**Transcriber's Name Maria Priya**

**Applied Econometrics**

**Prof. Tutan Ahmed**

**Vinod Gupta School of Management**

**Indian Institute of Technology, Kharagpur**

**Module No. # 02**

**Lecture No. # 19**

## Covariance Rule

Hello and welcome back to the lecture on Applied Econometrics. So, we are talking about expectation, variance and covariance. And in this lecture we are going to talk about the rules of variance and covariance. Okay. So, we will start with covariance rules. (refer time: 00:37) So, before we begin, let me actually give you a teaser. So, some sort of uh a problem that we want to solve together. Let us say I want to take the covariance of 2 independent random variables. So, basically, one is not dependent on other.

So, intuitively, we can make sense what the covariance is going to be for that. But let us actually mathematically see how we can actually derive that. So, let us say I have the previous; let me take a different colour; I have the previous formula of expectation. So, covariance of X Y is equal to expectation of X minus X bar and Y minus Y bar. Now, in the same way, we we sort of expand this and we get expectation of XY minus of X bar Y bar. So, that is the formula we derived previously, (refer time: 01:29) right, without assuming anything.

So, we got the same formula here. Now, I said that these 2 variables X and Y, they are actually independent of each other. So, their values are not dependent. So, if that is the case, we can actually write like we did for our 2 independent events when we actually dealt with the probability. So, we uh sort of uh say that probability XY is equal to P X and P Y. In the same fashion, we write here E X E Y minus X bar Y bar. So, E X E Y is nothing but X bar Y bar minus X bar Y bar. So, that is equal to 0. So, we intuitively make sense, right?

If 2 variables, 2 2 random variables are not correlated, if if there is no covariance between these 2, so, then in gen, in naturally, uh what we are going to get is that, we are going to get a 0 covariance between these 2, right. So, that is uh what makes, intuitively makes sense, but we can also see it from here. (refer time: 02:32) So, now we come back to the covariance rule. So, like we did for expectation, here also we will use 3 rules, and the rules are here: So, let us say I have a variable Y is equal to V plus W.

Now, if I want to take covariance of Y with another variable Cov Y and X, so, I am going to have covariance of; basically I can replace Y with V plus W; V plus W and X. Okay. And if I expand it, I am going to, what I am going to get is basically covariance of X V and covariance of X W. So, it really does not matter which way order, uh which order you write X and V, it is going to be the same. The second is, let us say I have Y as a sort of, I can express it uh with another random variable W multiplied by constant term a.

Now, if I take a covariance of these, covariance of let us say X Y; so, that will be covariance of X W. Okay. And here, what we do is, we take a as constant, is it comes out of the covariance term; so, it is basically a covariance of X W. So, this is the second rule of covariance. And the third rule of covariance is going to be; so, let us say I have a constant. So, Y is constant here; Y is some a. Now, if I want to take covariance of a with, you know, like uh, covariance of Y with another uh random variable X.

So, what will happen, we can make sense intuitively because, since Y is constant, Y is always having the value a. So, no matter what the value of X is, it is going to be constant. So, there there is no, there is really no co, they they really do not covary. So, so, then, I will have covariance of X Y is equal to covariance of X comma a; and that is going to be equal to 0. Okay. So, that is, uh these are the 3 rules of covariance that we need to sort of keep in mind. And with this, we are actually going to see the last topic here, is the variance rule.

So, let us actually write down for the previous. This is the covariance rules. And I am going to talk about variance rules. (refer time: 04:53) So, here we have, let us say, we will be using the results that we have got previously. Let us say I have a variable just uh like the previous case, let us say X, okay. And X could be represented as a, I do not know, V plus W plus Z, let us say. Now, I; so, this is the rule number 1. Now, I want to take a variance of X, V of X. So, what is going to be the resulting variance?

And let us say, what I can actually write, I can actually keep things simple. I; oops; keep things simple. And I can actually use uh 2 variables instead of 3, it will, it will make our live simple. So, if I take variance, so, I, what I will have is V of V of W. Okay. So, here, so, just note that. So, I can actually uh; just note that, that this V is (vari) variable, and this is a term for variance, okay; they they are different. So, what I will have is V of V plus V of W plus Cov V W. Okay. Now, uh; this is a 2; 2 Cov V and W. So, how I actually get this result?

So, essentially, I can actually represent V of X is nothing but Cov X X. Okay. So, it is, when I take a variance, it is nothing but covariance of the same variable with itself, right. So, if that is the case, so, then I will write Cov V W comma V W; V plus W. Okay. So, now, if if I go back to our covariance rules, we can use this as Cov, so, let us say V square plus VW plus WV plus W square. Okay. Now, if I expand it, Cov V square Cov VW Cov WV and Cov W square. Right.

So, now Cov V square and Cov W square, they are just going to give us the variance terms. So, is nothing but variance of V. And this term is nothing but variance of W. And these 2 terms are going to give me 2 Cov VW. Okay. So, this is a proof of the first rule when I express my random variable in terms of 2 other random variables. Okay. Now, the second rule of variance is, (refer time: 07:46) if I have, let us say Y is equal to some aX, okay, and I want to take variance of Y. So, what I will get is, I will get a square variance of X.

So, how how did I get that result? sa Similarly, you can again, like the previous one, you can actually use Cov Y Y. Variance of Y, variance of Y is nothing but Cov Y Y. So, if you do that, so, you are going to have Cov aX comma aX, right. And when you have Cov aX comma aX; so, you can actually go back to the formula here, where you can actually, where we took the constant term out. So, basically, we are going to have 2 constant terms. So, a square Cov X comma X. So, which is basically a square variance of X. Okay. Okay.

So, uh the third rule of variance is, if I have, (refer time: 08:47) if I have, let us say Y is a constant term, okay. If I have Y is a constant term, so, what would be the variance? Variance of Y is going to be 0; because, for a constant term, there is no question of variance; so, it is simple and intuitive. Okay. The fourth, if I have Y is equal to some random variable plus some constant term. Okay. So, how do I actually get the variance of this? So, variance of Y is going to be variance of X plus b. Okay.

So, similarly, you can apply the covariance rules, and you can see this is equal to Cov Y Y. And you are going to have Cov X plus b comma X plus b. So, you are going to have Cov of X square, which is nothing but the variance of, variance of X. And then, you are going to have Cov of b X plus Cov of X b plus Cov of b square. Now, you will, you note that this term, these 2 terms Cov of b X and Cov of uh X b from this rule here. If we have covariance with a constant term, covariance of a constant term, this is actually; uh covariance of X with a constant term is actually 0, right.

So, then, both these values, both these values are going to be 0, both these values are going to be 0. And this is quite obviously 0, because this is a constant term. So, essentially, what you are left with is variance of X. Okay. So, this is the fourth rule. So, if you have Y expressed in terms of random variable plus a constant term, so, the variance of Y, the variance of the, you know, the the random variable on the left-hand side is going to be the variance of random on the right-hand side.

So, these are the uh few rules for variance. So, with this, we sort of end this lecture on uh rules for variance and covariance. So, we are kind of done with the basic tools that we need to have to understand the probability distributions. So, in the next lecture, we are going to see some examples of probability distributions. Thank you.