Behavioral and Personal Finance

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Module – 02
Personal Finance
Lecture – 36

Loans and Amortization (Contd.)

Hello. Welcome back to the course Behavioral and Personal Finance. And in the module of personal finance; we have been discussing in previous few lectures about the sources of consumer credit and as part of consumer credit we have discussed previously. The features and mathematical understanding of loans and how loans are amortized for principal and interest payment. So, that the loan can be paid off by that borrowers and lenders receive the interest and principal amount.

In this session we will try to understand the basic mathematics behind the amount that is to be paid by borrower to lenders at every compounding interval.

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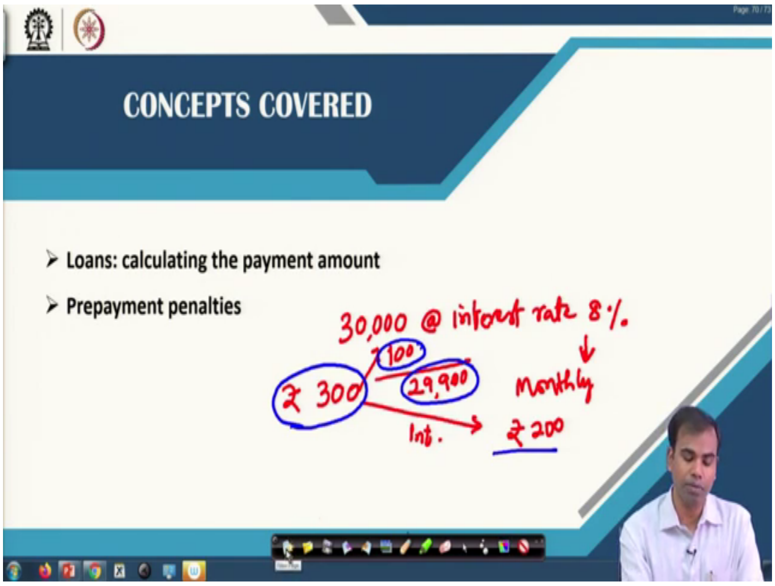

Basically, we try to understand how do we calculate the payment amount and if there are any prepayment penalties that can be considered in that formula itself.

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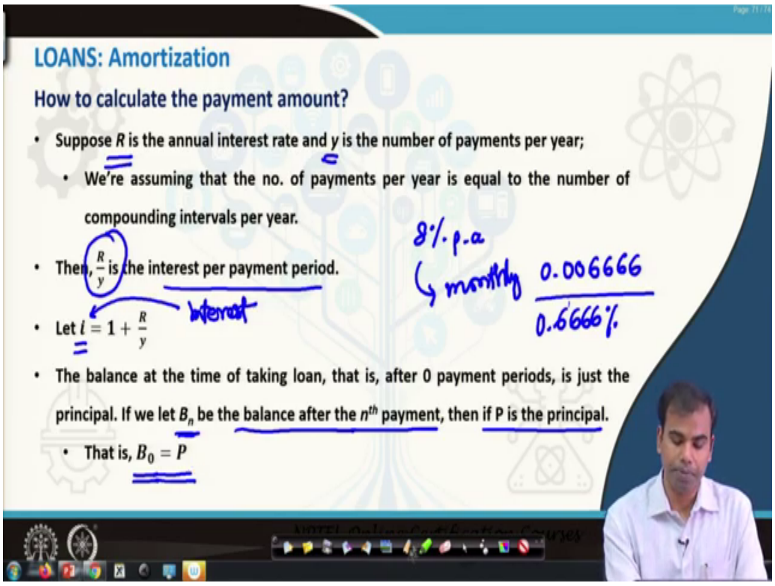

So, previously, we have understood that if we have taken a loan of 30000 rupees which is charged at an interest rate of 8 percent and that interest rate is compounded monthly. We pay 200 rupees of interest in the very first month. And if we choose to pay 300 rupees as first installment of this 300, 200 will go towards interest payment and 100, remaining 100 will go towards recovery of this 30000 of principal amount that will leave us with the principal amount in loan account to the extent of 29900.

Now, the question here comes as; how do we determine this 300 rupees to be paid that comprises of interest and principle repayment? In this session; we will try to discuss with the help of some basic understanding of mathematical features. First of all, let us start with the basic assumption. Suppose that R given here is the annual interest rate which is basically the rate of interest that is charged on in loan account and y is the number of payments per year.

Essentially, here we are assuming that the number of payments per year is equal to the number of compounding intervals per year.

If we recall in previous session we discussed that this is the typical feature of any loan repayment condition terms and conditions, where the number of payments per year would be equal to the number of compounding intervals per year. So, suppose if you have taken a loan that compounds monthly; the number of payments that you will be making will be 12, because the number of compounding intervals is 12 as well.

So, if R is the annual interest rate and y is the number of payments per year, then basically the payment that you are making in terms of interest rate is R by y that is interest per year. In previous example, we know that 8 percent interest rate per annum compounded monthly becomes 0.006666 in terms of percentage, which is basically 0.6, this much percentage.

So, here this interest payment which is basically given as i. So, this i is basically interest amount or interest payment as given as 1 plus R by y. Now, if we try to know the typical terms and conditions associated with a loan, the balance at the time of taking loan that is when you have not made any payment which is basically no payment has been made towards recovering the loan, it is just the principal amount. If you recall from previous example it was 30000 in all cases. If we let B n which is basically the amount of balance after nth period then if principal P is the principal, then B 0 which is basically balance after 0th period is nothing but P.

Now, we have set the basic terms and condition and notations. Let us move further. So, we have known that R is the rate of interest and y is basically the number of payments.

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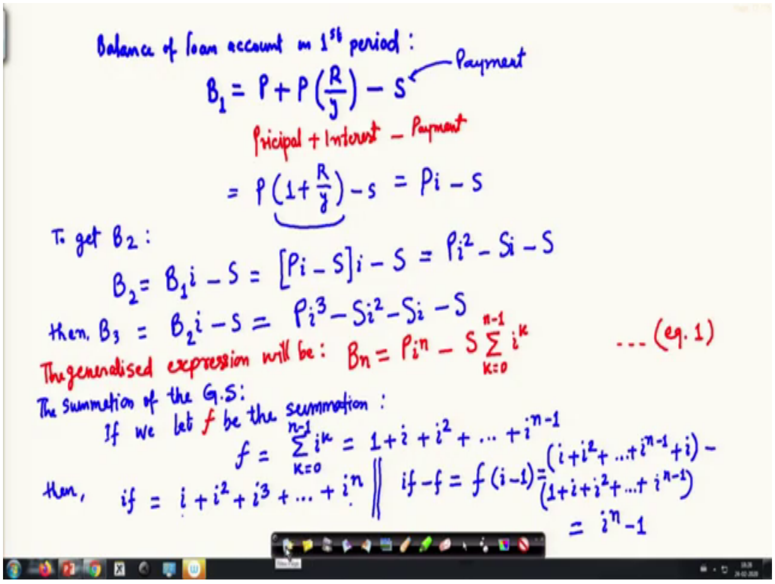

So, the payments that we will be making or if we go the to the balance of loan account in 1 th period or 1 st period which is the next after 0 will be nothing but B 1 is equal to principle payment plus principle into R that is rate of interest by y minus the payment that we have made.

So, essentially it is principle interest minus payment that you have made. If you try to simplify this, it will be P into 1 plus R by y minus S and previously we have defined this as i. So, basically this becomes P i minus S. To go to the next level, so basically to get B 2, we get B 2 is B 1 into i minus S which is nothing but, P i minus S into i minus S or we can also write this as P i square minus S i minus S.

Similarly, if we go to B 3 will be B 2 i minus S which is basically payment. Before you forget this s is payment which we have already defined is equal to P i cube minus S i square minus S i minus S. We are extending the same argument here. So, a generalized function will be or generalized expression of this particular sequence will be B n is P i n minus S sum of all i for K period, where K is 0 and here n minus 1. So, let us consider this as equation 1.

Now, we know that this particular summation is a geometric progression or geometric series. So, the sum of the geometric series will be, let us say if we let in a different color code f be the summation , then f will be sum of all i for K, where K is 0 and it would be n minus 1. Which is also expressed as 1 plus i i square and so on till the point when i to the power n minus 1. And then we can say that i into f is equal to i plus i square i cube and so on, till i to the power n.

Taking this expression further; we know that if i f minus f, this can be expressed as f i minus 1, which is which can be written as i plus i square which is the previous case; plus i cube and so on i n minus 1 plus i n. This is the first expression minus 1 plus this previous expression, i plus i square and so on till i n minus 1. This is the second part of the expression and this can be expressed at the end as i n minus 1.

Now, so far, we have seen where f has been the summation of geometric series as shown earlier. In this case also here we have taken f as the summation of all i s for given k that is the period for which it is compounding and it is shown as 1 plus i, i square, i cube and so on till i n minus 1. And then if we multiply on both side we get i f is equal to i plus i square i cube i 3 to the power 4 and so on, till i n. And if we have i f minus f which is f taken common i minus 1, then we have this expression taken separately which is leading us towards i n minus 1. Now, if we try to simplify this particular expression we go towards.

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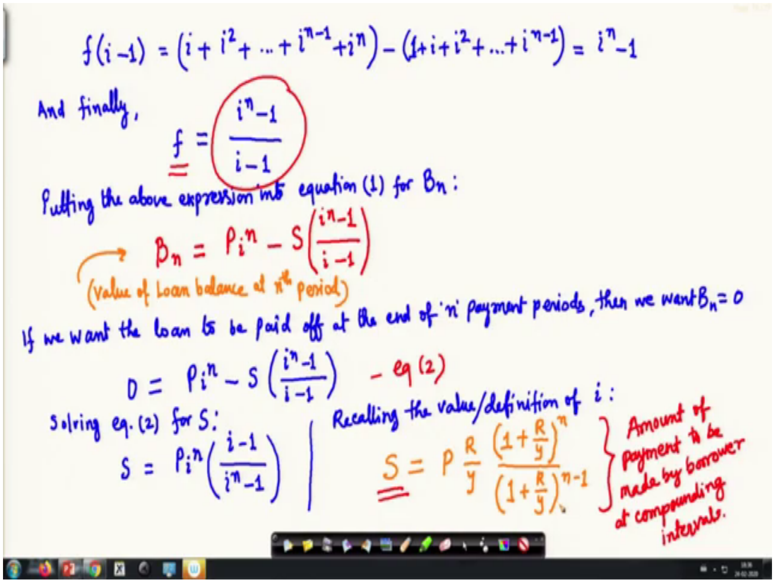

If we rewrite the same thing which is basically, f i minus 1 is given as i, i square, i cube and so on till i n minus 1. And i n minus i, i square and so on till i to the power n minus 1, which is i to the power n minus 1. So far, if we take this further to a final expression; this will be f is equal to i to the power n minus 1 divided by i minus 1 right.

Similarly, if we plug this particular expression of f which is basically summation of all i s in the equation putting the above expression into equation 1 for B n. Then the expression will become B n is equal to P i n minus S i n minus i by i minus 1. Because, that is what the summation of all is was given as f and f is expressed as this particular function.

Now, this particular thing tells us the value of loan balance for nth period. If we try to write it here value of loan balance at nth period is given as principal interest and the payment as a function of all is. If we say that, if we want the loan to be paid off fully at the end of n period, then the value of B n should be 0 right. Because, you want to pay off all the amount that is balance in loan account at the end of nth period. So, if you want the loan account to be 0 at the end of nth period so B n will be 0.

Now, if B n is 0, then this above function which is let us say 0 s in place of B n as a value of P i n minus S i n minus 1, i minus 1 and this is our let us say second equation. If we solve this for x, solving equation 2 for S which is basically the payment we get S is equal to principal in interest and then i minus 1 divided by i to the power n minus 1. This is what we get if we solve this for x and if you recall the value of i. So, plugging in recalling the value of a value or definition of i and plug in here. We know that S S is equal to P, i was defined as R by y and 1 plus R by y, 1 plus R by y to the power n, here to the power n minus 1.

This particular function basically the amount of payment to be made by borrowers at compounding interval. This is the amount of money that is to be paid at every interval of compounding by the borrower to the lender, which is basically P as the amount of principal which is the loan account into i; that is R by y into 1. This is one and these two values are taken from the previous expressions. So, this expression tells us how much amount has to be paid by the borrower to the lender as payment at every interval of compounding.

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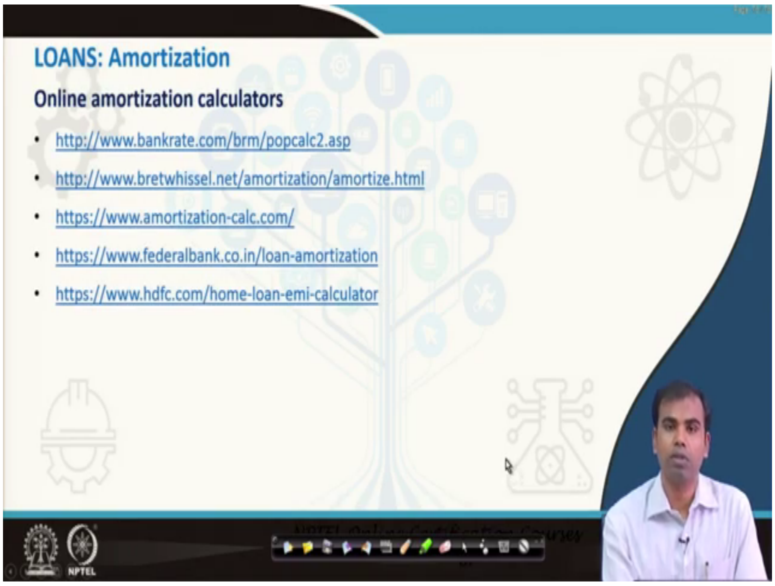

Once we understand this, this helps us the solution to the problem of amount of money to be paid at every compounding interval. Although this formula as is very mathematically intense, but when we try to understand a more simplified approaches to determine the amount of money that is to be paid by borrowers to the lenders in terms of loan amortization or any other amortization of loans or any other credit that the consumer has taken. There are several sources online available which can be used for determining the amount of money to be paid by the borrowers to the lender at every interval.

Here are some sources of amortization calculator which you can use if you have certain information such as the amount of money that you are planning to borrow. The rate of interest that is being charged the tenure and the type of compounding or frequency of compounding. You can use these online calculators to understand the amount of money to be paid and then compare across different loan alternatives that you have to determine which is most suitable or most fitting into your budget and financial planning.

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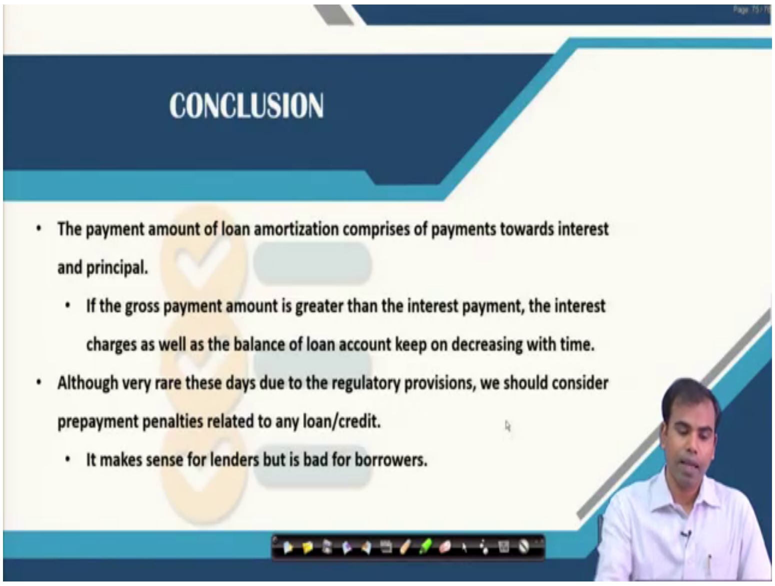

To conclude this session; we have discussed that the payment amount of loan amortization comprises of payments towards interest and principal and it is always better to have higher than interest charges as payment because that will contribute towards interest payment as well as towards the payment of principal amount. And if it is higher than it is always going to reduce your due principal amount on which that interest will be calculated. And that is how the interest charges will also lower down with every passing period.

Although there are several sources of online amortization calculator are available, but you can always try to understand what is most suitable to your financial needs and monthly spending and earning pattern that will take determine the amount of money to be paid for loan amortization.

Here I would like to point out that although it is very rare these days that prepayment penalties are charged to borrowers, but we should always try to consider this prepayment penalties related to any loan or credit basically these are the penalties when you pay the loan amount before the maturity.

Basically, if you have taken a loan for let us say 4 years and for some reason you have got a more amount of money before 4 years you would like to pay off all the loans and in the process certain financial institutions or sources of loan can charge you prepayment penalty because this is perfectly making sense for their business model.

Although, this is discouraged by most of the regulators in India and in some other countries as well, but it is always bad for the borrower to be charged for prepayment penalty. That is why when you are planning of our loan and repayment schedule you always consider this prepayment penalty as part of your financial planning.

For now this is it.

Thank you very much.