Applied Econometrics
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Lecture – 100
Asymptotic Property

Hello and welcome back to the lecture on applied econometrics.

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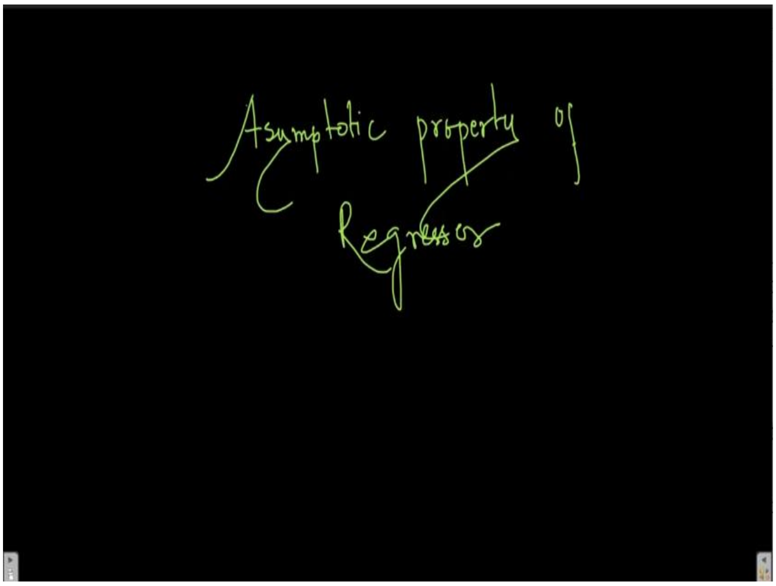

We have been talking about instrumental variable. And in this lecture, we are going to deal with a particular concept called asymptotic property of regressor. And we are going to see why exactly this asymptotic property of regressor is helpful and before that we actually will see you know what exactly is this asymptotic property of regressor? Now, before I proceed to explain this idea. Let me first write down a formula that we have been writing all throughout.

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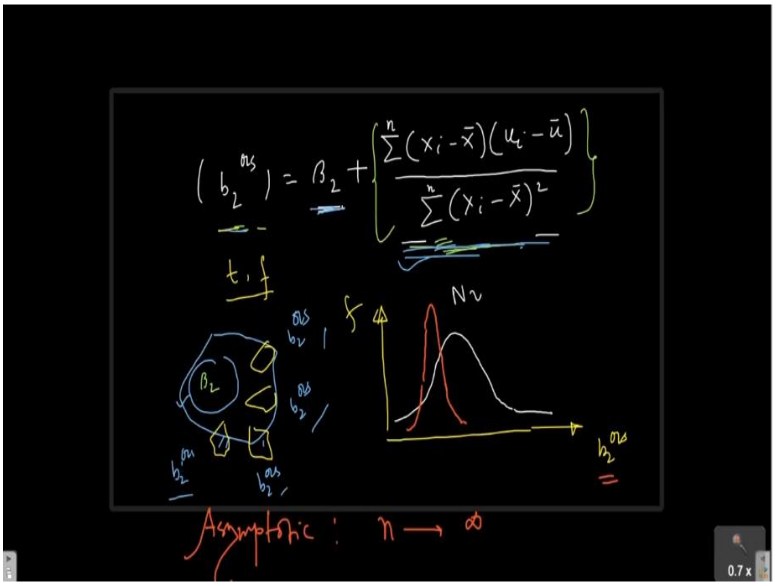

And that is b 2 ols is equal to beta 2 + summation x i – x bar into u i – u bar divided by summation x i – x bar whole square and this all numerator as well as the denominators is over all n. Now, we are familiar with this formulation and this formulation is important particularly this error term is really important in the sense that this error term actually gives a normal distribution of b 2 ols.

This error term this component of error is actually coming from the regression error and we know that regression error is actually, it follows a normal distribution as per the Gauss Markov assumption and we know, it follows normal distribution because the regression error is actually influenced by many, many, many regression you know many, many variables which are not in the regression but actually influences the error.

And since none of the variable has a in you know dominating influence so then we will have a normal distribution of the error term according to 1 variant of the central limit theorem. Now, since I have a part of this error term here so, this is also a normal distribution. Now, when I have this error term normal here and I know that beta 2 is a population parameter and that is basically a constant.

So, essentially this b 2 ols is also going to have a normal distribution. Now, why is it important? It is important because if and only, if I have beta 2 you know beta 2 ols, b 2 ols is normal then I can run things like t test, f test and so forth. Now, having said that we are going to actually see why exactly, how exactly is the asymptotic property and why exactly that is helpful? Now, let us think the population as something like this.

So, let us think how beta 2 and b 2 ols are related? Let us say this is our population and let us say there is a population parameter b 2. Now, you do not have the population, you do not have entire population with you neither you have a specific value of b 2. So, what you can do? You can draw samples, you can draw these different samples. Let us say and you get the values let us say b 2 ols, b 2 ols here.

And you get a b 2 ols here and we get a b 2 ols here, b 2 ols here and you get a b 2 ols and so forth. Now, never mind this part which has come out of this circle essentially, it means that the samples are drawn from this population. Now, I have all this b 2 ols but I will have this b 2 ols values essentially they will be different and the reason why they will be different is the error term.

Because the error term is present there for each of this different you know estimation of b 2 ols. We are going to have this different random error and that is going to give a different value. So, b 2 ols will not have a fixed value like beta 2. Of course b 2 ols contains beta 2 but along with that it contains this error component. Now, if that is the case now, if I plot so, essentially what I will get is a b 2 ols, a distribution of b 2 ols.

So, because this error component is normal the distribution of b 2 ols is also going to be normal. I can say that safely that the distribution of b 2 ols is going to be a normal distribution. Let us say this is the, you can plot probability density or frequency in the x axis y axis and the value of b 2 ols in the x axis. Now, if we plot this we will get a normal distribution. Now, when we actually talk about the asymptotic property.

Let me reduce the size a little bit, when I talk about the asymptotic property by asymptotic means. What happens when N tends to infinity? Asymptotic property means what happens to the b you know regressor here when N tends to infinity? So, I will try to see that. Now, we know for a fact that if N is large or, if N is increasing so, what I will have by distribution of b 2 ols will come close and close to the true value of beta or beta 2 here.

So, if that is the case so, what I will have is that I will have a narrower and a narrower distribution of b 2 ols and which is actually going to be close to our true beta 2. Now, so then, it means that if I increase the number to infinity, I am actually going to get a better estimate.

Now, that is important. Now, if I want to now, actually use this formulation I cannot so, previously when I was estimating this b 2 ols. I was actually taking expectation value on both the sides.

But here when N is infinity and i and actually the you know with more complex relationship I will have a formulation of limit. I will take limit instead of expectation on both the sides.

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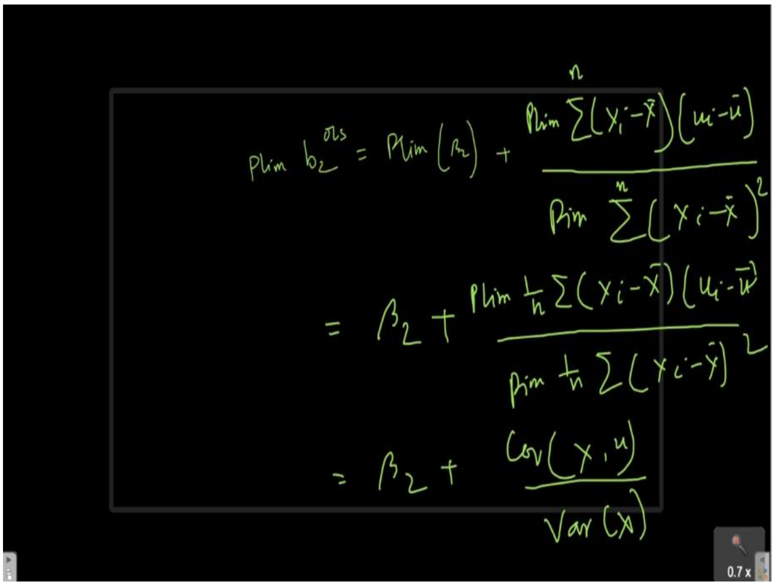

So what I will do? I will write P lim b 2 ols is equal to you can take P lim beta 2 and you will take, let us reduce the size a little bit, P lim on the numerator, summation of x i – x bar and u i – u bar. In the denominator you again take P lim but in this case you are going to take summation of x i – x bar whole square. Now, what I will get? So, P lim of beta to beta 2 is constant so, it is going to be beta 2.

Now, what I get in the numerator denominator? We do not get anything from this formulation because as i increase i, i as i keep on increasing N so, what will happen is that I will have larger and larger values. So, I will not come to any convergence but I want some convergence. Now, if you see, if I can actually multiply both numerator and denominator with 1 by N so, I can actually come to some result.

If I can multiply with 1 by N so, I will come to some result and the result on the numerator is going to be the covariance between x and u. Whereas the result in the denominator is going to be the variance of x. So, I will write that 1 by N summation of x i – x bar. So, essentially that is going to be beta 2 plus covariance x u and variance of x. So, this is the application of P lim and this is the result.

Now, you may wonder previously we used expectation rule and we got result same as what we got using P lim. But why exactly we bother about using P lim? Now, the reason is that as I mentioned previously is that sometimes what happens is that the when I go into the paradigm of stochastic regressor and the formulation becomes complex, we need with the expectation rule does not work anymore.

So, we need something more powerful, more robust and that is why we use P lim. Now, previously we have been using a non-stochastic regressor and it was absolutely fine to use the expectation rule. But now, we are into the domain of stochastic regressor and we see the problem of endogeneity and when we talk about instrumental variable we will solve the problematic endogeneity which is essentially a complex relationship between the regressor as well as the error term.

We are going to talk about the endogenous in the next lecture. So, essentially because of this complex relationship that we have between the x i's and the error terms so, we need to use p lim. So that is basically the reason why we need to use P lim and we will see the application of asymptotic property of the regressor going forward. So, with this we end this lecture here. Thank you.