

**Marketing Analytics**  
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**Lec 30**  
**Marketing Mix Models and Advertising Mix Model**

Hello, everybody. Welcome to Marketing Analytics Course. This is Doctor Swagato Chatterjee from VGSOM, IIT Kharagpur, who is taking this course for you. And today, we are in week six, and we are discussing about Marketing Mix Model and Advertising Models. So in the last video, we have discussed about a, how the various items of marketing mix actually interplay with each other. But in this particular video, they have, we have taken the Linear Model, Simply Linear Model and we tried to find out that how the effects are different. But in this particular video, we will try to do a little bit more, we will try to bring in a little bit of complexity non linearity in the model, and the effects of the past behavior also.

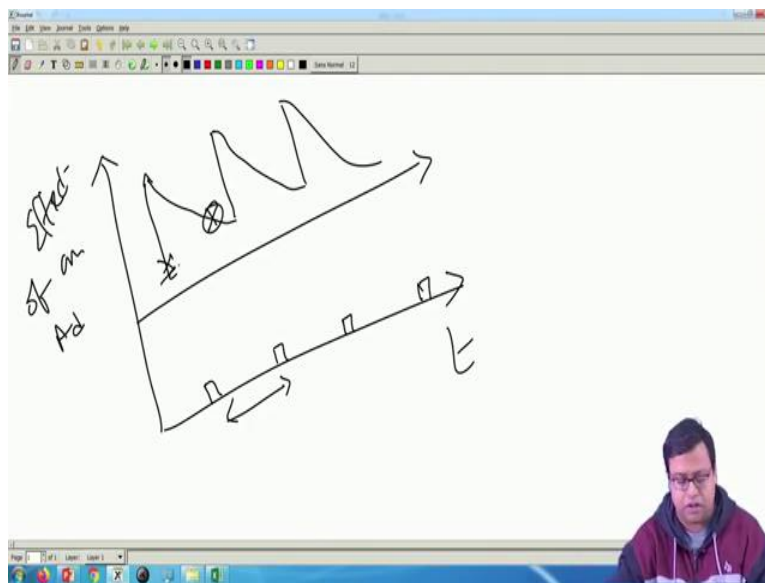
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Quarter of Year	Quarter	Ads	Adstock	Price	forecast	actual	APE	const
1	1	45		42,000		2,639		1
2	2	79		43,000		3,486		2
3	3	60		35,000		6,156		3
4	4	74		37,000		12,561		4
1	5	46		38,000		7,263		
2	6	44		37,000		6,354		mean
3	7	36		37,000		8,030		
4	8	30		44,000		13,892		
1	9	78		45,000		7,566		
2	10	49		42,000		6,761		
3	11	59		38,000		9,074		
4	12	67		46,000		16,655		
1	13	55		40,000		10,517		
2	14	42		43,000		8,647		
3	15	57		44,000		11,271		
4	16	23		35,000		29,031		
1	17	77		41,000		10,030		

So, if you see the data set that we have, in weeks six season, season 2 I think, the data set looks like this. And if I just zoom it up. So, they are quarterly data, quarter of the year, quarter number and the advertisements spend, price that you have done. And this is the actual sales that had happened. So I will first, do not look at the other things, we will discuss about that, first focus on column number C, D, E, G, and I.

So, quarter of the year, this is varies from one, two, three, four, one, two, three, four, one, two, three, four, and so on. And the next come is quarter number. So there is a, so quarter number means, how much data point I have? So, that's real number is given here, the advertisement's expenditure, the price that I have, and the actual sales value that I have. So, these are the five data points that I have currently. Now, I have to decide that, what kind of advertisements, so how, if you remember, the first thing that we decide is, how the sales is dependent on the price and the advertisement that you have done.

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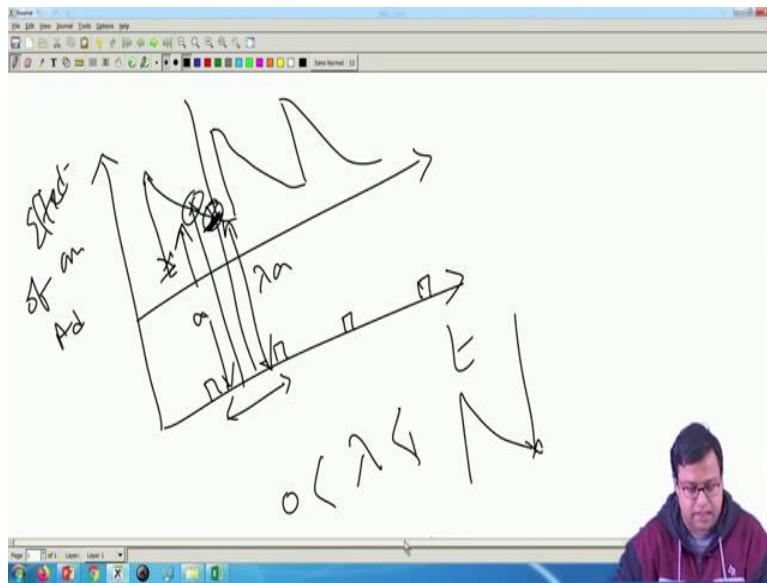
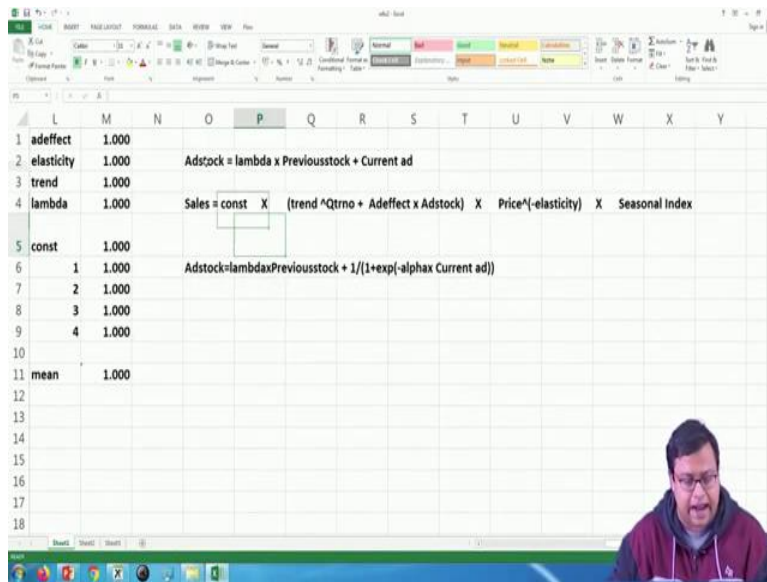


Now, advertisement is oftentimes is assume to be like this. So for example, we do not give on, keep on giving advertisements. If this is time period, and this is the effect of an ad, and let us say, you give advertisement here, here, here and here, the corresponding sales or the effect of the advertisement will first shoot up. And then, in this period, when you have not given any advertisement, people still remember about you, but slowly that memory comes down. So memory decays, the decay of memory happens. So it happens like this.

And then again, you gives some ad, again it goes up, again it happens like this, and again, it goes up and again it happens like this and so on. Now, how the decay will happen, whether it is a linear decay or non linear decay or curve related decay is something that is still a area of research, but this kind of things happen. So, all I am trying to say is that at this point, at this point, the effect of the ad is dependent on how much ad you have given here and how much

decay has happened. So, based on that assumption, there are lots of equations that has been created.

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So one of them is like this, where there is something called ad effect. So, if you carefully see the formulas here, it has been written here that Adstock, there is a something but Adstock. So, I am saying that, the ad effect gets in my memory as a stock, it is like, it is behaves like a inventory. Whatever there in the inventory models that we do, we do the selecting here, but Adstock is lambda into the previous stock.

So, whatever previous stock was there in the last time period that will decay little bit. So, last time period, probably let say, the, the measurement was like this. At the last time period, the measurement was like this. Lambda into this comes here. So this particular measurement is basically if this is A, the second one is lambda into A. The next time period it will be lambda is basically between 0 and 1. So that is lambda into previous stock, plus the adspent

Now, if you give certain ad plus the current ad. So, if you give certain ad at this point, if you give certain ad in this point it will shoot up. So, it will decay, decay, decay. If you give certain ad here, it will again shoot up. So, that is something this particular function is writing about. So, which is Adstock is equal to lambda plus previous stock plus current ad. So, this is something that we are trying to do here.

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The screenshot shows an Excel spreadsheet with the following content:

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	adeflect	1.000												
2	elasticity	1.000												
3	trend	1.000												
4	lambda	1.000												
5	const	1.000												
6	1	1.000												
7	2	1.000												
8	3	1.000												
9	4	1.000												
10														
11	mean	1.000												
12														
13														
14														
15														
16														
17														
18														

Formulas visible in the spreadsheet:

- Row 2:  $Adstock = \lambda \times Previousstock + Current\ ad$
- Row 4:  $Sales = const \times (trend^{Qtrno} + Adeflect \times Adstock) \times Price^{-(elasticity)} \times Seasonal\ Index$
- Row 6:  $Adstock = \lambda \times Previousstock + 1 / (1 + \exp(-\alpha \times Current\ ad))$

$$\epsilon = -\frac{\Delta D/D}{\Delta P/P}$$

$$\ln D = C - \epsilon \ln P$$

$$\ln D = C + \ln P^{-\epsilon}$$

$$\epsilon \frac{\Delta P}{P} = -\frac{\Delta D}{D}$$

$$\frac{\Delta D}{D} = -\epsilon \frac{\Delta P}{P}$$

$$\left(\frac{dD}{D}\right) = -\epsilon \left(\frac{dP}{P}\right)$$

$\approx \log_b \log_b a$

$$\epsilon = -\frac{\Delta D/D}{\Delta P/P}$$

$$\ln D = C - \epsilon \ln P$$

$$\ln D = C + \ln P^{-\epsilon}$$

$$D = e^{C - \epsilon \ln P}$$

$$= P^{-\epsilon} \cdot e^C$$

$$c = 0$$

And then, the sales has different components in it. So, I will explain one by one, the various components of the sales. So, the sales has different component in it. The first component of the sales is, let say, the price elasticity. So, I am keeping everything else constant. So, if everything else remains constant in this equation, so sales is equal to price to the power minus elasticity

So, why that is coming up? What is elasticity? Elasticity is basically, delta D by D minus, for D is the demand and delta P by P that is elasticity, there's a formula.

So if you remember that, that is the formula of elasticity. So then, I can write, delta P by P is equal to, is equal to minus delta D by D. Can I write this? Or can I write, delta D by D is equal to

minus  $E \Delta P$  by  $P$ . Fair enough. Now, can I then write this that  $\Delta D$  by  $D$  is equal to minus  $E \Delta P$  by  $P$ . I can write this.

So, if I write this and then if I do, try to do a integration in the both sides, I will get, basically  $\ln D$  is equal to some constant minus  $E \ln P$ . Am I, fair enough. So  $\ln D$  is equal to constant into  $\ln P$ .

I assume that constant to be 0. So then,  $D$  is equal to basically,  $\ln D$  is equal to  $C$  minus  $\ln P$  to the power minus  $E$ . There is a formula. So,  $A \log B$  can be written as  $\log B$  to the power  $A$ . So that particular formula I have used here. Fair enough. So then  $D$  is basically, is equal to, so if  $C$  is equal to 0, then  $E$  to the power  $\ln P$  to the power minus  $E$  basically,  $P$  to the power minus  $E$ . So that is why, all I am getting is demand is some constant into price to the power minus elasticity. Now, that part is clear.

So that is why, all I am getting is demand is some constant into price to the power minus elasticity. Now, that part is clear.

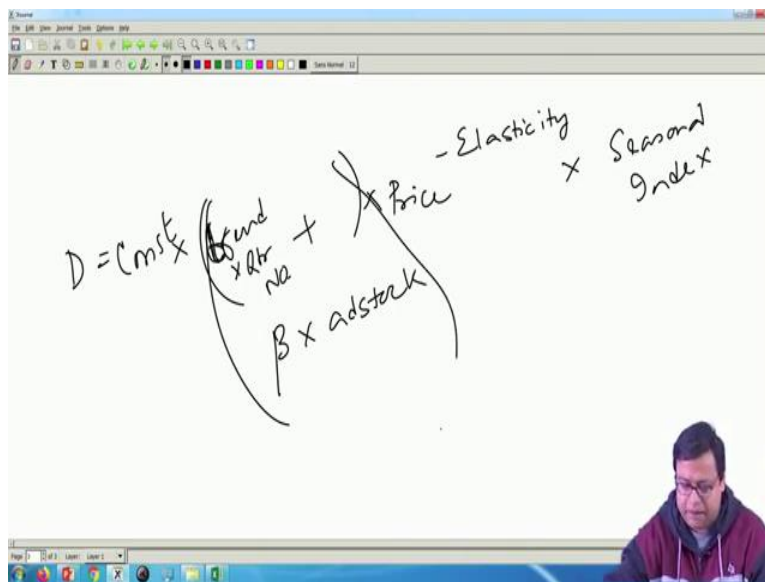
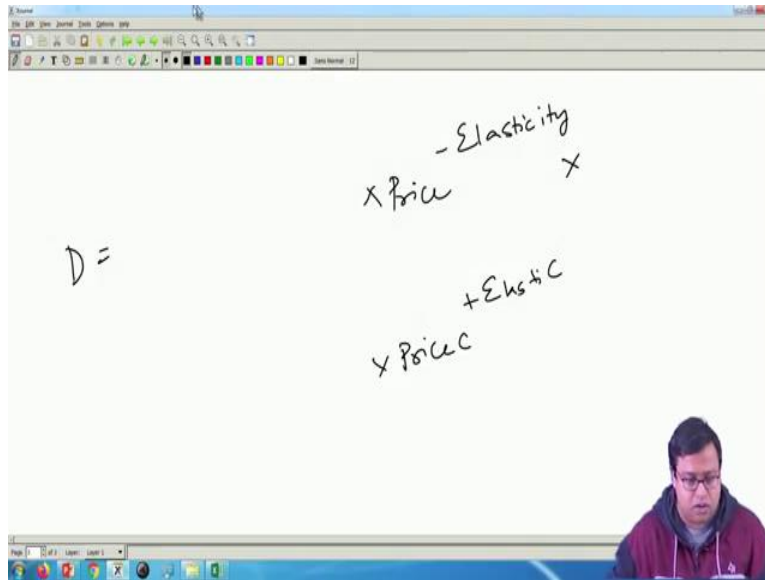
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The screenshot shows an Excel spreadsheet with the following data:

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	adefect	1.000												
2	elasticity	1.000												
3	trend	1.000												
4	lambda	1.000												
5	const	1.000												
6		1	1.000											
7		2	1.000											
8		3	1.000											
9		4	1.000											
10														
11	mean	1.000												
12														
13														
14														
15														
16														
17														
18														

Formulas visible in the spreadsheet:

- Row 3:  $Adstock = \lambda \times Previousstock + Current\ ad$
- Row 4:  $Sales = const \times (trend^{Qtrno} + Adefect \times Adstock) \times Price^{-(elasticity)} \times Seasonal\ Index$
- Row 6:  $Adstock = \lambda \times Previousstock + \frac{1}{1 + \exp(-\alpha \times Current\ ad)}$



So the price effect on demand is basically, price to the power minus elasticity.

Assuming that it is your price. If it is somebody else's price. If it is a customer's, competitor's price, which where the price increases your demand goes up, competitor's price increases your demand goes up. Then that would have been price competition into minus, plus elasticity of competition.

Because, this is called cross elasticity. The elasticity of somebody else's on your demand, which is a positive effect. But I am not taking into account that part right now.

So I will probably delete that part. So this is what I have. So then price to the power minus elasticity. And then the demand will have a seasonal component, we trend component. The seasonal component is basically, a seasonal components, seasonal index. This will be multiplied by some, some particular constant. And then there will be some trend component. Now, in this particular function, the trend has been defined as a multiplicative trend. So, trend to the power quarter number, you can also write into quarter number, no issues.

So trend, it is into quarter number. Okay? And then there will be constant at the front. Now, the only part that is left is how the advertisement will effect? Now advertisement will not effect on price. Advertisement will not effect on seasonal index. Advertisement will effect on the trend. So, any it will either trend is not, whether it is going up or going down. That going up or going down with the push for that up or for that for down, something like that is at the advertisement will do. So that is some beta parameter, into the Adstock.

That is what will come here. So this beta parameter into Adstock, we have writings in sets. Ad effect into Adstock. So I will change this part as quarter number into this thing you can do multiplication order. So this is something is my overall sales. Now I, I what I do not know? I do not what is my previous stock, I do not know what is lambda. I do not know this constant. I do not know the, the trend component, the ad effect component, the elasticity, the seasonal index. All of these things are unknown and I have to estimate.

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Quarter of Year	Quarter#	Ads	Adstock	Price	forecast	actual	APE	const
1	1	45	55.000	42.000	1.333	2.639	0.495	1.000
2	2	79	134.000	43.000	3.163	3.486	0.093	2.000
3	3	60	194.000	35.000	5.629	6.156	0.086	3.000
4	4	74	268.000	37.000	7.351	12.561	0.415	4.000
1	5	46	314.000	38.000	8.395	7.263	0.156	
2	6	44	358.000	37.000	9.838	6.354	0.548	mean 1.000
3	7	36	394.000	37.000	10.838	8.030	0.350	
4	8	30	424.000	44.000	9.818	13.892	0.293	
1	9	78	502.000	45.000	11.356	7.566	0.501	
2	10	49	551.000	42.000	13.357	6.761	0.975	
3	11	59	610.000	38.000	16.342	9.074	0.801	
4	12	67	677.000	46.000	14.978	16.655	0.101	
1	13	55	732.000	40.000	18.625	10.517	0.771	



**Solver Parameters**

Set Objective:  To:  By Changing Variable Cells:

Subject to the Constraints:

- <=
- <=
- <=
- <=
- <=

Make Unconstrained Variables Non-Negative

Select a Solving Method:  Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for Linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Quarter of Year	Quarter#	Ads	Adstock	Price	forecast	actual	APE	const
1	1	45	55.000	42.000	1.333	2.639	0.495	1.000
2	2	79	134.000	43.000	3.163	3.486	0.093	2.000
3	3	60	194.000	35.000	5.629	6.156	0.086	3.000
4	4	74	268.000	37.000	7.351	12.561	0.415	4.000
1	5	46	314.000	38.000	8.395	7.263	0.156	
2	6	44	358.000	37.000	9.838	6.354	0.548	mean 1.000
3	7	36	394.000	37.000	10.838	8.030	0.350	
4	8	30	424.000	44.000	9.818	13.892	0.293	
1	9	78	502.000	45.000	11.356	7.566	0.501	
2	10	49	551.000	42.000	13.357	6.761	0.975	
3	11	59	610.000	38.000	16.342	9.074	0.801	
4	12	67	677.000	46.000	14.978	16.655	0.101	
1	13	55	732.000	40.000	18.625	10.517	0.771	



Quarter of Year	Quarter#	Ads	Adstock	Price	forecast	actual	APE	const
1	1	45	91.597	42.000	1.245	2.639	0.528	1
2	2	79	170.597	43.000	1.773	3.486	0.491	2
3	3	60	230.597	35.000	6.156	6.156	0.000	3
4	4	74	304.597	37.000	14.646	12.561	0.166	4
1	5	46	350.597	38.000	7.263	7.263	0.000	
2	6	44	394.597	37.000	7.712	6.354	0.214	mean
3	7	36	430.597	37.000	9.128	8.030	0.137	
4	8	30	460.597	44.000	10.747	13.892	0.226	
1	9	78	538.597	45.000	5.504	7.566	0.273	
2	10	49	587.597	42.000	6.761	6.761	0.000	
3	11	59	646.597	38.000	12.264	9.074	0.352	
4	12	67	713.597	46.000	13.816	16.655	0.170	
1	13	55	768.597	40.000	12.866	10.517	0.223	

So, to start with, I have thought that each of them is one, everything an initial stock is 10. So, how much is the Adstock then? The Adstock is here initial stock plus, initial stock into lambda plus current ad. And this here, it is pervious Adstock into lambda plus current ad.

And here, before I jump the lambda should have been F4. So this is what mine Adstock is. Fair enough. Adstock looks like this. Then what is the sales forecast?

The sales forecast is basically, carefully you see, the sales forecast is this constant into the trend component I will right something here, into the price to the power minus elasticity component I will write something here, and into the seasonality component. I will write something here. So let us write the seasonally component. Seasonality is what? vlookup, what vlookup of this quarter number. In this particular table, and 2 comma FALSE.

And this L6 and M9 should be in a, in a table form. So, that is what I have created. Vlookup C6, C6 is means, the quarter of year, in which table L6 to M9. So this table. This table and then 2 comma FALSE. And what is the price to the power minus elasticity? This is price to the power, and within another bracket, here I have to write minus elasticity, minus this elasticity. And elasticity also should be in F4.

And then, so you can pause at various point of this particular video and see. And then, the quarter number into the trend, quarter number into the trend, and trend should be F4 plus ad effect into Adstock, and ad effect should be F4. So that is my forecast. And if I just drag it, this is

the forecasted sales that I am getting. And what is my average percentage error? This is my forecasted minus actual divided by actual, something like this, absolute. So, I will take ABS.

And mean absolute, it already is basically average of these values. Average of these values. Why it is coming 0 here? Okay. So, M5 should also be in F4. Okay. So mean absolute error, this is the thing, and this is the guy, whom I have to minimize. So, I go to data solver, and I try to minimize this particular person. So I will say that, set this by marking directory here. Yeah, sorry. Set my objective on that cell J4. And then minimize this. And whom will I change?

I change all the parameters that I have to estimate. I will change these people comma and my initial stock, which I do not have an idea. If I have an idea, I will put that. Otherwise, if I do not have the idea put that he there. Now all of this guys will have certain constants. For example, let us say the Adstock. The initial stock. The initial stock will be greater than 0. And initial stock will be smaller than hundred, something like that because you will see that, all the ad expenditure is between 0 to hundred. So, I am keeping it 0 to, between 0 to hundred. Fair enough.

And then, let us say, the, these values should also be greater than 0. See, the effect of advertisement on sales has to be positive. The elasticity we have assume to be positive that is why we have take minus elasticity. That trend component can be whatever I do not know, negative, positive. So I, I will not to put the trend component in this thing. And then, let us say the lambda or let us say trend also positive, because it is going up and it leaves a plot, you will see the trend is also going up.

The lambda it is between 0 to one. So, this is something that is important, we have to check. And this constant terms has to positive otherwise your sales will become negative. So, I will put everyone here, has to be greater than equal to 0. Okay? And let us say, everyone has to be smaller than equal to hundred. And this lambda decay. So, this has to be a smaller than equal to one. And anything else. I think that is fine. So, these are my some of the constant that I have taken. And I will use, let us say evolutionary method, and I will to try to solve it.

So all variables must have upper and lower bound. So, do I not have for every variable upper and lower one of the check. For example, okay. So this has been written as I1, sorry. So change this will be not I1 but H. Not H1 or I1, okay? H and then M1 to M9 is between 0 to one and M4 is

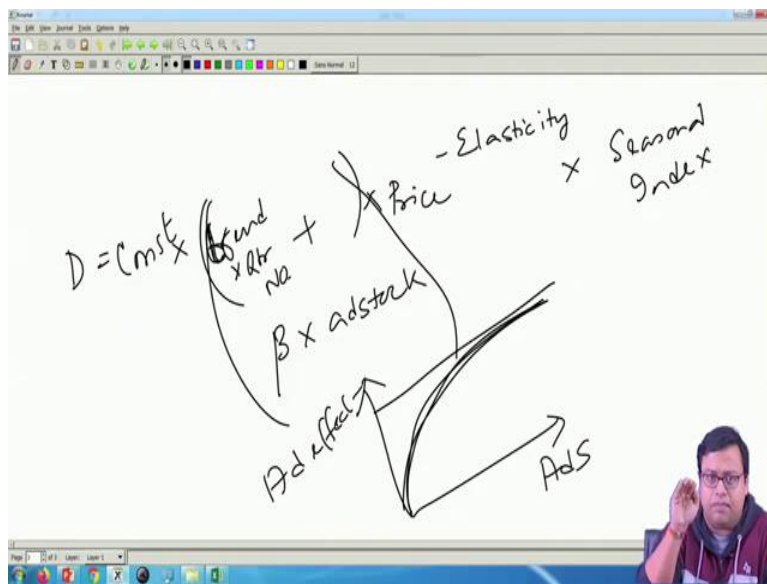
another one. Now I have to try to solve. So it is solving. And it has already reduce to 0.2, 1, 5 and it will take some time probably I will wait for one minute to see what happens. And then, I will, I will press an escape.

So, what it is doing? Is it is trying to change all these values and trying reduce this MAPE as lower as possible. When I get that, what I get is the function of sales or some label of non-linearity has been taking into account. In my picture, I, okay. So it is stuck at to one two. So I will just press an escape here. And I will stop, and I will say that okay, so keep my solver solution. And this is the values that I'm getting. So what, it is saying that, he could have done a little bit more probably. So, it is saying that this is, this is the ad effect.

You will see that the, for every this thing it is going up by 51, for every increased Adstock and the elasticity is like this and the trend is like this, and so on, the values for all, that lambda value is coming up to be one. Lambda value coming at to one means, absolutely full ad effect in the next, which should not be the case. We, so, if you had run this (almost) some more time would have hot a better lambda value. There is no decay at all. So this is something that is being shown, which might not be the case properly. This is something that we have done till now.

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	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1			adefect	51.459										
2			elasticity	4.189		Adstock = lambda x Previousstock + Current ad								
3	MAPE		trend	34.607										
4	0.212		lambda	1.000		Sales = const X (trend x Qtrno + Adefect x Adstock) X Price <sup>-elasticity</sup> X Season								
5	APE		const	37.830										
6	0.528		1	43.743		Adstock=lambda x Previousstock + 1/(1+exp(-alpha Current ad))								
7	0.491		2	36.886										
8	0.000		3	39.979										
9	0.166		4	90.868										
10	0.000													
11	0.214		mean	52.869										
12	0.137													
13	0.226													
14	0.273													
15	0.000													
16	0.352													
17	0.170													
18	0.223													



Now, another addition of these thing is, see Adstock here we are writing lambda into previous stock plus current ad. Now oftentime, the researchers suggests that the effect of ad is not linear. Then your ad expenditure goes up and your ad effect this guy, will have a, this kind of relations. So which is increasing but the marginal increase drops. That means it will look like a, I would say, look like S curve or something like that. So what, it will, it will finally, actual saturate at some level.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
19			2	14	42	810.597	43.000	8.453	8.647	0.022				
20			3	15	57	867.597	44.000	8.906	11.271	0.210				
21			4	16	23	890.597	35.000	54.221	29.031	0.868				
22			1	17	27	917.597	41.000	13.865	10.929	0.269				
23			2	18	40	957.597	41.000	12.204	9.856	0.238				
24			3	19	25	982.597	42.000	12.274	12.661	0.031				
25			4	20	30	1012.597	43.000	26.057	26.056	0.000				
26			1	21	67	1079.597	46.000	10.080	13.730	0.266				
27			2	22	27	1106.597	42.000	12.758	13.513	0.056				
28			3	23	53	1159.597	42.000	14.489	17.161	0.156				
29			4	24	56	1215.597	42.000	34.521	43.227	0.201				
30			1	25	40	1245.597	45	12.7567358						
31														
32														
33														
34														
35														
36														
37														

Now once that I have you this results, what do I do? Let say, it is a next time period. In the next time period, this one is 1, this one is 25, and let say, I do an expenditure of 32, then how much is my Adstock? My Adstock is something like this. Let say the price is, let say around forty one, and then this is my expected forecasted sales, in the next time period. So, I might want to know that at what level of price? And what level of ad expenditure, this particular value which is a forecasted sales, maximizes.

With this forecasted sales we can do the forecasted revenue also as we have forecasted profit also as we have done for video 1, week one, sorry, week six season 1 video, we can do that kind of calculation also. If my job is already to sales maximization, I have to find out. That now if I change these, if I try to optimize this based on the formula I go these. If I change 32 to 30 this value changes. If I change 41 to 45 this value changes. So what combination of this two, will give you the best result.

So, all I am trying to do here, in comparison to the previous excel, previous video a previous station is that we have created of function on formula which is a little bit difficult to estimate which is bigger than the previous one were, here the non-linearity components were also included in the model and based on that we are trying to predict how much will be the, this thing. And lambda is right now I am getting 1, actually real life situation lambda is something smaller than that, you can actually estimate lambda outside the data set also and you can put that value.

And then you will be finding out the decay effect of advertisement. In the, if you have run this particular thing for somewhat time probably you have also seeing the decay effect. And that is how the advertisement and sales promotion ones. Now till now, till this video, we are still discussing advertisement and sales that means advertisements and pricing as only one single component. So, there is only one type of ad, and there is only one type of price.

But we have not considered multiple types of price promotions that you can give at multiple pricing schemes that you can give. At the in-combination with multiple types of ads you give. This is the collective ad expenditure but the effect of ad expenditure might be different for TV, the decay will be different for TV, different for radio, different for other people which is still not being same in this particular video at a greater depth. So, in the next videos, where we will discuss about the marketing mix and et cetera.

We will go in greater depth about how various types of ads in various channels will actually effect. And we will also bringing the other component of marketing mix, as I told that marketing mix has the product component also and the place component also. So you have to decide that okay if you have a marketing budget, how much money. Now, this marketing budget sometimes comes into account with the, with the supply chain budget as we well. But still if I think overall as an organization, they can put money on the supply chain side, on the margin and et cetera side also.

So, where sales and your branding managers, these are the two people where in most of the organization they are competing with each other. The sales guys say that the, “Marketing guy gives me or branding guy gives me unrealistic assumptions or branding guy promises such-such things which cannot be delivered.” And the branding guys often tell that, “This sales people are not performing well, there are not actually pushing themselves and et cetera, et cetera.”

So these two people who are sales guy, who actually talks about, who are in the field, who talks with the retail stores, so gives the margin. And then there are certain supply chain managers also who gives a margin to the, who gives certain range to the travelers, transporters and et cetera. These are all place related costs, spread related, place related investments. Now, how those investments interplay with advertisement expenditure in multiple channels that you are doing?



And how all of these things also are related with this R&D expenditure or product development expenditure that we are doing, or product management expenditure that we are doing. This is something we are slowly going to a little bit more complex, complex models. Here, we have brought in a little bit of non-linearity for further more complexity can be brought in our data set. So, we will do that all of these things in the next videos, in this week six only. Thank you for being with me. I will meet you in the next video.