitted to hospital, I will be given medicine, which is not good for me. So I will get wrong treatment, wrongly treated, I will be wrongly treated. Whereas if I do not have COVID, and if I am tested as negative, so that is good, that is good, I am free, right? I do not have to do anything. I do not have COVID.

And the last case, if I do not have COVID sorry if I if I am tested negative whereas I have COVID. So that is very dangerous. So I have I am actually not treated, but I should have been treated, should have been treated, should have been treated. So you see the thing here is that this one the false positive and false negative, they either wrongly treated or should have been treated.

So both are dangerous, right? In both the cases, the person could have some repercussion, right? In the previous example, when we said the tiger and the shepherd, so if the boy when it is a false negative, the sheeps are all killed. When it is a false positive, the villagers ended up being very angry with the guy and the villagers will not come back again if the boy actually shouts next time.

So this both are both the errors, these are basically the errors, whenever we are talking about false negative or false positive, these are errors. So I really want to avoid these errors okay, in both the types of error, false positive as well as the false negative. So and equally, you can also think about this example. If this is a thief, and he is not jailed, so what will happen a thief is roaming around freely, and he might do any, you know further nuisance, right.

And if it is not thief and he is jailed, is a false positive, is a very bad thing. Because someone you know like without committing a crime, he is just wasting his life inside the jail, right? So that is also not desirable. So both the cases, these are both of these are errors, and we want to sort of minimize both the errors when we do the hypothesis testing. Now I go back to the hypothesis testing.

And essentially, the idea here is that we sort of conceptualize this errors, and we will see how we, when we talked about the decision rules and when we talked about all these different concepts, so using these concepts, how we try to actually minimize these errors, okay. So the reason I spoke about all these different concepts is that to give you an idea about the errors.

(Refer Slide Time: 19:29)

Handwritten definitions:

- Type I error \rightarrow False positive
- Type II error \rightarrow False Negative

	T	F
P	3	3
N	6	2

Interpretations:

- Denying the status quo wrongly
- Denying the alternative hypothesis wrongly

And we will see going forward, basically what we will be trying to do is essentially to minimize the errors, okay. So essentially, we spoke about this different types of error and now I will connect this concepts with type I error and type II error. So type I error

is essentially nothing but the false positive cases, false positive, false positive cases. And type II error is essentially the false negative cases, false negative cases, okay.

So type I error, so we can sort of keep, you know remember it or we can actually conceptualize it slightly differently, but the you know the concept remains the same and that is in type I error, we essentially see we want to reduce the type I error because, we want to reduce the type I error because what is happening here is that you actually denying the status quo, denying status quo wrongly, right?

So I am saying that you know like I do not have COVID but my test is saying that you are positive. So basically the status quo is I am negative. I you know I am as I was, you know, but the thing is the test is saying that I have COVID right. So and that is why it is wrongly denying the status quo. And we know that we always try to when we talked about null hypothesis and alternative hypothesis, we know that we want to maintain the status quo, right?

So that is why type I error is not good. It is we want to reduce it. Whereas, when you talk about the false negative cases, the false negative cases someone you know is known to be a not thief let us say the last example we take and we basically we basically, but he is actually a thief, right? So what is happening is that we are denying, let me actually not write it.

So essentially the point is we are denying the alternative hypothesis wrongly, right? So the alternative hypothesis was that that person is thief because it is actually disrupting the status quo. So that is why it is alternative hypothesis, and we are saying that he is not thief. So that is why it is wrongly sort of diagnosed. So basically both the cases we try to reduce the errors, okay.

And we will see how this type I error and type II error are related and how do we actually take care of this, you know if there is a tradeoff and how we actually take care of both the errors, okay. So alright. So with this, we sort of end this lecture. We have understood, you know we have sort of introduced some concepts and then we have explained in detail about what is you know true positive, true negative, false positive, false negative.

And we also sort of connected those ideas with type I error and type II error. With this, we end this lecture and in the next lecture, we are going to talk about the concepts that we need to come to a decision, all the six concepts that are spoken about, we are going to explain in the next lecture. Thank you.