Modelling and Analytics for Supply Chain Management
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Lecture 29
Multiple Items Inventory Models (Contd.)

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Hi, welcome to our discussion on multiple items inventory models. Today in this particular session, we are going to discuss inventory models, where there are constraints on space as well as in the next section we will discuss inventory modeling for cases where there are constraints on the total number of.

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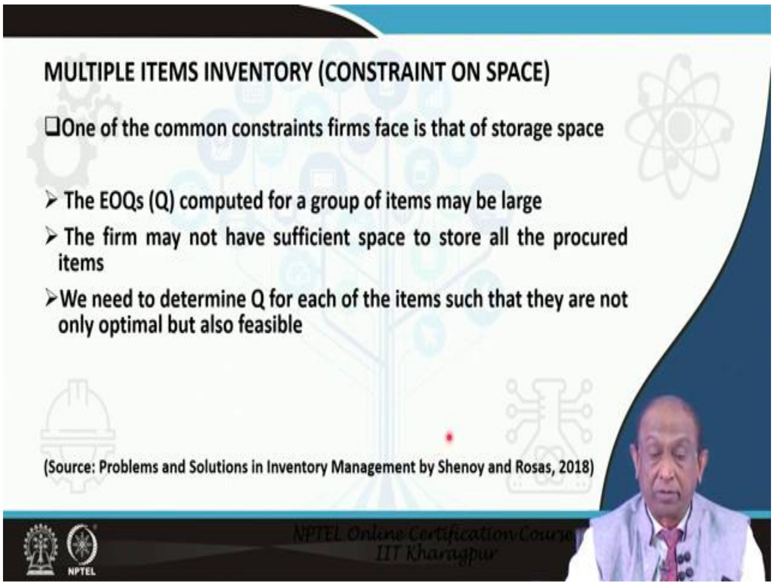

One of the common constraints, large organizations face is that of storage space. Even in small firms, we see this particular situation because of the size of the warehouse there is constraint on storage space. The optimal order quantities Q computed for a group of items, if we see then you know the total of this the size or the space occupied by this large number of items so huge that it cannot be accommodated in the warehouse.

The firm may not have sufficient space to store all the procured item. In that case, we need to determine Q or optimal order size for each of the items such that they are not only optimal but also feasible.

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We will discuss one such case consider an inventory system with total number of items equal to small n, let capital S be the maximum amount of space that is available to stock these items. Qj we denote as the optimal order size for the j th item, the ordering cost for the j th item is denoted by Coj, the annual demand for the j th item let it be equal to Dj, small i is the inventory carrying charge or rate per year and Cj denotes the unit cost for the j th item in the system. So, these are the notations that we are going to use for modeling this kind of problem.

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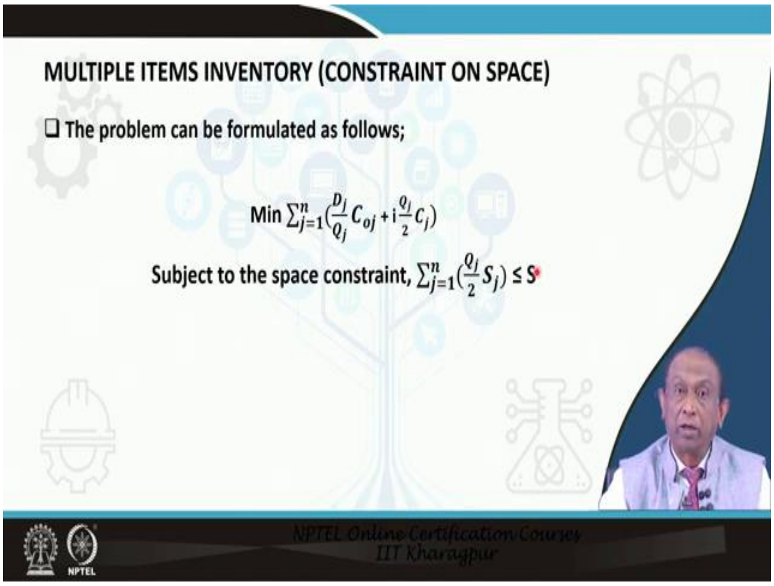

So, in this case the problem can be formulated as minimize the total cost which is the sum of ordering cost plus inventory holding cost, that is minimize j equals 1 to n Dj by Qj into Coj plus i into Qj by 2 into Cj this is the total cost for one item, summed over all n items. These are ordering cost component and this is the inventory holding cost component.

This is subject to the space constraint Q by Qj by 2 multiplied by Sj summed over j equal to 1 to n, which is the average space occupied by all the items that should be less than equal to capital S that is the maximum space available to accommodate this procured items.

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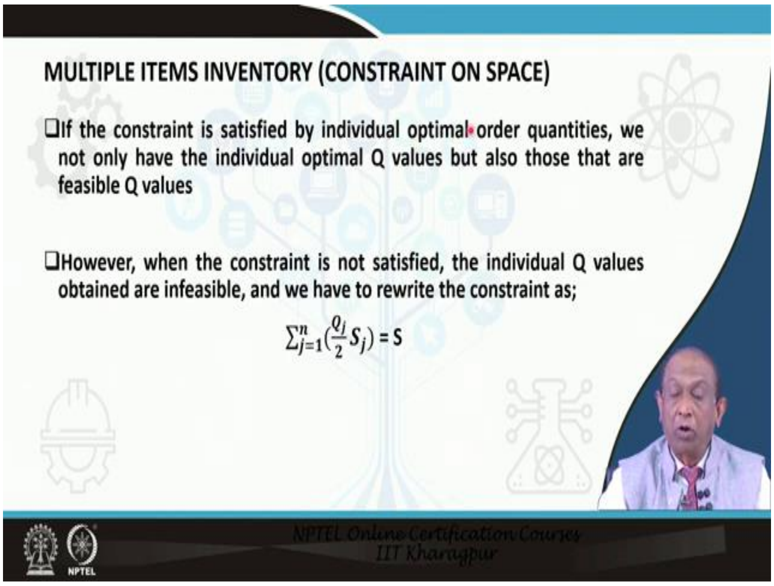

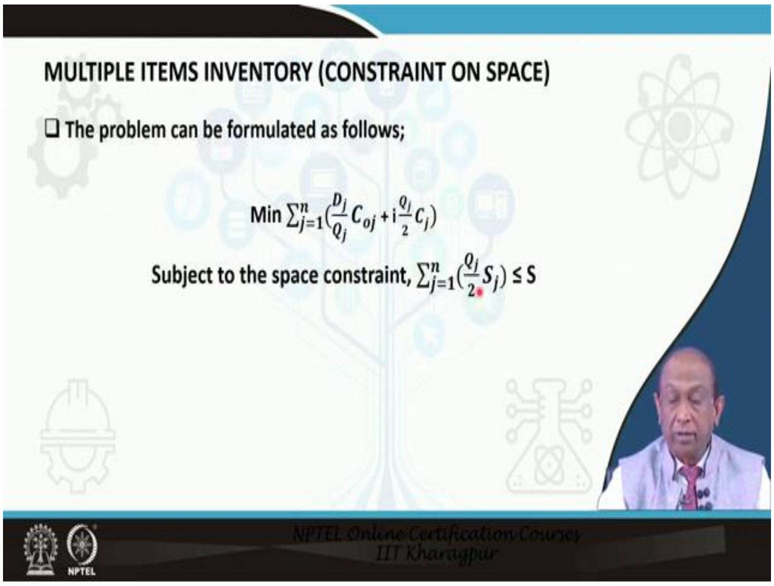

If the constraint that is Qj by 2 into S j less than equal to S, if this particular constraint is satisfied by individual optimal order quantities, we not only have the individual optimal Q values, but also those that are feasible Q values and we need not to worry. However, when the constraint is not satisfied, the individual Q values obtained are infeasible and we have to rewrite the constraint in this case as sum over j equal 1 to n Qj by 2 into S j equals S.

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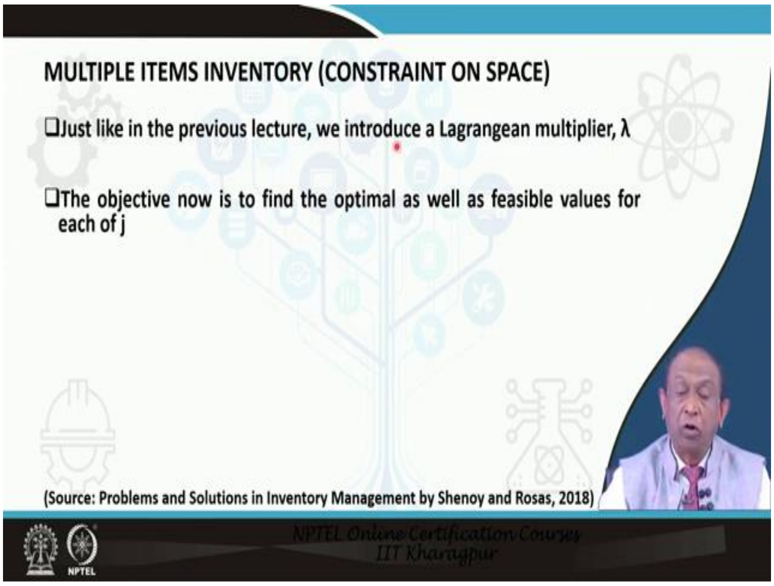

Just like in the previous lecture, here in we also introduced a Lagrangean multiplier, lambda then the objective now is to find the optimal as well as feasible values for each of j.

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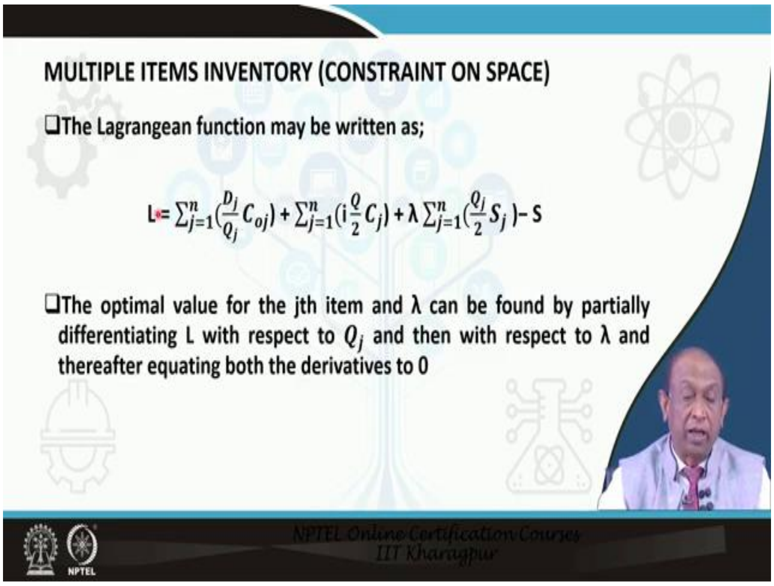

The Lagrangean function L can be expressed as sum over j equal to 1 to n Dj by Qj into Coj, the ordering cost component for the j th item plus this is the inventory holding cost, i into Q by 2 into Cj for the j th item and this needs to be summed over all the n items j equal to 1 to n plus the Lagrangean multiplier lambda multiplied by j equal to 1 to n Qj by 2 into Sj minus S.

The optimal value for the j th item as well as the Lambda can be found by partially differentiating L with respect to Qj first, and then with respect to lambda and thereafter, equating both the derivatives to 0.

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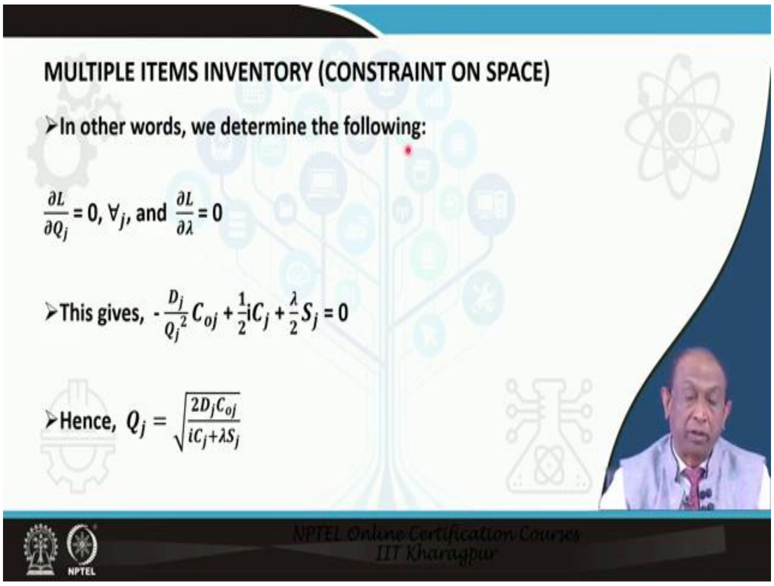

In other words, we determine the following, del del Qj of Lagrangean function L equals 0 for all j. And del del lambda of L equals 0. So, from the first equation, we get minus Dj by Qj whole square into Coj plus half i into Cj plus lambda by 2 Sj equals 0. Hence, we get an expression for Qj which is nothing but the square root of twice Dj into Coj divided by i into Cj plus lambda into Sj.

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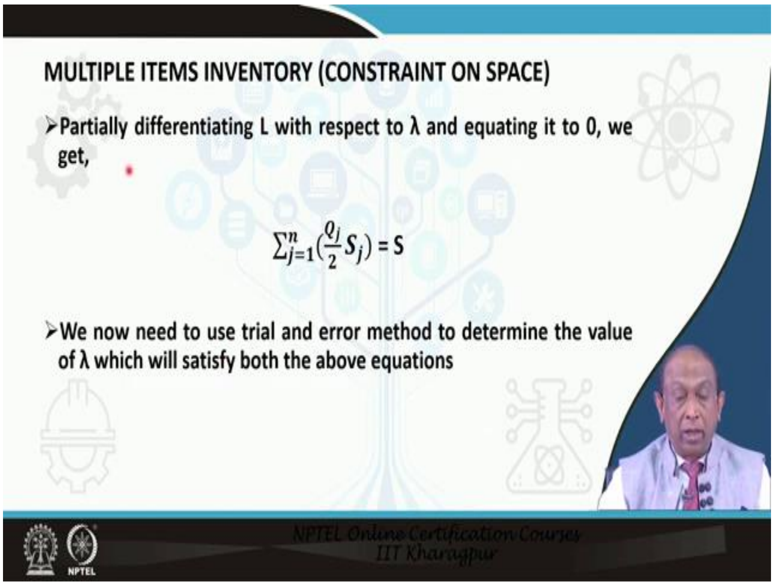

Partially differentiating L with respect to lambda and then equating it to 0 we will get Qj by 2 into Sj equals S. Now, we need to use trial and error method to determine the value of lambda which will satisfy both the above equations.

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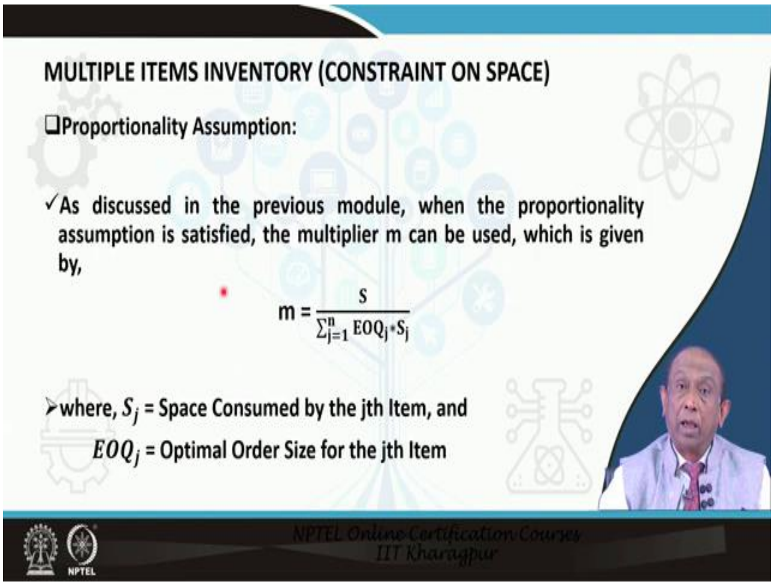

As discussed in the previous module, when the proportionality assumption is satisfied, we can find a multiplier m, which is equal to S by j equal to 1 to n EOQ j star S j in this case.

Where S j is the space consumed by the j th item and EOQj is the optimal order size for the j th item. If we can find out this value of m, then solving such problems become very easy.

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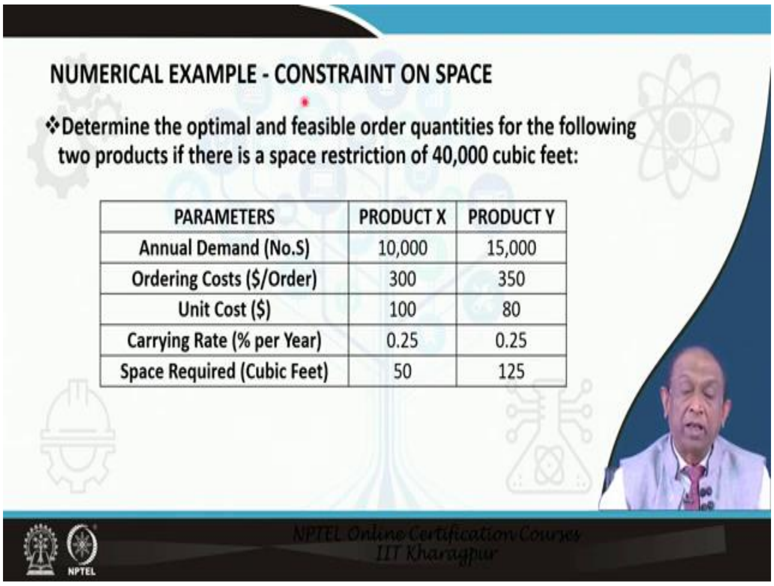

We will take one numerical example which basically illustrates a situation where there is constraint on space and how do we find the optimal Q values in such situation the problem is determine the optimal and feasible order quantities for the following two products if there is a space restriction of 40,000 cubic feet.

So, there are two products product X and product Y, annual demand for product X is 10,000 unit for product Y 15,000 units, ordering cost per order it is 300 dollar for product X and 350 dollar for product Y, unit cost for product X is 100 dollars and for product Y it is 80 dollar, the carrying rate percentage per year 0.25 here also 0.25. So, the interest rate or the carrying rate is the same for both the products X and Y and space required per unit of product X is 50 cubic feet and for product Y it is 125.

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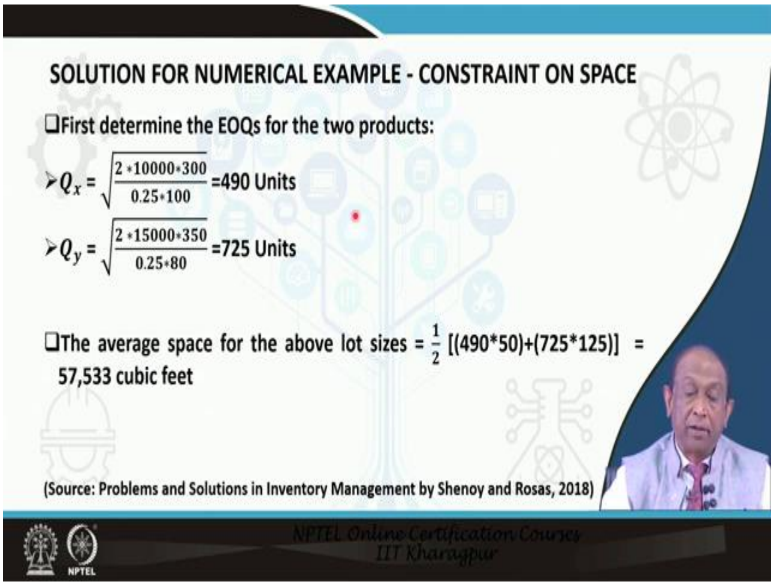

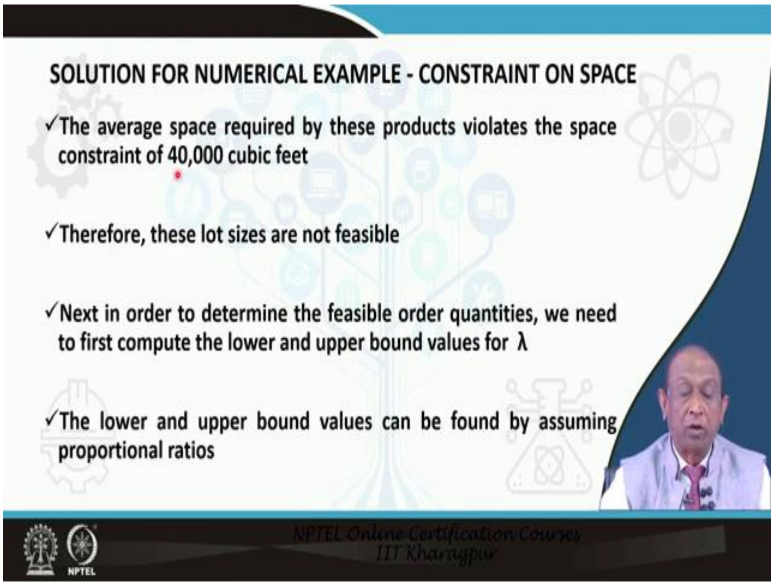

So, in order to solve this example first will show that if we determine the optimal lot size by the simple formula of which we use in case of EOQ. So, EOQ for product X works out to be 490 units, 2 into annual demand into ordering cost divided by holding cost. Similarly Qy works out to be 725 units. So, if you take these two values for Q x and Q y the average space for the above lot sizes then works out to be 57,533 cubic feet.

Obviously, these violates the constraint, what is the constraint that the total available space for accommodating these two products is 40,000 cubic feet and we required here 57,533 cubic feet. So, the constraint is violated. Therefore, these lot sizes that is 490 units and 725 units are not feasible.

So, in order to determine the feasible order quantities, we need to first compute the lower and upper bound values for lambda. The lower and upper bound values can be found very easily by assuming that proportional ratios.

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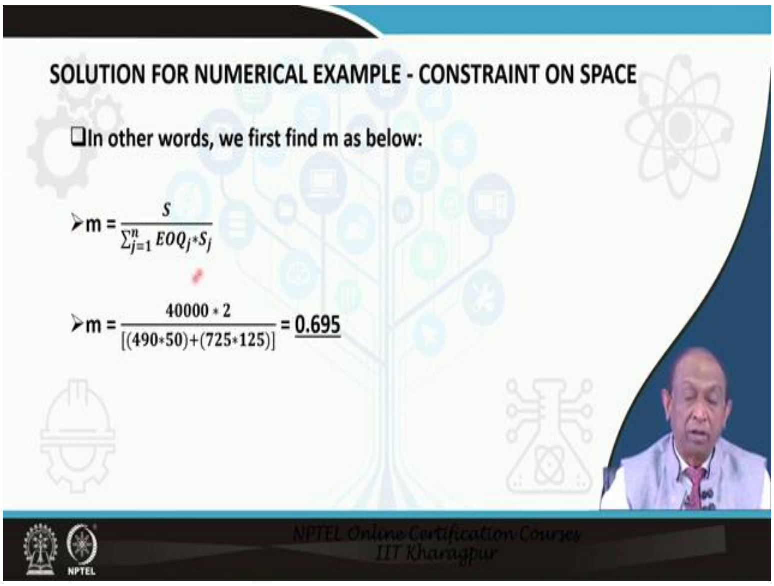

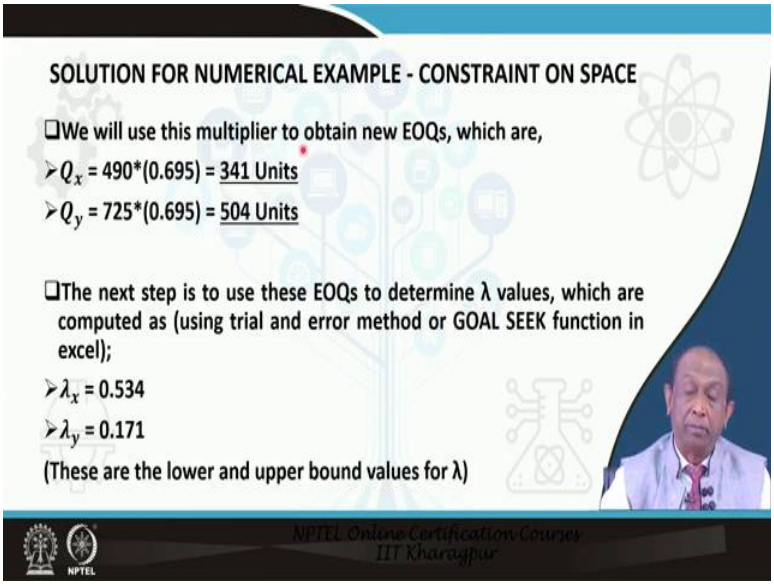

In that case what we will do, we will first find out the value of m from these by these expression. And we work out to this works out to be m equals 0.695. We will use this multiplier m to obtain new EOQs which in this case for Q x it is 341 units and for Q y it is 504 units having worked out this Qx and Qy values.

The next step will be to use this EOQs to determine lambda values and these lambda values can be obtained either by trial and error method or we can use the GOAL SEEK function in

Excel and if we do that, we will find lambda x to be 0.34 and lambda y 0.171. So, these are the, this is the lower bound and this is the upper bound values for lambda.

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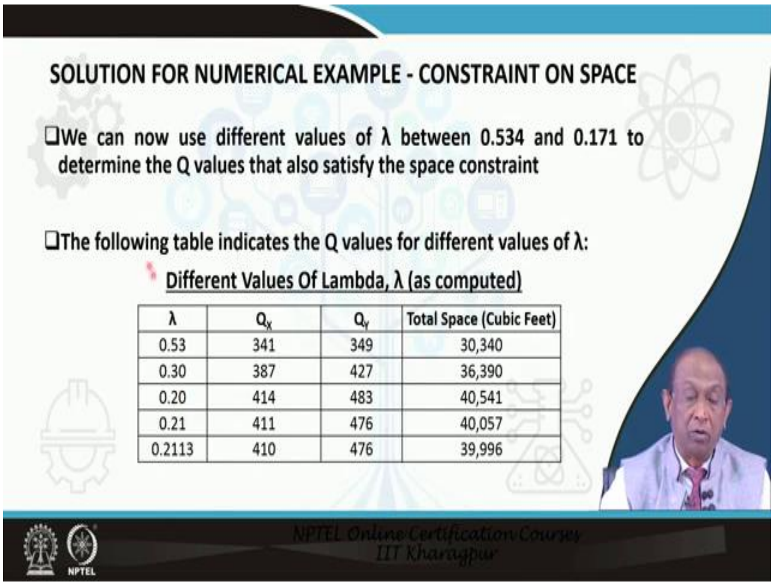

We can now use different values of lambda between 0.534 and 0.171 to determine the Q values that also satisfy the space constraint. So, here we show you a table in you getting Q values for different values of lambda. For lambda equal 0.53, Q x is 341 and Qy is 349 total space is 30,340.

If we take lambda equal 0.30 the corresponding values of Qx and Qy works out to be this total spaces like this, note this particular thing. You see here lambda if we take 0.2113 then Qx works out to be 410 and Qy works out to be 476 and under such situation, the total space required is 39,996 cubic feet.

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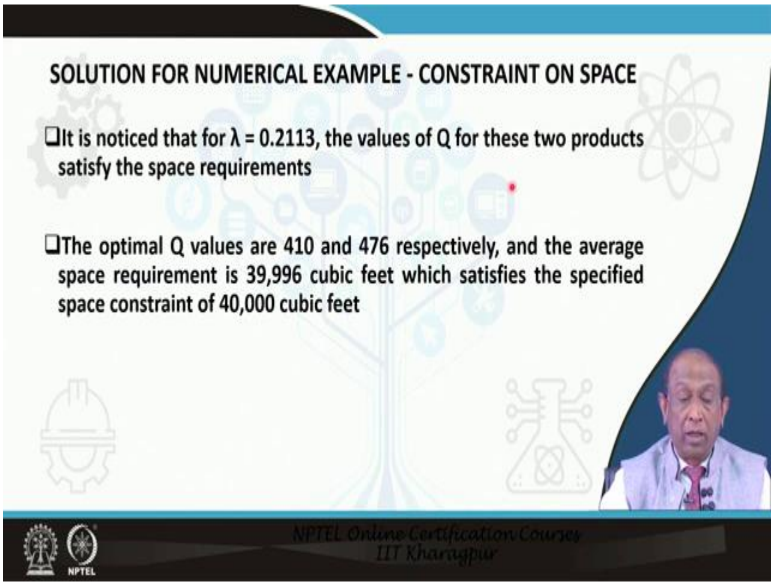

So, this satisfies the space requirement. So, for this particular example, the optimal Q values are 410 and 476 respectively for QX and QY. And the average space requirement under this circumstance is 39,996 cubic feet which basically satisfies the specified space constraint of 40,000 cubic feet.

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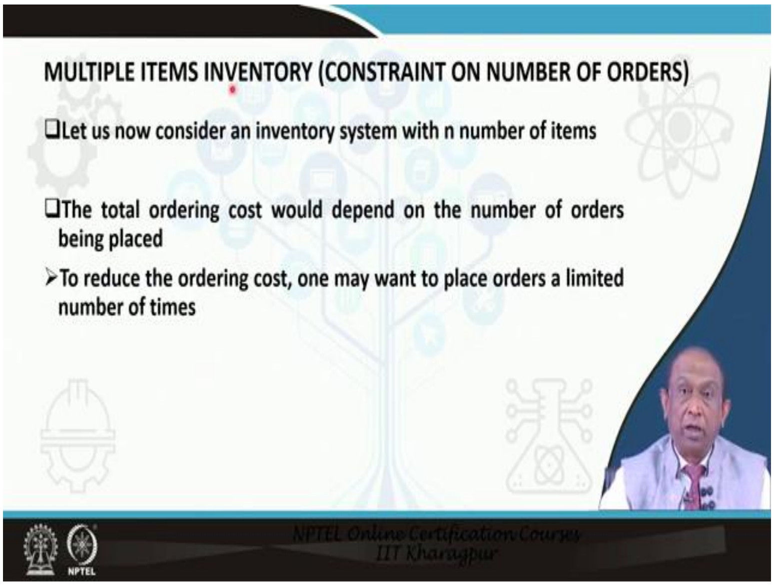

Now, we move on to the second section of our lecture, which is inventory modeling for multiple items considering constraint on the number of orders total number of orders. So, let us now consider an inventory system with n number of items. The total ordering cost would depend on the number of orders being placed and to reduce the ordering cost and organization may want to place orders a limited number of times.

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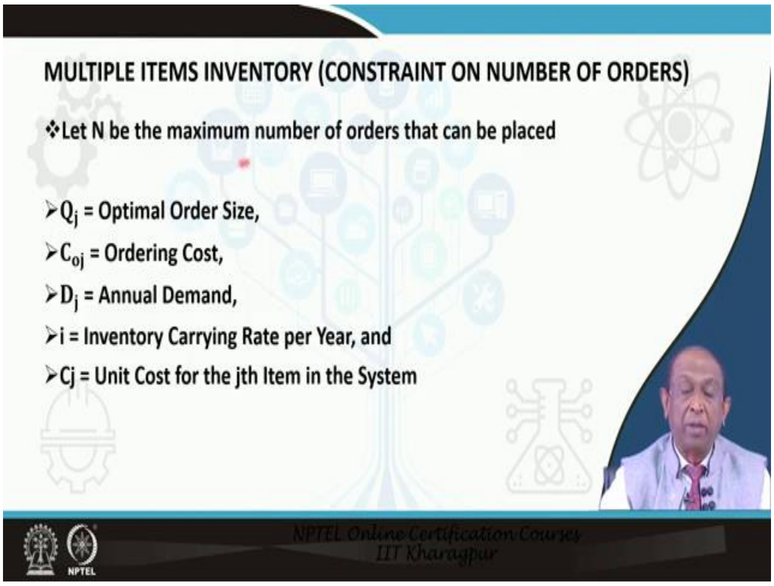

So, let N be the maximum number of orders that can be placed, this is the constraint let Qj is a optimal order size, for the j th item. Coj is ordering costs for the j th item, Dj representing the annual demand for the j th item, i equals the inventory carrying rate per year and Cj is a unit cost for the j th item in the system like the last you know, illustration, these are the notations that we will used.

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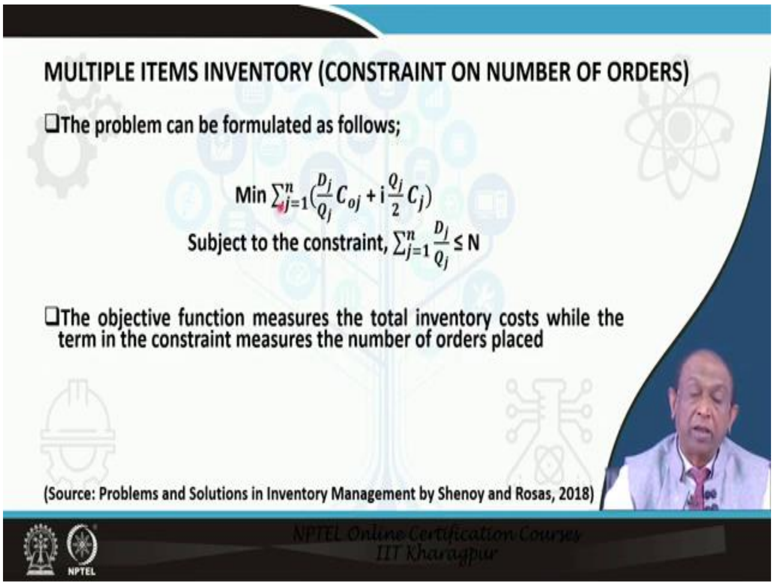

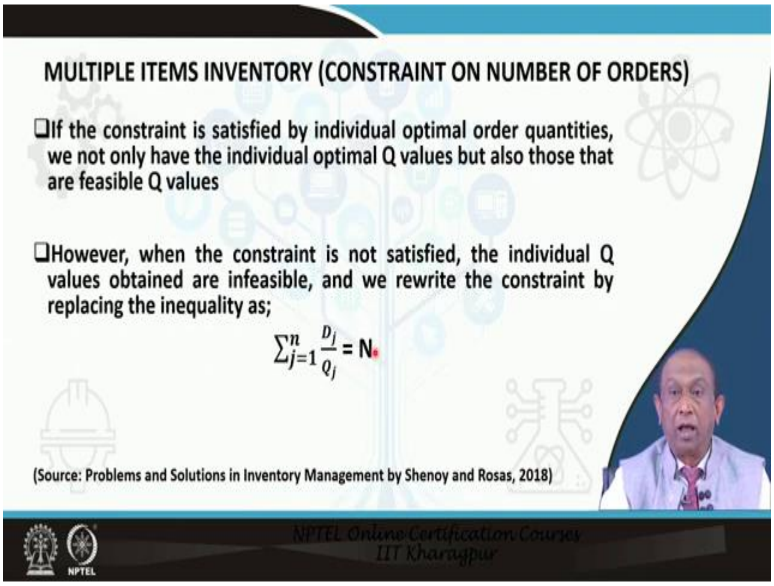

So, here also the problem can be formulated as minimization of total costs which is the cost of ordering Dj by Qj is the total number of orders over the year for the j th item multiplied by the ordering cost per order. So, Dj by Qj into Coj is the ordering cost for the j th item. And i into Qj by 2 into Cj is the average inventory holding cost for the j th item. And this has to be summed over all the n items that will give you the total expression for the cost.

This cost we need to minimize subject to the constraint that j equal 1 to n Dj by Qj is less than equal to N Dj if the annual demand for the j th item and Qj is the lot size for the j th item. So, Dj by Qj is the number of orders that we are placing for the j th item this when summed over all the items j equal to 1 to n is the total number of orders. And we have a constraint here that the total number of orders cannot exceed capital M, which is the constraint.

So, the objective function thereby measures the total inventory cost, while the term in the constraint measures the total number of orders placed. Now, if this constraint is violated, then the individual Q value of obtained will be infeasible and we rewrite the constraint by replacing this inequality as an equality and then bring in a Lagrangean multiplier for solving this constraint optimization problem.

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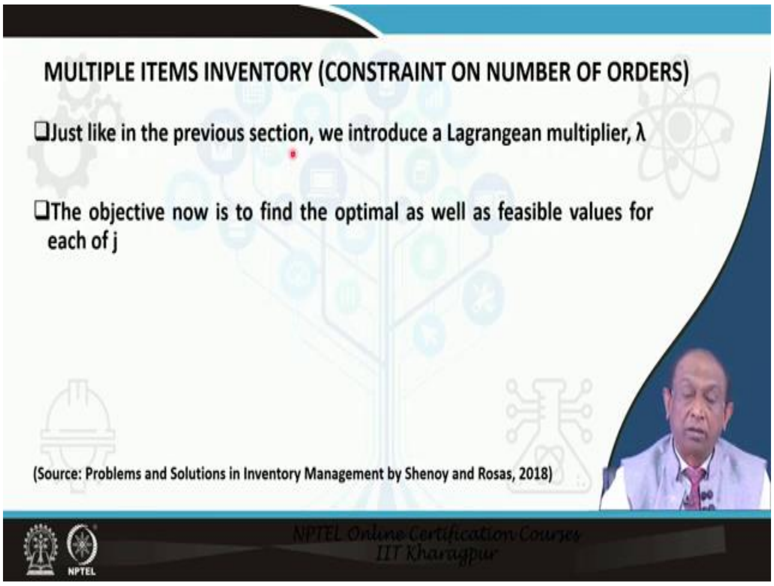

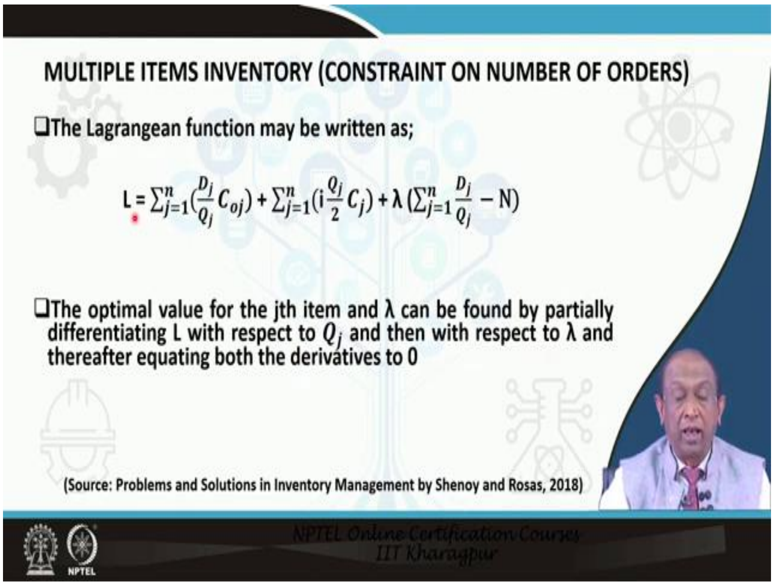

Just like in the previous section, here why we introduced a Lagrangean multiplier lambda and then the Lagrangean function L can be written as summation over j equals 1 to n the total cost for all the items ordering cost plus carrying cost plus lambda times lambda here is the Lagrangean multiplier, lambda times the equality constraint of j equal 1 to n Dj by Qj minus N.

The optimal value for the j th item and lambda as before can be found by partially differentiating this expression L with respect to Qj first and equating to 0 and then next, again we differentiate L with respect to lambda and then equate it to 0.

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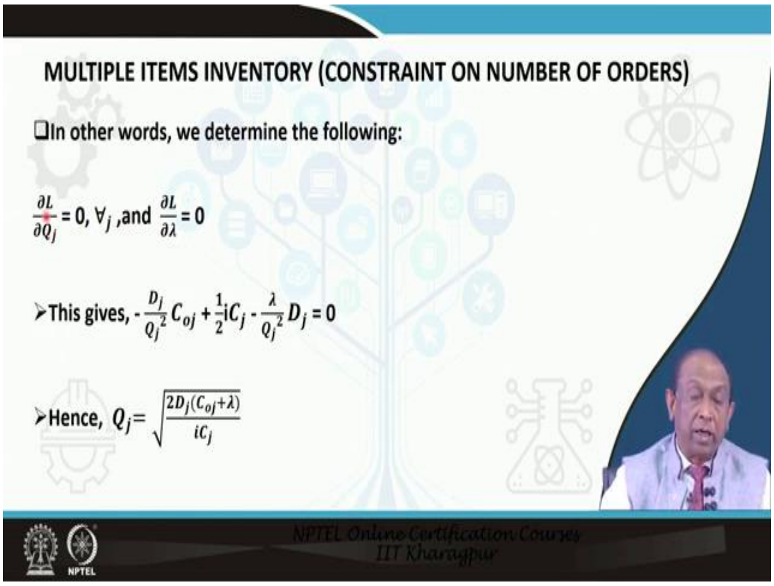

So, in other words del del Qj of this Lagrangean function L equals 0 for all j and partially differentiating L with respect to lambda and equating it to 0 is a second step from the first step, we get minus Dj by Qj square into Coj plus half i into Cj minus lambda into Qj the whole square into Dj equals 0. Now, from this particular equation with a little bit of manipulation, we can get the expression for Qj as the root over of 2 Dj into Coj plus lambda divided by i into Cj.

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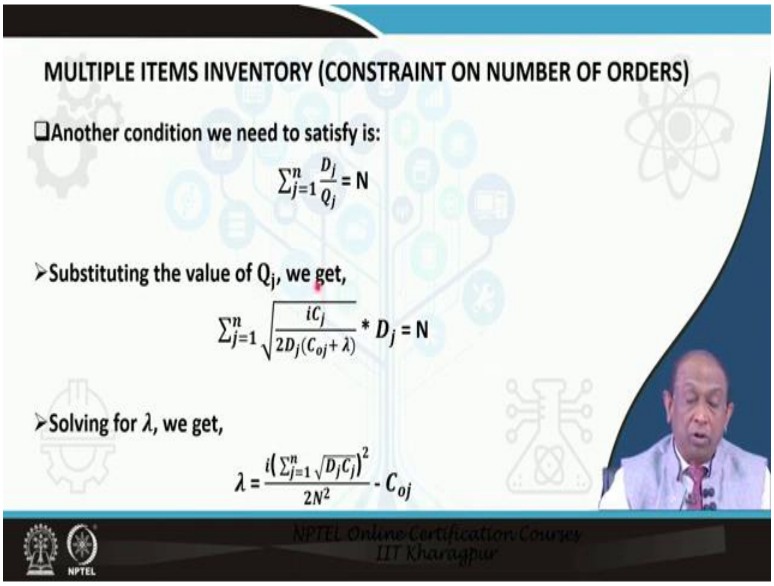

This expression for Qj we will substitute in this particular equation. So, if we do that, then we get this particular expression sum over j equal to 1 to n root over of i into Cj by this star Dj equals N. This particular condition we have derived by partially differentiating L with respect to lambda.

So, now if we solve for lambda from this particular equation, we will get lambda equals i times summation of j equal to 1 to n under root Dj Cj this whole square by 2 N square minus Coj, this is the expression for lambda and once we find the value of lambda, then the rest of the thing is very easy.

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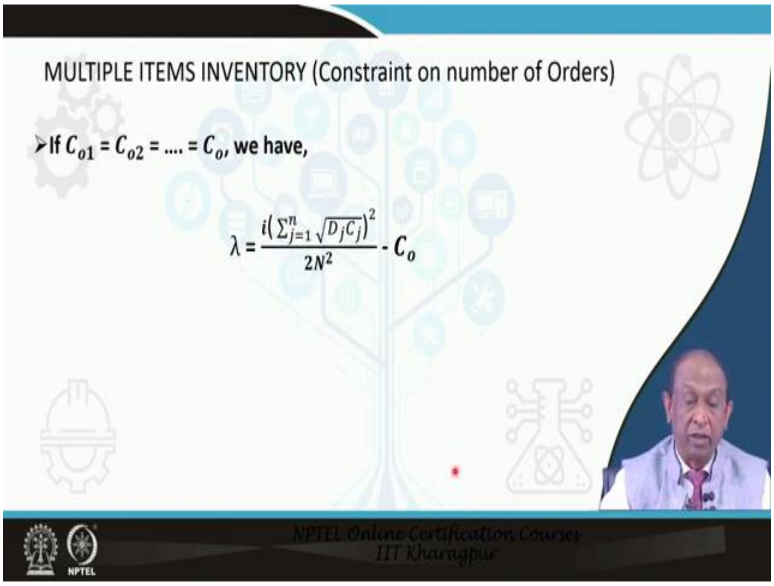

Now, in many cases, you see the ordering costs for these multiple items. Maybe the same, the ordering cost per order may not vary with respect to the individual items. So, under such circumstances, if Co1 equals Co2 equal to Co then this expression for lambda can be simplified to lambda equals i multiplied by j equal to 1 to n root over of Dj into Cj, whole square by 2 N square minus Co. If Co represents the common ordering costs for all the items.

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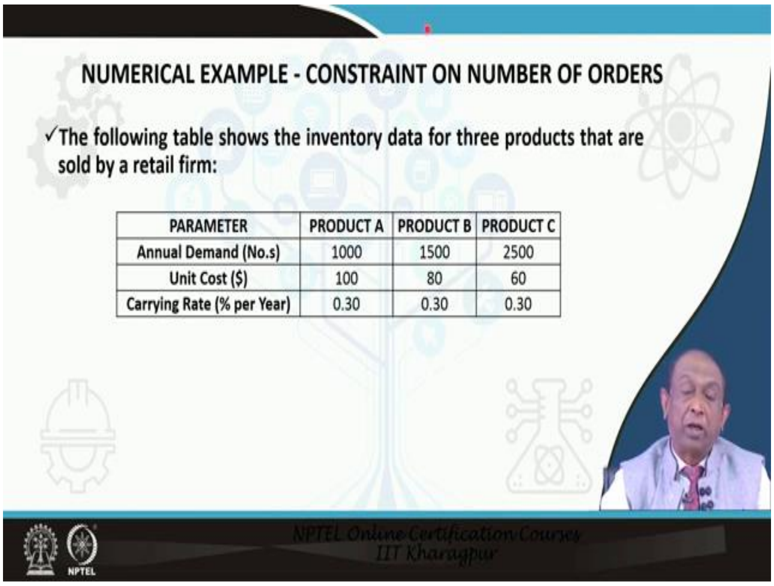

Now, this particular situation we will illustrate with the help of a numerical example. We discuss a particular problem where there are two products, three products product A, B and C that has sold by a retail firm the annual demand for product A is 1000 units for product B is 1500 units and for product C is 2500 unit. Unit cost in terms of dollars expressed in terms of dollars are 100 for product A, 80 for product B and 60 for product C and carrying rate carrying cost is 0.30 for product A these also same interest rate or carrying rate per year for all the items.

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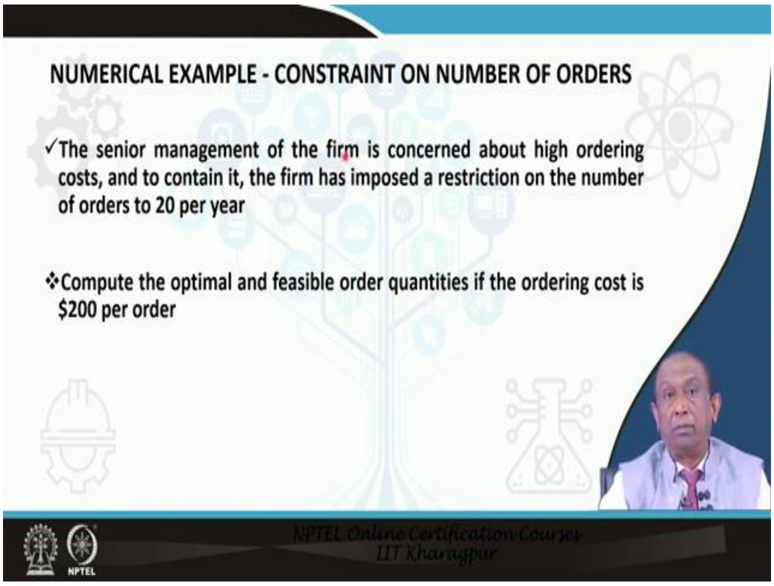

So, the problem is that the senior management of the firm is concerned about high ordering costs and to contain that cost the firm has imposed a restriction on the total number of orders and that total number of orders cannot exceed 20 per year that is the constraint. So, under such circumstances the problem is to compute the optimal and feasible order quantity if the ordering cost is dollar 200 per order and if it is common across all the items very simple.

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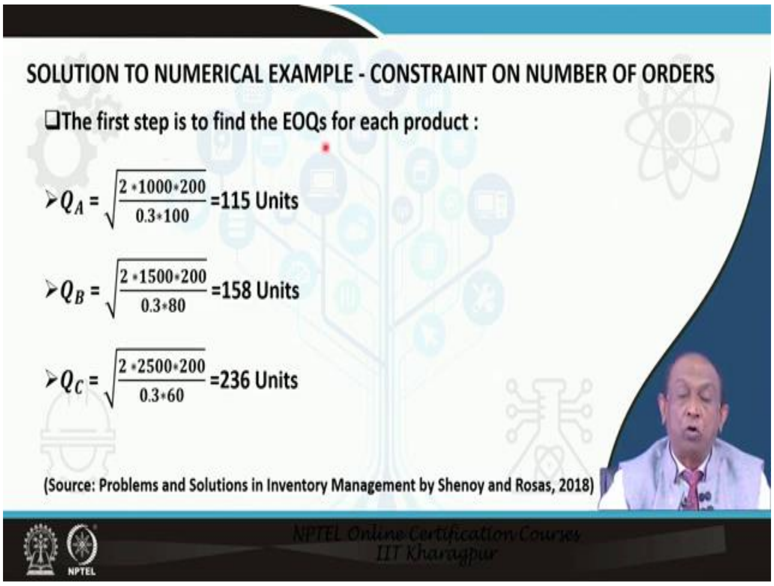

First to illustrate that if we find the EOQs for each product from the given data, then for product A, optimal lot size is 115, for QB is 158, and for QC it is to 236 units.

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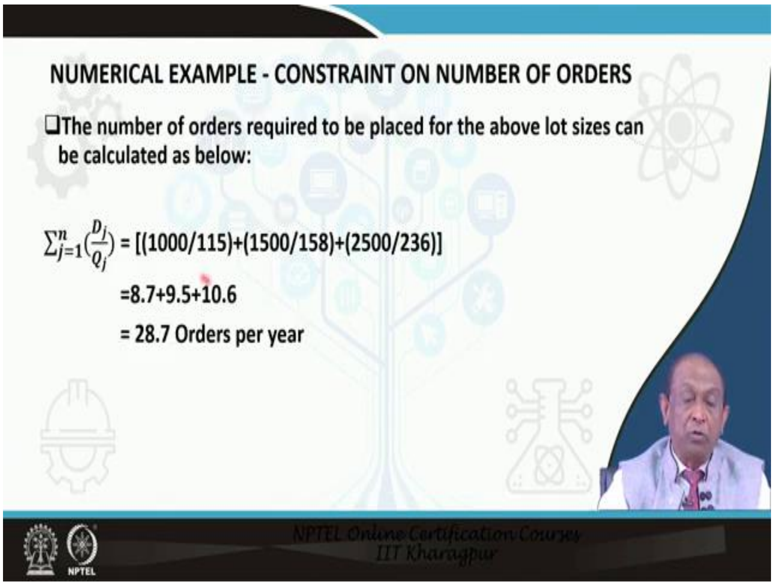

By doing that, the number of orders required to be placed for the computed lot sizes is 28.7 orders per year, which exceeds the constraint of total number of orders to be placed cannot exceed 20. So, here the constraint is violated with these EOQs. So, now we have to find out the optimal lot sizes or rather the feasible lot sizes which will not violate this constraint on the total number of orders.

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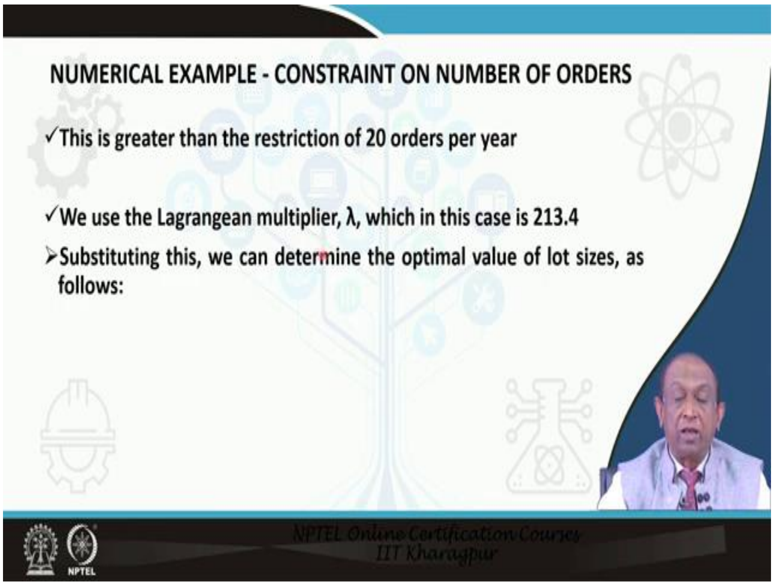

So, what we do as before we use the Lagrangean multiplier lambda and using the expression that I had discussed that if the cost is common across all the items, ordering costs rather, if the ordering cost Co is the same for all the items, then lambda can be computed from this expression.

So, using these expression what we have done that we have computed the value of lambda which has worked out to be 213.4 and this lambda we can substitute in this expression for this lot sizes in terms of lambda. So, we have worked out lambda. So, we can easily substitute that value.

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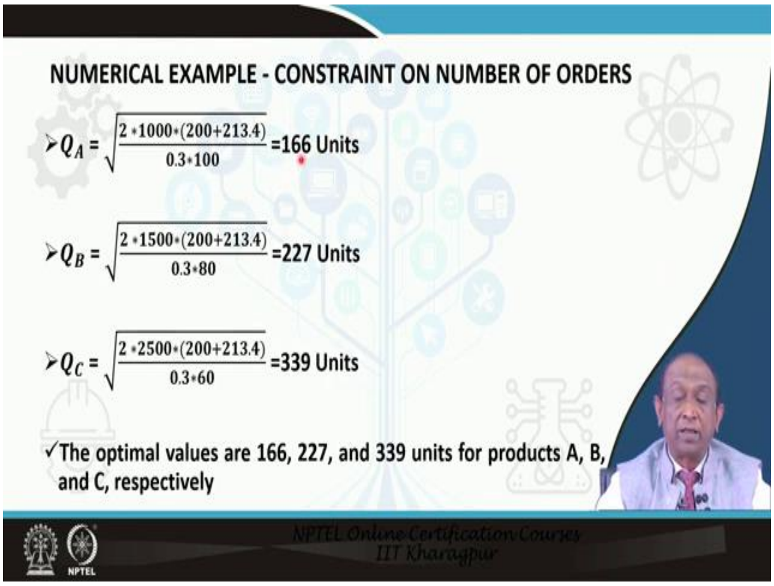

So, what we will get is QA will work out to be 166 units, QB will work out to be 227 units, and QC will work out to be 339 unit. So, these optimal values are 166, 227 and 339 units for products A, B and C respectively.

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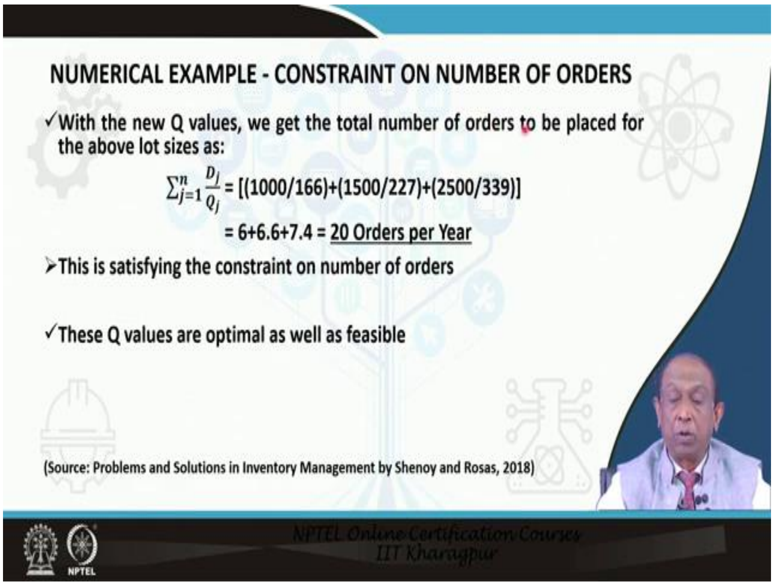

With this new Q values if you compute the total number of orders to be placed, you will find that the total number of orders per year equal to 20. So, this satisfies the constraint on the number of orders. So, these Q values are not only optimal, but also they are feasible values subject to the restriction that the total number of orders over the year is not exceeding 20. Thank you all for this particular have patience and these are the references that I have used for this particular session.