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Applied Econometrics

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Module No. # 02

Lecture No. # 18

Expectation, Variance, Covariance (Contd.)

Hello and welcome back to the lecture on Applied Econometrics. So, we had been talking about expectation, variance and covariance. So, in the previous lecture, we gave you the idea of expectation. (refer time: 00:37) Now, before I get into covariance, variance and covariance, let me give you some rules of expectation that we need to keep in mind. Okay. So, the rules of expectation: So, there are essentially 3 rules that we need to keep in mind, and the rules are following. So, let us say a variable is represented by, let us say X is equal to Y plus V plus Z. Okay.

Now, the first rule says, the expectation, if I take expectation, expectation of X is equal to expectation of Y plus V plus Z. So, then, that is going to be expectation of Y plus expectation of V plus expect expectation of Z. So, what do we call it, the expected value of the sum of several, you know, if, you know, if we, uh you know, uh explain it, so, the expected value of the sum of the several variables is going to be the sum of the expected value of the variables. Okay. So, this is the first rule.

So, if you have 3 variables representing a random variable, so, we can basically get the expectation of each of these random variables and sum them up. So, that is basically going to represent the expected value of X. Okay. Now, the second rule says, let us say I have a constant term B. So, and I want to take; and that is multiplied with a random variable. So, let us say I have a X, and I want to take an expectation of the whole thing. Then, what we will going to have is, that it is basically the product of the constant term and the expectation of the random variable.

So, basically it says that, if there is a constant term inside, you can take that constant term out. Okay. And third uh rule is that, if you eh if you take a expectation of a constant, or let us say expectation of a, okay; so, you can easily guess. So, if you take an average of the constant term, or you know, of of the same values across, you know, a n number of times, so, what you will get is nothing but the same value. Okay. So, these 3 rules we are going to remember, and we are going to see the application of these rules.

Now, let me now go back to the concept of variance. So, variance is a very uh, you know, widely used concept. Whenever we deal with empirical data, whenever we try to describe a population or sample, variance is one of the ma main characteristics that we need to deal with. Now, what is a variance? So, we know, when we when we deal with the frequency distribution; uh I I presume you already know how to calculate variance. (refer time: 03:21) So, we usually write sigma x square. And that we write 1 by n summation x i minus x bar square, right.

So, that is how we write. So, it basically says that the sum total of all; basically we know, you sa, you take the sum of all the dispersions from the mean value. Or, you know, if you have x's, different x's, you take the uh their their difference from the mean value, and then you sum them up. Now, in the language of uh, for a random variable, in in case of a probability distribution, what we what we write it as va V of X is equal to expectation of; this is the same way, but here we write X minus mu whole square. Okay.

So, it is nothing but the same sa same thing, but we are representing in terms of the expectation term, right. Now, if we sort of expand it, if we expand this whole thing, so, it will be expectation of X square minus 2 mu plus mu square. Okay. And expanding it further, what we will get is, expectation; now, we go back to the expectation rules, so, where we had the first rule. You can actually, if you have these 3 random variables representing X, so, you can actually have the expectation as a sum of the expectation of individual random variables.

So, if we use this, so, we are going to have expectation of X square minus 2 into expectation of X. And as we know, this is a constant term. So, as per the second rule, we can write 2 mu X, and third expectation of mu square, right. Now, we know that expectation of X, that is the mean, that is going to be mu. So, we can write expectation of X square. And then we can write 2; mu into mu, mu square. And then again, expectation of constant term; so, that is our third rule of expectation. So, that is going to be the same, that is mu square. Right.

Or we can write expectation of X square. And if we just add this up, so, I will have expectation of X square minus mu square. Or sometimes we also write it as expectation of X square minus of expectation of X, which is mu, and that whole square. Okay. This is a standard formula for variance that we uh use. Okay. And this is the formula that we will see, we often use and we, it is good to remember this particular form of this formula. Okay. Now, let us do an example, and let us calculate the expectation.

So, we will again use a previous example where we actually threw 2 dice. (refer time: 06:23) Okay. So, this this place is really cluttered, but let me see if I can actually do something here. uh So, we had our uh expectation of X, which is 7, right? Now, if I want to get (expectat); so, basically, X minus let us say mu. So, whenever I want to get the variance, as per this formula, whenever I want to get a variance as per this formula, so, I need to get the value of X minus mu, right.

And the value of X minus mu are going to be here; so, this is going to be 1 minus 7, which is minus 6. This is going to be 2 minus 7, minus 5. Then 3 minus 7 is going to be minus 4. I think it is becoming very small; or I should actually use a thinner pen. So, 1 minus 7 is minus 6; then 2 minus 7 is minus 5; 3 minus 7, minus 4; 4 minus 7, minus 3; 5 minus 7, minus 2; 6 minus 7, minus 1; 7 minus 7 is 0. Then 8 minus 7, 1; then 9 minus 7, 2; and 10 minus 7, 3; 4 te, 11 minus 7, 4; and 12 minus 7 is 5. Okay.

Now, if I want to square this term, so, I am going to have 36, 25, because I need to have X minus mu, X minus mu square. So, that is going to be 16, then 9, then 4, then 1, 0, 1, 4, 9, 16, 25. Okay. Now, all we have to do is, we have to multiply these values with the corresponding probabilities, okay. So, we have to; so, if I write it down here; so, variance of X is nothing but X minus mu whole square into; or let us say, if I write in smaller small x, then P x, okay; or we can write in capital terms also, just to represent the formula.

Now, here, we can actually now substitute the values. We can have; so, this one does not have any uh implication here; but here, this one has; so, 25 into; so, the corresponding probability is going to be 2 by 36; this is for 2. Then we will have 36; uh sorry, this is 16. 16 into 3 by 36. Then I will have 9 into 4 by 36. Then I will have 4 into 5 by 36. Then I will have 1 into 6 by 36. Then I will have 0. Then I will have, again; it is really looking a little cluttered, so, let me just create a box around this, so that I do not mix things up.

And then here, we will have, again 1 into 5 by 36. And then we will have 4 into 4 into you will have 9, corresponding is 4 by 36. Then I will have 9 into 3 by 36. Then I will have 16 into 2 by 36. And then finally, I will have 25 into 1 by 36. Really, really messy; what we have done, we should have used a separate page, but that is how it is, the formula is. This, so, we are basically using this value X minus mu whole square and corresponding probability. Okay. So, this is how we calculate this.

Now, we can also, we could have also used this formula. Instead of this, we could have also used expectation of X square minus expectation of X of whole square. Okay. So, we have already, we already have the value of expectation of X, we already have the value of expectation of X square. So, all we need to do is to sort of substitute these values. And we will, if we do that, we will see, in both cases, we are going to get the same result. Okay. All right. So, that is that is the uh idea of a variance.

And we have already shown you uh how how we can actually derive this formula from uh, you know, from this one. So, that is really important. We are going to use that. uh With this, we sort of end the uh discussion on variance. Now, let me talk about covariance. (refer time: 11:46) So, we will we will talk about the rules of variance and covariance later. But before that, let me actually talk about covariance. Now, covariance, as we know, we already perhaps know what is covariance, right?

So, till now, I am I was talking about 1 random variable. But when I talk about covariance, so co, so, I am talking about more than 1 random variable, right. So, I am talking about covarying things. So, when they are covarying, so, that means, of course there are more than 1 random variable. So, how we write about covariance is the same way we uh, you know, like uh, the way we have written about variance. So, covariance, we used to write covariance X Y is 1 by n summation x i minus x bar into y i minus y bar, right, over all i's; so, all the values.

So, what do we do is, you take the difference of each x values from corresponding uh x bar; and similarly, you take the difference of all the y values from the corresponding y bar; and you multiply them; and you sum them up; and then you take the average. Now, in terms of expectation, Cov X Y, what you write is, expectation of X i minus X bar into Y i minus Y bar. Okay. Or you can actually write, you can, you can simply, when you are talking about a random variable; so, perhaps you do not have to mention the i's, you can simply represent this way.

Okay. Now, if we uh expand this, what we get is; we can actually expand this, right? So, we can get expectation of let us say XY X Y bar X bar Y plus X bar Y bar. Okay. So, if we use the expectation rule, what we will get is, expectation of XY minus expectation of X Y bar minus expectation of X bar Y, and then finally, expectation of X bar Y bar. So, what we will get, expectation of XY. And here, again, you know that Y bar is constant, so, we can take the Y bar out as per the second rule of expectation, so, Y bar expectation of X.

And then you can take X bar out; expectation of Y. Let me reduce the screen size. And then I will have; now, I have 2 constant terms here, right? So, expectation of X bar Y bar is nothing but X bar Y bar, right. Now, what I what I get is expectation of XY minus. This is going to give me expectation of X is nothing but X bar; so, it is going to be X bar Y bar. And expectation is, expectation of Y is going to be, going to give me a Y bar; so, that is going to be X bar Y bar. And then I have a X bar Y bar.

So, it means, expectation of XY minus X bar Y bar. So, that is basically the uh, that is basically the formula of covariance. Okay. So, uh with this, we sort of end the discussion on variance and covariance. And in the next lecture, we are going to see some rules of variance and covariance. So, with this, we end the lecture here.