Modelling and Analytics for Supply Chain Management
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Lecture 40
Analytical Hierarchy Processing (AHP) Method

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Hi, welcome to our course on Modelling and Analytics for Supply Chain Management. Today, we will start with Module 09, Lecture 40; that is Analytic Hierarchy Processing Method.

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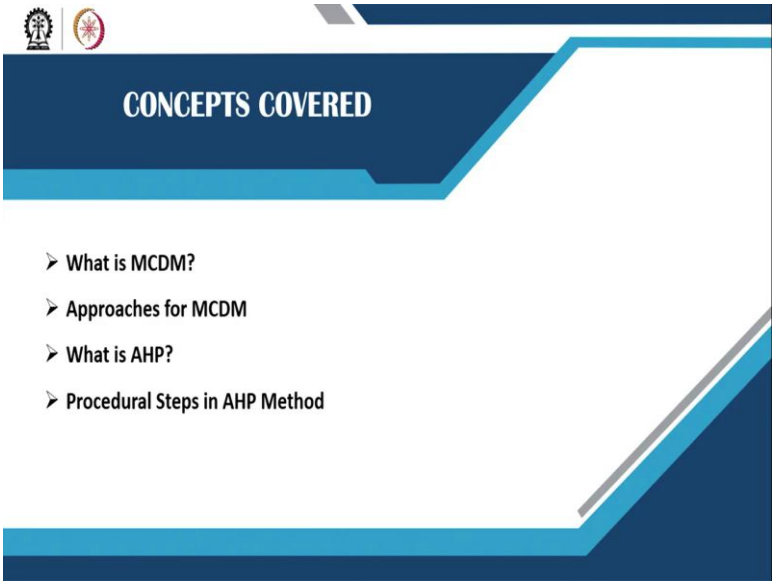

This Analytical Hierarchal Processing Method, which is also popularly known as AHP Method is one of many such techniques, which are used for 'multi criteria decision making'.

So, in today's lecture, we are basically going to cover what is 'multi criteria decision making', which in short form is known as MCDM. So, what is MCDM, approaches for Multi-criteria decision-making, what is AHP and basically the procedural steps in AHP Method; with an example, that is what is today's agenda.

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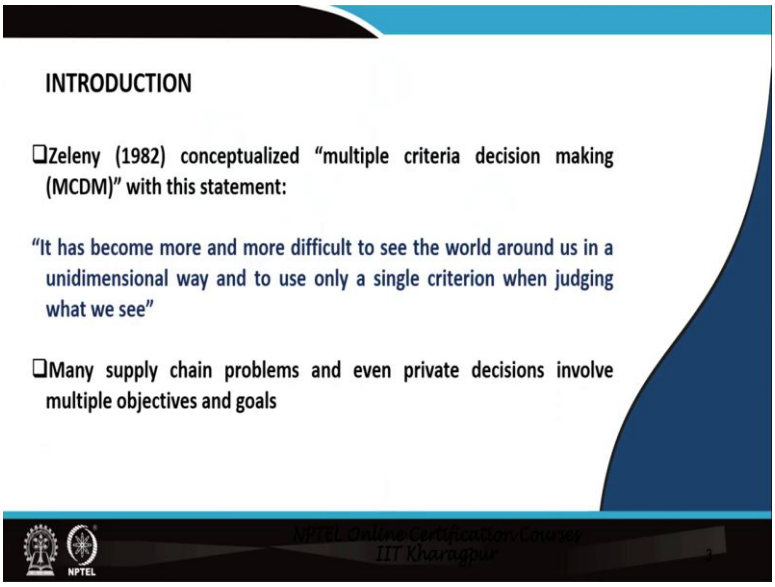

Now, what is decision-making? Decision-making involves choosing one of several alternatives, that exist to solve a particular problem at hand. Now "multiple criteria decision making" means that there are several criteria, multiple criteria. And there are some alternatives. And the decision maker has to choose one among the several alternatives, which satisfies all these criteria and gives the best result among the several other alternatives.

So, Zeleny conceptualized "multiple criteria decision making" with a statement, that it has become more and more difficult to see the world around us in a unidimensional way and to use only a single criterion when judging what we see. But in real life, lots of criteria are involved, when we are going to select one alternative among the several many.

Typical example is that, suppose you are going to (sel) select one of the suppliers among 4 or 5 competing suppliers, satisfying several criteria; multiple criterias may be; say quality of supply, the price that the supplier has quoted, conformance to delivery schedule; both; conformance to delivery schedule quantity, as well as adherence to delivery schedule date.

Then the infrastructure, quality of infrastructure at event; whether the supplier has got some facilities for quality assurance or not. Maybe you can also involve the supplier's integrity and

honesty as part of several criteria. Now, given these so many criteria, suppose there are 4 or 5 suppliers and we have to choose one among them.

So, many such problems in supply chain area, including say selection of best supplier. Then with respect to product; in the area of product analytics also, AHP has got multiple applications. For example; with (resp) suppose there are 4 or 5 different kinds of product and given multiple criteria such as reliability of the product, safety of the product, the features, designe features of the product, say the aesthetics related to the product. So, suppose there are several such criteria.

Now, which product you will choose? So, that is why, many supply chain problems and even private decisions involve multiple objectives and goals. And hence, in real life the application of multiple criteria decision-making techniques have been widely appreciated.

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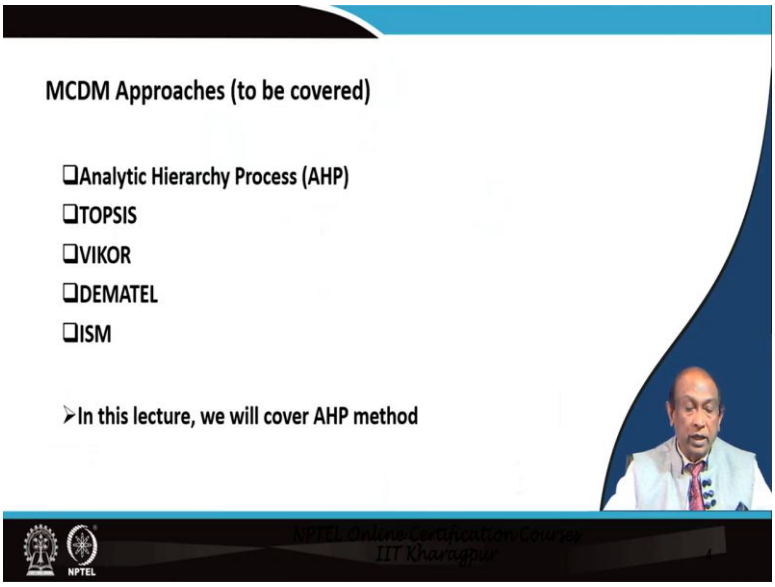

We will cover in our course, several such approaches and the popular ones are the Analytic Hierarchy Process (AHP), TOPSIS, VIKOR, DEMATEL and Interpretive Structural Method. So, in this lecture, we will cover Analytic Hierarchy Process method; AHP.

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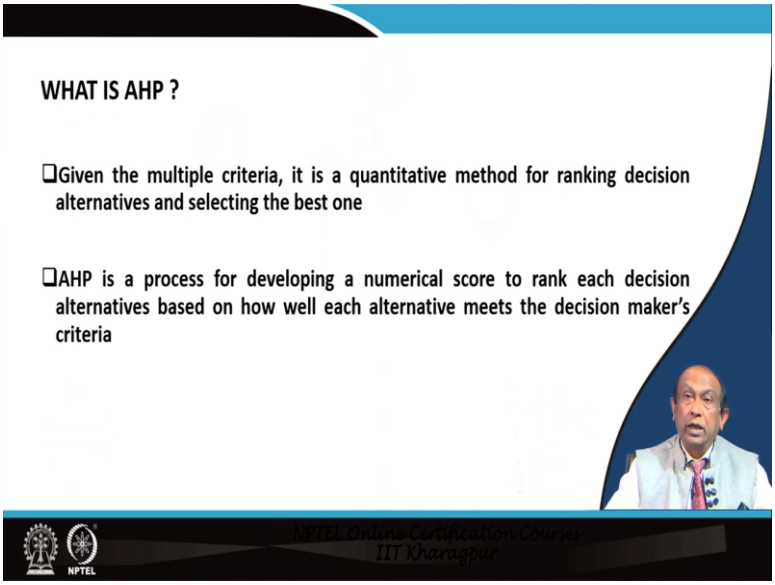

So, what is AHP? Given multiple criteria, it is a quantitative method for ranking alternatives; decision alternatives and selecting the best one. So, AHP is a process for developing a numerical score, to rank each decision alternatives based on how well each alternative meets the decision maker's criteria. That means, against each alternatives, we will develop a score and we will choose that particular alternative as the best one, which is having the maximum score. Now how to score this alternative?

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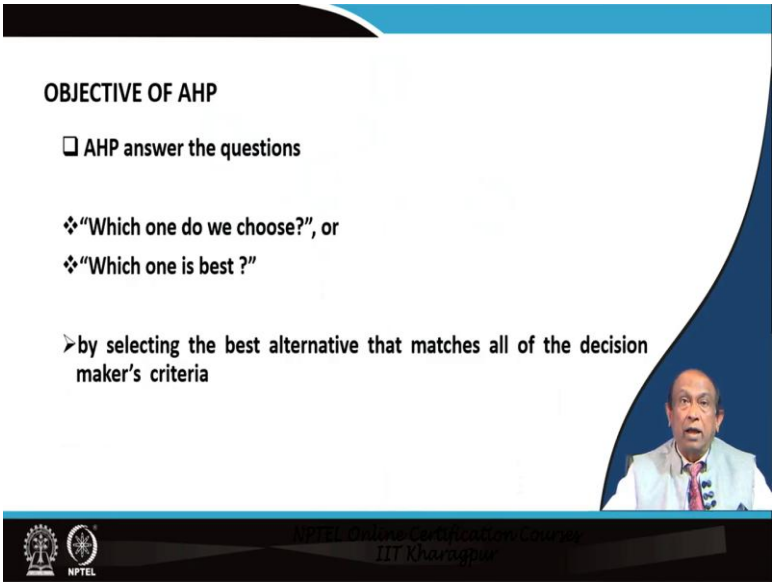

So, AHP basically answers the questions. Which alternative do we choose or which alternative is the best among the several many? By selecting the best alternative, that matches all of the decision maker's criteria. Maybe there are several alternatives and each one of them

might satisfy the decision maker's criteria. But we are going to select that particular alternative, whose score is maximum.

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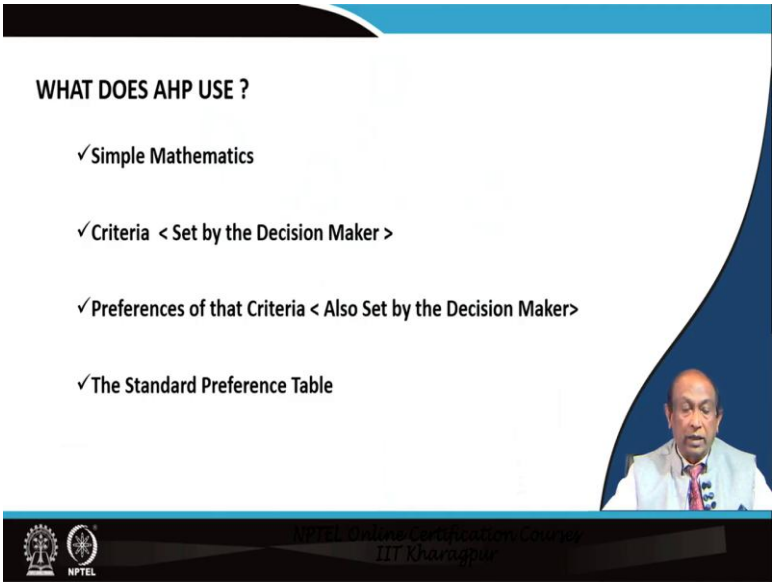

So, what does AHP use? Simple Mathematics. And the criteria are given; these criteria are set by the decision maker. And the relative importance of the several criteria, which are basically the preferences of that criteria are also set by the decision maker or sometimes we conduct a questionnaire survey among 4 or 5 decision makers. Wherein we do a pair-wise comparison among these criteria and then we, in a standard manner, we determine the relative importance of these criteria.

So, while determining the relative importance of the criteria or with respect to a criteria, when we try to determine the relative importance of alternative, we take the help of Standard Preference Table.

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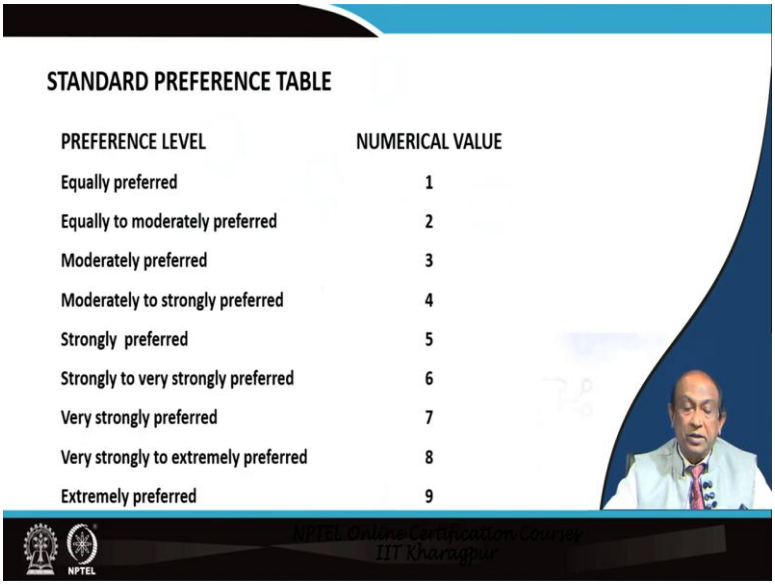

Now, what is this Standard Preference Table? See, if two alternatives are equally preferred and when we do a pair-wise comparison between these alternatives, we assign a numerical value '1'. If one of the criteria is equal or moderately preferred over the others, it gets a score or numerical value of 2. Like this, if 1 criteria is moderately preferred over others, then with respect to that criterion, the score that the other criteria which is better gets the numerical value '3'. Like this, we refer to a preference table; wherein the scalings are like this.

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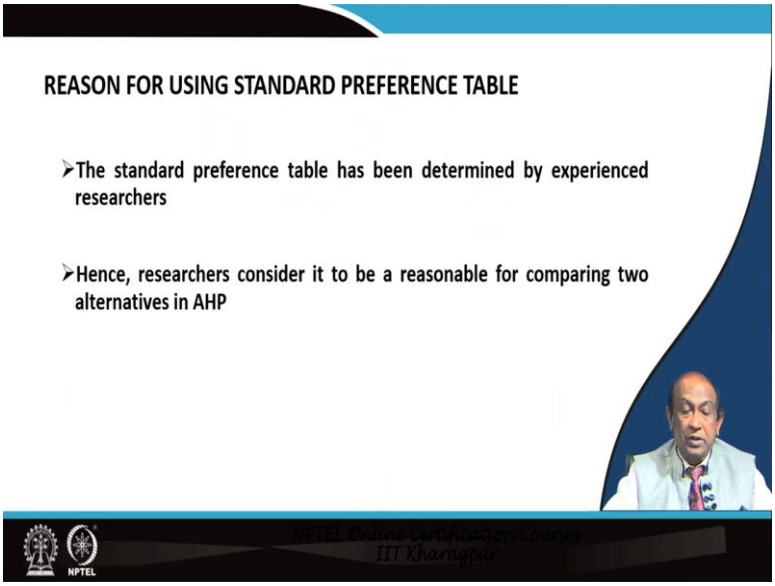

And this Standard Preference Table, which I have just shown you, has been determined by experienced researchers. Hence, researchers consider it to be reasonable for comparing two alternatives in AHP.

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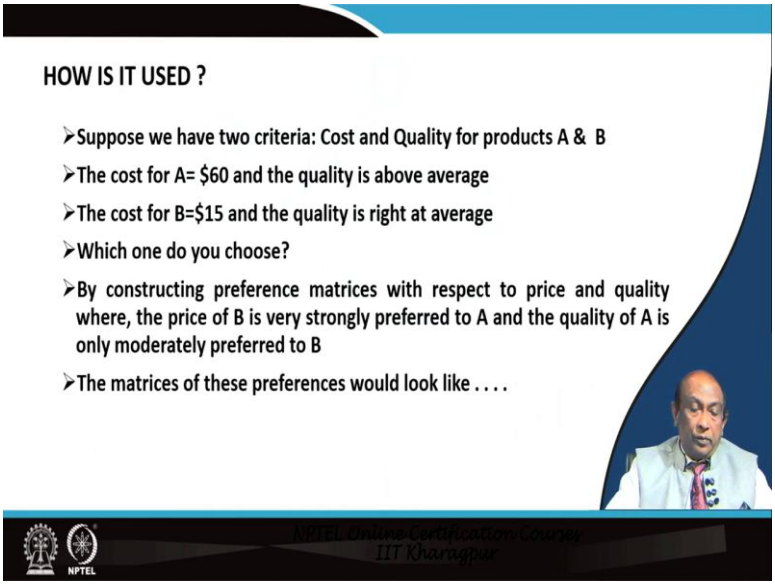

How is it used? Suppose we have two criteria; cost and quality for products A and B. And suppose the cost for, you know A is dollar 60 and the quality of that product A is above average. The cost for product B is say dollar 15 and the quality of that product is right at average.

Now, the question is; which one do you choose. By constructing preference matrices with respect to price and quality where the price of B is very strongly preferred to A and the quality of A is only moderately preferred to B, the preference matrix or preference matrices would look like, like this.

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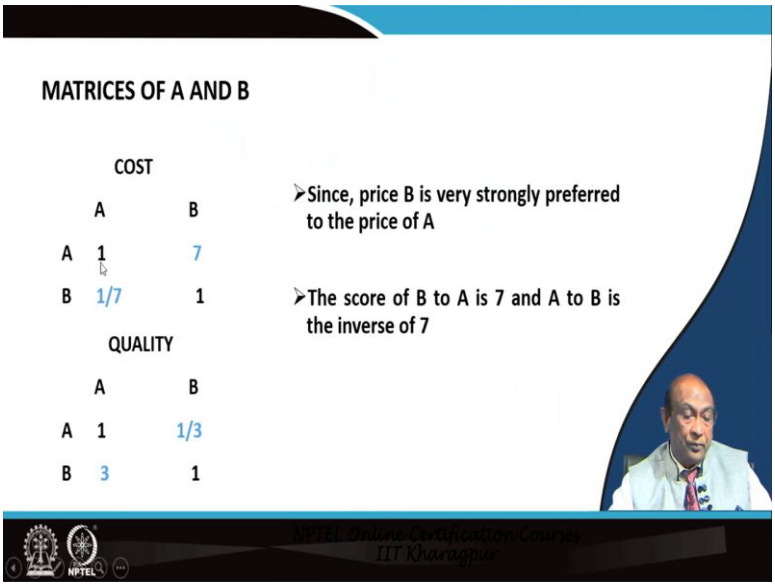

In this case, you see the diagonal elements will be 1. The criteria is cost. With respect to cost, B is strongly preferred over A. And hence, with respect to A, B gets a score of 7. Therefore, this particular element will get a score of 1 by 7, which is inverse of 7. Similarly, when we do a pair-wise comparison between A and B, with respect to the criteria quality, then what happens?

We have set the quality of A is moderately better than B. So, B's quality is less than A. And B gets a rating of 1 by 3. If this element gets 1 by 3, automatically in this matrix, B will have a value of 3. Like this, with respect to a particular criteria, we do a pair-wise comparison between the alternatives and construct a preference matrix.

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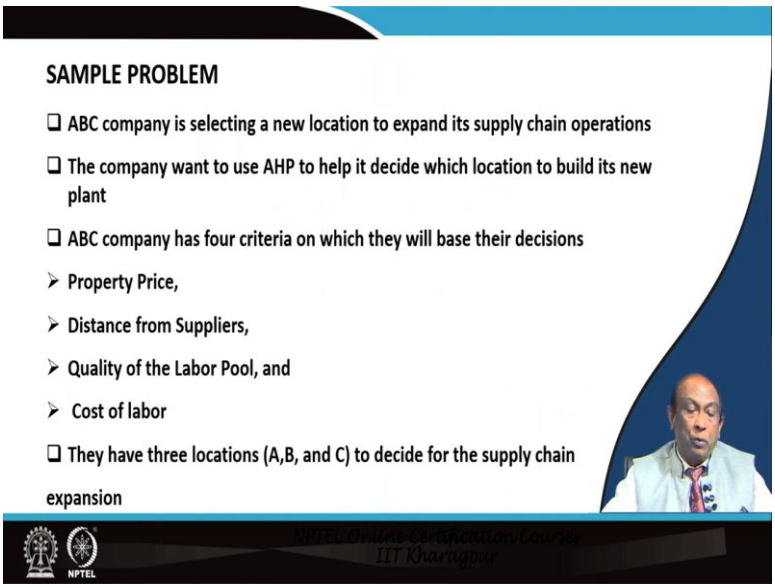

And here, we have a problem. ABC company is selecting a new location, to expand its supply chain operations. The company wants to use AHP to help it decide which location to build its new plant. Now, ABC company has 4 criteria on which they will base their decisions. What are these 4 criterias?

Property Price is one. The second one is 'distance from suppliers'; the distance from supply source. The third criteria is 'quality of the labor pool'. And the fourth criterion is 'cost of labor'. And they have 3 locations; A, B and C. And they have to decide; where they will, which location they will choose for their new plant to build the supply chain operation.

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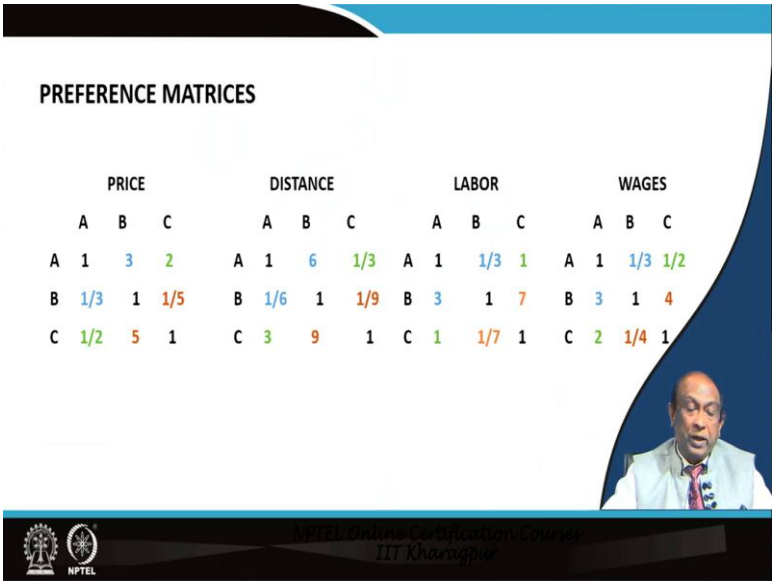

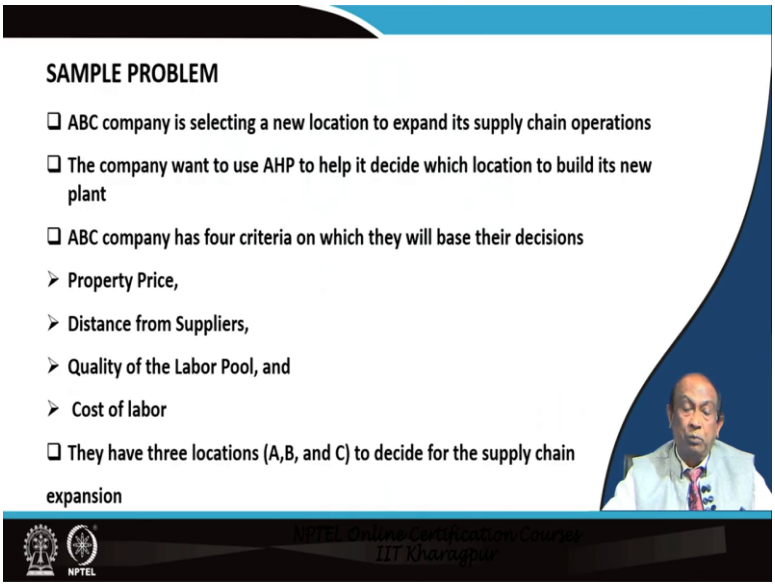

So, what we do; with respect to each of these criteria; for example, price, then distance from suppliers, then quality of the labor pool and cost of labor. With respect to each one of these criteria, we build the preference table. For example; with respect to the criterion 'price', we have 3 alternatives; that is the 3 locations.

We (hav) we have taken opinion from 3 or 4 experts and arrived at their average values or ratings. And I have found out, we have found out that this location B is moderately preferred over A. So, it gets a score of 3. C is equally or slightly more preferable than A. So, it gets a score of 2. Like this, we have built this preference table.

So, if this gets 3, correspondingly this value will be 1 by 3. If this value is 2, then this value will be 1 by 2. If this value is 1 by 5, then this value is 5. Like this; we build a preference

table with respect to price. In fact, if you see, if we look at this upper triangular matrix, then it is more than sufficient. Automatically you can construct the lower triangular matrix.

With respect to distance, the preference table that we have constructed will look like this. Similarly, with respect to labor; similarly, with respect to wages. Now many-a-times, if so happens, that the unit of measure, which is used to compare among the alternatives with respect to a criteria, this unit of measure will vary with respect to this criteria.

For example; distance is measured in kilometer. Wages will be measured in terms of dollars or rupees. So, the first thing that we do, is we try to normalize this preference matrix with respect to each of this criteria. And how do we normalize it?

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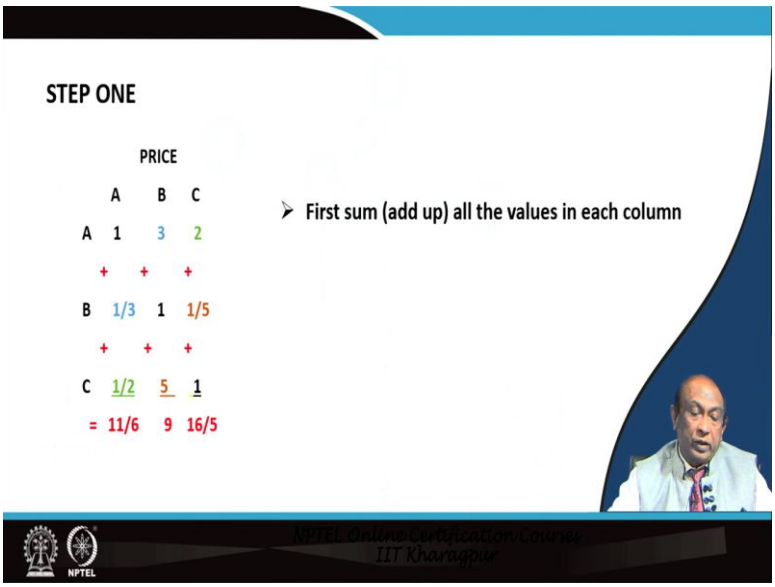

For example; with respect to price, we had this original preference table. In order to normalize this, we will first add all the elements in this matrix; column-wise. So, the first step is first sum or add-up all the values in each column. So, column-wise, you construct this column sums. Done? Then what we will do? Each of these elements in this matrix has to be divided by this column sum. For example; 1 divided 11 by 6 will be 6 by 11; like this.

1 by 3 will have to be divided by 11 by 6. 1 by 2 will be divided by 11 by 6. Similarly, 3 will be divided by 9. 1 will be divided by 9 and 5 will be divided by 9. For column C, 2 will be divided by 16 by 5. This one will be divided by 16 by 5.

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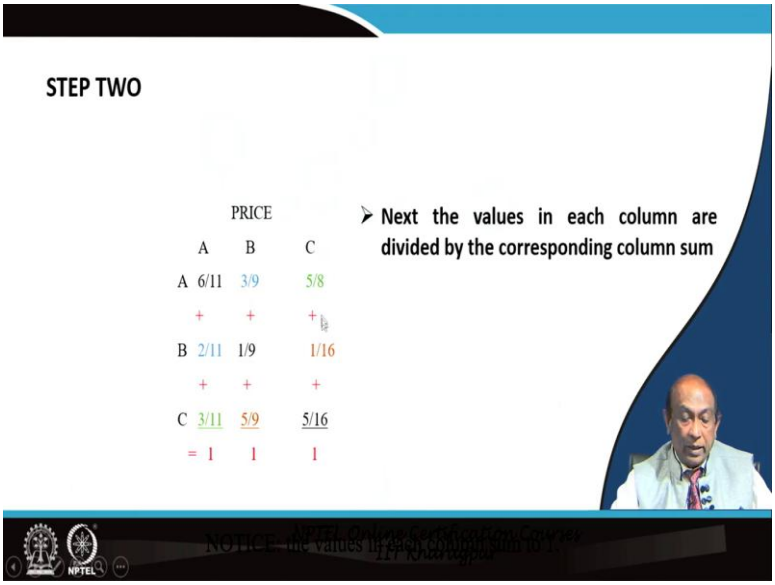

And like this, what we will do; we will get this particular table. Next; the values in each column are divided by the corresponding column sum. So, you will get this matrix.

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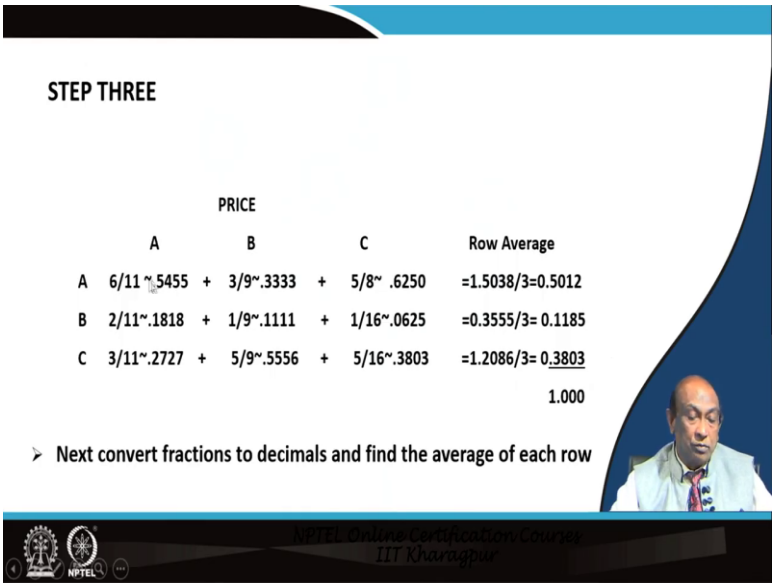

Next, what you do? You convert these values to decimals. Convert the fractions to decimals. For example; this becomes 0.5455, 2 by 11 becomes 0.1818, 3 by 11 becomes 0.2727; like this. With respect to each column, the fractions are converted to decimal values.

Then simply find out the average of these values that are there corresponding to a row, which we basically called ‘the row average’. So, for the first row A, the row average will be first 0.5455 plus 0.3333, plus 0.6250, which makes a total of 1.5038, divided by 3, equal to

0.5012. So, like this, compute the row averages corresponding to each and every row in the matrix.

And then ensure that this particular sum is 1. So, this particular column that you have got gives the relative importance of each of these locations with respect to the criteria 'price'. Those of you who are well conversant with matrix algebra, they must have by now realized, that this particular column is nothing but the Eigenvector corresponding to that matrix.

Remember; in order to determine the eigenvector, you first set up this characteristic equation. If A is the matrix, the determinant of A minus lambda i will be equal to 0. That is the characteristic equation, you solve for different values of lambda; those are eigenvalues. And corresponding to the maximum eigenvalue, you can get that corresponding eigenvector. So, those who are conversant with matrix algebra; if they can find out the eigenvector corresponding to that particular preference matrix, this is not necessary. But, for industry people, it is very easy to determine the eigenvector in this particular manner.

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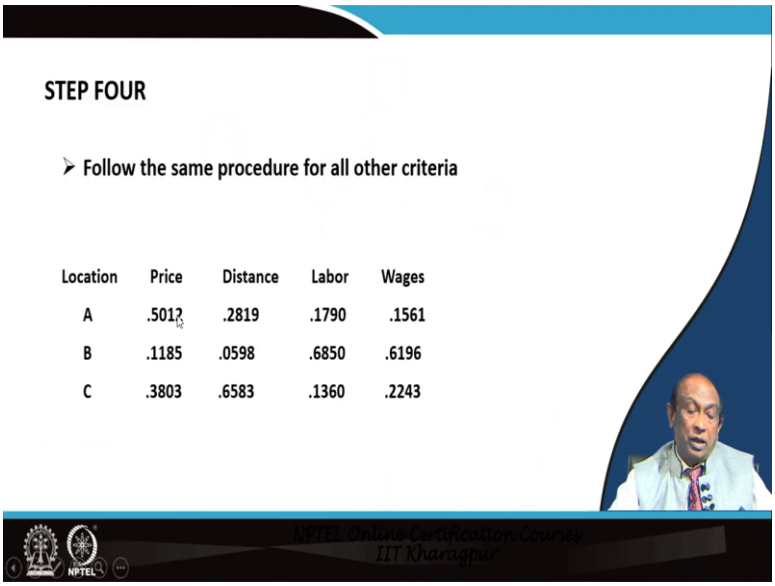

So, what we do, we determine the relative importance of all these locations, with respect to each of these criteria; price, distance, labor and wages. And we get this particular matrix. And you will say that this is basically a 3 by 4 matrix, which basically signifies the relative importance of these locations with respect to these criteria.

And you notice one thing, that with respect to price, location A is the best. But with respect to labor availability, quality labor available or not, you see location B is more preferable compared to the others. Similarly, with respect to distance from the supply source, this

particular location will be preferred over others. But we have to choose one among the 3 locations, for setting up our plants. And overall, that location should be preferred compared to the others, considering all the given criteria.

But, before we come to arrive at that particular decision, we have to also find out the relative importance of these criteria; in the same way as we determined the relative importance of the locations with respect to a criteria. So, how do we determine that? We will follow exactly the same method, that we had adopted earlier.

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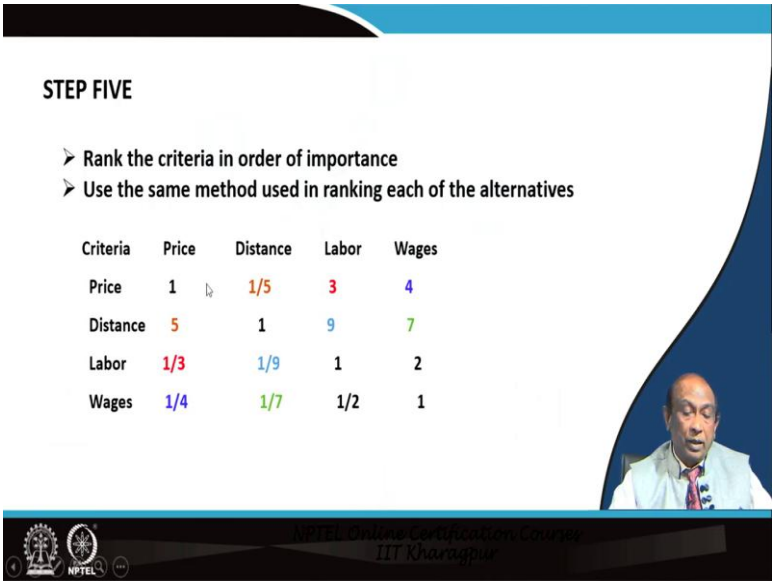

So, what we do? We first do a pair-wise comparison among this criteria by consulting some experts and taking their average value. For example; say, in here, what we find; that the importance of the wages that needs to be paid is much more than the importance given to price, or the importance of this (crit) criteria ‘labor’ with respect to distance is much more. So, we construct the preference table or the preference matrix with respect to the criteria.

And then, in the similar manner, what we had adopted earlier, we determine the eigenvector of this particular matrix.

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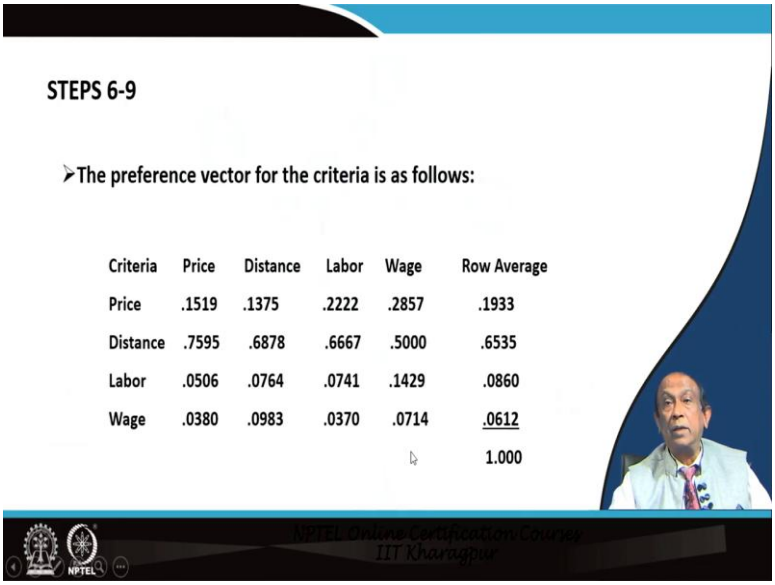

So, what we do, we first do the column sum and then divide each element of that matrix by those column sum and then for each row, we compute the row averages. And we finally arrive at this particular vector which gives us the relative importance among the criteria. So, if you look at this particular vector, or particular this column, you find that distance from the supply source has got much more weightage compared to any other criteria.

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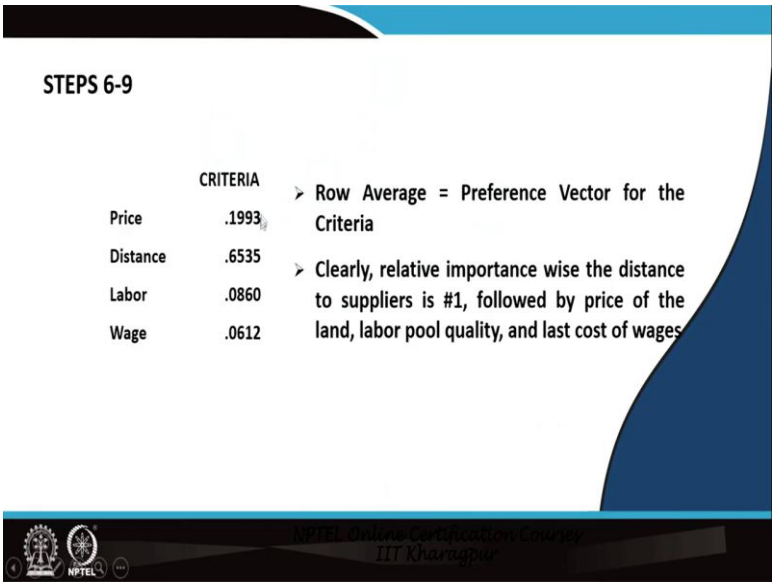

But, having got these weightages, so this is the eigenvector corresponding to the preference matrix that we have built, with respect to the different criteria. Now, you see, relative importance-wise, the distance to suppliers is having the maximum importance followed by price of the land, then followed by the labor pool quality and finally the cost of wages.

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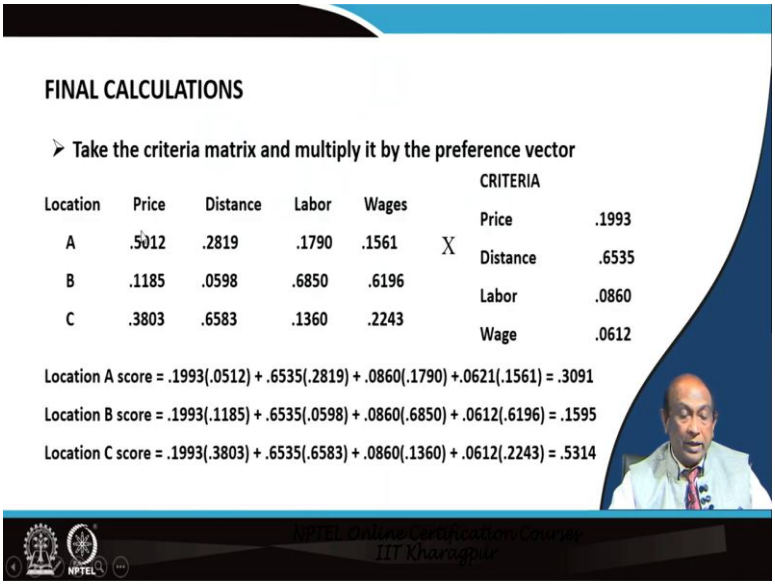

We take the criterion matrix, which is basically 3 by 4 and multiply it by this eigenvector, which is a matrix of 4 by 1. And hence, the resultant matrix will be a 3 by 1 matrix, whose elements will be as given 0.3091 is the location A score. Location B score is this and location C score is this.

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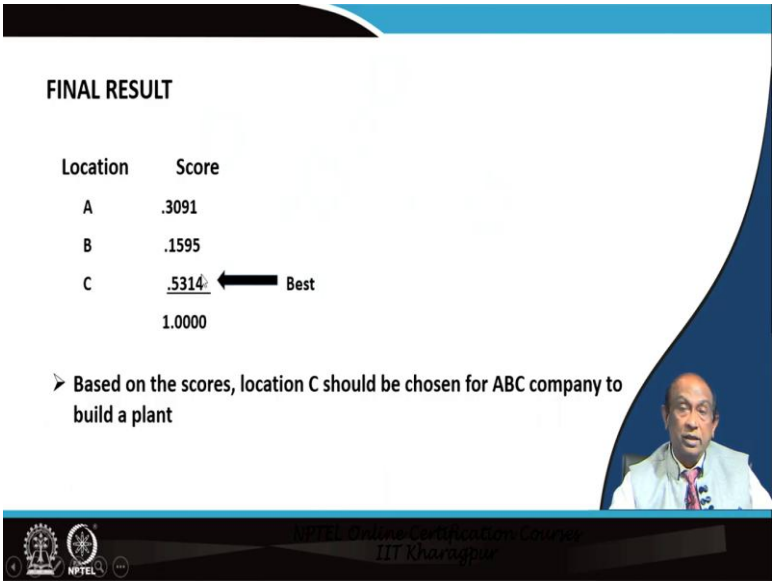

So, if that is so, which one is the best location? Location C, because that particular location has got the maximum score. Based on the scores, location C should be chosen by the ABC company to build a plant, to expand its supply chain operations. Like this, in industry, there are many uses of this technique; for selection of best suppliers, for selection of the best

product. And many such applications exist in the academic world, where AHP has been applied.

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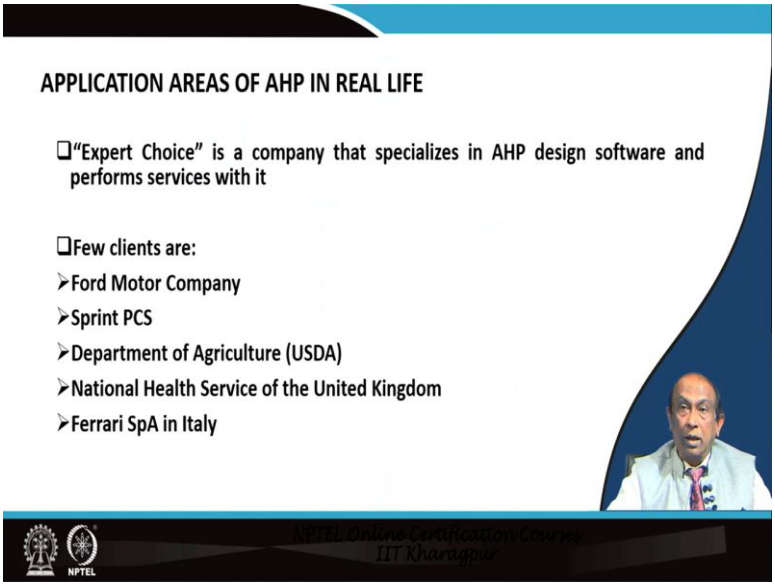

The references followed will be given. There are plenty of literatures available on AHP. And if you search the literature, you will find that Expert Choice is a company, that specializes in AHP design software and performs services with it. Few clients of Expert Choice are Ford Motor Company, Sprint PCS, Department of Agriculture, National Health Service of UK, Ferrari SpA in Italy. And there are plenty.

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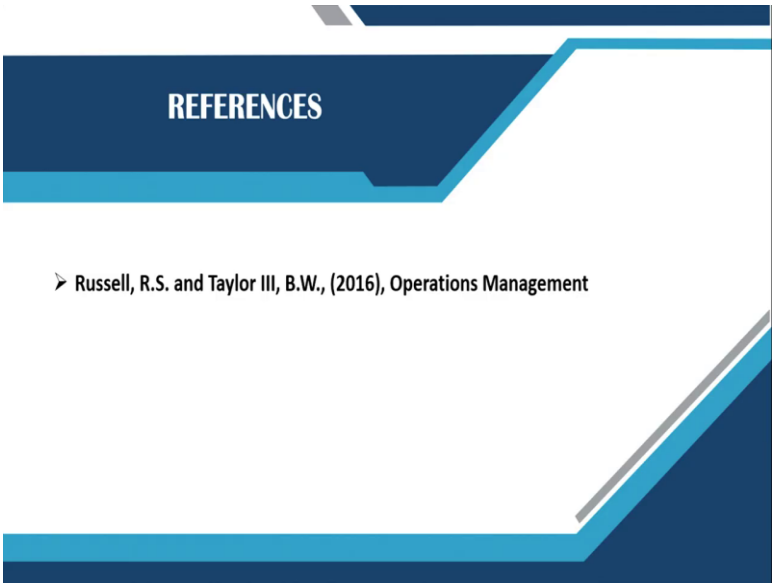

We have used this particular book as a reference material for applying AHP. Thank you all for your patience.