

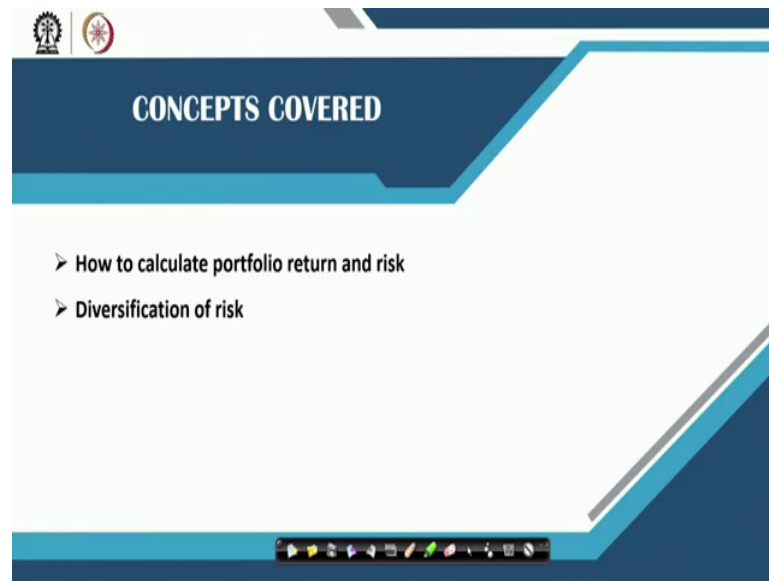
Behavioral and Personal Finance
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Module – 01
Behavioral Economics and Finance
Lecture – 22
Portfolio Return and Risk (Contd.)

Hi there, welcome back to the course Behavioral and Personal Finance. Previously, we were discussing about how to calculate return and risk associated with a single asset. In this session, we will touch upon the methods and formulae to calculate the return and risk associated with more than one assets. Previously we have discussed how combining different assets, basically more than one assets together construct a portfolio and this portfolio can be constructed to mitigate or minimize the risk and maximize the return.

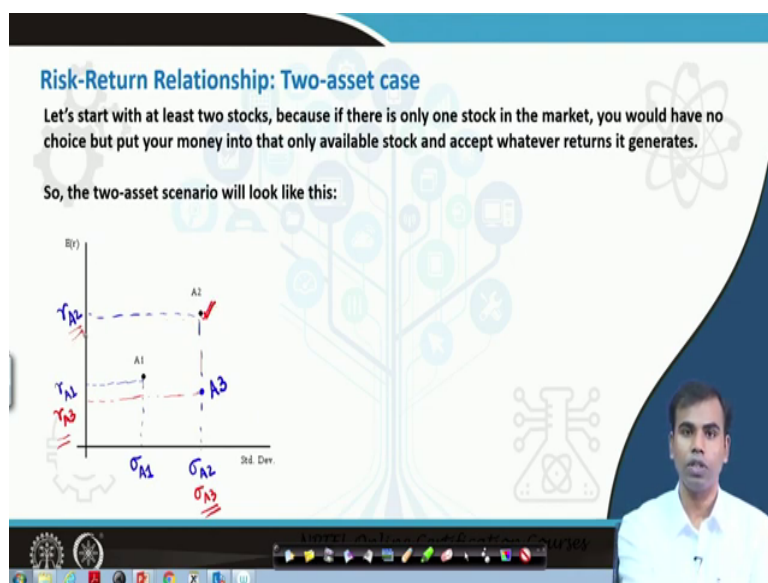
Basically, this is how we diversify our risk to us achieve higher rate of return from our investment. In this session we will touch upon the two topics.

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Mainly, we will focus on how to calculate a return and risk associated with a portfolio of assets. Here, we will deal with two asset case, where a combination of two assets will be used for showcasing the portfolio return and risk calculation. We will also touch upon the diversification concept, where we will try to understand how diversifying the risk through investing in different assets of unique characteristics can achieve the objective of risk minimization.

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If, we talk about risk and return relationship particularly in two asset case, it becomes very important to understand the relationship between these two assets, because if we have only one asset in the market. We do not have our any other choice, but to put our all investment and savings in the same assets. Suppose, the scenario, where you have just one investment a venue that is bank fixed deposit. Whatever money you will save will be deposited in that same fixed deposit because you have no other choice.

So, if there is only one asset in the market or in economic, all the savings all the investment would go to that particular asset only, but since the world is more complicated. So, let us start with a two asset scenario, where you assume that there are at least two assets, you can call it stock and bonds or you can call it asset one asset two or stock or bonds or any other

combination of two assets. So, the two asset scenario if we try to present here, would look like this where there are two assets A 1 and A 2 with different risk and return attributes.

So, when we talk about risk and return attributes basically we indicate that asset A has a return of this level. So, this is basically return for asset 1 and this is the risk associated with asset 1. So, risk is denoted by sigma which is the standard deviation and return is denoted by r associated with investment or asset 1. Similarly, for asset 2 we have returned for asset 2 as given and associated risk associate with the asset 2.

Now, if there are more assets scenario, where you have more than two assets and the asset would look something like this, so, for example, if there is an asset which lies here. So, let us call it asset 3. Typically, this asset might not exist in the market because we know that for the same level of risk, which is basically the risk for asset 3, we have lower return from the asset 3.

So, no investor would want to invest money in asset 3, because it offers lower rate of return for the same amount of risk. Because, for the same amount of risk that is sigma A 2, the investor has a choice to invest in asset 2 where he can get a higher rate of return. So, in this scenario the asset 3 would not exist. So, it will disappear from the market. Basically, this is how market information efficiency and the competitiveness of the market is determining the availability of investment choices that we have. So, when we have two assets asset 1 and asset 2 and associated return and risk characteristics, we can also indicate this through a simple numerical example.

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Risk-Return Relationship: Two-asset case

Let's start with at least two stocks, because if there is only one stock in the market, you would have no choice but put your money into that only available stock and accept whatever returns it generates.

So, the two-asset scenario will look like this:

Now, suppose that:

Stock	$E(r)$	σ	Weights
A1	10%	5%	0.5
A2	20%	10%	0.5

So, for example, here we have the numbers associated with these two assets. So, suppose asset 1 has a return of 10 percent with a sigma or risk of 5 percent and asset 2 has a return of 20 percent with a sigma or the risk of 10 percent. These are just arbitrary numbers this could be anything else, but for the sake of simplicity we have used these numbers for illustrate the relationship between risk and return on a portfolio of two assets.

So, if we try to create a portfolio of these two assets we can assign weights. So, for example, if we have 100 rupees to invest and we have choice to invest in either A or B or a combination of A or B, we have multiple scenarios and the scenarios could be we have a choice. So, scenario one would be, we have a choice to invest in a 100 rupees and A 2 0 rupees. Similarly, scenario two would be we have investment in A 1 0 and A 2 100 and then

we have multiple scenarios starting from in A 1 we have 10 rupees, in A 2 we have 90 rupees and so on.

So, essentially it indicates the weight. So, for the sake of simplicity we assume that we invest 50-50 percent of our investable money in each of the two assets. So, the weights assigned to each of the two assets are 50 percent for investment asset A and 50 percent for asset B.

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Portfolio Return: Two-asset case

Stock	E(r)	σ	Weights
A1	10%	5%	0.5
A2	20%	10%	0.5

- The return of the portfolio consisting of stocks A1 and A2 will be:
 - $R_p = w_1R_1 + (1-w)R_2$ or $R_p = \sum w_i R_i$
- Since r_1 and r_2 (the respective returns from stocks A1 and A2) are random variables, so we represent them as R_1 and R_2 , and
- The R_p , R_1 and R_2 are linearly related;
 - Again because they are random variables.
- The portfolio return, R_p is the weighted sum of R_1 and R_2 .

Handwritten calculations:

$$r_p = (.50 \times .10) + (.50 \times .20)$$
$$r_p = .15 = 15\%$$

The graph shows a coordinate system with return (r) on the vertical axis and standard deviation (σ) on the horizontal axis. A horizontal line is drawn at 10% on the r-axis. A line labeled 'A1' starts at 10% on the r-axis and goes up and to the right. A line labeled 'A2' starts at 20% on the r-axis and goes up and to the right. A dashed line from 15% on the r-axis meets the 'A1' line at a point where the weight is 0.50. A point on the 'A2' line is also marked with a weight of 0.50. The horizontal axis has markers at 5% and 10%.

Now, if you try to calculate the return on such a portfolio of two assets the return can be calculated using the weighted average rate of return for the two assets, which means if we put 50 percent of our money in asset A and 50 percent of our money in asset B. A has 10 percent of return and B has 20 percent of return. We can calculate the weighted average rate of return for the portfolio of asset A and B. The same can be used for calculating the return for a portfolio of any number of assets.

So, if we try to calculate the return here the return on this portfolio, return on this portfolio would be 50 percent weight in investment A 1 into 10 percent of return for investment A 1 and remaining 50 percent of weight in A 2 into 20 percent of return in A 2. So, we know that that rate of return on portfolio for these two assets would be 15 percent, this is the rate of return that we can calculate using the numbers given in the example.

If we try to illustrate through the graph method we have already shown that, the risk and return can be plotted on a two dimensional graph and we know that this is A 1 and this is A 2 where this has 10 percent of risk, this has percent of risk, this has 10 percent of return and this is 20 percent of return. If, we invest our money in A 1 and A 2 somewhere 50 percent so, the portfolio will be giving us a return of 15 percent in average.

So, this is what it happens when we invest our asset in 50 percent in A 1 and remaining 50 percent in A 2. So, if you know the weights to be assigned to different assets you can calculate the return on the portfolio in this particular approach. So, suppose you want to change the weight of the assets to a different combination. So, now, you want to invest less amount of money in A because it is giving you less amount of return. So, the amount of money you have invested in A is 20 percent and the remaining 80 percent is invested in B,

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Portfolio Return: Two-asset case

Stock	E(r)	σ	Weights
A1	10%	5%	0.5
A2	20%	10%	0.5

- The return of the portfolio consisting of stocks A1 and A2 will be:
 - $R_p = w_1R_1 + (1-w)R_2$ or $R_p = \sum w_i R_i$
- Since r_1 and r_2 (the respective returns from stocks A1 and A2) are random variables, so we represent them as R_1 and R_2 , and
- The R_p , R_1 and R_2 are linearly related;
 - Again because they are random variables.
- The portfolio return, R_p is the weighted sum of R_1 and R_2 .

$R_p = (0.20 \times 0.10) + (0.80 \times 0.20)$
 $R_p = (0.02) + (0.16) = 0.18$
 $= 18\%$

Then, the calculation of return would take you to a different value of return on portfolio. And, that will be 17 percent or actually it is basically, if we try to calculate this, this will be this much of return for investment A 1 and this much of return from investment B. So, basically the total return would be 18 percent from the investment that we are making.

So, this is how you can change the return by changing the weights associated with each of the investment avenues that you have. So, since the returns are random variables. So, we can calculate the return by linearly aligning them and combining them to the proportion that we prefer. Suppose, you want to invest in 5 different assets, let us say stocks of a company bond of a company and gold and let us say fixed deposit in a bank and you want to have some cash.

So, all these assets will have certain amount of return and you have to decide how much money has to go in each of these investment avenues and subsequently you can calculate the rate of return for the entire portfolio that you are holding.

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Portfolio Return: Two-asset case

- The return of the portfolio consisting of stocks A1 and A2 will be:
 - $R_p = w_1 R_1 + (1-w)R_2$ or, $R_p = \sum w_i R_i$
- Since r_1 and r_2 (the respective returns from stocks A1 and A2) are random variables, so we represent them as R_1 and R_2 , and
- The R_p , R_1 and R_2 are linearly related;
 - Again because they are random variables.
- The portfolio return, R_p is the weighted sum of R_1 and R_2 .

Stock	E(r)	σ	Weights
A1	10%	5%	0.5
A2	20%	10%	0.5

Handwritten notes on the slide:

100% \Rightarrow R_i

Stock	Bond	FD	Gold	Cash
0.30	0.10	0.20	0.20	0.20
r_s	r_b	r_f	r_g	r_c

Return on Portfolio = $\sum w_i r_i$

So, the example that I am trying to show here is if you have 5 different assets let us say stock, that is share of a company bond and then fixed deposit, then you want to have some money in terms of gold and then cash.

So, these are 5 assets in your portfolio. So, you have to decide how much of total investment should go to stock. So, let us say 30 percent of your money should go to stock, 10 percent of your money go to bond, 20 percent should go to fixed deposit, 20 percent should go to gold or any other precious metal, and remaining 20 percent you want to keep in cash. So, each of the assets that you have will have some amount of return over the period that for which you are

holding and then you can try to calculate the average rate of return for the investment that you have made.

Typically, cash does not have returned because cash you are holding is not yielding you any return or any benefit. In some cases it might cost you some money because you have to spend some money on maintaining the cash; other investment avenues might be giving you some return. So, if you have the return for example, if you have the return on stock return on bond, return on FD, return on gold and then if you have any return at all in cash you can combine these with the associated weights, for the inverse for the each of the investments and calculate the portfolio return using this approach.

So, portfolio return is basically or sum of weighted average sum of each of the return of your portfolio. Now, this is simple, because we know that returns are random variables and we can linearly be added to find the weighted average rate of return on portfolio of different assets.

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Portfolio Risk: Two-asset case

- Risk of two-asset portfolio is dependent on the covariance between the returns of two assets.
- So, the variance of a two-asset portfolio is:
$$\text{Var}(R_p) = w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2w_1(1-w) \text{Cov}(R_1, R_2)$$

Handwritten annotations on the formula:
- w_1^2 : Weight A_1
- σ_1^2 : Risk A_1
- $(1-w)^2$: Weight A_2
- σ_2^2 : Risk A_2
- $2w_1(1-w)$: Covariance term

But, what if the risk associated with these assets or this portfolio of different assets can be calculated as well. So, we know that a risk of a two portfolio, two asset portfolio is dependent on how these assets are related or how the covariance of these assets are related with each other?

So, we can try to calculate the variance of a two asset portfolio using this formula which is basically variance of return on portfolio is the weight of. So, we know that this is our weight in asset 1 this is a risk of asset 1; weight of asset 1 risk of asset 1 and, then this is $1 - w$, which is weight of asset 2 and this is risk of asset 2. What is more important is the covariance of returns on the two assets. What it implies is when we have two different assets in our portfolio; let us say stock and bond.

We need to understand how these two assets are correlated which means if I have some amount of money invested in stock and some amount of money invested in bond. If, my investment in stock is going up I need to understand whether the investment in bond is also going up or it is going down. If, it is going up then we can say that this is positively correlated, because both are moving in same direction. If it is going down which is the opposite direction of the return direction on of stock, then we can say that this is negatively correlated, because it is going in the opposite direction.

So, the relationship between assets that we have in our portfolio determines how they are correlated and, whether the total risk of the asset can be reduced or increased.

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Portfolio Risk: Two-asset case

- Risk of two-asset portfolio is dependent on the covariance between the returns of two assets.
- So, the variance of a two-asset portfolio is:
 - $Var(R_p) = w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2 * w_1 * (1-w) * Cov(R_1, R_2)$
- This can be re-written as:
 - $Var(R_p) = w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2 * w_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$
 - Where, ρ_{12} is the correlation between returns on stocks A1 and A2.
- The correlation coefficient, ρ_{ij} remains between -1 and $+1$.
- That is, $-1 < \rho_{ij} < 1$.
 - $\rho_{ij} = 1$: Perfectly correlated securities,
 - $\rho_{ij} = 0$: non-correlated securities, and
 - $\rho_{ij} = -1$: Perfectly negatively correlated securities.

So, if we take this particular approach further, we can rewrite that variance of the two asset portfolio case can be rewritten as the weight of individual assets and sigma square of

individual assets along with the correlation expressed as the covariance factor, which is rho here. If, you see the factor that we have here is in terms of rho which is basically the correlation between a return on asset A 1 and asset A 2.

And, that is where we can use this correlation or the relationship between 2 assets A 1 and A 2 here as it spans between minus 1 to plus 1, which is basically the two assets are extremely negatively correlated or extremely positively correlated or anything in between. So, if we have two assets A 1 and A 2 which are correlated to the extent of minus 1 to plus 1; if it is minus 1 we know that, it is perfectly negatively correlated; if it is plus 1, it is perfectly positively correlated and if it is 0, it is not correlated at all.

Taking this example in the real life, if we know that you have some money invested in stock market and thus some other amount of money invested in gold. Typically, we observe that when gold prices go up stock market goes down or vice versa. And, that is why we say that stock, and stock market and gold investment are negatively correlated. So, when you have in your investment distributed across stock market and gold investment then you have your correlation factor to be negative.

This essentially helps us in minimizing or reducing the risk, because we know that when we have two assets, the correlation will determine the total risk of the portfolio.

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Portfolio Risk: Two-asset case (cont.)

	Stock A1	Stock A2
Stock A1	$w_1^2 \sigma_1^2$	$W_1 * (1-w) * \sigma_1^2$ $= W_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$
Stock A2	$W_1 * (1-w) * \sigma_1^2$ $= W_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$	$(1-w)^2 \sigma_2^2$

Inputs required:
 → Weights of investment
 → Sigma of return on investment
 → ρ_{ij} = Correlation coefficient b/w the assets

- The variance of the two-asset portfolio is the sum these four boxes.
- That is, $\text{Var}(R_p) = w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2 * w_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$
- Therefore, the standard deviation of the portfolio, σ_p is: Squared Root of $\text{Var}(R_p)$.
- $\sigma_p = \sqrt{w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2 * w_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2}$ ← Portfolio Risk

In this way, if we try to understand it we can show case that the two-asset case, let us say stock A 1 and stock A 2 these are two assets for which we have calculated or we have expressed, how we can calculate the risk associated with each of the scenarios. And, if we sum it up together we have get the total variance of the portfolio of two assets as such given in the formula. And, if we square root it we know the sigma of the portfolio which is basically the risk of portfolio.

So, this basically indicate portfolio risk so, when we have a sigma of the portfolio calculated using the components. So, the input required to calculate for inputs, that we need to calculate the risk of portfolio are basically weights. So, weights assigned to each of the investments, we need to understand sigma or the risk, sigma of investment returns on investment. We also

need to know the correlation between which is basically the correlation coefficient between the assets.

So, if we have two assets, we will have one correlation factor, if you have more than two assets we will have more than one correlation factor. So, for example, this is the two assets scenario case where you have asset A 1 and A 2 and you can calculate the sigma of the portfolio of asset 1 and 2 by using this formula given as the portfolio risk. If, you have sigma associated with each of the, for each of the assets in the portfolio, you have weights associated with each of the assets in the portfolio and you have the correlation factor given for the two assets.

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Portfolio Risk: Two-asset case (cont.)

	Stock A1	Stock A2
Stock A1	$w_1^2 \sigma_1^2$	$W_1 * (1-w) * \sigma_1^2$ $= W_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$
Stock A2	$W_1 * (1-w) * \sigma_1^2$ $= W_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$	$(1-w)^2 \sigma_2^2$

•The variance of the two-asset portfolio is the sum these four boxes.

•That is, $Var(R_p) = w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2 * w_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2$

•Therefore, the standard deviation of the portfolio, σ_p , is: Squared Root of $Var(R_p)$.

• $\sigma_p = \sqrt{w_1^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2 * w_1 * (1-w) * \rho_{12} \sigma_1 \sigma_2}$

Handwritten notes:

r	σ	w
A1 10%	5%	.5
A2 20%	10%	.5

$$\sigma_p = \sqrt{(0.5)^2 \times (5\%)^2 + (0.5)^2 \times (10\%)^2 + 2 \times 0.5 \times 5\% \times 10\% \times 0.5}$$

$$\rho_{A1,A2} = +1$$

$$\rho_{ij} = -1$$

$$\rho_{ij} = 0$$

So, if we take the example that we had started in the beginning, we had two assets A 1, A 2 for which we had returned as 10 percent, and 20 percent, sigma as 5 percent and 10 percent

and weight as 50-50. So, if we try to calculate a portfolio of these two assets and the associated risk, basically what we can do is we can calculate it as weight, which is basically the 50 percent weight of asset A 1 into 5 percent of return sorry risk, then we have 50 percent of weight for asset 2 into 10 percent of sigma for asset 2. And, then we have 2 into weight 1 sigma 1, weight 2 which is again 50 percent sigma 2 into the factor of correlation coefficient A 1, A 2. If, we get this value we can calculate the sigma of portfolio as for the given data.

Now, if we assume that this correlation is plus 1. So, if we assume here that the correlation between A 1 and A 2 is plus 1, this will add more risk to the individual risk of the assets. If we assume that this is going to be minus 1, this factor is if it is minus 1, then this entire factor will become a negative or the reduction for from the total risk of each of the assets individual risk. And, if it is 0, which means the entire factor will be removed and it will be as much as the risk associated with each of the individual assets. This is how this sigma can be minimized or increased depending on how correlated the assets in your portfolio are.

So, this leads to the concept called a diversification, where you try to include the assets in your portfolio, which are mostly negatively correlated or not correlated at all. Now, practically there are no assets which are not correlated or which have 0 correlation with each other. However, we can try to have assets in our portfolio that will have less positive correlation or at max negative correlation so, that our risk or the sigma of the portfolio can be minimized or reduced.

This particular diversification concept essentially leads to the saying that goes as do not put all your basket x in the same basket, which implies that if you have assets of similar nature in terms of riskiness and correlation, then if one asset goes down other asset will also go down. And, subsequently your total return on the portfolio and total will go down and total risk of the portfolio will increase.

If, you have assets in your portfolio which are negatively correlated, it implies that if one asset goes down, the other asset would go up and the return on the other asset will

compensate for the loss on the first asset. That is how we can use this correlation factor to understand, the riskiness of the portfolio of different assets.

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Characteristics of a Multiple-asset Portfolio

- The expected portfolio return will be:
 - Weighted average of the expected returns of each stock in the portfolio.
- If we have N stocks in our portfolio, we'll have:
 - N variance terms, and
 - $(N^2 - N)$ Covariance terms.
- Since the $\text{Cov}_{ij} = \rho_{ij} \sigma_i \sigma_j$, the number of correlation coefficients will also be:
 - $(N^2 - N)$.
- As N (the number of stocks) becomes very large, the portfolio variance tends toward the average covariance.
 - For example, a 10-stock portfolio, there will be 10 variance terms, 90 covariance terms, and 90 correlation coefficients.

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Before, we wind up the technical tools for calculating portfolio risk and return we should highlight the expected return on the portfolio is basically weighted average rate of return of individual assets. And, if you try to calculate return on portfolio of N assets, we will have N variance term and N square minus N as covariance terms. So, it implies that if we have more number of assets in our portfolio, we will have more number of variance and covariance terms.

So, if the number of stocks or number of assets in the portfolio is very large then the portfolio variance tend toward the average covariance. For example, if there are 10 stock portfolios, there will be 10 variance terms and 90 covariance term and 90 correlation coefficients, which

means you will have a very complicated computation that cannot be possibly done by individuals.

So, when you have a portfolio of multiple assets or assets in huge numbers, then the calculation of this variance and covariance term is very complicated and the ultimate effect on your decision making might be very significant. To sum this session up we would highlight by saying that having understood behavioral economics and finance concepts, and learning the basic tools and techniques of present value calculation of future cash flows, and risk and return associated with individual and portfolio of assets.

We can move towards how the personal finance decisions can be taken keeping in mind the behavioral economic concept and the traditional financial tools and techniques. We will take up these issues in next session.

Thank you very much.