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Applied Econometrics

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Lecture No. #11

Bayesian Theorem (Contd.): Repeated Trial

Hello and welcome back to the lecture on Applied Econometrics. So in the previous lecture, we got some familiarity with Bayesian theorem and we did some hands on with the Bayesian theorem. Now in this lecture, we are going to see something very interesting. And that is how with the repeated trials, we keep on updating our beliefs. So let us look at let us continue with the problem that we have been doing in the previous lecture. (refer time: 00:46)

So the problem we had in hand is that somebody is detected to be COVID positive. Now the question is, what is the probability that that person might actually have COVID. Now we have done only one prior, and we found the probability to be actually only 9.02%. Now now if suppose the doctor is prescribing or that person to have another trial, and so basically repeated trials. (refer time: 01:13)

So let me write down, repeated trial trial, and the person is diagnosed to have COVID again, okay. So he is repeatedly diagnosed as COVID. Like, in the first test is diagnosed as COVID, in the second test is diagnosed as COVID. Now the question is, what exactly what is the actual, now what is the probability that the person is having COVID and this is what you are going to see in this problem. (refer time: 01:44)

Now if you remember, in the previous case, we will actually consult the previous diagram that we already had. And in this diagram, what we had is this blue zone, this blue zone, these are the people who would be diagnosed to have COVID, okay. So these people like this part of the people, these are actually the people who are having COVID.

And these people here, they they do not have COVID, but they are diagnosed to have COVID because of the faulty, you know test instrument, okay. So then the total number of people who would be uh who would be my sample space is this people here, is this people here, and this people here, okay. This people here and this people here.

So what will change here is this total number of people here. So that is, here you will see the total number will be 10 and here I have 99.9. So my N is changing to be 10 + 99.9, which is 109.9. Now if my N is this, I will now try to understand what is the probability that a person who is tested, uh you know to have COVID, what is the probability that this person is having COVID?

So let us see what are the things that has changed because of the change in the end. So one thing that will change is the probability of COVID. Now my initially, this probability of COVID was 1 by 1000. But now since my sample space has changed, it is only 109.9. So my denominator is going to be 109.9. My numerator is going to be 10. Because this is the, you know the correct number of people who actually has COVID, right?

So probability that the, you know that a person has COVID, in this new sample space is uh is 10 by 100, or 109.9. Now probability of someone not COVID, not COVID is going to be, you know in numerator you will have these people who actually do not have COVID, right? So this by the total number of people, right? Here is 109.9. Now so this is the updated uh sort of you know probabilities that I have in the second case, when I am going to do the repeated trial.

And if you see that earlier, it was 1 by 1000. And it was 999 by 1000 right? Now other things, now the question is these are the these are the things that changed. Now what are the things which has not changed? That is another important question here. And what has not changed here is your test probability, right? Your uh probability of test accuracy. So your test accuracy remains the same.

So let us say your probability of this probability, and this probability because your equipments are remaining same, we are not changing the equipment. So the probability that someone will be tested as positive, someone will be tested as positive given someone have COVID is going to remain 99 by 100 because that is the accuracy rate, right?

And the probability someone will be tested as positive given that the person has no COVID is going to be the same as 1 by 100, all right? So these are the new probabilities and which I got. And if I plug this new probabilities in the formula that I have derived here, in the formula that I have derived here, I will actually get the updated probability and let us just do that.

(refer time: 05:08)

The new probability is going to be, so what I want to get is probability that the person is having COVID given that he is test positive, the same thing, the same thing that we got previously. This is what we want to get. Probability I have COVID given that I am tested positive. So this is what I want to get. Now what are the values I have to plug here?

I have to plug these values, probability of COVID, probability of positive given COVID. So these values are this probability of COVID is 10 by 10 by 109.9 into I have probability of positive given COVID. So which is 99 by 100. So it will remain just as it is. And what I will have in the denominator? I will have in the denominator probability that a person is who does not have COVID P of NC, which is basically 99.9 by 109.9 into that the person tested positive who does not have COVID.

So 1 by 100 into 1 100 plus the thing that I have in the numerator, which is basically you remember the same A and B, right? So if I add this component here, it will be 10 into 99 by 109.9 by 100, right? So essentially what I have, so you see the term on the numerator, so both have 109.9 into 100 in the denominator and same here. So we can basically cancel these things out.

So what I will have finally is 10 into 99. And then here I will have 99.9 plus 10 into 99, which is 990. So essentially 990 by 990 plus 99.9. It should be something like it is pretty close to 1 actually. If I just add these things up 1089 990. So if I do that, it should be something around around 0.9. It would be around 0.9.

So you can see the jump in the value. So initially, we had in the first test if someone was detected to have COVID it is only, the right probability is only 9%. But whereas if you are detected to have COVID in the second test as well, so your chance is going to be 0.9 or or 90% or above actually. I mean, you do the calculation, but it should be somewhere around 90%.

So this is how you now when you see the second time you have COVID so this is how you update your belief. So initially, you believed that you have COVID and that the probability was 9%. But now you believe that your uh your chance of having COVID is 90% or more. So this is how we keep on updating your belief. And with this, we end this lecture and in the next lecture, we will do another problem on Bayesian theorem.

And we will do the whole exercise together. And we will try to kind of grasp the whole idea of how to actually calculate using the Bayes theorem. Thank you.