

Modelling and Analytics for Supply Chain Management
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Lecture 32
Continuous Review System (Contd.)

Good afternoon and welcome, to our session on Continuous Review System related to Inventory Modelling.

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CONTINUOUS REVIEW SYSTEM (CASE II)

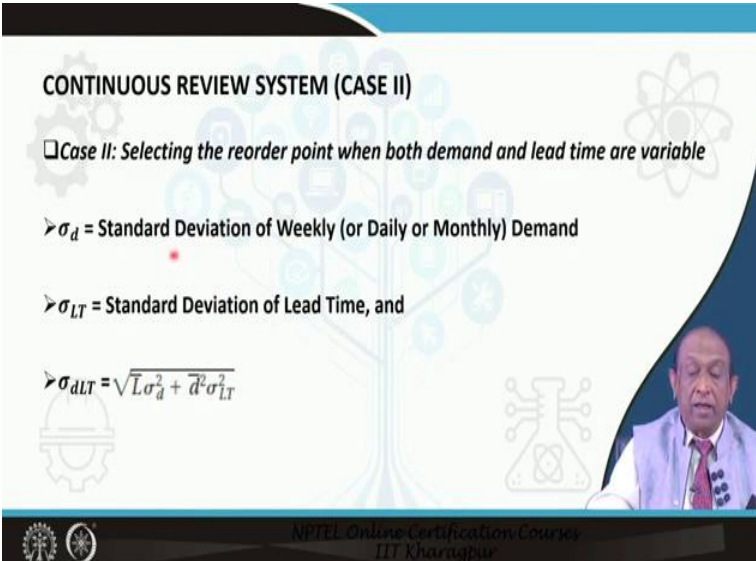
□ *Case II: Selecting the reorder point when both demand and lead time are variable*

❖ Safety Stock (SS) = $Z * \sigma_{dLT}$

➤ $R = (\text{Average Weekly Demand} * \text{Average Lead Time in Weeks}) + \text{Safety Stock}$
 $= \bar{d}\bar{L} + SS$

where, \bar{d} = Average Weekly (or Daily or Monthly) Demand
 \bar{L} = Average Weekly (or Daily or Monthly) Lead Time

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CONTINUOUS REVIEW SYSTEM (CASE II)

□ *Case II: Selecting the reorder point when both demand and lead time are variable*

➤ σ_d = Standard Deviation of Weekly (or Daily or Monthly) Demand

➤ σ_{LT} = Standard Deviation of Lead Time, and

➤ $\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$

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So, again in such a case also we can determine the reorder point and discussed earlier. Now, we will be discussing about the reorder point when both lead time and demand both of them are

varying. So, first case we had seen that the demand is varying and the lead time is constant, second case we have seen that the other way around lead time is varying demand is constant, and the third which is the most common is both lead time and demand both of them are varying.

In that case, also the reorder point will be nothing but, \bar{d} into \bar{L} where \bar{d} is average weekly or daily or monthly demand and \bar{L} is average weekly or daily or monthly lead time expression for that and the Standard Deviation of weekly demand or say daily or monthly demand as an when whatever value we get if it is σ_d and standard deviation of lead time if, it is σ_{LT} then standard deviation of demand over lead time is given by the expression which is square root of \bar{L} into σ_d square plus \bar{d} into σ_{LT} square.

Now, here you might ask me that why is this \bar{d} square coming into this see, this particular position is very clear that where demand is varying and lead time is constant their the standard deviation was σ_d into root L . Now, this expression is the variance of demand during lead time in the situation where demand is varying and lead time is constant, this particular expression is the variance of lead time which is σ_{LT} square multiplied by \bar{d} square.

Now, see σ_{LT} is in terms of days but, this expression here it is in quantity. So, we have multiplied by average consumption per day or average consumption per period to make this particular expression in terms of quantity we cannot add apples with oranges we have to add quantity with quantity. So, if we add and then take the square root we get $\sigma_d \bar{L}$ which is nothing but the demand, standard deviation of demand over lead time.

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NUMERICAL EXAMPLE ON DEMAND UNCERTAINTY (CASE I)

□ *Case I: Selecting the reorder point when demand is variable and lead time is constant*

- ✓ Demand for an item is normally distributed with a mean of 2000 units a year and standard deviation of 400 units
- ✓ Unit cost is Rs. 100
- ✓ Reorder cost is Rs. 200
- ✓ Inventory holding cost is 20 percent of value a year, and
- ✓ Lead time is fixed at 3 weeks

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)

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Now, let us take one problem which will clarify this particular situation in better way. For example, suppose demand for an item is normally distributed with a mean of 2000 units a year and standard deviation of demand is 400 units per year, unit cost of the item is rupees 100, reordering cost is rupees 200, inventory holding cost is 20 percent of value a year and lead time is fixed at 3 weeks. So, here demand is variable lead time is constant.

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NUMERICAL EXAMPLE ON DEMAND UNCERTAINTY (CASE I)

□ *Case I: Selecting the reorder point when demand is variable and lead time is constant*

➤ Questions to be answered:

- ❖ Describe an ordering policy that gives a 95 per cent service level
- ❖ What is the cost of the safety stock?

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)

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NUMERICAL EXAMPLE FOR DEMAND UNCERTAINTY (CASE I)

Case I: Selecting the reorder point when demand is variable and lead time is constant

Solution:

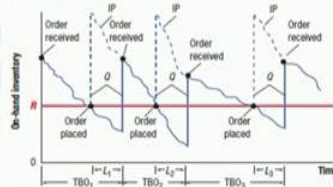
$\bar{d} = 2,000$ a year

$\sigma_d = 400$ units

Unit cost = $C = \text{Rs. } 100$ / unit

Ordering Cost = $S = \text{Rs. } 200$ an order

Holding Cost = $H = 0.2 * 100 = \text{Rs. } 20$ a unit a year



(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)



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That the questions to be answered is describe an ordering policy that gives a 95 percent service level and what is the cost of the safety stock? The solution is, demand is given 2000 units per year, standard deviation of demand is 400 units, unit cost rupees 100 per unit, ordering cost given 200 an order, holding cost is nothing but 0.2 star 100 is rupees 20 a unit a year.

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NUMERICAL EXAMPLE FOR DEMAND UNCERTAINTY (CASE I)

Case I: Selecting the reorder point when demand is variable and lead time is constant

Lead time = $L = 3$ weeks = $3/52$ years

Reorder size = $\sqrt{(2 \times 200 \times 2000 / 20)} = 200$ units

Reorder level (R) = $L \times \bar{d} + \text{Safety Stock}$

$$\begin{aligned} \text{Hence, } R &= 3/52 \times 2000 + Z * \sigma_{dLT} \\ &= 3 \times 2000 / 52 + Z * \sigma_d \sqrt{L} \\ &= 115 + 1.64 * 400 * \sqrt{(3 / 52)} \\ &= 115 + 158 = 273 \text{ Units} \end{aligned}$$

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)



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Lead time is expressed as 3 weeks which is nothing but 3 by 52 years, reorder size then come out to be root of 2 into annual demand into ordering cost divided by holding cost which is 200 units this reorder size is nothing but q and reorder level is L multiplied by d bar plus safety stock.

L is nothing but 3 by 52 in terms of year multiplied by average yearly demand plus Z into sigma d LT that is Z in this case is for corresponding to the service level specify this 1.64 multiplied by 400 that is the standard deviation of demand into root over of R of 3 by 52 which gives 273 units.

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NUMERICAL EXAMPLE FOR DEMAND UNCERTAINTY (CASE I)

□ *Case I: Selecting the reorder point when demand is variable and lead time is constant*

□ **Solution:**

- ✓ The ordering policy is to order 200 units whenever stock declines to $(115 + 158) = \underline{273 \text{ units}}$
- ✓ Orders should arrive, on average, when there are 158 units left
- ✓ The expected cost of the safety stock = $\text{Rs. } 158 * 20 = \underline{\text{Rs. } 3160 \text{ a year}}$

The graph shows 'On-hand inventory' on the y-axis and 'Time' on the x-axis. A horizontal red line represents the reorder point (R). Vertical dashed lines indicate when orders are placed (O) and received (IP). The time between placing an order and receiving it is the lead time (L). The inventory level fluctuates between R and 0. The expected inventory level when an order is received is 158 units.

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)

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So, the ordering policy in this case is to order 200 units whenever the stock declines to 273 units that is the reorder point, order should arrive on average when there are 158 units left, the expected cost of the safety stock is 158 star 20 with multiplied by 20 which is rupees 3160 a year.

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NUMERICAL EXAMPLE ON DEMAND UNCERTAINTY (CASE II)

□ *Case II: Selecting the reorder point when both demand and lead time are variable*

- ✓ The average demand for a popular ball-point pen is 12,000 pens per week with a standard deviation of 3,000 pens
- ✓ The current inventory policy calls for replenishment orders of 156,000 pens
- ✓ The average lead time from the distributor is 5 weeks with a standard deviation of 2 weeks

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)

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Let us, look at another example where both demand and lead time are variable, the average demand for a popular ball point pen is 12000 pens per week with a standard deviation of 3000 pens, the current inventory policy calls for replenishment orders of 156000 pens, the average lead time from the distributor is 5 weeks with a standard deviation of 2 weeks.

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NUMERICAL EXAMPLE ON DEMAND UNCERTAINTY (CASE II)

□ *Case II: Selecting the reorder point when both demand and lead time are variable*

➤ Question to be answered:

❖ If management wants a 95 percent cycle-service level, what should be the reorder point ?

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)

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NUMERICAL EXAMPLE ON DEMAND UNCERTAINTY (CASE II)

□ Case II: Selecting the reorder point when both demand and lead time are variable

□ Solution: We have,

➤ $\bar{d} = 12,000$ a week

➤ $\sigma_d = 3000$ units

➤ Lead Time = $\bar{L} = 5$ weeks

➤ $\sigma_{LT} = 2$ weeks

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)



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If management wants a 95 percent cycle-service level, what should be the reorder point? Here problem is very simple the average demand is given 12000 per week, standard deviation of demand is 3000 units, lead time is 5 weeks and standard deviation of lead time is 2 weeks.

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NUMERICAL EXAMPLE ON DEMAND UNCERTAINTY (CASE II)

□ Case II: Selecting the reorder point when both demand and lead time are variable

□ Solution:

➤ $\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2} = \sqrt{(5 * 3000^2) + 12000^2 * 2^2} = 24919.87$ pens

➤ For 95% service level, $SS = Z * \sigma_{dLT} = 1.65 * 24919.87 = 41,118$ pens

✓ Reorder point = $R = \bar{d}\bar{L} + SS = 12000 * 5 + 41,118 = 101,118$ pens

(Source: Operations Management: Processes and Supply Chains by Krajewski et al., 2019)



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REFERENCES

- Cachon, G.P., and Terwiesch, C., (2019), Matching Supply With Demand: An Introduction to Operation Management
- Krajewski, L.J., Larry P. Ritzman, L.P., and Malhotra, M.K., (2019), Operations Management: Processes and Supply Chains

So, in this case standard deviation of demand during lead time is given by this expression and when we substitute the values we get σ_{dLT} equals 24919.87 pens which can be rounded of for a 95 percent service level the Z value is 1.645 or 1.65 you can take that multiplied by σ_{dLT} that is the value of safety stock which is 41,118 pens and with this the reorder point works out to be average demand into average lead time plus the safety stock which is 101,118 pens.

This particular problem is being given in this book Operations Management by Krajewski and I have already sited all the references so, you can consult these books for a detail study on the topic. Thank you all.