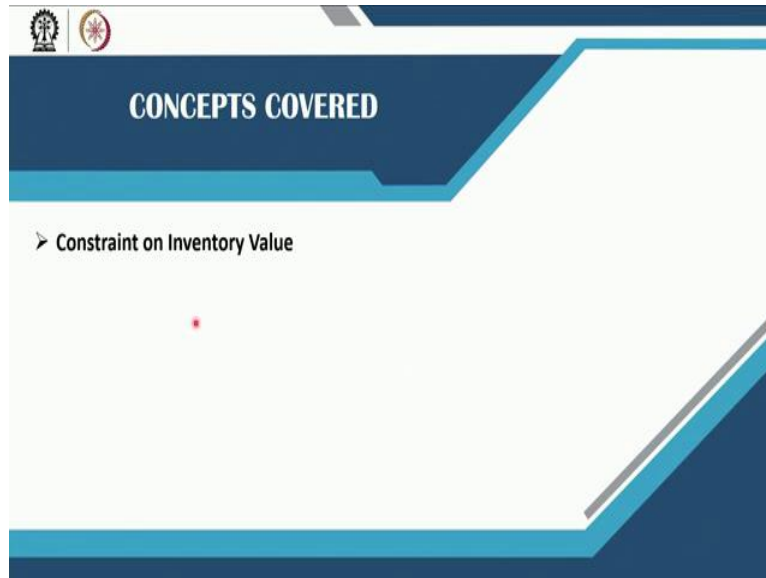


Modelling and Analytics for Supply Chain Management
Professor Kunal Kanti Ghosh
Vinod Gupta School of Management
Indian Institute of technology, Kharagpur
Lecture 28
Multiple Items Inventory Models

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Welcome to modeling and analytics for supply chain management course. Today in this particular session, we will be dealing with multiple items, inventory models and the concept that will be dealt in this particular session will be how to determine the inventory level for multiple items subject to a restriction on the total inventory value. So, that is why we have written constraint on inventory value.

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The slide has a white background with a blue border on the right side. The title 'MULTIPLE ITEMS INVENTORY (Constraint on Inventory Value)' is at the top. Three bullet points are listed below the title. In the bottom right corner, there is a small video inset showing a man in a white shirt and tie speaking. The background of the slide features faint icons of gears, a tree, and a circuit board. At the bottom, there are logos for NPTEL and IIT Kharagpur.

MULTIPLE ITEMS INVENTORY (Constraint on Inventory Value)

- In multiple item problems, we will consider more than one item at a time.
- In previous sessions, we dealt with inventory models that are applicable to each item separately.
- However, when there is a constraint that limits the ordering quantity due to various reasons, multiple item models require different kind of analysis.

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So, multiple items inventory with the constraint on the total inventory value. In multiple item problems, we will consider more than one item at a time. In our previous sessions, we dealt with inventory models that are applicable to each item separately. However, whenever there is a constraint that limits the ordering quantity for each item due to various reasons, multiple item models require different kind of analysis.

This various reasons maybe constraint on inventory value. Constraint may be on the total number of orders, constraint maybe on the total storage space available and like that, and there can be two or more constraints operating at the same point in time. We may need to find out whether any solution exists at that point or not.

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The slide features a light blue background with faint icons of gears, a tree, and a person. The title is 'MULTIPLE ITEMS INVENTORY (Constraint on Inventory Value)'. Below the title are two bullet points, each starting with a square checkbox icon. A small video inset in the bottom right corner shows a man in a white shirt and red tie speaking. The footer contains the NPTEL logo and the text 'NPTEL Online Certification Courses IIT Kharagpur'.

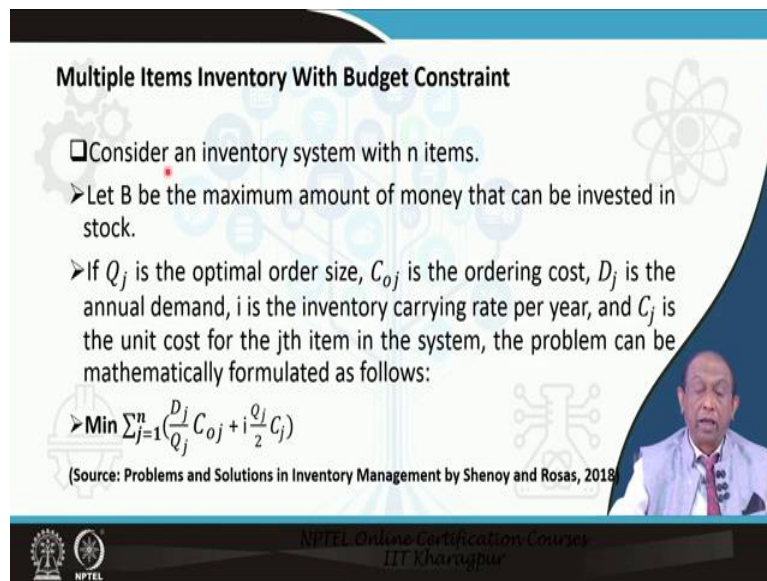
MULTIPLE ITEMS INVENTORY (Constraint on Inventory Value)

- In real life, we have more than thousands of items in a factory and each of them has a certain inventory resulting in a very large amount of money tied up in stocks.
- An organization might want reduce the total money value of inventory to a given value B to reduce its need for working capital.

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In real life, we have more than thousands of items in a factory and each one of them has a sudden inventory resulting in a very large amount of money tied up in stocks. So, an organization might want to reduce the total money value of inventory to a maximum limit or a given value B to reduce its need for working capital.

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Multiple Items Inventory With Budget Constraint

- Consider an inventory system with n items.
- Let B be the maximum amount of money that can be invested in stock.
- If Q_j is the optimal order size, C_{oj} is the ordering cost, D_j is the annual demand, i is the inventory carrying rate per year, and C_j is the unit cost for the j th item in the system, the problem can be mathematically formulated as follows:

$$\text{➤ Min } \sum_{j=1}^n \left(\frac{D_j}{Q_j} C_{oj} + i \frac{Q_j}{2} C_j \right)$$

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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We will consider an inventory system with say, n items and let B capital B be the maximum amount of money that can be invested in stock. This is a constraint that has been imposed on this problem by the managers in the organization. If Q_j denotes the optimal order size for an item j , C_{oj} is the corresponding ordering cost for the item. D_j represents the annual demand for the item j , i is the inventory carrying rate per year.

And if C_j denotes the unit cost for the j th item in the system, then the problem can be mathematically formulated as minimize sum of j equals 1 to n D_j by Q_j , D_j by Q_j is a number of orders for the j th item multiplied by the corresponding ordering costs C_{oj} . So, this portion gives you the ordering cost for the j th item plus i times Q_j by 2 is the average inventory for the j th item.

When we are ordering lots, by amount of Q_j multiplied by C_j which is the carrying cost for the item j and this has to be done for all the n items that you have installed. So, the total cost expression becomes minimization of j equals 1 to n sum of j equal 1 to n , the total cost expression as we have explained.

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Multiple Items Inventory With Budget Constraint

➤ Subject to, $\sum_{j=1}^n \left(\frac{Q_j}{2} C_j\right) \leq B$

➤ The objective function measures the total inventory costs while the term in the constraint $\sum_{j=1}^n \left(\frac{Q_j}{2} C_j\right)$ restricts the investment in inventory to B.

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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But this is subject to the restriction that sum over j equal to 1 to n Q_j by 2 into C_j less than equal to B what is Q_j by 2? This is the average inventory value for the j th item that multiplied by C_j that is a carrying cost for the j th item which is nothing but the unit cost of the item multiplied by interest rate. This in here this C_j represents the cost for the j th item and this has to be summed over j equal to 1 to n, which is less than equal to B.

B is the total amount of investment that you can make for these n items in inventory. So, the objective function measures the total inventory cost while the term in this constraint restricts the investment in inventory to a value of B.

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Multiple Items Inventory With Budget Constraint

➤ If the constraint is satisfied by individual optimal order quantities, we not only have the individual optimal Q values but also those that are feasible Q values.

➤ However, when the constraint is not satisfied, the individual Q values obtained are infeasible, and we rewrite the constraint by replacing the inequality as

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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Now, if this constraint that is this one is satisfied by individual optimal order quantities. That means, we compute the EOQ for all these n items individually and then find out the expression for this. And then if this sum is less than equal to B that is what I am saying that, if the constraint is satisfied by individual optimal order quantities. We not only have the individual optimal Q values but these are all feasible Q values.

However, when the constraint is not satisfied, that means, we compute this expression and we find that it exceeds B then the individual Q values obtained are infeasible. And then, under such situation we need to rewrite the constraint by replacing the inequality, the inequality was here.

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Multiple Items Inventory With Budget Constraint

- $\sum_{j=1}^n \left(\frac{Q_j}{2}\right) C_j = B$
- To solve this problem, we need to introduce a Lagrangean multiplier, λ .
- The objective now is to find the optimal as well as feasible values for each of j.
- The Lagrangean function can be written as
- $L = \sum_{j=1}^n \left(\frac{D_j}{Q_j}\right) C_{oj} + \sum_{j=1}^n \left(i\frac{Q_j}{2}\right) C_j + \lambda \left(\sum_{j=1}^n \left(\frac{Q_j}{2}\right) C_j - B\right)$

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By representing this inequality as sum of j equal to 1 to n. Q_j by 2 into C_j equals B with an equality. So, the problem is minimize the total cost subject to this particular restriction. In order to solve this kind of problem, we need to introduce our Lagrangean multiplier, which is denoted by lambda. If we introduce this Lagrangean multiplier lambda. Then the objective now is to find the optimal as well as feasible values for each of these j items.

The Lagrangean function can be written as L equals this is nothing but this total cost expression that we have already discussed plus lambda times j equals 1 to n Q_j by 2 into C_j minus B, which is the constraint here. This is the normal procedure in which we solve this kind of constraint optimization problems.

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Multiple Items Inventory With Budget Constraint

- The optimal values for Q_j and λ can be found by partially differentiating L and equating it to 0.
- In other words, we determine the following: $\frac{\partial L}{\partial Q_j} = 0, \forall j$
- and $\frac{\partial L}{\partial \lambda} = 0$
- So we get, $-\frac{D_j}{Q_j^2} C_{oj} + \frac{1}{2} C_j + \frac{\lambda}{2} C_j = 0$

$$Q_j = \sqrt{\frac{2D_j C_{oj}}{C_j(i+\lambda)}}$$


Multiple Items Inventory With Budget Constraint

- $\sum_{j=1}^n (\frac{Q_j}{2} C_j) = B$
- To solve this problem, we need to introduce a Lagrangean multiplier, λ .
- The objective now is to find the optimal as well as feasible values for each of j .
- The Lagrangean function can be written as
- $L = \sum_{j=1}^n (\frac{D_j}{Q_j} C_{oj}) + \sum_{j=1}^n (i \frac{Q_j}{2} C_j) + \lambda (\sum_{j=1}^n (\frac{Q_j}{2} C_j) - B)$



The optimal values for Q_j and λ can be found by partially differentiating this expression L with respect to Q as well as partially differentiating this expression with respect to λ and equating both these expressions with 0. That means, $\frac{\partial L}{\partial \lambda} = 0$ and $\frac{\partial L}{\partial Q_j} = 0$. So, if we find out the $\frac{\partial L}{\partial Q_j} = 0$ we will get this expression. So, minus D_j by Q_j^2 into C_{oj} plus with respect to Q_j I am differentiating.

So, i by 2 into C_j plus λ by 2 into C_j . And then we equate it to 0 for minimum value of the cost. This when we simplify, we get the expression for Q_j as root over of 2 times D_j into C_{oj} divided by C_j into $i + \lambda$. So, you see this is the optimal order in quantity for the j th item, but in this expression there is this Lagrangean multiplier λ appearing. So, we have to find out this expression or values of λ . So, where from will get the other thing that you need to do is $\frac{\partial L}{\partial \lambda} = 0$.

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Multiple Items Inventory With Budget Constraint

- Another condition we need to satisfy is: $\sum_{j=1}^n (Q_j C_j) = B$
- Substituting the value of Q_j , we get
- $\sqrt{\frac{2D_j C_{oj}}{c_j(1+\lambda)}} (C_j/2) = B$
- Solving for λ , we get $\lambda = \frac{1}{2B^2} \left(\sum_{j=1}^n \sqrt{C_{oj} D_j C_j} \right)^2 - 1$

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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So, if I do that, then we will get this particular equation or condition because in this expression for L when differentiated with respect to lambda, I will get this, this one, 0 this one is 0, this lambda del del lambda of 1, so we will get this expression. Now, in this expression, if we substitute the value of Qj.

We will get I will found out I will substitute the value of Qj from here this is the value of Qj. So, this has been put in here. So, this multiplied by Cj by 2 gives us the value of gives us this is equal to B. Now, if we solve for lambda, this equation we get lambda equal to, 1 upon 2 B square sum over j equal 1 to n root over of Coj into Dj into Cj, is entire whole square minus 1. This is one aspect.

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Multiple Items Inventory With Budget Constraint

- ☐ Proportionality Assumption:
- Consider a situation where the amount of money invested in an item held in inventory is in proportion to the overall budget.
- In other words, we assume that $\frac{C_1}{h_1} = \frac{C_2}{h_2} \dots = \frac{C_n}{h_n} = \frac{C}{h}$
- If this assumption is valid, then we can rewrite
- $Q_j = \sqrt{\frac{2D_j C_{oj}}{h_j}} \sqrt{\frac{1}{1+\lambda \frac{C}{h}}}$
- It should be noted that $h = iC$.

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The problem can be simplified with an assumption which is basically the proportionality assumption. Consider a situation where the amount of money invested in an item held in inventory is in proportion to the overall budget in that case, we assume that C_1 by h_1 equal to C_2 by h_2 like this C_n by h_n is equal to C by h . If this assumption is valid, then we can rewrite Q_j as, root over of twice D_j into C_{oj} by h_j into 1 by $1 + \lambda$ into C by h . So, this h is nothing but the carrying cost i into C .

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Multiple Items Inventory With Budget Constraint

- There are two terms in the previous equation, The first term is EOQ, while the second one can be set to m , the multiplier.
- Q_j can thus be re written as $Q_j = EOQ_j * m$
- Where $m = \sqrt{\frac{1}{1 + \lambda \frac{C}{h}}}$
- It is noticed that the term m is independent of λ .
- It is easy to compute m since we need only the EOQ and the budget
- This method can be only used if the proportionality assumption is met

Multiple Items Inventory With Budget Constraint

Proportionality Assumption:

- Consider a situation where the amount of money invested in an item held in inventory is in proportion to the overall budget.
- In other words, we assume that $\frac{C_1}{h_1} = \frac{C_2}{h_2} \dots = \frac{C_n}{h_n} = \frac{C}{h}$
- If this assumption is valid, then we can rewrite

$$Q_j = \sqrt{\frac{2D_j C_{oj}}{h_j}} \sqrt{\frac{1}{1 + \lambda \frac{C}{h}}}$$

- It should be noted that $h = iC$.

Now, there are two terms in this expression for Q_j , the first term is the expression for Q_j is a EOQ simply the EOQ, while the second one can be set to some value m which we basically called the multiplier. So, Q_j can now be written as EOQ for the j th item multiplied by m where m is this, that means, this expression, 1 upon square root of 1 upon $1 + \lambda$ into C by h .

It is noticed that this term m is independent of λ and it is easy to compute m . Since we need only the EOQ and the budget. This method can be only used if the proportionality assumption is met.

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Numerical Example: Budget Constraint

❑ Question:

- Inventory parameters for three products – A, B, and C – sold by a retailer are shown as below:

Parameter	Product A	Product B	Product C
Annual demand	1000	1500	2500
Ordering cost	\$30	\$35	\$50
Unit cost	\$50	\$100	\$150

- The retailer uses an inventory carrying rate of 25% per annum.
- What would be the economic lot size if the retailer does not want to invest more than \$10,000 in the average inventory of these three products?

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Now, we will illustrate this particular thing with the example with an example on budget constraint. Say there are three products A B and C which is being sold by a retailer. The annual demand for each of the products are known. For product A the annual demand is 1000 units, this is 1500 units for product C it is 2500 units. The ordering cost for product A is 30 for product B is 35 dollars and for production C is 50 dollars.


It may be in a real life situation, that this ordering cost maybe the same for all the items. Unit costs for product A is dollar 50 for product B is dollar 100 for product C is dollar 150 and the retailer uses an inventory carrying rate of 25 percent per annum. The problem is what would be the economic lot size if the retailer does not want to invest more than dollar 10,000 in the average inventory of these three products? So, that means there is a constraint that the maximum investment in inventory is dollar 10,000 which is nothing but B.

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Numerical Example: Budget Constraint

➤ Solution:

➤ Using the standard EOQ formula we can determine the EOQs for three products :

$$Q_A = \sqrt{\frac{2 \cdot 1000 \cdot 30}{0.25 \cdot 50}} = 69 \text{ Units}$$
$$Q_B = \sqrt{\frac{2 \cdot 1500 \cdot 35}{0.25 \cdot 100}} = 65 \text{ Units}$$
$$Q_C = \sqrt{\frac{2 \cdot 2500 \cdot 50}{0.25 \cdot 150}} = 82 \text{ Units}$$


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Numerical Example: Budget Constraint


❑ Question:

➤ Inventory parameters for three products – A, B, and C – sold by a retailer are shown as below:

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➤ The retailer uses an inventory carrying rate of 25% per annum.

➤ What would be the economic lot size if the retailer does not want to invest more than \$10,000 in the average inventory of these three products?



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So, now if we use the standard EOQ formula, we will determine the EOQ for these three products, Q_A , Q_B and Q_C . For Q_A it is 2 into annual demand that is 1000 multiplied by the ordering cost dollar 30 by i into C , the unit cost is given as 50, which comes out to be 69 units. Similarly for Q_B it is 2 times 1500 into its ordering costs 35 multiplied by i into its unit cost is 100, 65 units and for Q_C it is root over of 2 into 2500 into 50 divided by 0.25 into 150, 69 units, 65 units and 82 unit.

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Numerical Example: Budget Constraint

- The average investment in inventory for the above lot sizes can be determined as below :
- Average investment = $\frac{1}{2} [(69*50)+(65*100)+(82*150)] = 11,125$
- It is noticed that the investment in average inventory violates the specified budget constraint of \$10,000
- Therefore, these lot sizes are not feasible.
- We need to determine the value of λ to compute feasible lot sizes that satisfy the budgetary constraint



(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)



Numerical Example: Budget Constraint

- The average investment in inventory is $= \frac{1}{2} [(62*50)+(58*100)+(74*150)] = 10,000$
- With the revised EOQs, the investment in average inventory satisfies the specified budgetary constraint.
- Therefore, the economic and feasible lot sizes for products A, B, and C are 62, 58, and 74 units, respectively

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)



So, the average investment in inventory for this above lot sizes are the that we have determined equals Q by 2 into C_j which is 69 into 50 plus 65 into 100 plus 82 into 150 divided by 2, the C_j 's are given 50, 100 and 150 and this is Q_A Q_B this is Q_C . So, Q_A by 2 is average inventory multiplied by the cost. So, this becomes 11,125.

So, you can noticed that the investment in average inventory violates the specified budget constraint of dollar 10,000. Therefore, these lot sizes are not feasible. Hence, what do we have to do under such condition we need to determine the value of λ that is the Lagrangean multiplier to compute the feasible sizes that satisfy the budgetary constraint.

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Numerical Example: Budget Constraint

➤ $\lambda = \frac{1}{2B^2} (\sum_{j=1}^n \sqrt{C_{oj}D_jC_j})^2 - i = 0.058$

➤ We can now compute the revised EOQ values by substituting the value of λ

➤ $Q_A = \sqrt{\frac{2 \cdot 1000 \cdot 30}{(0.25 + 0.058) \cdot 50}} = \sqrt{\frac{2 \cdot 1000 \cdot 30}{0.308 \cdot 50}} = 62 \text{ Units}$

➤ $Q_B = \sqrt{\frac{2 \cdot 1500 \cdot 35}{(0.25 + 0.058) \cdot 100}} = \sqrt{\frac{2 \cdot 1500 \cdot 35}{0.308 \cdot 100}} = 58 \text{ Units}$

➤ $Q_C = \sqrt{\frac{2 \cdot 2500 \cdot 50}{(0.25 + 0.058) \cdot 150}} = \sqrt{\frac{2 \cdot 2500 \cdot 50}{0.308 \cdot 150}} = 74 \text{ Units}$

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And as you have noted that we had determine the value of lambda as $\frac{1}{2B^2}$ multiplied by this minus i . Now, if we substitute the corresponding values and compute lambda will come out to be 0.058. Once we find out the value of lambda, we can now compute the revised EOQ values by substituting the value of lambda you see, the expression for Q_j was in this expression for Q_j , if we put the value of lambda then we will find out that Q_A will work out to be 62 units. Q_B will work out to be 58 units and Q_C will work out to be 74 units.

Under these circumstances, the average investment in inventory if we compute, then it will come out to be 10,000. With the revised EOQs, the investment in average inventory specify the specified budgetary constraint so the constraint is satisfied. Therefore, the economic and feasible lot sizes for products A, B and C are 62, 58 and 74 units respectively.

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Numerical Example: Budget Constraint

❑ Question:

- Determine the optimal quantities in the previous exercise assuming that the ratio of unit cost of the product to its carrying cost is constant. In other words,
- $\frac{C_A}{h_A} = \frac{C_B}{h_B} = \frac{C_C}{h_C}$ where C_A, C_B and C_C are the unit costs and h_A, h_B and h_C are the carrying costs for products A, B, and C, respectively

(Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)

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The second question is determine the optimal quantities in the previous exercise, assuming that the ratio of unit cost of the product to its carrying cost is constant that is the proportionality assumptions that we had discussed. In other words, in here, C_A by h_A equals C_B by h_B equal to C_C by h_C , where C_A, C_B and C_C are the unit costs for the item A, B and C. h_A, h_B and h_C are the carrying costs for the product A, B, and C respectively.

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Numerical Example: Budget Constraint

❑ Solution:

- The initial EOQs are 69, 65, and 82 units respectively, and the average investment in inventory is \$11,125 which violates the budgetary constraint.
- If proportionality condition is assumed, then we have a simple solution on hand.
- We do not need to compute the Lagrangean factor
- Instead, we would need to determine the multiplier, m as below

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So, the initial EOQ were 69, 65 and 82 units in the previous exercise 69, 65 and 82 units and the average investment in inventory with this initial EOQ values were 11,125 dollars that violated the budgetary constraint. Now, if proportionality condition is assumed, then we can use a simple solution we do not need to compute the Lagrangean factor or multiplier under that circumstance. Instead, we would need to determine the multiplier m .


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Numerical Example: Budget Constraint


$$m = \frac{B}{\sum_{j=1}^n EOQ_j * C_j}$$

- In this problem, the budgetary constraint B is \$10,000
- And $\sum_{j=1}^n EOQ_j * C_j = 11,125$

➤ (Source: Problems and Solutions in Inventory Management by Shenoy and Rosas, 2018)




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Numerical Example: Budget Constraint

- Therefore we get, $m = \frac{10000}{11125} = .899$
- Multiplying the initial EOQs with m, we get
- $Q_A = 0.899 * 69 = 62$ Units
- $Q_B = 0.899 * 65 = 58$ Units
- $Q_C = 0.899 * 82 = 74$ Units

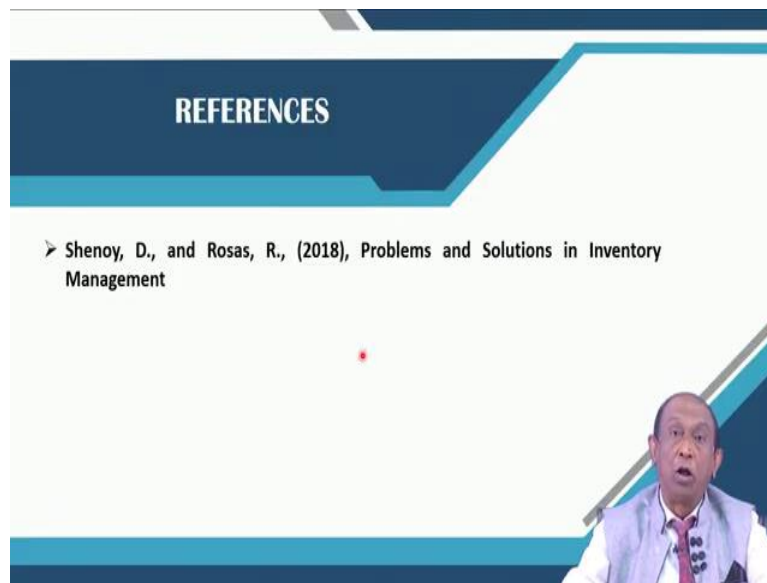


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m is B by EOQ j into Cj, this particular expression in this problem the budgetary constraint is B is dollar 10,000 and this comes out to be this one. So, we get m equals 0.899. Multiplying the initial EOQs with m, we get QA QB and QC as 62, 58, and 74 units, which tallies with what we had worked out earlier, but in this case, the solution is much simpler. Thank you all.

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This is the reference that I have used for explaining you this particular situation that is constraint inventory problem where there is a restriction on the total amount of investment in inventory. Thank you