Applied Econometrics
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Lecture – 102
Simultaneous Equation Model

Hello and welcome back to the lecture on applied econometry.

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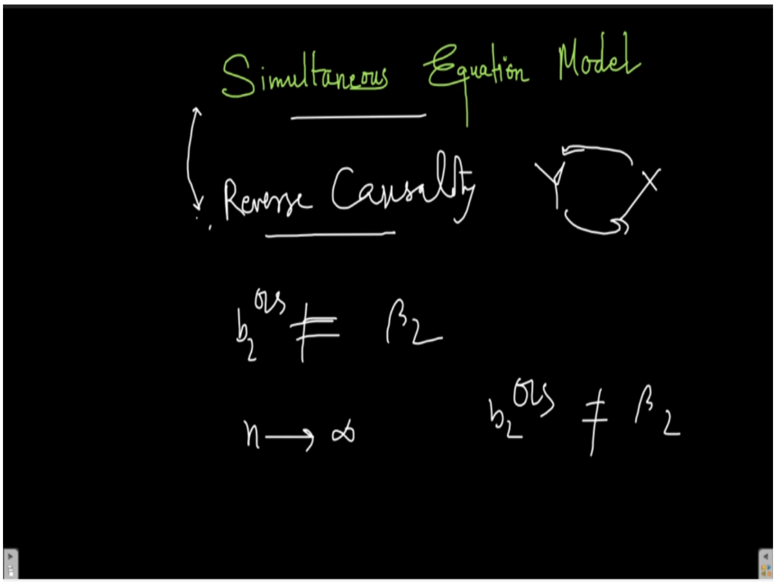

We are talking about instrumental variable and in the previous lecture we spoke in detail about different sources of endogenicity and one of these different sources is reverse causality. Now, by reverse causality, we mean where we have both X and Y present in the model and X influencing Y and at the same time Y is also influencing X. So, there is certain amount of circularity involved in the modeling.

Now, we do not want that. We actually want to avoid this kind of circularity and we will see how to really avoid it. We basically will see that with instrumental variable we can actually avoid this kind of circularity. Now, the reverse causality is something that is often present in simultaneous equation model. And we are going to see what is a simultaneous equation model and how the reverse causality is present?

So, let me actually talk about the, what happens, if we have reverse causality? So, we will have a b 2 ols which is a biased estimator of beta 2 and essentially the other: So, the bias is a problem and the other problem is the inconsistency. Why inconsistency? We mean, if n tends to infinity so, b 2 ols is not going to be beta 2. So, these problems we are going to see when I have reverse causality problem. Now, when we; talk about this simultaneous equation model. So, let me take an example.

Previously, we spoke about the child labour problem where child labour and family income we have seen that both can be used as X and Y variable. And here we will take another example where we will talk about the price rate and wage rate. And I took this example from Christopher Doug at his textbook.

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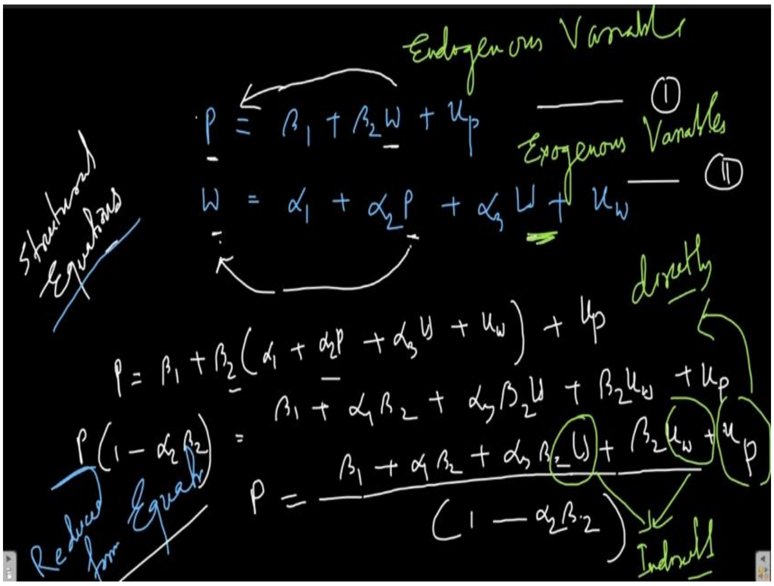

Now, let me give, let me write down a couple of equation and then I will explain the economics behind this equations. The equation 1, lets the price rate is basically price rate is equal to let us say beta 1 + beta 2 wage rate and let us say there is a error term here u p. And the second equation is the wage rate and the wage rate is let us say alpha 1 + alpha 2 price rate and alpha 3 unemployment and then there is an error component.

Let us say I am noting it as u w. So, let me explain the economics behind these 2 equations. So, the first equation, it is basically saying that price rate is actually dependent on the wage rate. Now, how can I explain? How an economist will think this? So, essentially, if the wage rate is increasing so, people will have more money at the disposal. So, they can buy more things with that new you know additional money.

And the moment you can buy more things, there will be more demand in the market as compared to the supply and then the price will increase. So that is essentially wage rate is influencing the price rate. The second equation that can we; can also explain the second equation. So, let us say the prices are increasing. So, when the prices are increasing what will happen is that people working in different organization will ask for a raise.

They will want their wages to increase because they need to, they want to maintain some standard of living. Now, so, basically the price upward pressure on price would also, you know because of the high bargaining power or given bargaining power of the labourer, the wage rate will also feel a upward pressure. Now, on the other hand; if there is a high unemployment levels in the market.

Now, high unemployment will reduce the bargaining power of the labour. So, essentially that will cut down on the bargaining power and then even, if the price rate will increase because of the high unemployment, people may just not you know try to let us say you know pressurize the management to give a raise. So, essentially unemployment is also important so far as wage rate is concerned.

So, these are my 2 equations and we will see how this reverse causality endogenity problem is present here? Now, of course we can see here that in the first equation the w is influencing p, whereas in the second equation, it is p which is influencing w. So, quite clearly we can see you know that the reverse causality problem is present here. Now, let me we can also call this equations actually structural equations.

Now, let us actually try to make these equations, the first equation free of w and second equation free of p. So, let us actually write down p in terms of the other variable and w in terms of the other variable. So, let me substitute equation 2 into equation 1. So, let me write down p is equal to beta 1 + beta 2 into w means alpha 1 + alpha 2 p then we have alpha 3 u and then you have u w + u p.

Now, if I let us say, if I multiply all these terms and then take this one in the left hand side. So, I will get p into 1 – alpha 2 beta 2 is equal to I will have beta 1 + alpha 1 beta 2 + alpha 3 beta 2 u + beta 2 u w and then I have my u p. Now, if I divide both sides with 1 – alpha 2 beta 2. So, I will have p is equal to this is going to be 1 – alpha 2 beta 2 = beta 1 + alpha 1 beta 2 + alpha 3 beta 2 u + beta 2.

This is small u w and the small u p.So, this is my equation of price and in my right hand side I have actually removed the w. And this equation is called reduced form equation. So, the first set of equation is called a structural equation and this is called reduced form equation. Similarly I can also express w in terms of basically by removing p from w.

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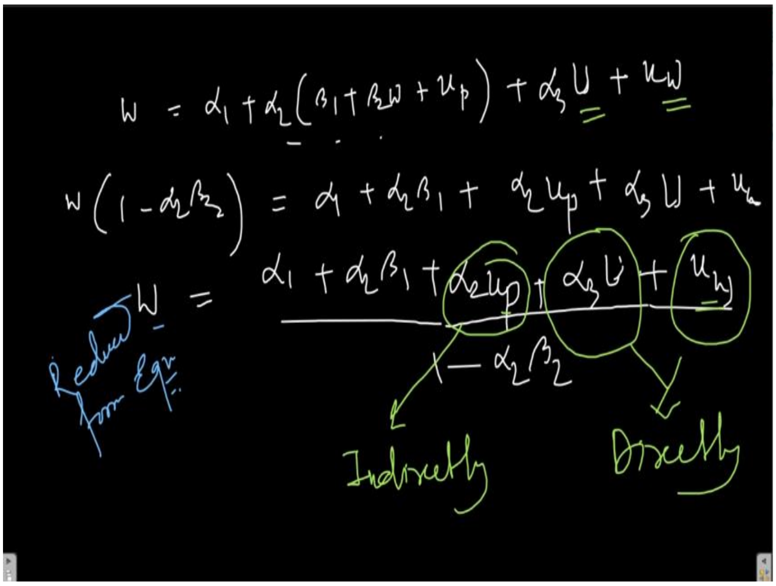

So, what I will do in the second equation I will substitute p and let me take a new page and I will write down w = alpha 1 + alpha 2 now, equation for p was beta 1 + beta 2 w + u w + u p. Because this is I am writing the equation of price and then I have alpha 3 then I will have alpha 3 into unemployment u and then finally I have u w. Now, again like the previous case we will take, we will multiply this alpha 2 with the other values.

And then I will take alpha 2 beta 2 on the left hand side. So, w into 1 – alpha 2 beta 2 is going to be is equal to alpha 1 + alpha 2 beta 1 + alpha 2 U p + alpha 3 u and then u w. Now, if I divide both side by 1 – alpha 2 beta 2. So, what I will get is w is equal to alpha 1 + alpha 2 beta 1 + alpha 2 u p + alpha 3 u + u w by 1 – alpha 2 beta 2. So, note here of course, we have removed p from the equation of w.

Now, this also this equation also known as the reduced form equation, just like the previous one. Now, one thing we need to note here is w is depending on which variables and p is depending on which variables and how the relationship is whether they are directly related or indirectly related? So, in this equation let us first start with w. So, we see that there is a w u w. So, this is there in the you know in the equation of w the first structural equation of w.

So, this one is influencing directly. Similarly u is also present in the structural equation this one is also influencing directly. So, this both are influencing directly. Whereas the u p, this is coming through the root of price. Now, this one is you know mediated through price and this is influencing indirectly. So, there are 3 variables which are or 3 yeah 3 variables u p, u w and u they are influencing w in either directly or indirectly.

Now, similarly for p we can see that we have u, we have u w and we have u p. So, they are either influencing directly or influencing indirectly. So, here U and so, capital U and u w they are actually directing, they are actually influencing indirectly. And this one is influencing directly because that is already there in the structural equation. Now, this variable, this w and p which we can see that there is this relationship of reverse causality.

We mentioned it previously but we mention it again they are called the endogenous variables here. The w and p are endogenous variables. Now, here we have one variable which is not endogenous. So, here w and p is determined by the model. So that is why we call them an endogenous variable. But look at the variable unemployment. So, unemployment is given exogenously, unemployment rate.

So, unemployment rate is not determined by the model. So, it is called exogenous variable. So, this is very important in the context of reverse causality. And you know the endogenic problem and when you actually use instrumental variable to address these problems. So, we will see the importance of exogenous and endogenous variable when we want to use the instrumental variable.

Now, we understood that which depends on what and you know how they are related directly or indirectly. Now, going forward we will try to see, we will go back to the structural equation and we will actually try to estimate get an estimate of the b 2 ols.

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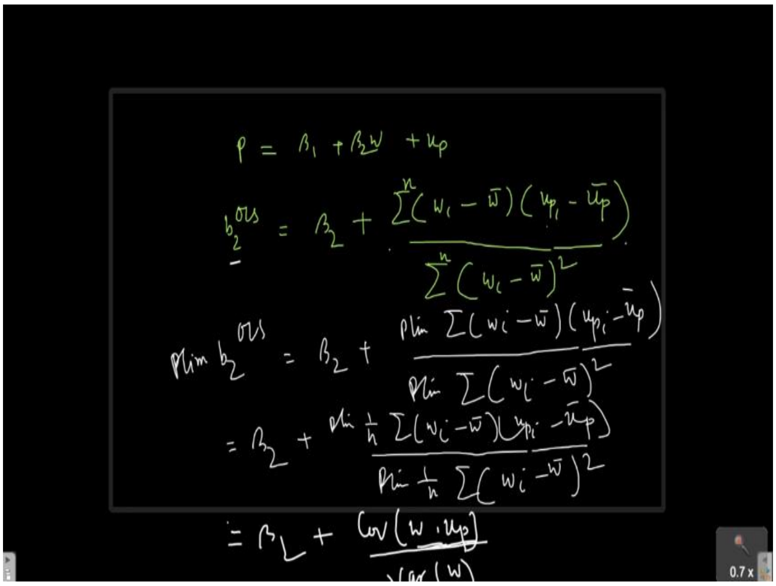

So, let us say P was beta 1 + beta 2 w and u p, small u, p. Now, if I want to get a beta 2 ols estimate for this? So, what I will get is you know following the previous you know we have learnt it previously is equal to summation w i – w bar into u pi – u p bar and in the denominator I will have summation w i – w bar square and this is over all n these are all n. Now, we have seen previously now, if we actually you know we have seen previously that the w is actually dependent on u p indirectly.

Now,, if I use the equation for our this error and take expectation, we will see that the we cannot make this term equal to 0 or if we take the P lim for both the sides, we will see that we cannot make this term is equal to zero. And we can do that let us say P lim, let me use a different color, P lim b 2 ols so, beta 2 is constant. So, it is going to be beta 2 and then I will take P lim on both numerator and denominator.

P lim all over n, w i – w bar and then u pi –u p bar. And then again in my denominator I will have P lim summation w i – w bar whole square. Now, we know that it will not make sense, if I do not divide it by n. So, if I divide it by n both numerator and denominator what I am going to have is P lim 1 by n summation w i – w bar u pi –u p bar and similarly in the denominator I will have P lim 1 by n summation w i – w bar whole square.

So, we know what it means in the numerator, it is basically the covariance term and in the denominator we are going to have the variance term. So, let me write it down beta 2 basically covariance w u p and variance of w. Now, we can, you know sort of substitute the value of w and see what happens to covariance but intuitively we can make sense. Because of the fact that w is actually having of term u p which is basically w is indirectly related to u p.

So, this term cannot be 0. So, once this term is not 0 our estimator is going to be bias. So that is basically the problem and even for P lim when we have seen for n tends to infinity we cannot reach to b 2 ols does not reach to beta 2. So that is the problem of consistency. That is essentially there is a inconsistency problem. So, that is basically h the thing that we see here. Now, you can think it as this going forward. Let us say you have this equation, it is an intuitive sense. How we can think of it?

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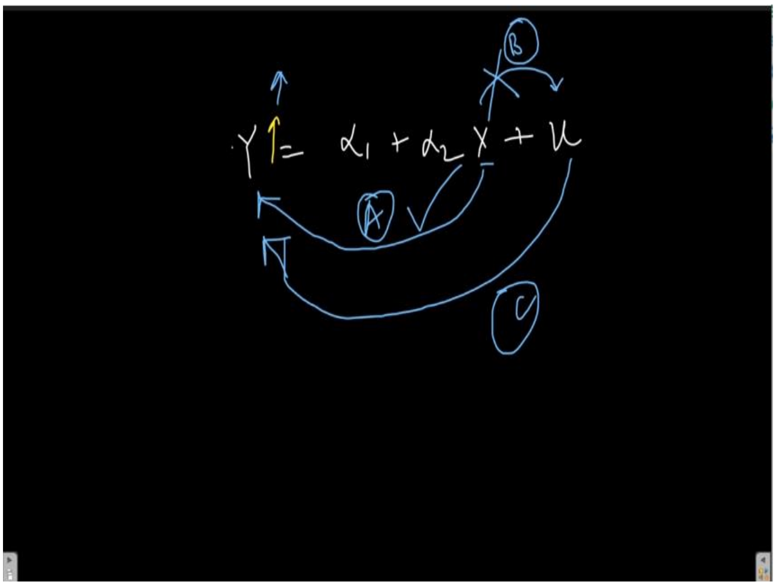

Let us say, I have an equation where Y is equal to let us say alpha 1 + alpha 2 X. And let us say u. Now, when I have this kind of thing here, when my x variable is not free from my error term. So, what will happen is that any change in X is going to influence the error term. So, change in X we are actually interested to know, if the change in X how the change in X is actually influencing the Y term?

So, let us say this is my a process and but we know that if the covariance X and u is not 0 this is also influencing the u term. So, let us say this is a b process and we know that this u term is also influencing the Y and because the u term is influenced by X so that is being compounded in the effect on X. So, essentially what is happening is this Y is going to get all these different influences because of X.

So, this is the problem we do not want to get all these extra influences. We would just want to get this influence here. So, we will try to see how using instrumental variable we can address this problem? And we will see by taking an instrument for which this relationship between X and u and their covariance is 0. We can actually get a result which we want. And that we will cover in the next lecture. So, with this we end this lecture here. Thank you.