

Modelling and Analytics for Supply Chain Management
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Lecture 29
Multiple Items Inventory Models (Contd.)

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Hi, welcome to our discussion on multiple items inventory models. Today in this particular session, we are going to discuss inventory models, where there are constraints on space as well as in the next section we will discuss inventory modeling for cases where there are constraints on the total number of.

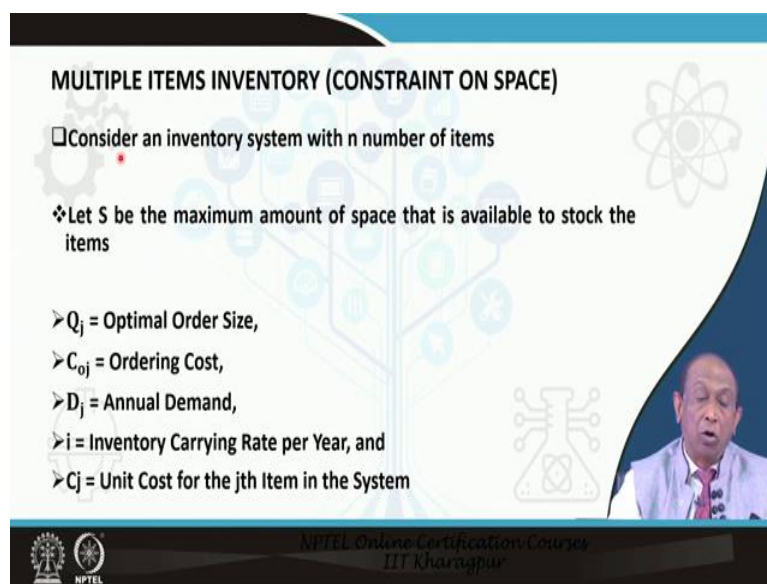
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The slide has a white background with a dark blue header containing the title 'MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)'. The main content includes a square bullet point: '❑ One of the common constraints firms face is that of storage space', followed by three right-pointing arrow bullet points: '➤ The EOQs (Q) computed for a group of items may be large', '➤ The firm may not have sufficient space to store all the procured items', and '➤ We need to determine Q for each of the items such that they are not only optimal but also feasible'. At the bottom left, there is a source citation: '(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)'. The slide also features several decorative icons: a gear, a lightbulb, a circuit board, and a stylized atom. In the bottom right corner, there is a small video inset of Professor Kunal Kanti Ghosh. The footer contains the NPTEL logo and the text 'NPTEL Online Certification Course IIT Kharagpur'.

One of the common constraints, large organizations face is that of storage space. Even in small firms, we see this particular situation because of the size of the warehouse there is constraint on storage space. The optimal order quantities Q computed for a group of items, if we see then you know the total of this the size or the space occupied by this large number of items so huge that it cannot be accommodated in the warehouse.

The firm may not have sufficient space to store all the procured item. In that case, we need to determine Q or optimal order size for each of the items such that they are not only optimal but also feasible.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

- Consider an inventory system with n number of items
- ❖ Let S be the maximum amount of space that is available to stock the items
- Q_j = Optimal Order Size,
- C_{oj} = Ordering Cost,
- D_j = Annual Demand,
- i = Inventory Carrying Rate per Year, and
- C_j = Unit Cost for the j th Item in the System

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We will discuss one such case consider an inventory system with total number of items equal to small n , let capital S be the maximum amount of space that is available to stock these items. Q_j we denote as the optimal order size for the j th item, the ordering cost for the j th item is denoted by C_{oj} , the annual demand for the j th item let it be equal to D_j , small i is the inventory carrying charge or rate per year and C_j denotes the unit cost for the j th item in the system. So, these are the notations that we are going to use for modeling this kind of problem.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

□ The problem can be formulated as follows;

$$\text{Min } \sum_{j=1}^n \left(\frac{D_j}{Q_j} C_{oj} + i \frac{Q_j}{2} C_j \right)$$

Subject to the space constraint, $\sum_{j=1}^n \left(\frac{Q_j}{2} S_j \right) \leq S$

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So, in this case the problem can be formulated as minimize the total cost which is the sum of ordering cost plus inventory holding cost, that is minimize j equals 1 to n D_j by Q_j into C_{oj} plus i into Q_j by 2 into C_j this is the total cost for one item, summed over all n items. These are ordering cost component and this is the inventory holding cost component.

This is subject to the space constraint Q by Q_j by 2 multiplied by S_j summed over j equal to 1 to n , which is the average space occupied by all the items that should be less than equal to capital S that is the maximum space available to accommodate this procured items.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

□ If the constraint is satisfied by individual optimal order quantities, we not only have the individual optimal Q values but also those that are feasible Q values

□ However, when the constraint is not satisfied, the individual Q values obtained are infeasible, and we have to rewrite the constraint as;

$$\sum_{j=1}^n \left(\frac{Q_j}{2} S_j \right) = S$$

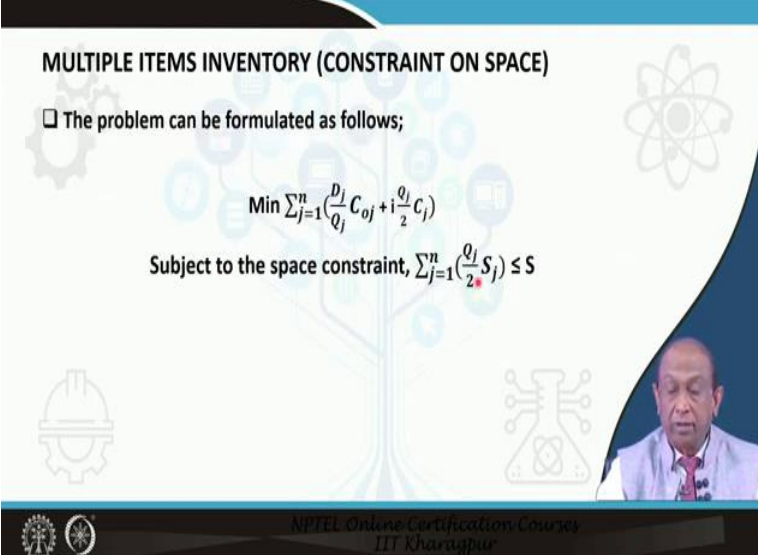
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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

□ The problem can be formulated as follows;

$$\text{Min } \sum_{j=1}^n \left(\frac{D_j}{Q_j} C_{oj} + i \frac{Q_j}{2} C_j \right)$$

Subject to the space constraint, $\sum_{j=1}^n \left(\frac{Q_j}{2} S_j \right) \leq S$



If the constraint that is Q_j by 2 into S_j less than equal to S , if this particular constraint is satisfied by individual optimal order quantities, we not only have the individual optimal Q values, but also those that are feasible Q values and we need not to worry. However, when the constraint is not satisfied, the individual Q values obtained are infeasible and we have to rewrite the constraint in this case as sum over j equal 1 to n Q_j by 2 into S_j equals S .

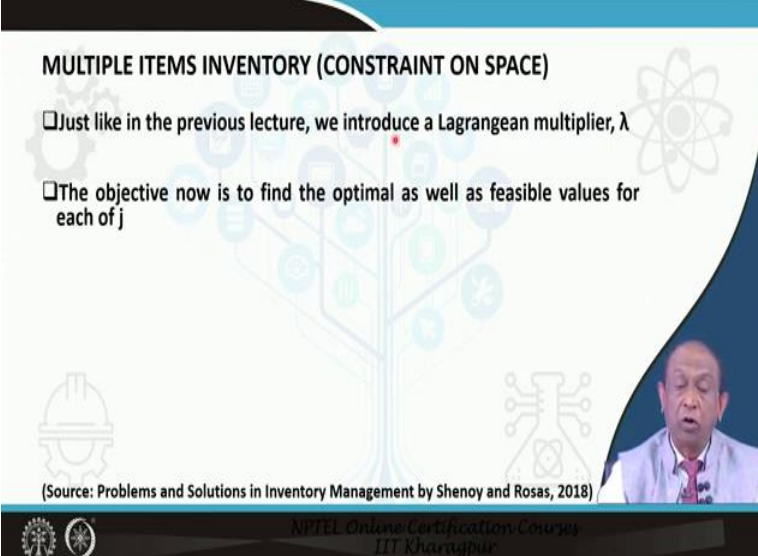
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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

□ Just like in the previous lecture, we introduce a Lagrangean multiplier, λ

□ The objective now is to find the optimal as well as feasible values for each of j

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)



Just like in the previous lecture, here in we also introduced a Lagrangean multiplier, λ then the objective now is to find the optimal as well as feasible values for each of j .

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

□ The Lagrangean function may be written as;

$$L = \sum_{j=1}^n \left(\frac{D_j}{Q_j} C_{oj} \right) + \sum_{j=1}^n \left(i \frac{Q_j}{2} C_j \right) + \lambda \sum_{j=1}^n \left(\frac{Q_j}{2} S_j \right) - S$$

□ The optimal value for the jth item and λ can be found by partially differentiating L with respect to Q_j and then with respect to λ and thereafter equating both the derivatives to 0

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The Lagrangean function L can be expressed as sum over j equal to 1 to n D_j by Q_j into C_{oj} , the ordering cost component for the j th item plus this is the inventory holding cost, i into Q by 2 into C_j for the j th item and this needs to be summed over all the n items j equal to 1 to n plus the Lagrangean multiplier lambda multiplied by j equal to 1 to n Q_j by 2 into S_j minus S.

The optimal value for the j th item as well as the Lambda can be found by partially differentiating L with respect to Q_j first, and then with respect to lambda and thereafter, equating both the derivatives to 0.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

➤ In other words, we determine the following:

$$\frac{\partial L}{\partial Q_j} = 0, \forall j, \text{ and } \frac{\partial L}{\partial \lambda} = 0$$

➤ This gives, $-\frac{D_j}{Q_j^2} C_{oj} + \frac{1}{2} i C_j + \frac{\lambda}{2} S_j = 0$

➤ Hence, $Q_j = \sqrt{\frac{2D_j C_{oj}}{i C_j + \lambda S_j}}$

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In other words, we determine the following, del del Q_j of Lagrangean function L equals 0 for all j. And del del lambda of L equals 0. So, from the first equation, we get minus D_j by Q_j

whole square into C_j plus half i into C_j plus λ by 2 S_j equals 0. Hence, we get an expression for Q_j which is nothing but the square root of twice D_j into C_j divided by i into C_j plus λ into S_j .

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

➤ Partially differentiating L with respect to λ and equating it to 0, we get,

$$\sum_{j=1}^n \left(\frac{Q_j}{2} S_j \right) = S$$

➤ We now need to use trial and error method to determine the value of λ which will satisfy both the above equations

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Partially differentiating L with respect to λ and then equating it to 0 we will get Q_j by 2 into S_j equals S . Now, we need to use trial and error method to determine the value of λ which will satisfy both the above equations.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON SPACE)

☐ Proportionality Assumption:

✓ As discussed in the previous module, when the proportionality assumption is satisfied, the multiplier m can be used, which is given by,

$$m = \frac{S}{\sum_{j=1}^n EOQ_j \cdot S_j}$$

➤ where, S_j = Space Consumed by the j th Item, and
 EOQ_j = Optimal Order Size for the j th Item

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As discussed in the previous module, when the proportionality assumption is satisfied, we can find a multiplier m , which is equal to S by j equal to 1 to n EOQ_j star S_j in this case.

Where S_j is the space consumed by the j th item and EOQ_j is the optimal order size for the j th item. If we can find out this value of m , then solving such problems become very easy.

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NUMERICAL EXAMPLE - CONSTRAINT ON SPACE

❖ Determine the optimal and feasible order quantities for the following two products if there is a space restriction of 40,000 cubic feet:

PARAMETERS	PRODUCT X	PRODUCT Y
Annual Demand (No.S)	10,000	15,000
Ordering Costs (\$/Order)	300	350
Unit Cost (\$)	100	80
Carrying Rate (% per Year)	0.25	0.25
Space Required (Cubic Feet)	50	125

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We will take one numerical example which basically illustrates a situation where there is constraint on space and how do we find the optimal Q values in such situation the problem is determine the optimal and feasible order quantities for the following two products if there is a space restriction of 40,000 cubic feet.

So, there are two products product X and product Y, annual demand for product X is 10,000 unit for product Y 15,000 units, ordering cost per order it is 300 dollar for product X and 350 dollar for product Y, unit cost for product X is 100 dollars and for product Y it is 80 dollar, the carrying rate percentage per year 0.25 here also 0.25. So, the interest rate or the carrying rate is the same for both the products X and Y and space required per unit of product X is 50 cubic feet and for product Y it is 125.

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SOLUTION FOR NUMERICAL EXAMPLE - CONSTRAINT ON SPACE



□ First determine the EOQs for the two products:

➤ $Q_x = \sqrt{\frac{2 \cdot 10000 \cdot 300}{0.25 \cdot 100}} = 490$ Units

➤ $Q_y = \sqrt{\frac{2 \cdot 15000 \cdot 350}{0.25 \cdot 80}} = 725$ Units

□ The average space for the above lot sizes = $\frac{1}{2} [(490 \cdot 50) + (725 \cdot 125)] = 57,533$ cubic feet

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)





SOLUTION FOR NUMERICAL EXAMPLE - CONSTRAINT ON SPACE

✓ The average space required by these products violates the space constraint of 40,000 cubic feet

✓ Therefore, these lot sizes are not feasible

✓ Next in order to determine the feasible order quantities, we need to first compute the lower and upper bound values for λ

✓ The lower and upper bound values can be found by assuming proportional ratios



So, in order to solve this example first will show that if we determine the optimal lot size by the simple formula of which we use in case of EOQ. So, EOQ for product X works out to be 490 units, 2 into annual demand into ordering cost divided by holding cost. Similarly Q_y works out to be 725 units. So, if you take these two values for Q_x and Q_y the average space for the above lot sizes then works out to be 57,533 cubic feet.

Obviously, these violates the constraint, what is the constraint that the total available space for accommodating these two products is 40,000 cubic feet and we required here 57,533 cubic feet. So, the constraint is violated. Therefore, these lot sizes that is 490 units and 725 units are not feasible.

So, in order to determine the feasible order quantities, we need to first compute the lower and upper bound values for lambda. The lower and upper bound values can be found very easily by assuming that proportional ratios.

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SOLUTION FOR NUMERICAL EXAMPLE - CONSTRAINT ON SPACE

□ In other words, we first find m as below:

$$m = \frac{S}{\sum_{j=1}^n EOQ_j \cdot S_j}$$
$$m = \frac{40000 \cdot 2}{[(490 \cdot 50) + (725 \cdot 125)]} = \underline{0.695}$$

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SOLUTION FOR NUMERICAL EXAMPLE - CONSTRAINT ON SPACE

□ We will use this multiplier to obtain new EOQs, which are,

$$Q_x = 490 \cdot (0.695) = \underline{341 \text{ Units}}$$
$$Q_y = 725 \cdot (0.695) = \underline{504 \text{ Units}}$$

□ The next step is to use these EOQs to determine λ values, which are computed as (using trial and error method or GOAL SEEK function in excel);

$$\lambda_x = 0.534$$
$$\lambda_y = 0.171$$

(These are the lower and upper bound values for λ)

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In that case what we will do, we will first find out the value of m from these by these expression. And we work out to this works out to be m equals 0.695. We will use this multiplier m to obtain new EOQs which in this case for Q x it is 341 units and for Q y it is 504 units having worked out this Qx and Qy values.

The next step will be to use this EOQs to determine lambda values and these lambda values can be obtained either by trial and error method or we can use the GOAL SEEK function in

Excel and if we do that, we will find lambda x to be 0.34 and lambda y 0.171. So, these are the, this is the lower bound and this is the upper bound values for lambda.

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SOLUTION FOR NUMERICAL EXAMPLE - CONSTRAINT ON SPACE

- We can now use different values of λ between 0.534 and 0.171 to determine the Q values that also satisfy the space constraint
- The following table indicates the Q values for different values of λ :

Different Values Of Lambda, λ (as computed)

λ	Q_x	Q_y	Total Space (Cubic Feet)
0.53	341	349	30,340
0.30	387	427	36,390
0.20	414	483	40,541
0.21	411	476	40,057
0.2113	410	476	39,996

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We can now use different values of lambda between 0.534 and 0.171 to determine the Q values that also satisfy the space constraint. So, here we show you a table in you getting Q values for different values of lambda. For lambda equal 0.53, Q x is 341 and Qy is 349 total space is 30,340.

If we take lambda equal 0.30 the corresponding values of Qx and Qy works out to be this total spaces like this, note this particular thing. You see here lambda if we take 0.2113 then Qx works out to be 410 and Qy works out to be 476 and under such situation, the total space required is 39,996 cubic feet.

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SOLUTION FOR NUMERICAL EXAMPLE - CONSTRAINT ON SPACE

- It is noticed that for $\lambda = 0.2113$, the values of Q for these two products satisfy the space requirements
- The optimal Q values are 410 and 476 respectively, and the average space requirement is 39,996 cubic feet which satisfies the specified space constraint of 40,000 cubic feet

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So, this satisfies the space requirement. So, for this particular example, the optimal Q values are 410 and 476 respectively for QX and QY. And the average space requirement under this circumstance is 39,996 cubic feet which basically satisfies the specified space constraint of 40,000 cubic feet.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

- Let us now consider an inventory system with n number of items
- The total ordering cost would depend on the number of orders being placed
- To reduce the ordering cost, one may want to place orders a limited number of times

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Now, we move on to the second section of our lecture, which is inventory modeling for multiple items considering constraint on the number of orders total number of orders. So, let us now consider an inventory system with n number of items. The total ordering cost would depend on the number of orders being placed and to reduce the ordering cost and organization may want to place orders a limited number of times.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

- ❖ Let N be the maximum number of orders that can be placed
- Q_j = Optimal Order Size,
- C_{oj} = Ordering Cost,
- D_j = Annual Demand,
- i = Inventory Carrying Rate per Year, and
- C_j = Unit Cost for the j th Item in the System

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So, let N be the maximum number of orders that can be placed, this is the constraint let Q_j is a optimal order size, for the j th item. C_{oj} is ordering costs for the j th item, D_j representing the annual demand for the j th item, i equals the inventory carrying rate per year and C_j is a unit cost for the j th item in the system like the last you know, illustration, these are the notations that we will used.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

❑ The problem can be formulated as follows;

$$\text{Min } \sum_{j=1}^n \left(\frac{D_j}{Q_j} C_{oj} + i \frac{Q_j}{2} C_j \right)$$

Subject to the constraint, $\sum_{j=1}^n \frac{D_j}{Q_j} \leq N$

❑ The objective function measures the total inventory costs while the term in the constraint measures the number of orders placed

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

- ❑ If the constraint is satisfied by individual optimal order quantities, we not only have the individual optimal Q values but also those that are feasible Q values
- ❑ However, when the constraint is not satisfied, the individual Q values obtained are infeasible, and we rewrite the constraint by replacing the inequality as;

$$\sum_{j=1}^n \frac{D_j}{Q_j} = N$$

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

So, here also the problem can be formulated as minimization of total costs which is the cost of ordering D_j by Q_j is the total number of orders over the year for the j th item multiplied by the ordering cost per order. So, D_j by Q_j into C_{oj} is the ordering cost for the j th item. And i into Q_j by 2 into C_j is the average inventory holding cost for the j th item. And this has to be summed over all the n items that will give you the total expression for the cost.

This cost we need to minimize subject to the constraint that $\sum_{j=1}^n \frac{D_j}{Q_j}$ is less than equal to N . D_j is the annual demand for the j th item and Q_j is the lot size for the j th item. So, $\frac{D_j}{Q_j}$ is the number of orders that we are placing for the j th item this when summed over all the items j equal to 1 to n is the total number of orders. And we have a constraint here that the total number of orders cannot exceed capital M , which is the constraint.


So, the objective function thereby measures the total inventory cost, while the term in the constraint measures the total number of orders placed. Now, if this constraint is violated, then the individual Q value of obtained will be infeasible and we rewrite the constraint by replacing this inequality as an equality and then bring in a Lagrangean multiplier for solving this constraint optimization problem.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

- ❑ Just like in the previous section, we introduce a Lagrangean multiplier, λ
- ❑ The objective now is to find the optimal as well as feasible values for each of j

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)



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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

- ❑ The Lagrangean function may be written as;

$$L = \sum_{j=1}^n \left(\frac{D_j}{Q_j} C_{oj} \right) + \sum_{j=1}^n \left(i \frac{Q_j}{2} C_j \right) + \lambda \left(\sum_{j=1}^n \frac{D_j}{Q_j} - N \right)$$

- ❑ The optimal value for the j th item and λ can be found by partially differentiating L with respect to Q_j and then with respect to λ and thereafter equating both the derivatives to 0

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)



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Just like in the previous section, here why we introduced a Lagrangean multiplier lambda and then the Lagrangean function L can be written as summation over j equals 1 to n the total cost for all the items ordering cost plus carrying cost plus lambda times lambda here is the Lagrangean multiplier, lambda times the equality constraint of j equal 1 to n D_j by Q_j minus N .

The optimal value for the j th item and lambda as before can be found by partially differentiating this expression L with respect to Q_j first and equating to 0 and then next, again we differentiate L with respect to lambda and then equate it to 0.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

□ In other words, we determine the following:

$$\frac{\partial L}{\partial Q_j} = 0, \forall j, \text{ and } \frac{\partial L}{\partial \lambda} = 0$$

➤ This gives, $-\frac{D_j}{Q_j^2} C_{oj} + \frac{1}{2} C_j - \frac{\lambda}{Q_j^2} D_j = 0$

➤ Hence, $Q_j = \sqrt{\frac{2D_j(C_{oj} + \lambda)}{C_j}}$

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So, in other words del del Qj of this Lagrangean function L equals 0 for all j and partially differentiating L with respect to lambda and equating it to 0 is a second step from the first step, we get minus Dj by Qj square into Coj plus half i into Cj minus lambda into Qj the whole square into Dj equals 0. Now, from this particular equation with a little bit of manipulation, we can get the expression for Qj as the root over of 2 Dj into Coj plus lambda divided by i into Cj.

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MULTIPLE ITEMS INVENTORY (CONSTRAINT ON NUMBER OF ORDERS)

□ Another condition we need to satisfy is:

$$\sum_{j=1}^n \frac{D_j}{Q_j} = N$$

➤ Substituting the value of Q_j , we get,

$$\sum_{j=1}^n \sqrt{\frac{iC_j}{2D_j(C_{oj} + \lambda)}} * D_j = N$$

➤ Solving for λ , we get,

$$\lambda = \frac{(\sum_{j=1}^n \sqrt{D_j C_j})^2}{2N^2} - C_{oj}$$

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This expression for Qj we will substitute in this particular equation. So, if we do that, then we get this particular expression sum over j equal to 1 to n root over of i into Cj by this star Dj equals N. This particular condition we have derived by partially differentiating L with respect to lambda.

So, now if we solve for lambda from this particular equation, we will get lambda equals i times summation of j equal to 1 to n under root Dj Cj this whole square by 2 N square minus Coj, this is the expression for lambda and once we find the value of lambda, then the rest of the thing is very easy.

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MULTIPLE ITEMS INVENTORY (Constraint on number of Orders)

➤ If $C_{o1} = C_{o2} = \dots = C_o$, we have,

$$\lambda = \frac{i(\sum_{j=1}^n \sqrt{D_j C_j})^2}{2N^2} - C_o$$

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Now, in many cases, you see the ordering costs for these multiple items. Maybe the same, the ordering cost per order may not vary with respect to the individual items. So, under such circumstances, if Co1 equals Co2 equal to Co then this expression for lambda can be simplified to lambda equals i multiplied by j equal to 1 to n root over of Dj into Cj, whole square by 2 N square minus Co. If Co represents the common ordering costs for all the items.

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NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

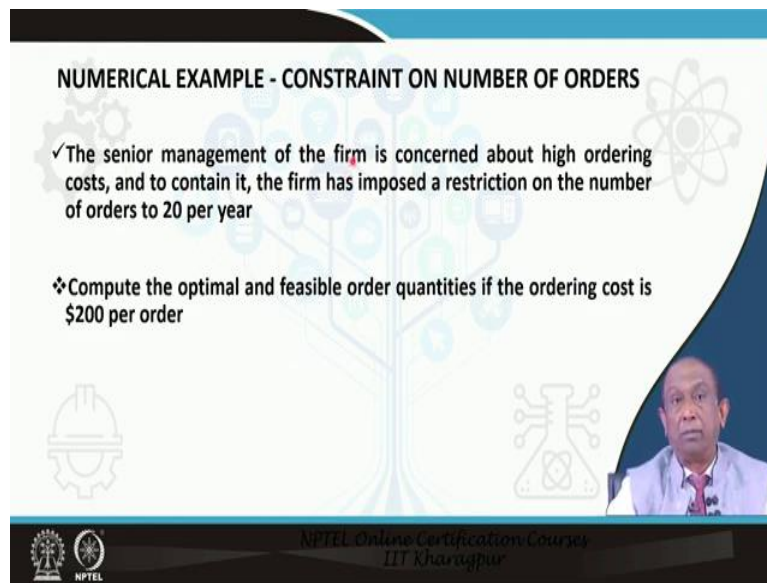
✓ The following table shows the inventory data for three products that are sold by a retail firm:

PARAMETER	PRODUCT A	PRODUCT B	PRODUCT C
Annual Demand (No.s)	1000	1500	2500
Unit Cost (\$)	100	80	60
Carrying Rate (% per Year)	0.30	0.30	0.30

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Now, this particular situation we will illustrate with the help of a numerical example. We discuss a particular problem where there are two products, three products product A, B and C that has sold by a retail firm the annual demand for product A is 1000 units for product B is 1500 units and for product C is 2500 unit. Unit cost in terms of dollars expressed in terms of dollars are 100 for product A, 80 for product B and 60 for product C and carrying rate carrying cost is 0.30 for product A these also same interest rate or carrying rate per year for all the items.

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NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

- ✓ The senior management of the firm is concerned about high ordering costs, and to contain it, the firm has imposed a restriction on the number of orders to 20 per year
- ❖ Compute the optimal and feasible order quantities if the ordering cost is \$200 per order

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So, the problem is that the senior management of the firm is concerned about high ordering costs and to contain that cost the firm has imposed a restriction on the total number of orders and that total number of orders cannot exceed 20 per year that is the constraint. So, under such circumstances the problem is to compute the optimal and feasible order quantity if the ordering cost is dollar 200 per order and if it is common across all the items very simple.

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SOLUTION TO NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

□ The first step is to find the EOQs for each product :

➤ $Q_A = \sqrt{\frac{2 \cdot 1000 \cdot 200}{0.3 \cdot 100}} = 115$ Units

➤ $Q_B = \sqrt{\frac{2 \cdot 1500 \cdot 200}{0.3 \cdot 80}} = 158$ Units

➤ $Q_C = \sqrt{\frac{2 \cdot 2500 \cdot 200}{0.3 \cdot 60}} = 236$ Units

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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First to illustrate that if we find the EOQs for each product from the given data, then for product A, optimal lot size is 115, for QB is 158, and for QC it is to 236 units.

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NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

□ The number of orders required to be placed for the above lot sizes can be calculated as below:

$\sum_{j=1}^n \left(\frac{D_j}{Q_j}\right) = [(1000/115) + (1500/158) + (2500/236)]$

$= 8.7 + 9.5 + 10.6$

$= 28.7$ Orders per year

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By doing that, the number of orders required to be placed for the computed lot sizes is 28.7 orders per year, which exceeds the constraint of total number of orders to be placed cannot exceed 20. So, here the constraint is violated with these EOQs. So, now we have to find out the optimal lot sizes or rather the feasible lot sizes which will not violate this constraint on the total number of orders.

(Refer Slide Time: 28:20)

NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

- ✓ This is greater than the restriction of 20 orders per year
- ✓ We use the Lagrangean multiplier, λ , which in this case is 213.4
- Substituting this, we can determine the optimal value of lot sizes, as follows:

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So, what we do as before we use the Lagrangean multiplier lambda and using the expression that I had discussed that if the cost is common across all the items, ordering costs rather, if the ordering cost C_o is the same for all the items, then lambda can be computed from this expression.

So, using these expression what we have done that we have computed the value of lambda which has worked out to be 213.4 and this lambda we can substitute in this expression for this lot sizes in terms of lambda. So, we have worked out lambda. So, we can easily substitute that value.

(Refer Slide Time: 29:29)

NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

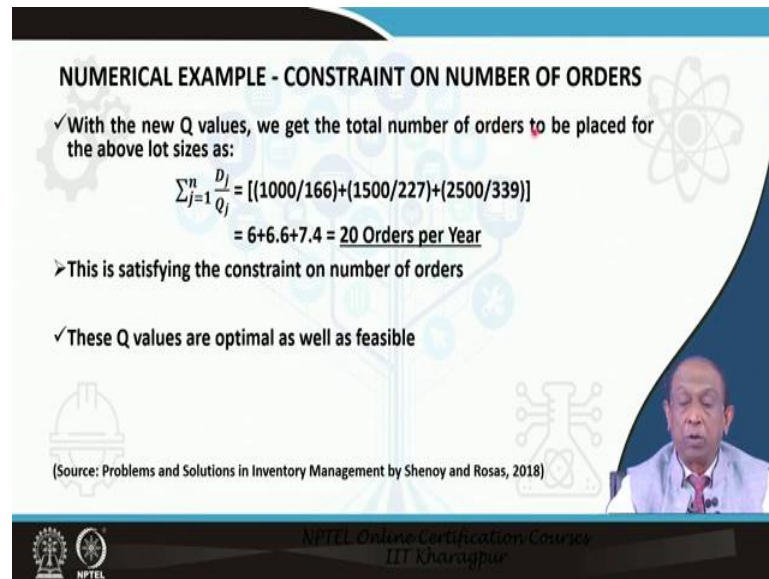
- $Q_A = \sqrt{\frac{2 \cdot 1000 \cdot (200 + 213.4)}{0.3 \cdot 100}} = 166 \text{ Units}$
- $Q_B = \sqrt{\frac{2 \cdot 1500 \cdot (200 + 213.4)}{0.3 \cdot 80}} = 227 \text{ Units}$
- $Q_C = \sqrt{\frac{2 \cdot 2500 \cdot (200 + 213.4)}{0.3 \cdot 60}} = 339 \text{ Units}$

✓ The optimal values are 166, 227, and 339 units for products A, B, and C, respectively

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So, what we will get is QA will work out to be 166 units, QB will work out to be 227 units, and QC will work out to be 339 unit. So, these optimal values are 166, 227 and 339 units for products A, B and C respectively.

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NUMERICAL EXAMPLE - CONSTRAINT ON NUMBER OF ORDERS

✓ With the new Q values, we get the total number of orders to be placed for the above lot sizes as:

$$\sum_{j=1}^n \frac{D_j}{Q_j} = [(1000/166)+(1500/227)+(2500/339)]$$
$$= 6+6.6+7.4 = \underline{20 \text{ Orders per Year}}$$

➤ This is satisfying the constraint on number of orders

✓ These Q values are optimal as well as feasible

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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With this new Q values if you compute the total number of orders to be placed, you will find that the total number of orders per year equal to 20. So, this satisfies the constraint on the number of orders. So, these Q values are not only optimal, but also they are feasible values subject to the restriction that the total number of orders over the year is not exceeding 20. Thank you all for this particular have patience and these are the references that I have used for this particular session.