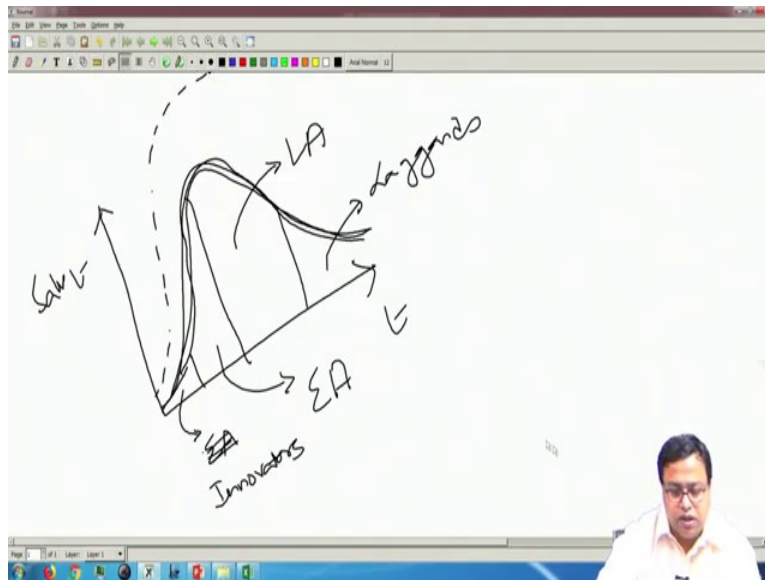


Marketing Analytics
Professor Swagato Chatterjee
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Lecture 21
Demand Forecasting and Pricing (contd.)

Hello everybody, welcome to Marketing Analytics course. This is Doctor Swagato Chatterjee from VGSOM, IIT Kharagpur, who is taking the course for you, we are in week four. And we are discussing demand forecasting and pricing. So, in this particular video I will talk about a special type of forecasting technique where the product is generally durable product which you buy only once in a life time or probably once in a particular model at least if not in the life time. And then you so the maximum possible market share is fixed in this kind of a situation what happens?

So, we call it a technology adoption model where bass diffusion comes into the picture, and where I say that there are for any product there is a product life cycle.

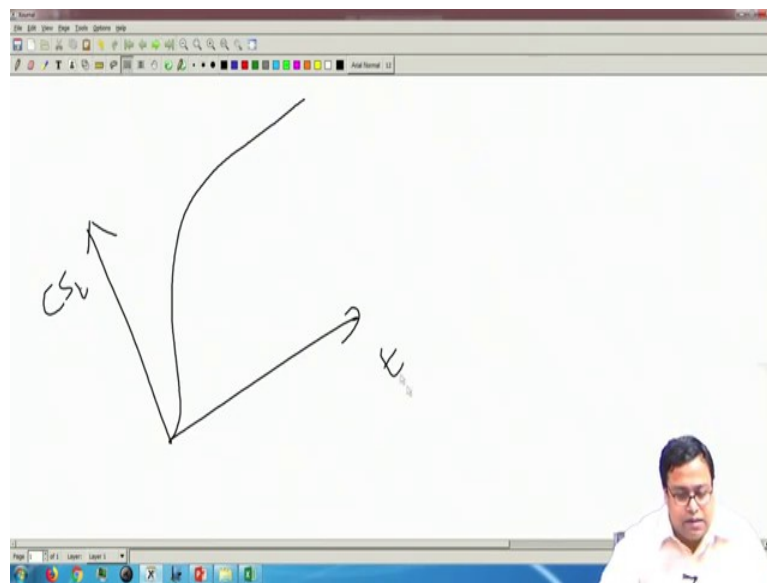
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And product life cycle the product life cycle if you have seen, there are see the product life cycle PLC curve those in marketing 101, looks like this, the PLC curve looks like this. And if this is the thing this is the sales, at time t and this is the time t. First the sales is low and then it picks up and then it reaches maturity and then decline.

So, this is the part who are basically early adopters, sorry so these are innovators. Then there will be early adopters, this part is probably so these are laggards. So, innovators, early adopters, late adopters, and laggards probably something like that. So, this is something that I try to look at. And this is same for, heavily same I would not say exactly same, for many technology products. Now, often time we are interested to actually estimate this particular curve. How this particular curve will look like. And if you also see that, if I try this as sales at time t , if I try to plot the cumulative sales. The cumulative sales looks like this.

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So, it goes up then gets saturated at a certain point of time. So, if it goes up, and get saturated at a certain point of time, it might look like a S curve. So, the this is how it looks like basically it goes up and get saturated at a certain part of time. So, this is the cumulative sales, time t and that is how it looks like. So, this is something that we are trying to model.

And then there was professor Bass in, I think 1960s when he has given this particular model. And he said that. So, there are early adopters, what happens in the market, there were some. So, it is also related to spreading of rumors spreading of virus infections, actually happening of a viral message or probably creating a revolution, all of this things are related to this kind of a model world.

There has to be a significant mass accumulated at the starting point. Only when that significant mass gets accumulated it will actually reach the infection point and then will go up. If it does not

get that significant mind of the starting point, then it will not actually reach the highest levels. So, that is why maximum amount of technology products fail because they fail early, and the early failure occurs because the significant mass is not getting generated.

Significant amount of demand is not getting generated at the starting point of their life. So, that is something which is common and you will see that epidemic is not very common, viral epidemic is not very common. It happens but it if if you can actually take care of in the starting point in a in a first 100, 200, 1000 patience if you can, if you can limit it within that limit then you are safe. You can survive, and if it goes out of the hand if the significant mass is getting generated in terms of number of patients at the starting point then it will reach epidemic point of view.

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Handwritten notes on a whiteboard showing the derivation of the Bass model:

$$F'(t) = f(t)$$

$$F(t) = P + qF(t)(1-F(t))$$

Annotations include:

- P is labeled as "coeff of innovation".
- q is labeled as "coeff of imitation".
- A note says "Total cumulative sales upto time t" with an arrow pointing to $(1-F(t))$.

So, something like that is this particular model where Bass told that, that let us say the accumulation, the extra sales $f(t)$ if the total sales possible is 1. The 100 %. Then the %sales happening at time period t , will actually depend on two things. There is some innovators, p innovators and then there are q adopters.

So, p is called coefficient of innovation, and q is called coefficient of adoptions. So, some guys who are innovators. So, there will be always some people who you will actually who will actually be there in a mass the huge mass who will be coming into coming in touch with the particular technology and buy.

Let us say p % age of the overall population, are innovators. What is the innovators mean, innovators means whenever they come in contact with the particular technology, they purchases the technology or whenever they come in contact with a particular virus, they gets affected by the virus.

Whenever they comes contact with a particular let us say viral message, they share the viral message. So, these are the guys who are innovators. So, at time period t , how many persons are left at time period t , $f(t)$ person's, $f(t)$ the small $f(t)$ persons are actually who are going to adopt this particular product at time period t , fair enough.

Then if the t is very small, then capital $F(t) = \int f(t) dt$ or it is the, in other words in normal language it is the total cumulative number of people, cumulative sales that up to time t .

So, up to time t total sales that has happened, or total number of people who has got adopted, is actually told by capital $F(t)$. Now, if capital $F(t)$ number of people, has already bought this particular product, up to time period t . Then how many people are left assuming the total is 1, $(1 - F(t))$ people are left this many people are left, and out of these many people p %, p proportion of people will actually come in contact and purchase.

Each every time period. So, $p(1 - F(t))$. These many people purchases at this time period, adopts on their own. Now, the rest of the people does not adopt on their own. They adopt based on whether other people have adopted or not. So, they actually are friends of the already people who have adopted.

So, what happens is this capital $F(t)$ people who have already adopted, they go and talk with others. The word of mouth, and because of this word of mouth there are some people gets affected. How much, which people are there, $q(F(t))$ this proportion number of people means the more the adoption happens, the more the adoption happens, the more is the chances of word of mouth. No, so if more number of people have actually experienced that product, then the word of mouth will happen more. So, this is the overall word of mouth this part is the overall word of mouth.

Now, these proportion of people come from where? The same the rest of the guys they will comes from the rest of the people no. They will not come from somebody else. So, this is my

total rest of the people, this is my total rest of the people. Now, out of this rest of the people, p proportion comes on their own and $qF(t)$ proportion comes by word of mouth.

They do not come on their own, they come by word of mouth. So, as $F(t)$ goes up, as already accumulated cells goes up, as adoption number of people who have already adopted. Let us say WhatsApp, WhatsApp how many people will actually adopt, so let us say today is 2020 and there will be how many people will adopt in let us say February 2020. How many people will adopt.

So, let us say if there are around I do not know, 10 million or 100 million customers possible who have internet access, they are possible hundred million customers. And out of them have 50 million has already adopted, and the rest is 50 million. So, as this rest of 50 million is higher, the chances of adoption is higher as this shrinks this rest of 50 millions shrinks.

The number of people who will adopt at a time period will also shrink. If instead of 50 million, if it was 5 million the rest of the people. If by chance instead of 50 million if it was 5 million then the number of people who will adopt in February 2020 will also be smaller in the second case. So, how many people will adopt in their... in the current time period, will depend on how many people is left aside till now. Who have not adopted till now. So, that is this part this part is that.

And how many people will adopt in the current time period, will depend on some amount which is actually the innovators, who depend on nobody which is the p and some amount which depends on the already adopted people. If the already adopted people is 25 million, what says the already adopted people is 5 million then the 25 million guy will have a higher word of mouth than the 5 million guy.

So, that part is actually taking care of this point. What we are talking about the imitation part. So, this is the p is called coefficient of innovation, coefficient of innovation and this q is called coefficient of imitation. So, and these two has huge meaning. So, by saying that what I am trying to say is that the currently the I would say just 1 minute.

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$$\frac{f(t)}{[1 - F(t)]} = p + q F(t)$$

$$S(t) = \frac{m \times f(t)}{p} = \frac{m (p+q)^2}{p} \times \frac{e^{-(p+q)t}}{\left(1 + \frac{q}{p} e^{-(p+q)t}\right)^2}$$

So, currently the what is what is the function then? The function is $f(t) / [1 - F(t)]$. This is actually the function of Bass diffusion model is $f(t) / [1 - F(t)] = p + q F(t)$. If you actually try to solve this is a differential equation and if you try to solve this. And what is what is sales at time period t , if total number of market share is m , the total market number of revenue that you create, $m \times$ small $f(t)$ is the total sales that can happen at time period t . $[S(t) = m \times f(t)]$.

And the functional formula for this is, if you solve that you will find, $[m (p+q)^2 / p]$. This is the part $\times e^{-(p+q)t}$. And this function I do not remember I am just writing it from a note that I have. So, it is difficult to remember, 1 + you can just google it up, $e^{-(p+q)}$ of that. So, you can just google it up. $[S(t) = (m (p+q)^2 / p) \times e^{-(p+q)t} / (1 + q/p e^{-(p+q)t})^2]$

I am not saying that you have to, so but I have actually copied, I have kept a note I have written it there. This is the function that is there. So, this p and q endures the compulsion formula. It has not no purpose. But there are some things why it helps. So, this is the sales function. So if you will use this particular sales function later.

So, you we have to remember this, this is the sales function. We do not need to remember this, but this we have note it down. So, that we can use it later that is number 1. What extra? So, you will see that there is a peak point, when the sales is peak. That peak sales point is called the function is $t^* = \ln q - \ln p / p + q$. And at this time period, the number of products that are sold, so $f(t)$ is basically $f(t^*) = 1/4q(p+q)^2$. So, you will see that at the highest time period, when the if the more is

the innovation and imitation the higher you will reach. And more is the imitation in comparison to innovation, it will come down. So, that is something that we will get.

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$$CS(t) = m \times F(t)$$

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}$$

And if I find to find out the cumulative sales, if I try to find out the cumulative sales that was a individual sales, that was the individual sales if I try to find out the cumulative sales the formula is, so F(t) the formula is basically and S cumulative sales $CS(t) = m \times F(t)$. And F(t) formula is.

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}$$

So, something like that is the formula. And will use this formulas to estimate the effect of the effect of innovation and imitation in case of our technology product.

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Year	laptops/100 predicted
1	12
2	17
3	20
4	24
5	28
6	37
7	48
8	60
9	71
10	78
11	82
12	87
13	90

Year	laptops/100 predicted
1	12
2	17
3	20
4	24
5	28
6	37
7	48
8	60
9	71
10	78
11	82
12	87
13	90

So, how do I do that? So, let us say I have a data set. Simple data set, this is the data set that various times and we will see that 100 is the maximum possible achievable sales. And I have 12 % of the maximum possible has been achieved in 2001, 17 % in 2002, this cumulative 20 % in 3, 24 % in 4 and etcetera, etcetera. That is the value that I got.

Now, let us say if I say that ok the cumulative value p is 1 and q is also 1. Then what is the cumulative value. The cumulated, so this is my predicted. The predicted cumulative value is the formula is written here 1 -, so is = just check what I am writing. 1 - exponential of what?

Exponential of something, what will I write it in this something, in the something I will write something \times the t, the t is this. And what is this? This is $p + q$. So, this + this and H2 and H3 I have to, I have to put a F4. So, carefully see what did I write that the equations, I am just writing the equation of the this equation basically I am writing this equation properly.

So, you have to find out that whether I am writing correctly. And then divided by $1 +$ what is here? And exponential of something here. What is here I will write q/p , so q/p means this divided by H2. H2 is and I will put a F4 sign here. F4 here and F4 here and exponential whatever I wrote inside this one will come up to here again. So, the same $p + q \times t$, I will put it here and then put a negative sign(-) in front of it.

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The screenshot shows an Excel spreadsheet with the following data:

Year	laptops/100 predicted	p	q
1	12	0.76159	
2	17	-52.6341	
3	20	-401.434	
4	24	-2978.96	
5	28	-22024.5	
6	37	-162753	
7	48	-1202602	
8	60	-8886109	
9	71	-6.6E+07	
10	78	-4.9E+08	
11	82	-3.6E+09	
12	87	-2.6E+10	
13	90	-2E+11	

Year	laptops/l	predicted	
1	2001	12	0.76159
2	2002	17	0.96403
3	2003	20	0.99505
4	2004	24	0.99933
5	2005	28	0.99991
6	2006	37	0.99999
7	2007	48	1
8	2008	60	1
9	2009	71	1
10	2010	78	1
11	2011	82	1
12	2012	87	1
13	2013	90	1

$$CS(t) = m \times F(t)$$

$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}}$$

So, if I just run this, I think I have wrote this one will be negative here. I think I have not run it correctly. So, this is something is the %. So this should be if I am not wrong, this should be - also just check I have forgot. So, this is something that is that is the formula that we get.

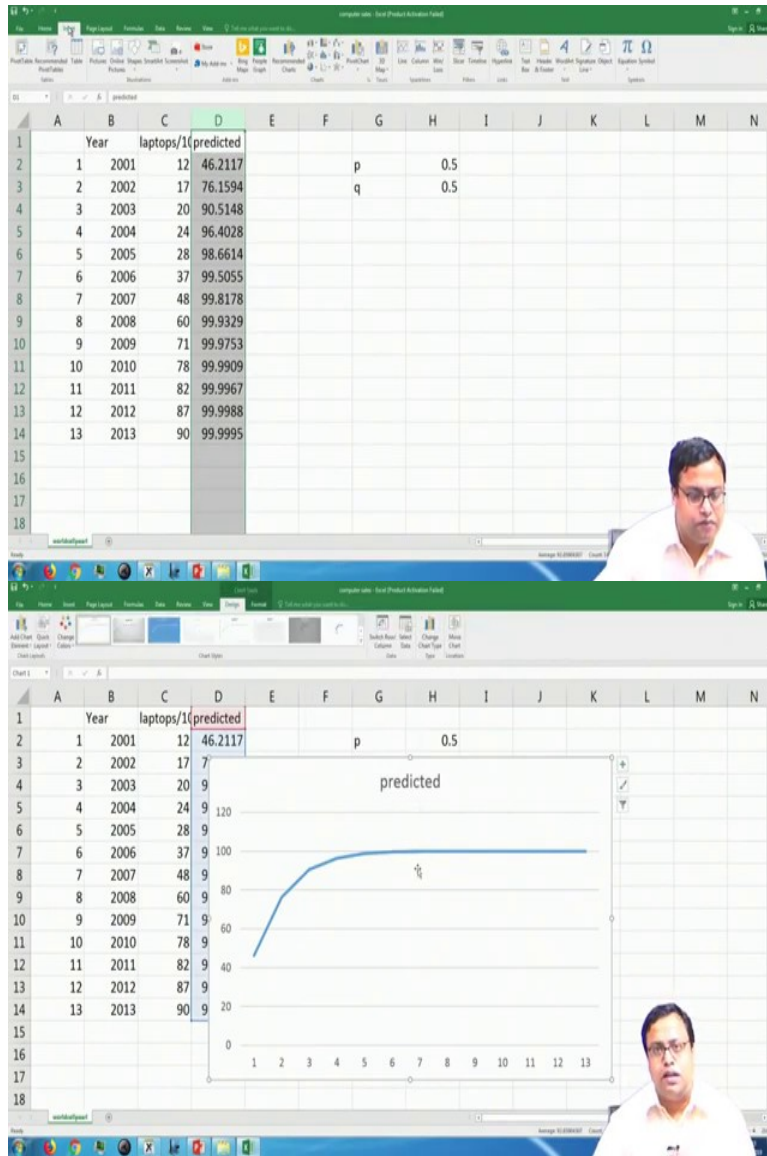
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Year	laptops/1	predicted			
1	2001	12	0.76159	p	1
2	2002	17	0.96403	q	1
3	2003	20	0.99505		
4	2004	24	0.99933		
5	2005	28	0.99991		
6	2006	37	0.99999		
7	2007	48	1		
8	2008	60	1		
9	2009	71	1		
10	2010	78	1		
11	2011	82	1		
12	2012	87	1		
13	2013	90	1		

Year	laptops/1	predicted			
1	2001	$12/42 * 100$	0.76159	p	1
2	2002	17	0.96403	q	1
3	2003	20	0.99505		
4	2004	24	0.99933		
5	2005	28	0.99991		
6	2006	37	0.99999		
7	2007	48	1		
8	2008	60	1		
9	2009	71	1		
10	2010	78	1		
11	2011	82	1		
12	2012	87	1		
13	2013	90	1		

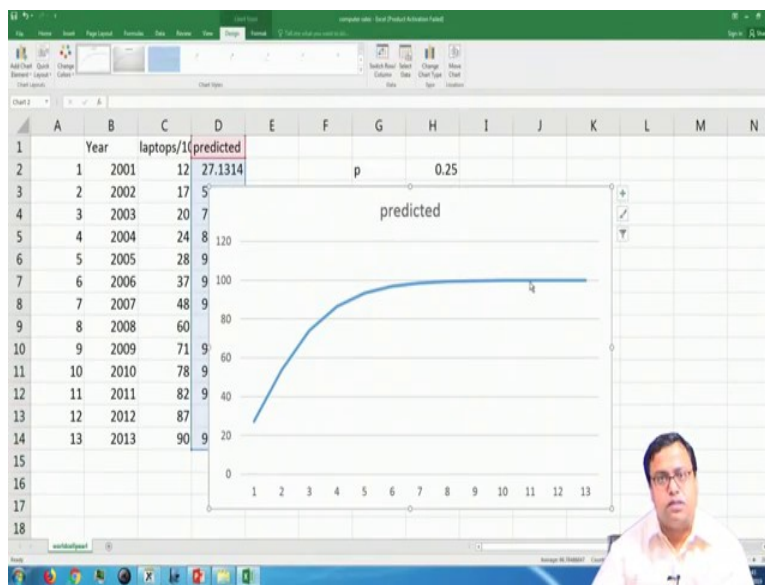
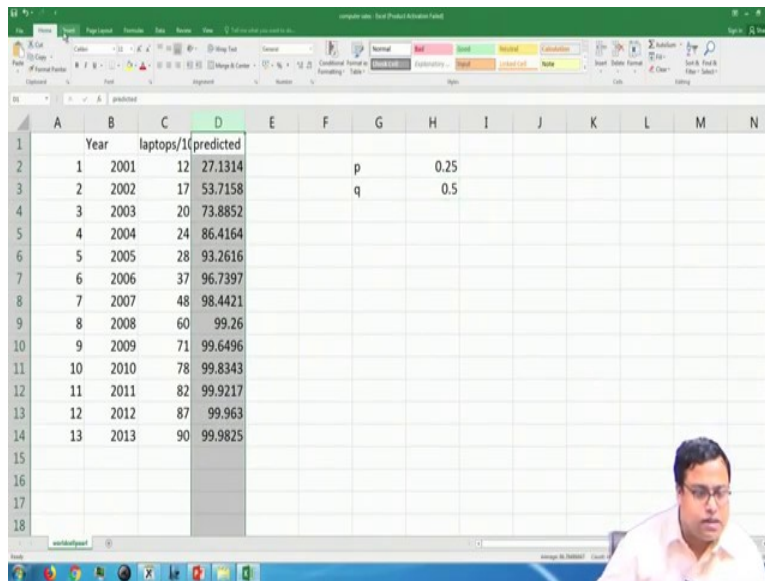
And if I try to, if now I try to, so this something and then this is this is in proportional term, this $\times 100$ is the maximum possible thing. So, this is what I am getting say 76 to 100.

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I will reduce it a little bit, let us say the this is 0.5 and this is also 0.5. Now, if I just try plot it, if you will see that if I try to plot this particular curve, this curve comes up to be as almost a S curve that I just told you.

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Let us say if I if I further reduce it let us say 0.25 and 0.5. Then and then try to plot it. It comes up to be a S curve. So, simple good looking S curve. But this values and this values are different because the actual pq value and the one that I started with will be different.

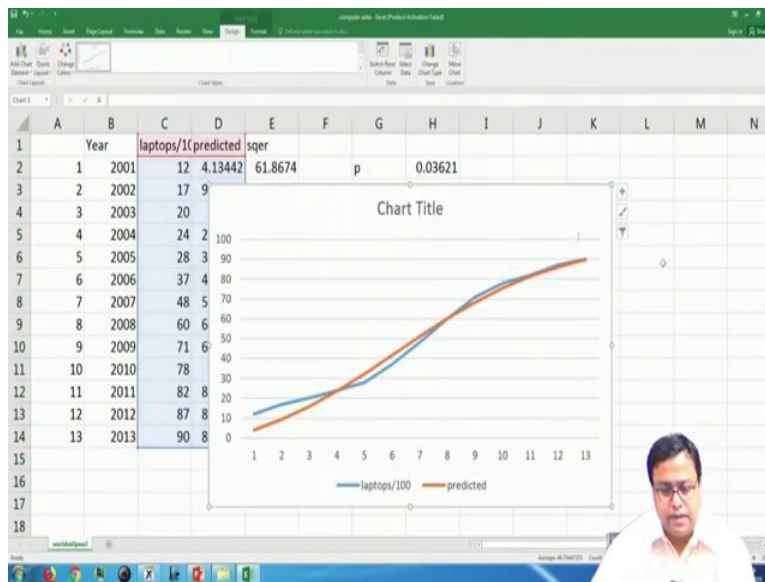
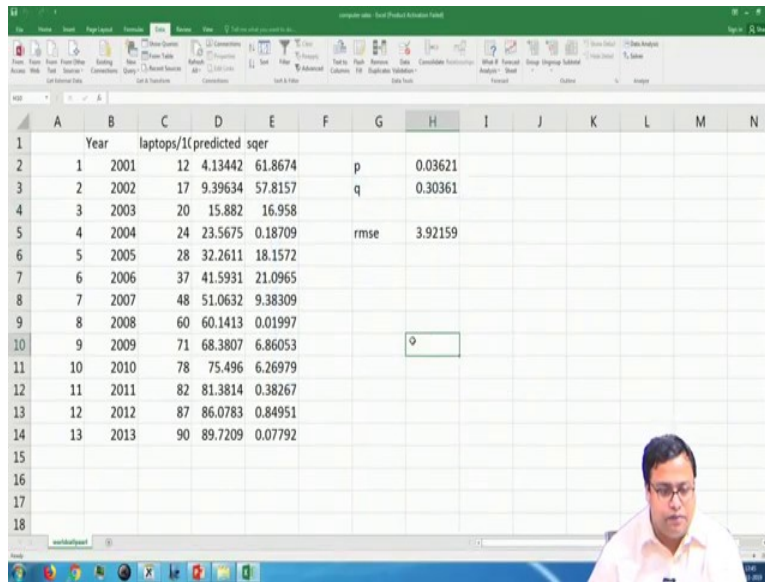
	Year	laptops/l	predicted	sqer			
1	1	2001	12	27.1314	228.959	p	0.25
2	2	2002	17	53.7158	1348.05	q	0.5
3	3	2003	20	73.8852	2903.61		
4	4	2004	24	86.4164	3895.81	rmse	41.3024
5	5	2005	28	93.2616	4259.08		
6	6	2006	37	96.7397	3568.84		
7	7	2007	48	98.4421	2544.4		
8	8	2008	60	99.26	1541.35		
9	9	2009	71	99.6496	820.797		
10	10	2010	78	99.8343	476.735		
11	11	2011	82	99.9217	321.186		
12	12	2012	87	99.963	168.039		
13	13	2013	90	99.9825	99.6506		

So, I will solve this. And it gives me, ok so p is zero, p cannot be 0 as well. So that is something that I have to also check, so sorry I forgot that. So I have to check that it cannot be 0. So solve and add that. This guys has to be greater than equal to 0. Add, so now if I have this one as my constraint, now if I try to solve. It is still coming up to be 0 just 1 minute. I have done something wrong.

So it might be the case that, p cannot be 0 because if p is 0 then this one comes to be infinite. So what did I do, I wrote greater than and equal to. That is something that I did wrong.

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	Year	laptops/l	predicted	sqer			
1	1	2001	12	27.1314	228.959	p	0.25
2	2	2002	17	53.7158	1348.05	q	0.5
3	3	2003	20	73.8852	2903.61		
4	4	2004	24	86.4164	3895.81	rmse	41.3024
5	5	2005	28	93.2616	4259.08		
6	6	2006	37	96.7397	3568.84		
7	7	2007	48	98.4421	2544.4		
8	8	2008	60	99.26	1541.35		
9	9	2009	71	99.6496	820.797		
10	10	2010	78	99.8343	476.735		
11	11	2011	82	99.9217	321.186		
12	12	2012	87	99.963	168.039		
13	13	2013	90	99.9825	99.6506		



So, this is not greater than equal to (\geq) change it and make it. So, greater than equal to (\geq) some very small value .001 now solve it. So, I have to change 0.25, 0.5 and in the solver I change it to 0.001, solve it. And now I am getting some value, fair enough.

So this is the predicted, this is the actual and my rmse is much lower. And these are the these are the cases that I am getting, so initially I am not getting results but as I go ahead I getting good pretty much pretty close 89, 90, 87, 81 and so on. The primary 2, 3 values are not very good. So, this is the table. So, if I just try to plot this two curves, see this are very close curves. Actually what I am getting. And that is how.

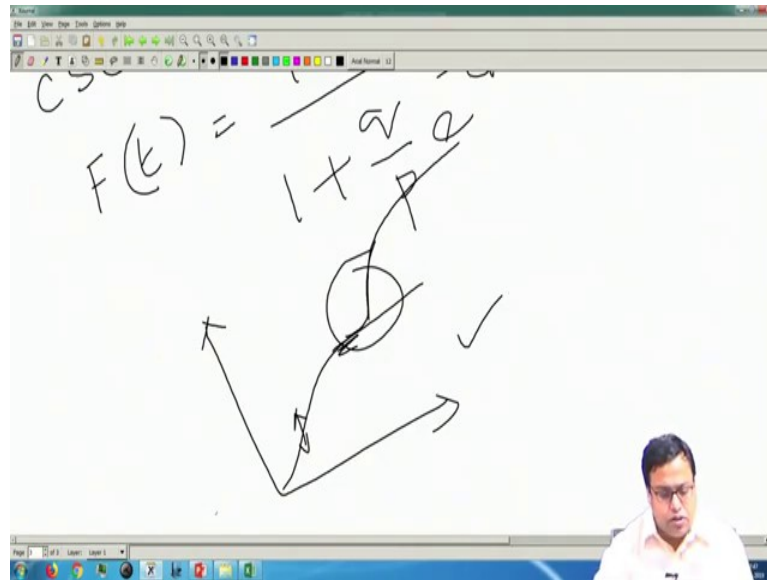
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	A	B	C	D	E	F	G	H	I	J	K	L	M	N
13		12	2012	87	86.0783	0.84951								
14		13	2013	90	89.7209	0.07792								
15		14		92.4834										
16		15		94.5434										
17		16		96.0602										
18		17		97.1665										
19		18		97.968										
20		19		98.5458										
21		20		98.9609										
22		21		99.2583										
23		22		99.471										
24		23		99.6229										
25		24		99.7313										
26		25		99.8086										
27		26		99.8636										
28		27		99.9029										
29		28		99.9309										
30		29		99.9508										

Now, if I know that, if I have to know that at what point I will reach 99 %age. When I will over more or less get everything. I will just simply use this one end probably dragging it up value upto 65 time period. I do not know. So 12, 13 and then I will drag it up upto 65 time period. And if I just double click on this, it will give me okay.

So, you are reaching 99 at 21st time period. So, another another probably 8 time periods and it will reach your maximum possible thing. Now, why this is important, why will I use it. If by chance I know that my pq is like this. At this point of time at 32, then I will know that at what time I will reach the maturity, and if I know that at what time period I will reach the maturity, in how many days I will reach the maturity.

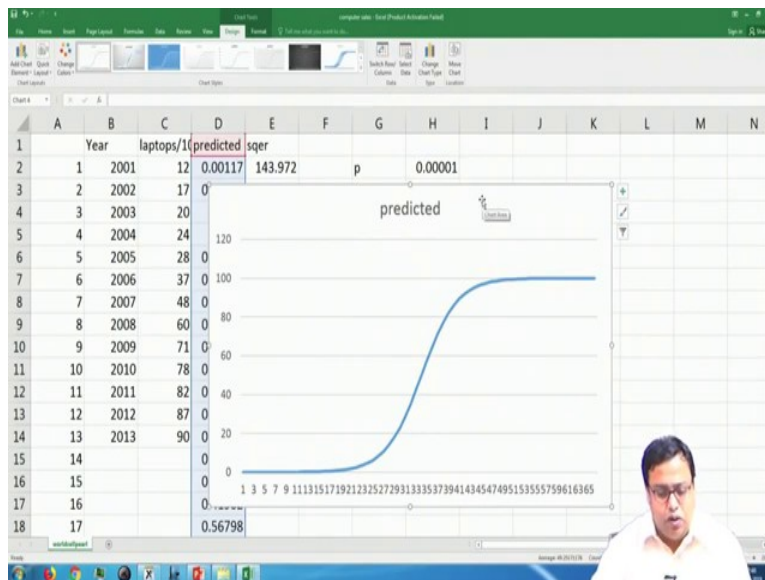
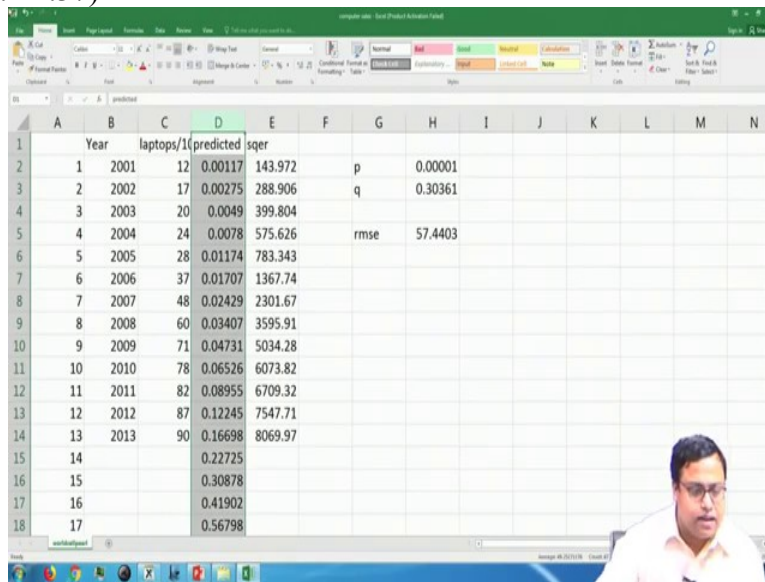
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I can pre plan, if you remember like iPhone 1 and iPhone 1 iPhone 2, so they keep on innovating. So, that they try to create a curve, sales curve which looks like this. So their PLC curve is like this, it goes out which is the maturity and then another curve comes up. From here itself another curve comes up and goes. So, they do not let it come down. They come up with a new newer products and etc.

So, this is something that is important. If I when I am standing here, if I know that ok I am reaching maturity or not, I at all if I am reaching maturity or not then this particular thing is ok. Now, you will also see that this p if it by chance if the p value is very low. By chance if the p value is let us say, I am just giving an example.

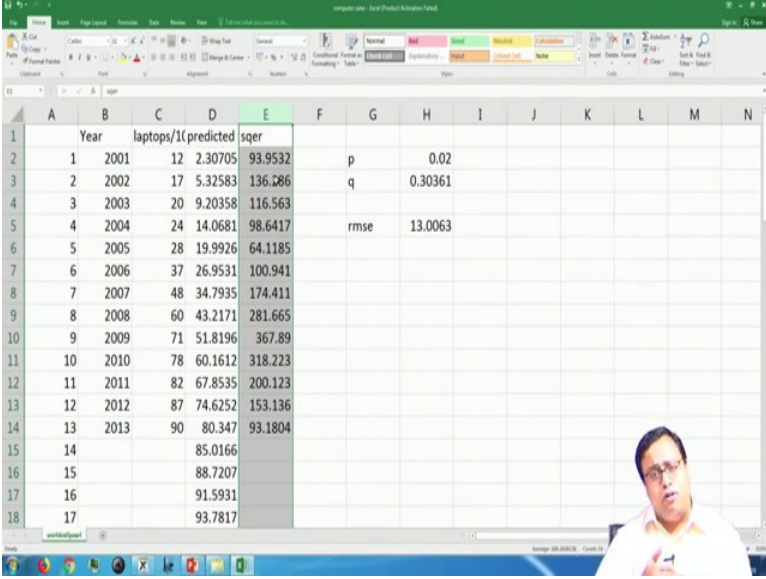
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By chance this P value is 0.00001, then how does the curve looks like. Just try to plot this curve, when the p value is very low. So, the p value is very low it will reach, it initially it will be absolutely 0, nobody will buy it initially. So, if nobody buys it and then even only after 30 months or 30 years somebody buys it, it is the yearly data.

So, 30 years, somebody buys it then actually it fails. It will fail 2, 3... So, that is something that is very important to understand that your p has to be higher.

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1		Year	laptops/10 predicted	sqer										
2		1	2001	12	2.30705	93.9532	p	0.02						
3		2	2002	17	5.32583	136.286	q	0.30361						
4		3	2003	20	9.20358	116.563								
5		4	2004	24	14.0681	98.6417	rmse	13.0063						
6		5	2005	28	19.9926	64.1185								
7		6	2006	37	26.9531	100.941								
8		7	2007	48	34.7935	174.411								
9		8	2008	60	43.2171	281.665								
10		9	2009	71	51.8196	367.89								
11		10	2010	78	60.1612	318.223								
12		11	2011	82	67.8535	200.123								
13		12	2012	87	74.6252	153.136								
14		13	2013	90	80.347	93.1804								
15		14			85.0166									
16		15			88.7207									
17		16			91.5931									
18		17			93.7817									

So, p has to be at least 0.02 to get something to get to reach upto a tenth level or something like that. So, that % should be there in your p , which is the coefficient of innovation. And then coefficient of adoption can be something that it is also very important. But this is something that is very important.

If it is lower, then your product does not reach the maturity level, it will actually die after second time period, third time period and max 10 time period it will die. So, that is something that will keep into account. So, that is all for sales forecasting, we called it Bass diffusion model. And later point of time people have actually included much other things.

So, for example now I have some error, I can also find out how these error which is not related to diffusion which is related to something else. So, I can find out the predictive model, but this error is predicted by the price and sales and advertisement expenditure and etc. So, how much price I have given on that particular month if or a particular year, if the price on the particular year was higher, the demand will coming down.

So, again all that rest of the part which is non-time series part can be explained by just putting a regression equation. With this error here it is $sqer$ error. We have to square root it up. So, with the error component and all other X variable that you have you try to create a regression equation. So that is all we have about forecasting.

We will go on and meet you in the next week and we will do, how to use this forecasting. So, how to use this demand function. Its various pricing problems. Will talk about that in week 5. Thank you very much for being with me.