

Behavioral and Personal Finance
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Module – 01
Behavioral Economics and Finance
Lecture - 20
Valuation of Financial Assets (Contd.)

Hi there, continuing from the previous session where we were discussing about different types of cash flows and we touched upon the approaches which we use for calculating the present value of cash flows of perpetual nature. Such as, the simple perpetuity cash flows and then we also discussed how we can evaluate the present value of delayed perpetual cash flows and subsequently, the cash flows which are growing in future and how to calculate the present value of these cash flows.

(Refer Slide Time: 00:53)

Delayed Perpetual CF.

Timeline: t_0 (CF), t_1 (CF₁), t_2 (CF₂), t_3 (CF₂ ...), ∞

$PV_{t_0} = \left(\frac{C}{r}\right) \frac{1}{(1+r)^t}$

$PV = \frac{C}{r}$

Growing Perpetuity:

Timeline: t_0 , t_1 (CF₁), t_2 (CF₁(1+g)), t_3 (CF₁(1+g)²), ∞

where $g = \text{growth rate of CF}$

$r > g$

$[X(1+r) - C]$ should be equal to $X(1+g)$

$X(1+r) - C = X(1+g)$
 $X + Xr - C = X + Xg$

$Xr - C = Xg$
 $Xr - Xg = C$

$X(r-g) = C$
 $X = \frac{C}{(r-g)}$

Taking the discussion further we understood that the formula that we have discussed here earlier was given as the present value of growing perpetual cash flow can be determined using this formula, where we should understand that r is the discounting rate and g is the growth rate of cash flows.

So, if the discounting rate is as far as greater than g it is always going to be continuing in future, but if the discounting rate becomes smaller than g then the cash flow will not continue for perpetuity. You take a simple example here, suppose you invest some amount of money today in an investment which is supposed to give you a perpetual income and that income is C , the investment that you are making today is X .

So, if you expect that the growth rate in your income is more than r which is g being greater than r , it implies that every period you are getting more money than your investment is generating, and that is why it will not continue for perpetuity in future. So, the basic assumption of this growing perpetuity approach is the growth rate should be less than the rate of discounting which we are using; which is given by our should be always greater than g .

Now, let us move on to the different type of cash flows that we have touched upon earlier that is annuity cash flows. Before, you move on to annuity cash flow let me jot down the formula that we have discussed.

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Annuity Cash Flows:

$P1$ t_0 t_1 t_2 t_3 ... t_{100} t_{101} ... ∞
 CF CF CF ... ∞
 $PV_{P1} = \frac{C}{r}$

$P2$ t_0 t_1 t_2 t_3 ... t_{100} t_{101} t_{102} ... ∞
 CF CF ... ∞
 $PV_{P2} = \frac{C}{r} / (1+r)^{100}$

$PV_{Ann.} = P1 - P2$

Annuity for 100 yrs
 $PV_{Annuity\ 100\ yrs} = \frac{C}{r} - \left\{ \frac{C}{r} / (1+r)^{100} \right\}$
 Present value of 100 yr. annuity
 $= \frac{C}{r} \left(1 - \frac{1}{(1+r)^{100}} \right)$
 $PV_{Ann.} = \frac{C}{r} \left(1 - \frac{1}{(1+r)^{n-1}} \right)$

Perpetual CF $PV = \frac{C}{r}$
 Growing Perpetuity $PV = \frac{C}{r-g}$
 Delayed perpetuity $PV = \frac{C}{r} / (1+r)^n$

So, we have known that perpetuity cash flow is; perpetual cash flow is given as present value is equal to C by r ; where, C is your cash flow r is your discounting rate and growing perpetuity is given as C by r minus g and delayed perpetuity is given by C by r by 1 plus, let me rewrite this. So, basically we have discussed earlier that a delayed perpetuity implies that the perpetual cash flow is occurring at a later date. So, C by r divided by 1 plus r to the power n , where n is the number of years for which it is delayed.

Now, let us move on to annuity cash flows. So, annuity cash flows are cash flows which are being generated for certain number of years only, it will not move on in the future for perpetuity. And, we have seen that it has a predefined maturity period or number of years for which we need to calculate examples could be investment in your fixed deposits, insurance premium or any other investment which has a fixed tenure such as maturable bond.

If you have investment in bound with certain number of years of maturity, you are expecting to generate a cash flow for certain number of years and then you can stop getting the cash flows. So, how do we find the present value of such annuity cash flows? So, instead of directly jumping to the formula of annuity cash flows, let me specify the formula using the methods that we have learned so far.

So, so far we have learnt methods to calculate the present value of perpetual cash flows given as C by r . We have also learned that if the perpetual cash flow is growing, we can calculate the present value given by C by r minus g and if the perpetual cash flow is delayed. We can use the formula C by r divided by 1 plus r to the power n to calculate the present value of such a delayed perpetual cash flows.

So, let us combine these methods to understand how we can calculate the annuity cash flows. Going back the same methodology, we will first have a time line example. So, suppose we have a time line example where the periods are defined as t_0 , t_1 , t_2 and so on and it continues till perpetuity. So, cash flows are also generated in a similar fashion, let us take a normal cash flows. Let us call this situation 1 that is project 1.

So, if this is project 1, let us have a project 2 where a similar time line for perpetuity given time given as t_1 , t_2 and so on, but this difference in this scenario is the cash flow is occurring after let say this is t_{100} . So, at $t_{100} + 1$ year; so, let us say this is given as t_{100} , t_{101} and so on. So, in project 2 the first cash flow is coming at t_{101} which is after 100 years.

Now, if we try to understand the difference between these two basically first project P 1 gives you a cash flow which is generating being generated at year 1 that is t_1 , t_2 and so on; in second situation which is project 2, the cash flow is delayed for first 100 years or 100 periods and after that the first cash flow originates. So, the second example is delayed cash flow of perpetual in nature and first exam first situation which is project 1 is the typical simple perpetuity cash flow.

Now, if we have already discussed the formula to calculate the present value. So, we can say that present value of project 1 will be C that is CF ; so, let us assume that this C is the CF by r . In case of present value of project 2, it will be C by r of course, divided by 1 plus r to the power 100 , right because this has delayed for 100 years and it is starting at 100 first year.

So, if we simplify this formula to understand the net present value of situations for which the perpetuity is gone and the cash flow is generated for certain number of years. So, it will be as simple as that. So, in this case the cash flow is occurring at perpetuity, in this case the cash flow is occurring from here till perpetuity, right.

So, the difference between these two cash flows will be basically the cash flow which is basically generated as t to $t + 1$ till $t + 100$. So, basically this will be an annuity for 100 years, right. So, when you try to understand the nature of cash flows for first 100 years, then it will be P_2 and P_1 the difference. So the basically, if you try to understand the present value of annuity it will be the present value of P_1 minus present value of P_2 .

Basically, present value of P_1 implies that you are getting a cash flow for the whole perpetual period present value of P_2 implies that you are going to get the present value of cash flows originated after 100 years; so, 100 first year and so on. So, when you try to calculate the present value of first 100 years it is basically giving you an annuity formula.

So, annuity formula will be if we try to simplify the formula present value of annuity for 100 years, in this case it will be C by r minus C by r divided by 1 plus r to the power 100 , because we are delaying it for first 100 years. So, the formula would be C by r into, if you simplify this you will get 1 minus 1 by 1 plus r to the power 100 here and this will be the present value of 100 year annuity.

If you try to understand this from this time line point of view, the present value of first 100 years of cash flows can be calculated using the present value of each of the cash flow that are being generated at t_1 , t_2 , t_3 in project 1 case, and that those cash flows are brought back to

the present time at time 0 which is basically the today's time; and, that is how we calculate the formula for present value of 100 years annuity.

So, basically if we simplify this formula, present value of annuity will be to keep it simple C by $r(1 - (1 + r)^{-n})$. And, if it is number of years given as 102 you have to take $n - 1$ because your first cash flow is generated at 101. So, $101 - 1$ will be number of years of 100.

So, this is the formula that you can use to calculate the present value of annuity cash flows for certain number of years. You have to be careful while calculating this present value of future cash flows, because this becomes very critical when you try to understand at what year the first cash flow is being generated which basically means that the number of years for which it has been delayed. So, this is for understanding how the annuity cash flows can be calculated in terms of present value.

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Growing Annuity

Timeline: t_0 , t_1 , t_2 , t_3 , ..., t_n

Cash flows: CF_1 , $CF_1(1+g)$, $CF_1(1+g)^2$

$$PV_{\text{Growing Ann.}} = \frac{C}{(r-g)} \left(1 - \left(\frac{1+g}{1+r} \right)^n \right)$$

PV of Cash Flows

- Perpetual
 - Simple = $\frac{C}{r}$
 - Delayed perpetuity = $\frac{C}{r} / (1+r)^n$
 - Growing perpetuity = $\frac{C}{(r-g)}$
- Annual
 - Simple Annuity
 - Growing Annuity

If you try to simplify the same formula using the growing annuity. So, if we go back to the growing annuity case where the time line would be something like this. So, you have time as defined as t_0 , t_1 , t_2 and so on and this is limited for certain number of years let us say n and cash flow are growing. So, basically in year 1, you are getting CF_1 , in year 2 you are getting CF_1 into $1 + g$ which is basically your growth rate, in year 3 you are getting cash flow $1 + g$ plus growth rate with increment and so on.

Then, the formula for this present value of growing annuity will be C by r minus z going by the same logic into $1 - \frac{1+g}{1+r}$ to the power n , and this should be giving you the present value of growing annually where the annual cash flows are increasing. And, this we have covered 2 approaches basically cash flows.

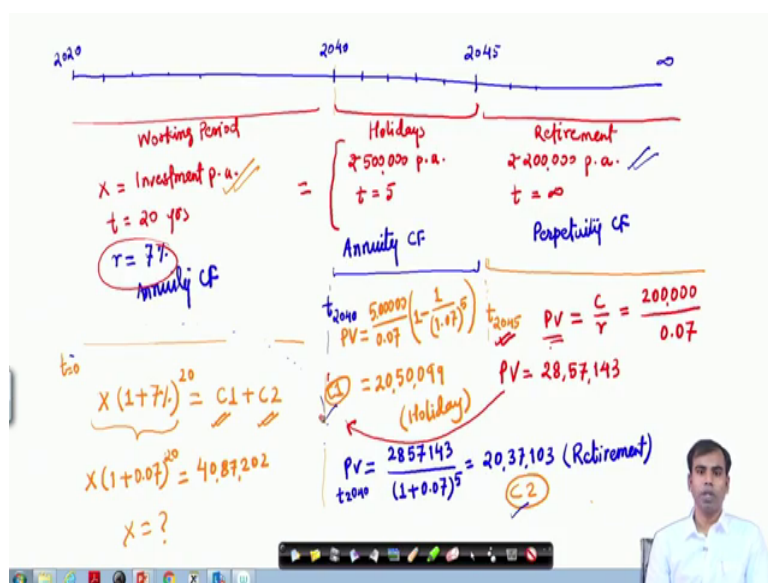
So, basically we are trying to discuss the present value of cash flows of perpetual in nature and annual in nature, in perpetual we discussed simple perpetuity which is basically given as C by r then we discussed delayed perpetuity. Where we discuss C by r to the power divided by 1 plus r to the power n for which it is delayed.

And, we discussed growing perpetuity where we came up with a came up with the formula C by r minus g where g is the growth rate, and then in annual we discussed simple annuity and growing annuity. Delayed will be the same case, you have to just delay it for certain number of years if it is given and this formula these 2 formula are already discussed in this session.

Now, having learnt these 2 methods or these approaches to calculate the present value of expected cash flows of different nature. We can also implement these methods to understand some very simplistic or realistic financial decision making problems. Particularly when it comes to personal finance decisions these approaches of calculating present value becomes more critical.

If you recall from one of the previous sessions, we presented you a situation where miss highflyer was about to make her financial planning decision and the situation was like this. So, I will go back and try to represent the situation here.

(Refer Slide Time: 18:37)



So, the situation was miss highflyer was about to plan for her future and it is she is going to graduate in 2020 which is that immediate year. And, she is expected to work for 20 years then see is planning for some vacation trips for next 5 years and after that she will be retiring.

So, basically we had defined earlier that this will be her working period which is going to be 20 years, this is going to be her vacation period or holidays and this is going to be her retirement. If you recall we said that in holiday period she will be needing 5 lakh rupees every year and in her retirement she will be needing 2 lakh rupees every year for her survival.

The question was how much amount of money to be saved every year investment per year for 20 years. So, that it should be equal to her holidays for of 5 lakh rupees per annum for 5 years and retirement requirement for of 2 lakh rupees per year for perpetuity.

Now, this is the situation and additional information was given as the rate of interest that can be used as borrowing or lending rate to be let us say 7 percent, if you recall this was a situation given to you. Now, if you try to calculate the present value of these cash flows using the methods that we had discussed in this session and earlier session. We know that this is an annuity situation where annuity cash flow, this is a perpetuity cash flow and this is again an annuity cash flow.

So, when you try to implement the tools that we have learned for calculating present value, you can simply use these functions to calculate the present value of future cash flows. The point here is for every set of cash flows you have to bring it to a common time line. For example, these 5 years of cash flows should be calculated at this point of time which is basically t being 2040 and for these cash flows, this will be the time line where you can calculate where t is going to be 2045 and for these cash flows t will be 0.

Now, when you try to apply the formula that we have learned the formula for this cash flow will be; so, present value of this cash flow will be C by r which is 2 lakh rupees of every year requirement by r . So, we are using the same r here 7 percent which is basically the rate of discounting. So, whatever present value you are going to get is at this point of time, now this has to be brought it to one common point of time.

So, if you calculate this value C by r using this 2 lakh rupees of cash flows and 7 percent of discounting rate, this will be let us say 28,57,143. So, this is the present value at t 2045, if you bring it back to this point of time which is basically again bringing back for 5 years. This will be basically calculated as so, present value will be 28,57,143 this is your present value, and has to be discounted with 1 plus 7 percent for 5 years which is going to be basically 20,37,103. Please cross check the calculations although I am trying to be as precise as possible.

So, this is the present value at t 2040 for this cash flow this retirement requirement. So, let us name is retirement, this is the present value of retirement fund; similarly, we can calculate the

present value of this holiday funds. So, holiday funds will be calculated present value of annuity cash flow of 5 lakh.

So, we have 5 lakh here that is C by r which is 7 percent into $1 - \frac{1}{1.07^5}$ which is $1 + r$ to the power n which is 5. So, the calculated value is given as 20,50,099; so, this let us call this holiday fund. So, as explained earlier if you are doing a financial planning for miss high flyer you know that at time t 2040 this which is basically this point of time, she will be needing cash flow 1 and cash flow 2, right. So, the requirement for her will be cash flow 1 and cash flow 2.

And, the total sum total of cash flow 1 and cash flow 2 should basically be total of this amount of money which is basically X into, X is the amount of money that she has invested into $1 + 7$ percent which is r to the power 20 years; which is basically, the compounded value of the investment that she has made and this should be equal to C_1 plus C_2 , right. Because, this is the value that will be the value of investment X that is to be met at this point of time and it should be equal to C_1 and C_2 . So, that her investment should be sufficient enough to cover her holiday requirement and in future for her retirement requirement.

Now, if you equate this value; so, X into $1 + 0.07$ for 20 years is equal to C_1 and C_2 , C_1 and C_2 sums up together to 40,87,202. So, if you equate this you should be able to find the value of X and it should be giving it to the value of investment that is to be met per annum to get the financial planning done. Or an alternative way could be to bring all these C_1 and C_2 back to present value that is today, and this instead of this compounding you can use it for discounting it to C by r into $1 - \frac{1}{1 + r}$ to the power n that is 20 years.

So, that will be an alternative approach. I hope you try to calculate the value and come up with some solution. We will discuss more similar examples and of financial planning in next session and till now we have already touched upon the calculation of present value of different type of cash flows including perpetuity and annuity cash flows. This example was illustrated to highlight the situations of personal finance problems that is it for now.

Thank you very much.