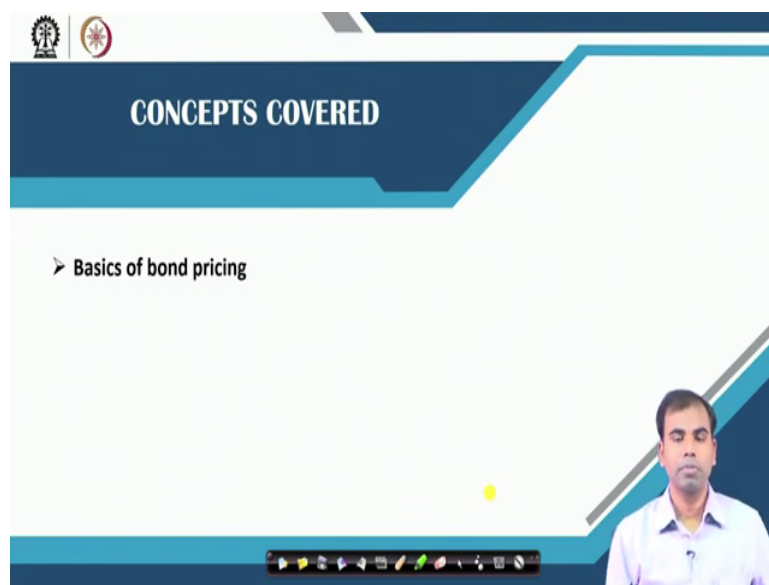


Behavioral and Personal Finance
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Module – 02
Personal Finance
Lecture – 32
Fixed Income Investments (Contd.)

Hi there, continuing with the previous discussion that we were having on fixed income securities, in this session; we will discuss more about the pricing of the bonds and how bonds are evaluated in terms of their value in our portfolio.

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The topic that we are focusing in this session is the pricing of the bond; basically, based on the argument that we have built earlier when we discussed about the present value discounting

method where we try to understand, how the present value of future cash flows can be determined with the help of a discounting factor or a rate which is used for discounting future values and how these methods or the method of discounting of future cash flows can be implemented in terms of valuation of bonds.

The idea of understanding the valuation of bonds is to make sure that when you try to diversify your investment portfolios and include fixed income securities, particularly bonds in your portfolio so that you can earn a decent amount of return or a fixed return from your investment without compromising much on the risk.

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Introduction to bond pricing

Three-step process

1. Obtain the cash flows that the bondholder is expected to get (over holding period)
2. Obtain the discount rates for the maturities corresponding to the cash flow dates
3. Obtain the bond price as the discounted value of the cash flows

$$PV(\text{Bond}) = \sum_{t=1}^T \frac{CF_t}{[1 + R(0, t)]^t} = \sum_{t=1}^T B(0, t) CF_t$$

Market Price ← *Cash Flows (coupon × FV)*
Discounting rate
Discounting factor

Simple principle → Sound practice??

- Where do we get the discount factors, $B(0,t)$, from?
- Do we use the above equation to obtain bond prices or implied discount factors?
- Can we deviate from this simple rule? Why?

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So, the discussion or the understanding of valuation of bonds is basically a three step process. So, the bond or as any similar financial asset can be evaluated by following these three steps and the steps include the calculation of cash flows associated with the bond or the instrument

that you are holding, basically, cash flows coming to the bondholder that he or she is expecting in terms of cash flows coming from the lender and expected to get over the holding period.

Basically, the cash flows that the investor is expecting from the bond investment over the holding period should be determined first, then it should be decided what are the maturities associated with different instruments or different bonds that he or she has invested in and with maturities; the discount rate should also be determined. Because, we know that cash flows in terms of coupon rate or interest rate are coming at different points of time in future. So, for every time of point we need to understand the maturity as well as the timeframe and associated discount rate.

And third step is to use the present value of all those future cash flows to obtain the bond price as a result of discounted cash flow value. To simplify the formula that we follow looks like as given on the screen. So, if you see, the formula says that there is a cash flow associated with the investment and then there is a rate. So, this basically indicates cash flow that as basically you are getting in terms of coupon rate or interest rate that you are receiving on your bond investment coupon into face value, because that is what you are going to get.

And then this is your discounting rate. When you try to use discounting rate for multiple of periods or multiple time points you get something known as discount factor. So, if you try to find the present value of all coupon payments that the investor is receiving over a period of time and the present value of all those future coupon payments are summed up together; you find the value of or you can call it the present value of the bond. And when this present value of bond is compared with the market price, you can decide whether the bond that is available for investment is worth buying or not.

So, the whole idea is depend depending on the present value of all future cash flows in terms of coupon payments that the investor is expecting to receive over holding period. Now, this is very simple. This idea is or the process is very simple in theory and principle whereas, if you try to convert this into a sound practice for the benefit of the investor. You need to discuss or understand three pertinent questions and these questions are the source of discount factors.

Basically, how do we calculate or how do we know the discount factors, because everything is depending on the discount factor as given in the function here. And second point that we consider is whether the use of equation as specified here is only meant for considering bond price or to calculate the implied discount factor. So, that we can calculate the present value of bonds. And finally, can we deviate from the simple rule and if yes why and if no again why we should not deviate from the simple rule.

The simple rule here is; we need to find the present value of all future coupon payments and those coupon payments can lead us to the present value of bonds and that present value of bonds can be compared with the market price or purchase price of that bond to make it a worthy investment.

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Time Value of Money
Discounting Method

$$P_0 = \sum_{t=1}^T \frac{CF_t}{(1+r)^t} = \frac{CF}{r} \left(1 - \frac{1}{(1+r)^T} \right)$$

Price Today ← Present Value of Annuity Cash Flows
 $CF_1 = CF_2 = CF_3 = \dots$

More generally, we obtain:

$$P_0 = \frac{C}{r} \left(1 - \frac{1}{(1+r)^T} \right) + \frac{N}{(1+r)^T}$$

← Coupon recd. ← Nominal Value
 ← Tenure
 ← Discounting rate

Let's have a BOND
 Annual Coupon rate = 5%
 Maturity = 10 yrs.
 FV = ₹1000
 All discounting rates = 6%
 5%

Timeline: t_0 to t_1 to t_2 to ... to t_{10}
 Cash flows: C at t_1, t_2, \dots ; $C+N$ at t_{10}

$$P_0 = \sum_{t=1}^9 \frac{50}{(1+6\%)^t} + \frac{1,050}{(1+6\%)^{10}} = ₹26.39$$

Price of Bond

Let us try to understand this from the framework of time value of money. If we try to use this to understand, the first concept that we should always keep in mind is time value of money, because we have agreed that the value of money changes over time and this concept must be considered when you are taking a financial decision that is spanning over multiple periods.

So, when you use time value of money concept to calculate the present value of future cash flow; basically, you follow discounting method or discounting approach of finding present value. So, the approach says that the price today of any financial security is basically the present value of cash flows that you are receiving discounted by a rate discounted for cash flow for a number of period that you are holding the asset and this should be sum up together to know the present value of all those future investment.

Now, this can also be rewritten as cash flow as by rate of discounting and $1 - \frac{1}{1 + r^T}$ as your function of finding the present value. If you recall that our earlier discussion; this basically indicates the present value of annuity cash flow with the assumption that CF_1 is equal to CF_2 is equal to CF_3 and so on.

So, all CFs are basically constant and you can find the present value of any cash flows associated with an investment and you can sum it up to know the price today. In a more generic framework; in a very simplest simplified, we can obtain or we can write it such that the price of any asset today is $C \times \frac{1 - \frac{1}{1 + r^T}}{r}$ that is discounting rate for number of years for which it has been held plus $\frac{N}{1 + r^T}$ divided by $1 + r$ to the power number of years for which it has been held for.

Now here, this is your nominal value or face value that you have invested in the beginning or the face value that is the bond or the investment is carrying T is your tenure or maturity period and r is discounting rate, C of course, is your coupon or interest that you are receiving. If you try to see this through an example, maybe we can take up an example where let us take a let us have a bond which has the following characteristic. So, a bond has annual coupon rate as 5

percent. Maturity is given for this particular bond in example is 10 years and face value or nominal value is let us say 1000 rupees.

If you remember, we showed in previous session; the example of a bond where I showed you a bond certificate that carries all these characteristic. The face value was 1000 rupees, maturity was in that case 10 years and coupon rate was in that example 7.1 for 4 percent, here we have 5 percent for example. And along with this, we also carries all discounting rate for our different maturity being 6 percent.

So, if we follow this particular this particular case or this particular example; remember the time line approach we discussed earlier. So, we have a time line that is t_0 t_1 t_2 and so on till t_{10} , which is basically number of years of maturity. And then you have cash flow coming in every period. Here, you get cash flow as well as the nominal or face value.

So, when you try to find the price of the bond today; you actually get some of all coupon which is 5 percent of 1000 rupees. So, 50 divided by $1 + 6$ percent; which is your discounting rate for t years, where t is going to be 9 plus in the end you get 1050, because you get 50 of coupon and 1000 of nominal value. So, you get 1050 discounted with $1 + 6$ percent; that is your discounting rate for 10 years.

If you follow this approach, you can plug in the formula given here. If you follow this approach you can use the same thing in alternate case; we can keep this for 10 years and we can reduce the amount to 1000 rupees here. So, this function gives you a value of, if you try to calculate the value of this particular function or this particular value of cash flows; you try to get 926.39.

So, we can say that a bond of 1000 rupees face value is worth 926.39 today. So, that is the market, that is the price of the bond based on the feature or the characteristic as explained here. If you are able to get this bond for lesser than this. If you are able to buy that bond for less than 926.39, it is always a wise investment decision. If you are getting it for more than

926 9, it is not a good investment to make. Suppose, we take the same example, but we replace this particular discount rate the 6 percent to be 5 percent.

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Bond, $FV = 1000$
 $c = 5\%$
 $T = 10 \text{ yrs.}$
 $r = \text{6\% } \rightarrow \text{5\%}$

Timeline: t_0 to t_{10} . Cash flows: 50 at $t_1, t_2, t_3, \dots, t_{10}$. Final cash flow at t_{10} is $50 + 1000$.

$$P_0 = \frac{50}{5\%} \left(1 - \frac{1}{(1+5\%)^{10}} \right) + \frac{1000}{(1+5\%)^{10}} = 2,100$$

Labels under the formula:
 - $\frac{50}{5\%} \left(1 - \frac{1}{(1+5\%)^{10}} \right)$: PV of coupon payments
 - $\frac{1000}{(1+5\%)^{10}}$: PV of MV at maturity

Rule: If Discounting Rate (r) equals to coupon rate (c), the bond value is equal to the Face Value.

So, let us take this example again, if we try to take this bond where we have face value of 1000, coupon as 5 percent, maturity as 10 years and discount rate as earlier it was 6 percent, but now instead of 6 percent let us have it 5 percent.

So, we will do the same exercise again. We will have a timeline, where we will plot the all cash flows that we are expecting to receive. This is my timeline; the coupon that I am getting is 50 till tenth year. And tenth year I am getting 50 plus 1000, which is my coupon plus face value. And if we try to find the price today; we can simply use either of the formula discussed earlier where 50 divided by 5 percent and 1 minus 1 by 1 plus 5 percent for 10 years plus 1000 rupee of this particular money coming back discounted for 1 plus 5 percent for 10 years.

If you try to use the this particular; approach which is basically, the present value of coupon payments that we receive and present value of nominal value that we are receiving at maturity. So, if we use this particular approach what we are going to get is; 1000 rupees and this gives us a remark or the rule that if discounting rate which is basically the rate at which you discount the future cash flows. Discounting rate are equals to coupon rate, which is basically the rate at which interest are paid. The bond value is equal to the face value.

So, this rule is very simple, if you have to make a decision; whether to invest in bonds or not. You need to find our discounting rate and a bond with some coupon rate that will give you an opportunity to calculate the present value of this bond and then you can make a decision whether to invest then in this particular bond or not.

Moving forward, if we take this example of value calculation for bond investment and we know that so far, we have discussed bonds which provide coupons paid annually. If you try to calculate value of a bond that pays coupon semi annually which is basically twice a year then we have to modify the formula that we consider.

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Bond pays Coupon SEMI-ANNUALLY $\left\{ \begin{array}{l} \# \text{ periods } (\times 2) \\ \% \text{ coupon } (\div 2) \end{array} \right.$

$$P_0 = \sum_{t=1}^{2t} \frac{N \times \frac{C}{2}}{\left(1 + \frac{r}{2}\right)^t} + \frac{N}{\left(1 + \frac{r}{2}\right)^{2t}}$$

Simplified to:

$$P_0 = \underbrace{\frac{N \times C}{r} \left[1 - \frac{1}{\left(1 + \frac{r}{2}\right)^{2t}} \right]}_{\text{PV of coupons recvd.}} + \underbrace{\frac{N}{\left(1 + \frac{r}{2}\right)^{2t}}}_{\text{PV of payment at maturity}}$$

YTM (Yield to Maturity)

Bond, $C = 8\%$, $T = 2$ yrs, $FV = 1000$, semiannually. sells for $103.23 \rightarrow 103 + \frac{23}{32} = 103.72\%$.

$$1037.20 = \frac{40}{\left(1 + \frac{r}{2}\right)} + \frac{40}{\left(1 + \frac{r}{2}\right)^2} + \frac{1040}{\left(1 + \frac{r}{2}\right)^2}$$

Using trial and error method, $Y = 3\%$, $r = 6\%$

So, suppose a bond that pays coupon which is basically the interest semi annually. So, semi annually means; the coupon payment that is coming to you is twice a year. So, if it is semi annually which results in the number of periods or the frequency of your coupon receipt is multiplied by 2 and the rate of coupon that you are expecting to receive is divided by 2 right because, the frequency of interest coming to you has increased twice and the coupon rate that was promised by the issuer is divided by 2, because you are receiving twice. So, half of the coupon rate will be paid after first 6 months and remaining half of the coupon will paid after second 6 months.

If you try to modify the formula that we have discussed just now, we can calculate the present value of a bond that pays and a coupon semi annually. So, n is the phase value into coupon rate by 2; that is our coupon payment $1 + r$ by 2 is your rate of discounting and t is your time period that you have considered and N is your final payment that you can discount with 1

by $1 + r/2$ to the power $2t$. Where, this should be summed t is going to be 1 and this is $2t$. Because, this t is number of years for which you have held this particular bond and it is it can be simplified into a formula which goes like this where you have phase value into coupon payment divided by r into $1 - 1/(1 + r/2)^{2t}$, which is basically twice that frequency plus phase value divided by $1 + r/2$ and $2t$.

So, this is your present value of all coupon payments, present value of all coupon payments that you are expecting to receive. And this is present value of payment received at maturity. So, this is how you can calculate the present value of bond which are paying you coupons twice every year. A related concept is known as yield to maturity or also known as YTM basically, this is yield to maturity the concept says that if you hold the bond for the period that you have invested in and you consider a rate or discounting rate or a rate of reference at which the present value of all coupon payments that you are expecting to receive is equal to the bond value that you have invested then this rate is supposed to be YTM or yield to maturity.

That is one of the criteria for investing in bonds you should always try to compare yield to maturity with different bond investments and you can use this as a decision criteria. So, suppose in this example; if I have a bond which gives me a coupon of 8 percent and maturity is 2 years, where phase value or nominal value is 1000 rupee the year which is paid as let us say semi annually and sells for 123, let us say let us revise the number 103.23 which is basically the coat price.

So, this corresponds to the rate which can be calculated as $103.23/72$. So, this gives you a rate which can be equal to 1.432083 percent which is basically a premium of 3.72 percent. So, we know that for every 1000 rupee of bond. We have a present value is equal to 1.03720 rupee is equal to the coupon payment that you are receiving twice.

So, 80 rupee, so coupon will become if we calculate coupon here it is 8 percent of 1000 rupees that is 80 divided by 2 then becomes 40. So, you have 40 rupee of coupon $1 + r/2$ to the power r divided by 2 $40/(1 + r/2)^2$ $40/3$ half $1 + r/2$ divided by 2

power 3. And then in the fourth half here, you receive 140 1040 rupees and discounted by r by 2 to the power 4.

So, if you use trial and error; the method of equating these 2 sides. If you use trial and error method the rate at which we can find this to these 2 sides of the equation is basically r by 2 will be 3 percent, where r is going to be 6 percent. So, this is my yield to maturity in this case. So, this particular 6 percent is my yield to maturity, which indicates the rate at which the future value of all these cash flows or coupon payments is equal to the present value of the bond.

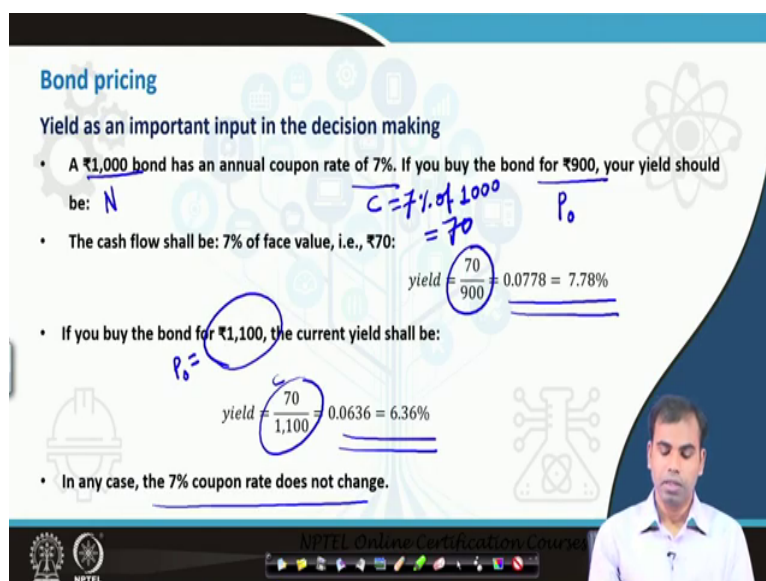
So, present value is basically coming from that code or that price that we have received from the market and this is how we calculate the present value of bonds and associated characteristic in terms of yield to maturity. Another example of this particular sequence of discussion on bonds and associated characteristics such as yield is given in terms of yield that is an important input for decision making.

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Bond pricing

Yield as an important input in the decision making

- A ₹1,000 bond has an annual coupon rate of 7%. If you buy the bond for ₹900, your yield should be: N
- The cash flow shall be: 7% of face value, i.e., ₹70:
 $C = 7\% \text{ of } 1000 = 70$
 P_0
 $\text{yield} = \frac{70}{900} = 0.0778 = 7.78\%$
- If you buy the bond for ₹1,100, the current yield shall be:
 $P_0 = 1100$
 $\text{yield} = \frac{70}{1,100} = 0.0636 = 6.36\%$
- In any case, the 7% coupon rate does not change.



We have already discussed. So, for example, if you have a bond which has 1000 rupee of face value and annual coupon rate of 7 percent. If you want to buy this bond for 900 rupees, the yield should be.

So, here this is your market value or rather you can say, which is nominal value, coupon rate and price. So, yield is always given as 70 by 900. So, 7 percent is basically, 7 percent of 1000 rupees which is 70 rupees. So, this is your 70 rupee of coupon. This gives you and yield of 7.78 percent.

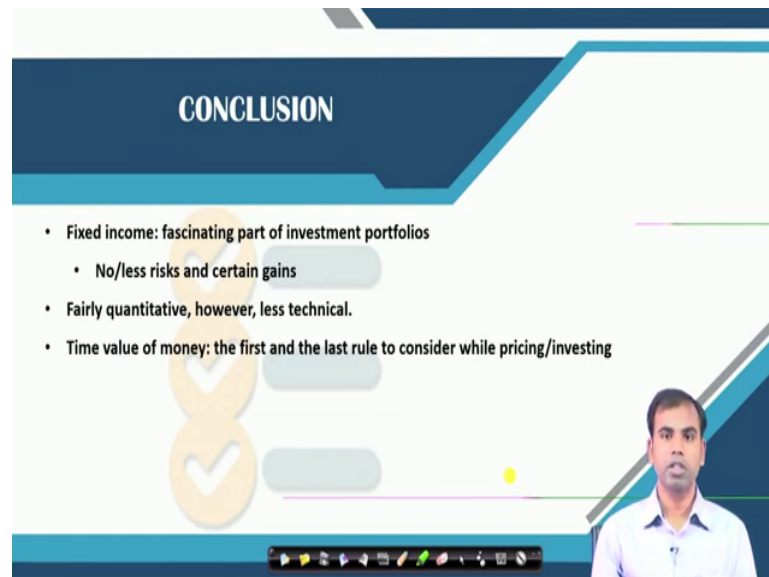
So, if you buy this bond for 1000 11 with 1100 rupees. So, instead of 900 you buy it for 1100 rupees; your yield coupon will remain same, this is your coupon. So, coupon will remain same

7 percent of 1000 rupees, but yield will go down. So, this is another input that you can consider while making decisions to invest in your bond portfolio so.

So, far we have understood including the discussion that we have had earlier is the possibility and advantage of including risky assets in your portfolio such as mutual fund and other equity instruments. We have also discussed how to find the value of bonds to be included in your portfolio and the advantage of including bonds and other money market instruments such as treasury bills commercial papers and certificate of deposits as a risk lower risk and assured return instrument.

These two these two approaches of including risky and risk free or rather a riskless asset in your portfolio helps you diversify your investment. And thereby you can optimize your risk return trade off subsequently, you can earn higher rate of return for a given level of risk.

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CONCLUSION

- Fixed income: fascinating part of investment portfolios
 - No/less risks and certain gains
- Fairly quantitative, however, less technical.
- Time value of money: the first and the last rule to consider while pricing/investing

The slide features a dark blue header with the word 'CONCLUSION' in white. Below the header, there is a list of three main bullet points. The first point is 'Fixed income: fascinating part of investment portfolios', which has a sub-bullet 'No/less risks and certain gains'. The second point is 'Fairly quantitative, however, less technical.' and the third is 'Time value of money: the first and the last rule to consider while pricing/investing'. In the bottom right corner, there is a small video feed of a man in a white shirt. At the bottom of the slide, there is a navigation bar with various icons.

So, to conclude this session; we know that fixed income instruments are fascinating part of investment portfolios. Because, it carries no or less risk with certain gains or assured returns it is fairly quantitative, but to keep it simple we have discussed some major concepts or ideas associated with fixed income securities such as the characteristics and valuation of different of fixed income securities such as bonds treasury bills commercial papers and certificate of deposits which are available for investment by individual investors as well.

We should always keep in mind the rule of time value of money, because this is the first and the last rule to consider when you try to include any financial assets in your portfolio. This is it for now.

Thank you very much.