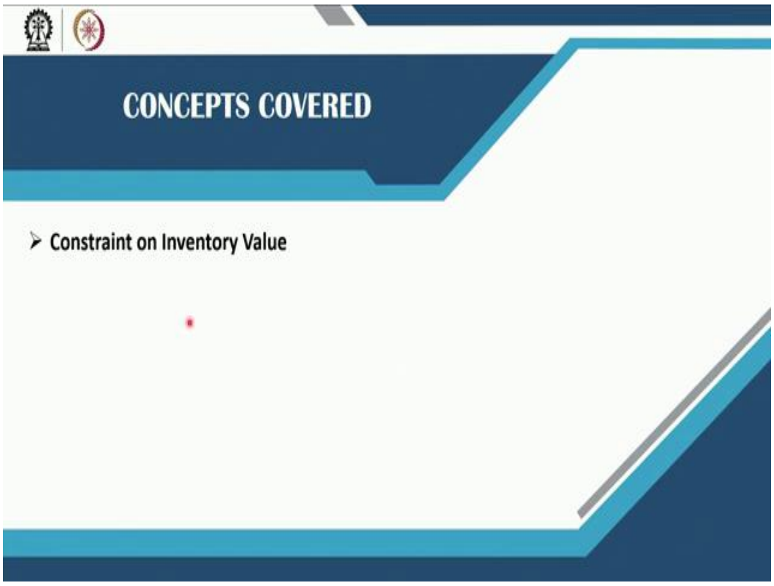
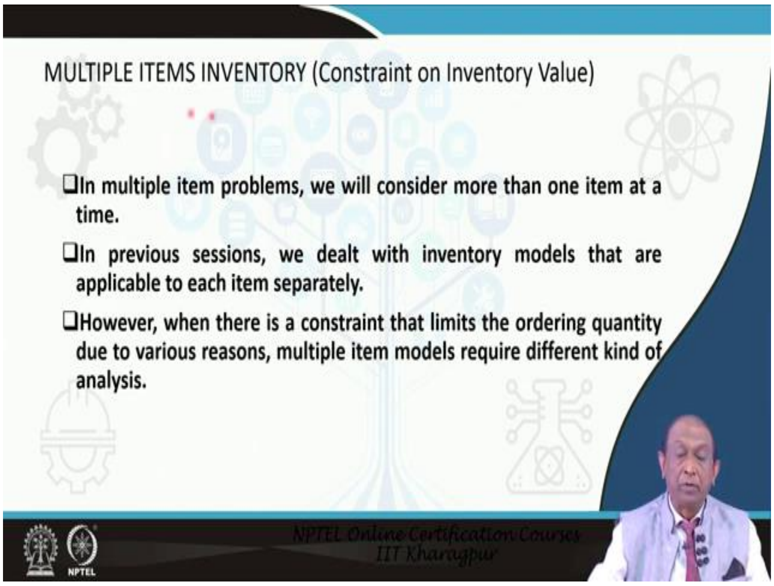
Modelling and Analytics for Supply Chain Management
Professor Kunal Kanti Ghosh
Vinod Gupta School of Management
Indian Institute of technology, Kharagpur
Lecture 28
Multiple Items Inventory Models

(Refer Slide Time: 00:33)



Welcome to modeling and analytics for supply chain management course. Today in this particular session, we will be dealing with multiple items, inventory models and the concept that will be dealt in this particular session will be how to determine the inventory level for multiple items subject to a restriction on the total inventory value. So, that is why we have written constraint on inventory value.

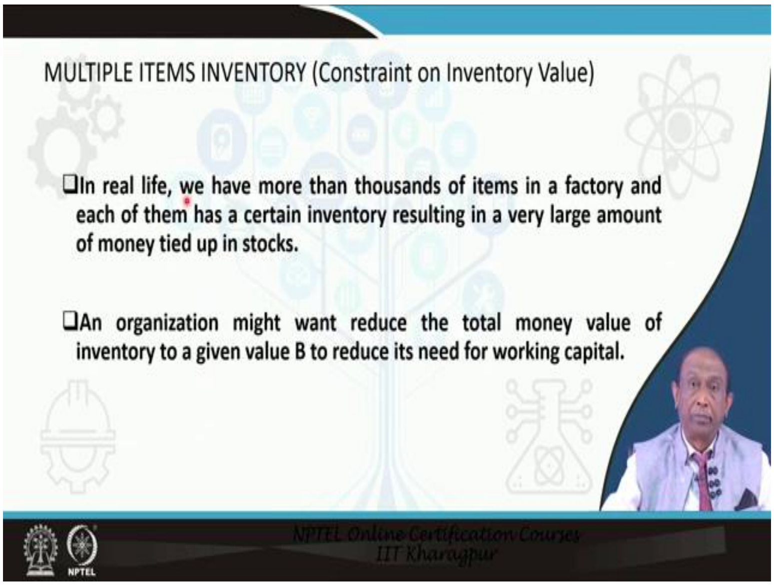
(Refer Slide Time: 01:01)



So, multiple items inventory with the constraint on the total inventory value. In multiple item problems, we will consider more than one item at a time. In our previous sessions, we dealt with inventory models that are applicable to each item separately. However, whenever there is a constraint that limits the ordering quantity for each item due to various reasons, multiple item models require different kind of analysis.

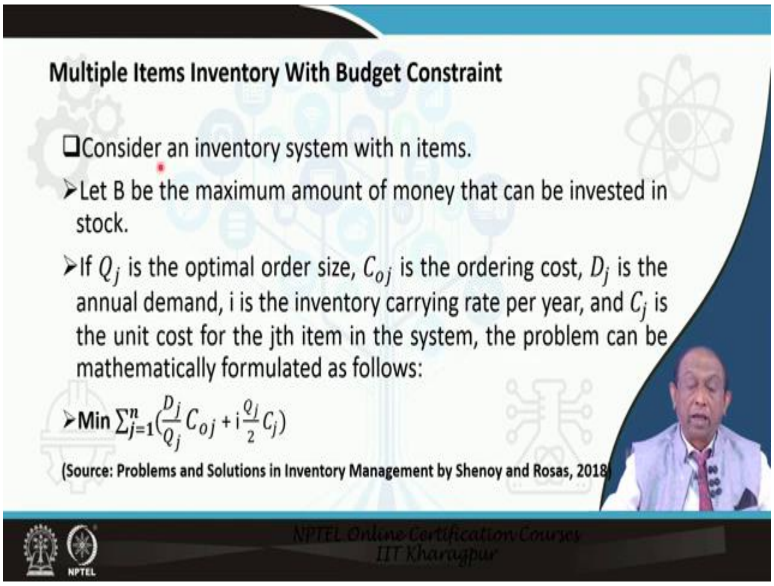
This various reasons maybe constraint on inventory value. Constraint may be on the total number of orders, constraint maybe on the total storage space available and like that, and there can be two or more constraints operating at the same point in time. We may need to find out whether any solution exists at that point or not.

(Refer Slide Time: 02:44)



In real life, we have more than thousands of items in a factory and each one of them has a sudden inventory resulting in a very large amount of money tied up in stocks. So, an organization might want to reduce the total money value of inventory to a maximum limit or a given value B to reduce its need for working capital.

(Refer Slide Time: 03:42)

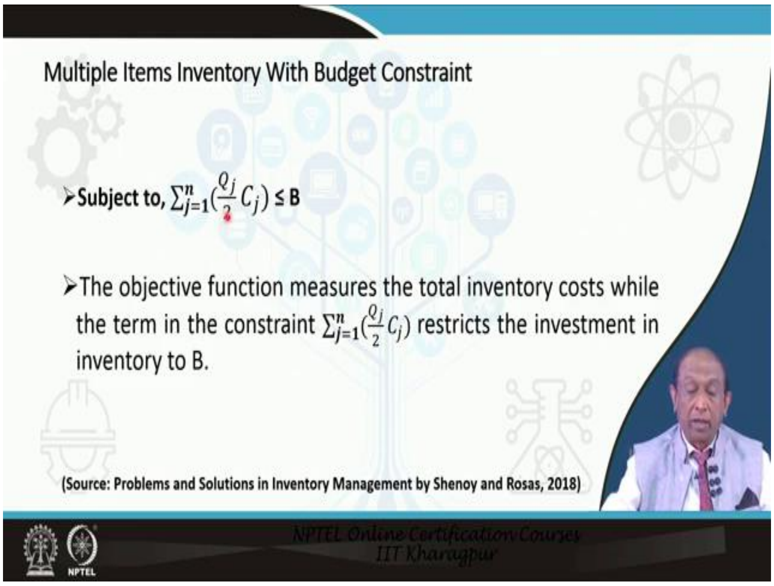


We will consider an inventory system with say, n items and let B capital B be the maximum amount of money that can be invested in stock. This is a constraint that has been imposed on this problem by the managers in the organization. If Qj denotes the optimal order size for an item j, Coj is the corresponding ordering cost for the item. Dj represents the annual demand for the item j, i is the inventory carrying rate per year.

And if Cj denotes the unit cost for the j th item in the system, then the problem can be mathematically formulated as minimize sum of j equals 1 to n Dj by Qj, Dj by Qj is a number of orders for the j th item multiplied by the corresponding ordering costs Coj. So, this portion gives you the ordering cost for the j th item plus i times Qj by 2 is the average inventory for the j th item.

When we are ordering lots, by amount of Qj multiplied by Cj which is the carrying cost for the item j and this has to be done for all the n items that you have installed. So, the total cost expression becomes minimization of j equals 1 to n sum of j equal 1 to n, the total cost expression as we have explained.

(Refer Slide Time: 06:33)



But this is subject to the restriction that sum over j equal to 1 to n Qj by 2 into Cj less than equal to B what is Qj by 2? This is the average inventory value for the j th item that multiplied by Cj that is a carrying cost for the j th item which is nothing but the unit cost of the item multiplied by interest rate. This in here this Cj represents the cost for the j th item and this has to be summed over j equal to 1 to n, which is less than equal to B.

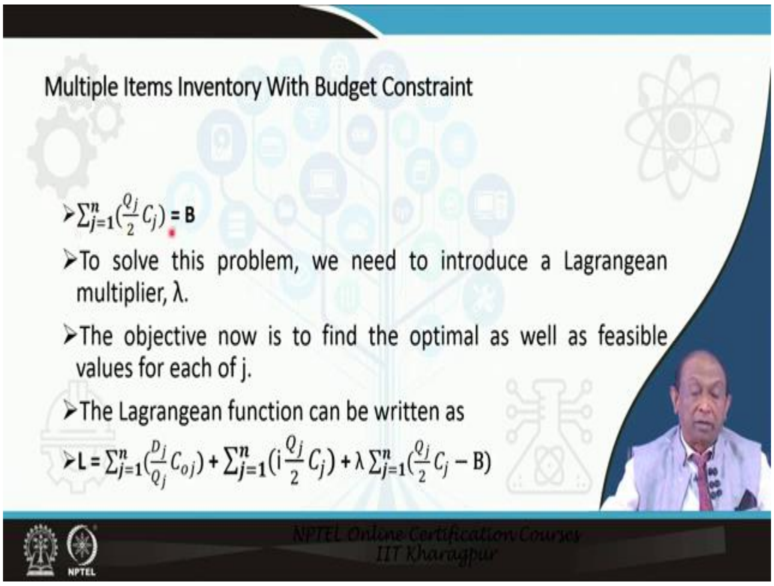
B is the total amount of investment that you can make for these n items in inventory. So, the objective function measures the total inventory cost while the term in this constraint restricts the investment in inventory to a value of B.

(Refer Slide Time: 08:05)

Now, if this constraint that is this one is satisfied by individual optimal order quantities. That means, we compute the EOQ for all these n items individually and then find out the expression for this. And then if this sum is less than equal to B that is what I am saying that, if the constraint is satisfied by individual optimal order quantities. We not only have the individual optimal Q values but these are all feasible Q values.

However, when the constraint is not satisfied, that means, we compute this expression and we find that it exceeds B then the individual Q values obtained are infeasible. And then, under such situation we need to rewrite the constraint by replacing the inequality, the inequality was here.

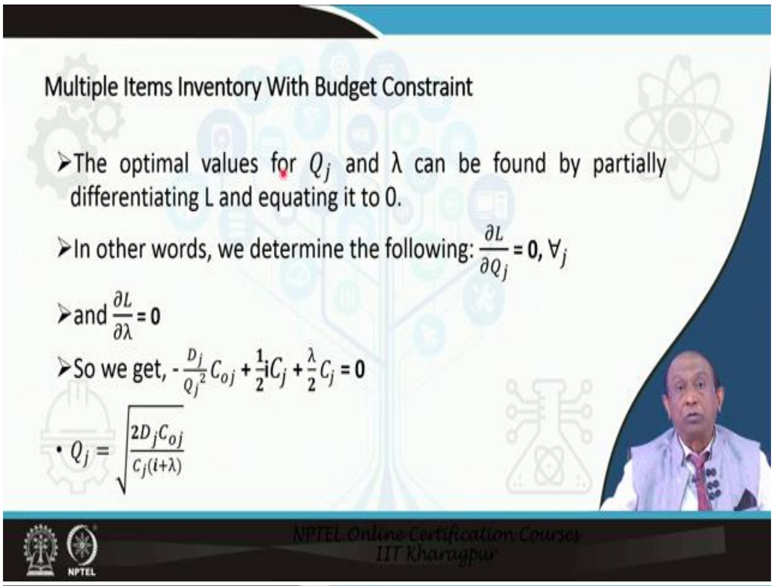
(Refer Slide Time: 09:52)

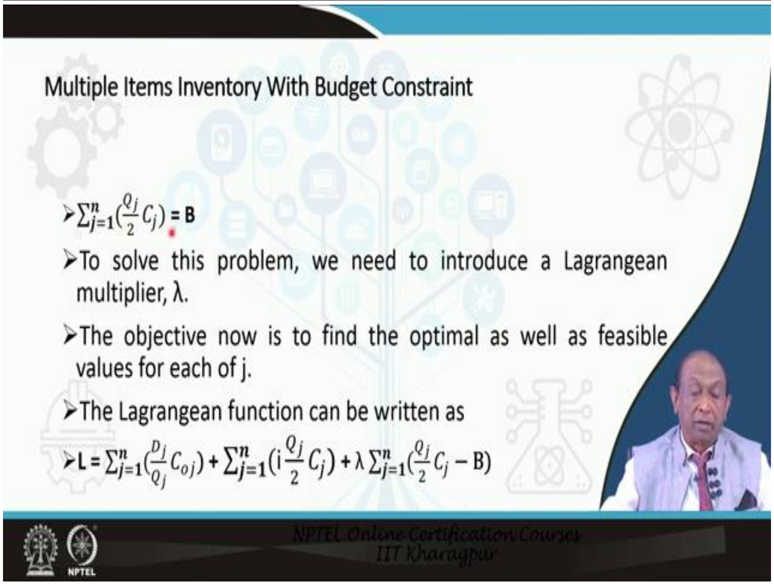


By representing this inequality as sum of j equal to 1 to n. Qj by 2 into Cj equals B with an equality. So, the problem is minimize the total cost subject to this particular restriction. In order to solve this kind of problem, we need to introduce our Lagrangean multiplier, which is denoted by lambda. If we introduce this Lagrangean multiplier lambda. Then the objective now is to find the optimal as well as feasible values for each of these j items.

The Lagrangean function can be written as L equals this is nothing but this total cost expression that we have already discussed plus lambda times j equals 1 to n Qj by 2 into Cj minus B, which is the constraint here. This is the normal procedure in which we solve this kind of constraint optimization problems.

(Refer Slide Time: 11:33)

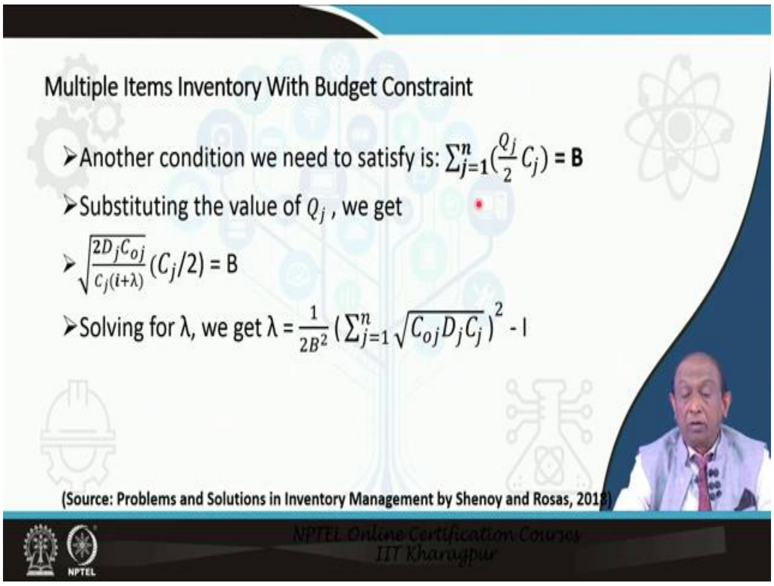




The optimal values for Qj and lambda can be found by partially differentiating this expression L with respect to Q as well as partially differentiating this expression with respect to lambda and equating both these expressions with 0. That means, del L del lambda equals 0 and del L del Qj equals 0. So, if we find out the del Qj of L equals 0 we will get this expression. So, minus Dj by Qj square into Coj plus with respect to Qj I am differentiating.

So, i buy 2 into Cj plus lambda by 2 into Cj. And then we equate it to 0 for minimum value of the cost. This when we simplify, we get the expression for Qj as root over of 2 times Dj into Coj divided by Cj into i plus lambda. So, you see this is the optimal order in quantity for the j th item, but in this expression there is this Lagrangean multiplier lambda appearing. So, we have to find out this expression or values of lambda. So, where from will get the other thing that you need to do is del del lambda of L equals 0.

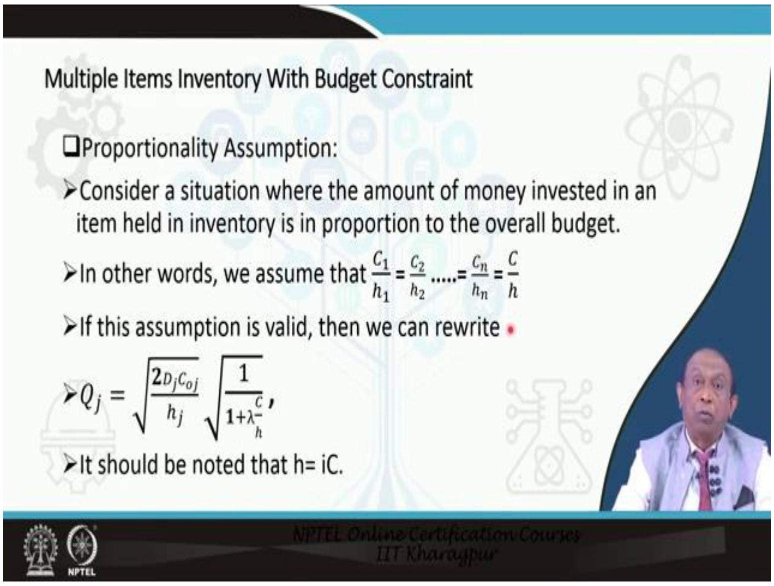
(Refer Slide Time: 14:46)



So, if I do that, then we will get this particular equation or condition because in this expression for L when differentiated with respect to lambda, I will get this, this one, 0 this one is 0, this lambda del del lambda of 1, so we will get this expression. Now, in this expression, if we substitute the value of Qj.

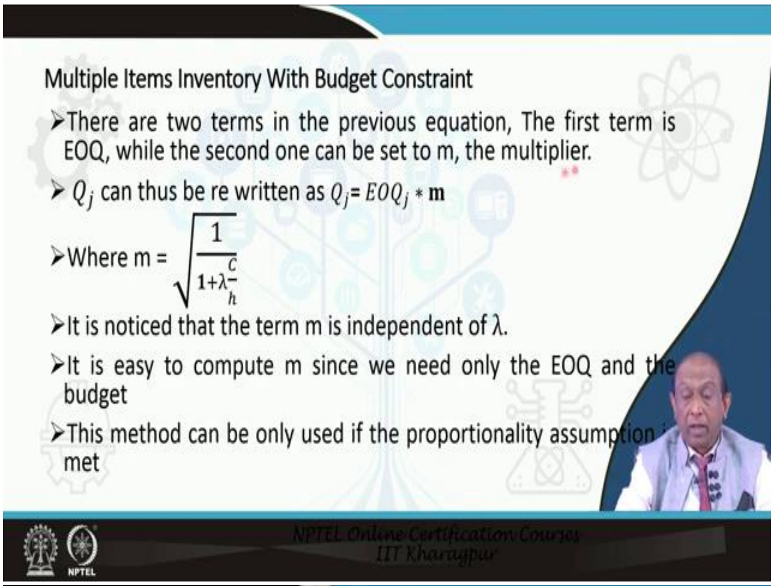
We will get I will found out I will substitute the value of Qj from here this is the value of Qj. So, this has been put in here. So, this multiplied by Cj by 2 gives us the value of gives us this is equal to B. Now, if we solve for lambda, this equation we get lambda equal to, 1 upon 2 B square sum over j equal 1 to n root over of Coj into Dj into Cj, is entire whole square minus 1. This is one aspect.

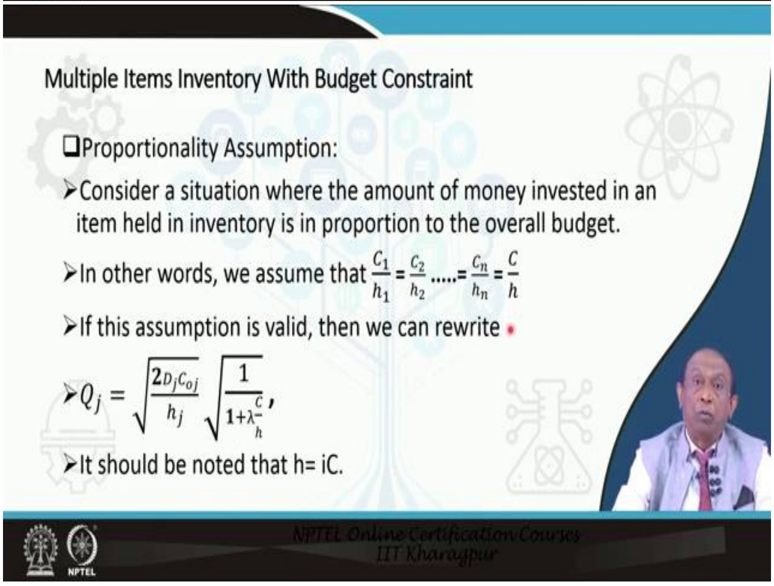
(Refer Slide Time: 16:35)



The problem can be simplified with an assumption which is basically the proportionality assumption. Consider a situation where the amount of money invested in an item held in inventory is in proportion to the overall budget in that case, we assume that C1 by h1 equal to C2 by h2 like this Cn by hn is equal to C by h. If this assumption is valid, then we can rewrite Qj as, root over of twice Dj into Coj by hj into 1 by 1 plus lambda into C by h. So, this h is nothing but the carrying cost i into C.

(Refer Slide Time: 18:04)





Now, there are two terms in this expression for Qj, the first term is the expression for Qj is a EOQ simply the EOQ, while the second one can be set to some value m which we basically called the multiplier. So, Qj can now be written as EOQ for the j th item multiplied by m where m is this, that means, this expression, 1 upon square root of 1 upon 1 plus lambda into C by h.

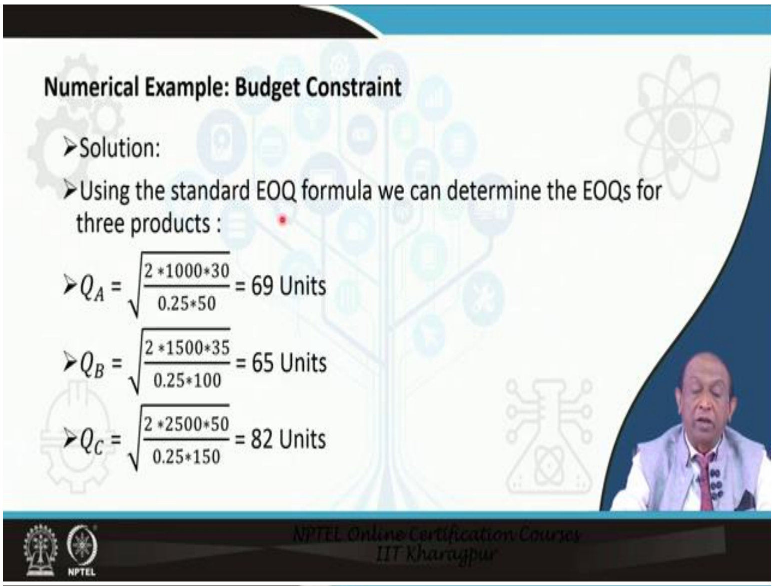
It is noticed that this term m is independent of lambda and it is easy to compute m. Since we need only the EOQ and the budget. This method can be only used if the proportionality assumption is met.

(Refer Slide Time: 20:05)

Now, we will illustrate this particular thing with the example with an example on budget constraint. Say there are three products A B and C which is being sold by a retailer. The annual demand for each of the products are known. For product A the annual demand is 1000 units, this is 1500 units for product C it is 2500 units. The ordering cost for product A is 30 for product B is 35 dollars and for production C is 50 dollars.

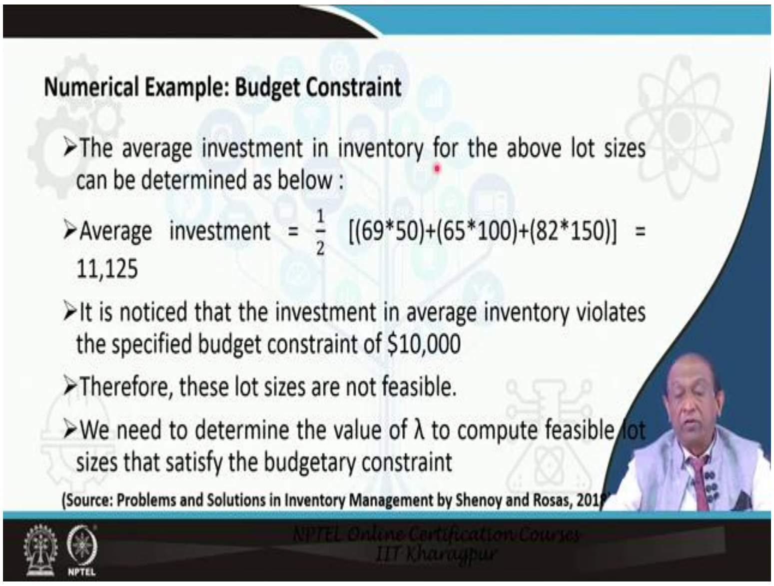
It may be in a real life situation, that this ordering cost maybe the same for all the items. Unit costs for product A is dollar 50 for product B is dollar 100 for product C is dollar 150 and the retailer uses an inventory carrying rate of 25 percent per annum. The problem is what would be the economic lot size if the retailer does not want to invest more than dollar 10,000 in the average inventory of these three products? So, that means there is a constraint that the maximum investment in inventory is dollar 10,000 which is nothing but B.

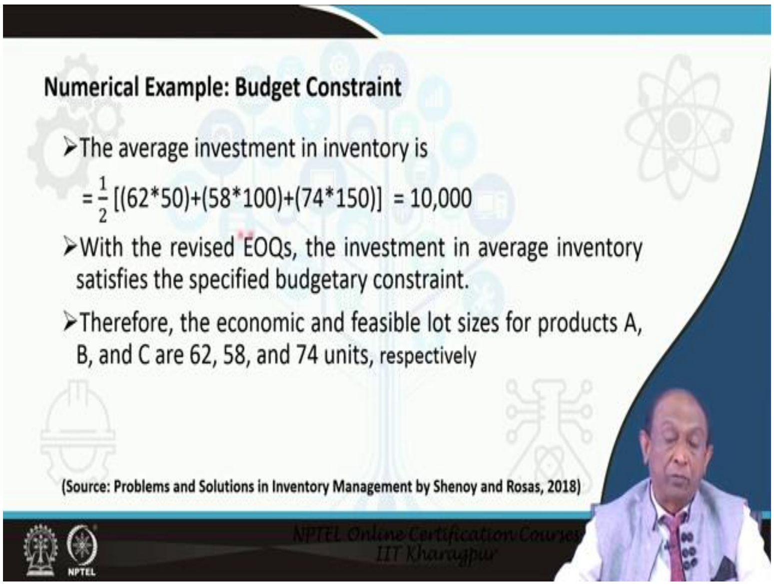
(Refer Slide Time: 21:58)



So, now if we use the standard EOQ formula, we will determine the EOQ for these three products, QA, QB and QC. For QA it is 2 into annual demand that is 1000 multiplied by the ordering cost dollar 30 by i into C, the unit cost is given as 50, which comes out to be 69 units. Similarly for QB it is 2 times 1500 into its ordering costs 35 multiplied by i into its unit cost is 100, 65 units and for QC it is root over of 2 into 2500 into 50 divided by 0.25 into 150, 69 units, 65 units and 82 unit.

(Refer Slide Time: 23:42)

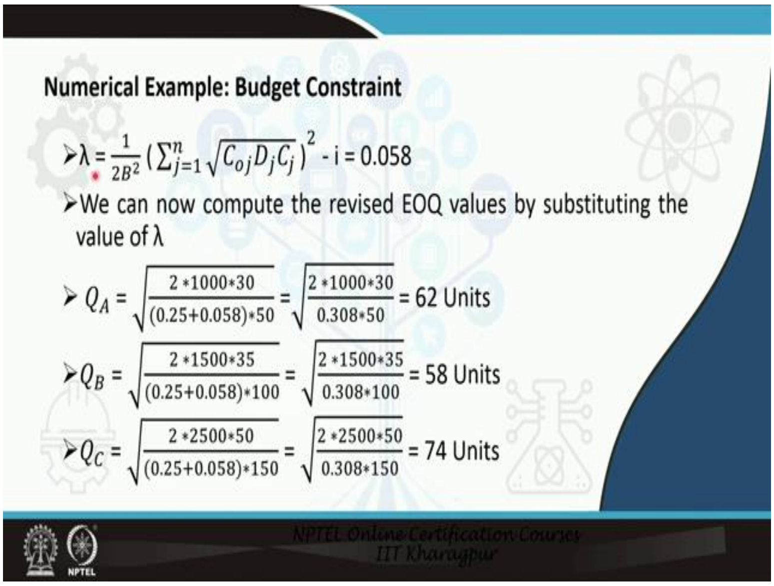




So, the average investment in inventory for this above lot sizes are the that we have determined equals Q by 2 into Cj which is 69 into 50 plus 65 into 100 plus 82 into 150 divided by 2, the Cj's are given 50, 100 and 150 and this is QA QB this is QC. So, QA by 2 is average inventory multiplied by the cost. So, this becomes 11,125.

So, you can noticed that the investment in average inventory violates the specified budget constraint of dollar 10,000. Therefore, these lot sizes are not feasible. Hence, what do we have to do under such condition we need to determine the value of lambda that is the Lagrangean multiplier to compute the feasible sizes that satisfy the budgetary constraint.

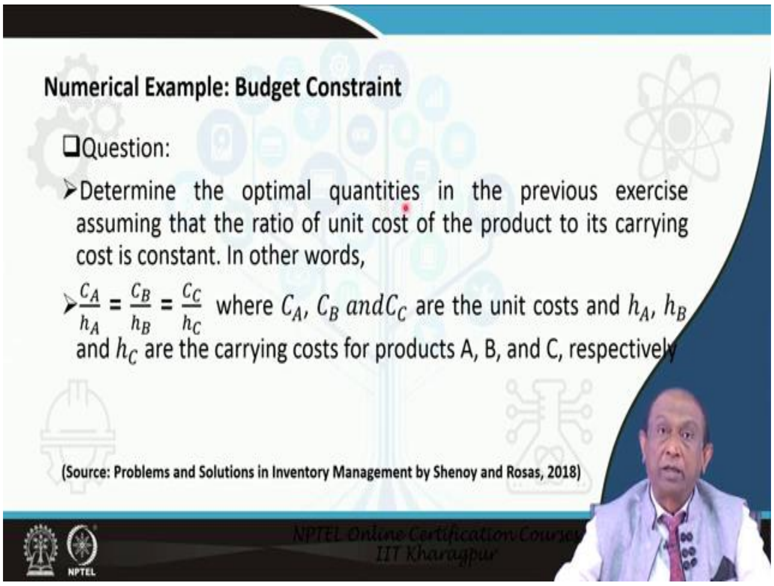
(Refer Slide Time: 25:43)



And as you have noted that we had determine the value of lambda as 1 by 2 B square multiplied by this minus i. Now, if we substitute the corresponding values and compute lambda will come out to be 0.058. Once we find out the value of lambda, we can now compute the revised EOQ values by substituting the value of lambda you see, the expression for Qj was in this expression for Qj, if we put the value of lambda then we will find out that QA will work out to be 62 units. QB will work out to be 58 units and QC will work out to be 74 units.

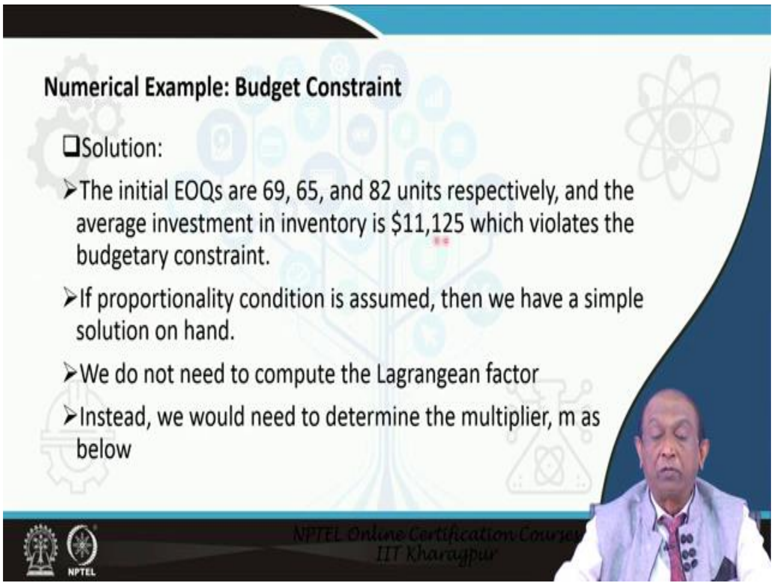
Under these circumstances, the average investment in inventory if we compute, then it will come out to be 10,000. With the revised EOQs, the investment in average inventory specify the specified budgetary constraint so the constraint is satisfied. Therefore, the economic and feasible lot sizes for products A, B and C are 62, 58 and 74 units respectively.

(Refer Slide Time: 27:59)



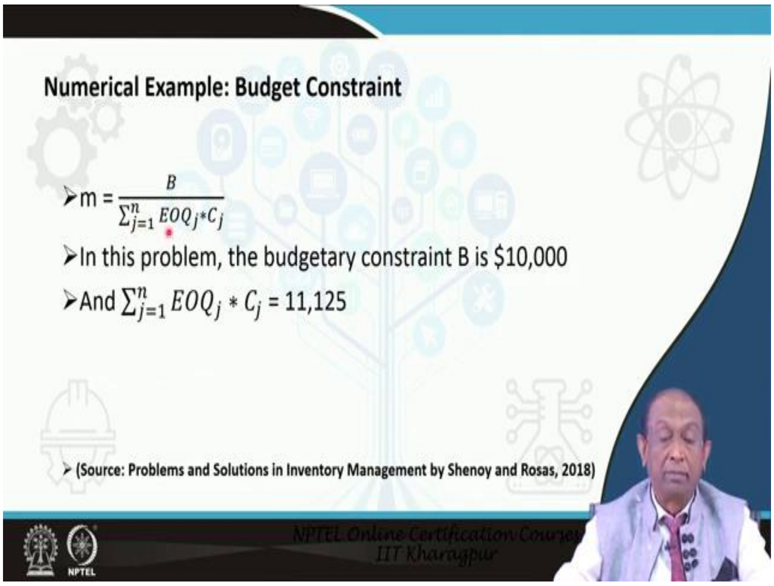
The second question is determine the optimal quantities in the previous exercise, assuming that the ratio of unit cost of the product to its carrying cost is constant that is the proportionality assumptions that we had discussed. In other words, in here, CA by h A equals CB by h B equal to CC by hc, where CA, CB and CC are the unit costs for the item A, B and c. hA, hB and hC are the carrying costs for the product A, B, and C respectively.

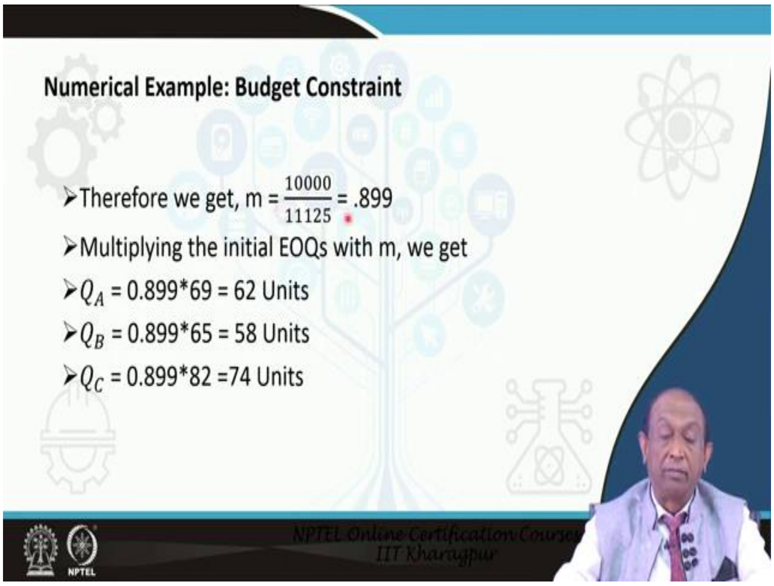
(Refer Slide Time: 28:49)



So, the initial EOQ were 69, 65 and 82 units in the previous exercise 69, 65 and 82 units and the average investment in inventory with this initial EOQ values were 11,125 dollars that violated the budgetary constraint. Now, if proportionality condition is assumed, then we can use a simple solution we do not need to compute the Lagrangean factor or multiplier under that circumstance. Instead, we would need to determine the multiplier m.

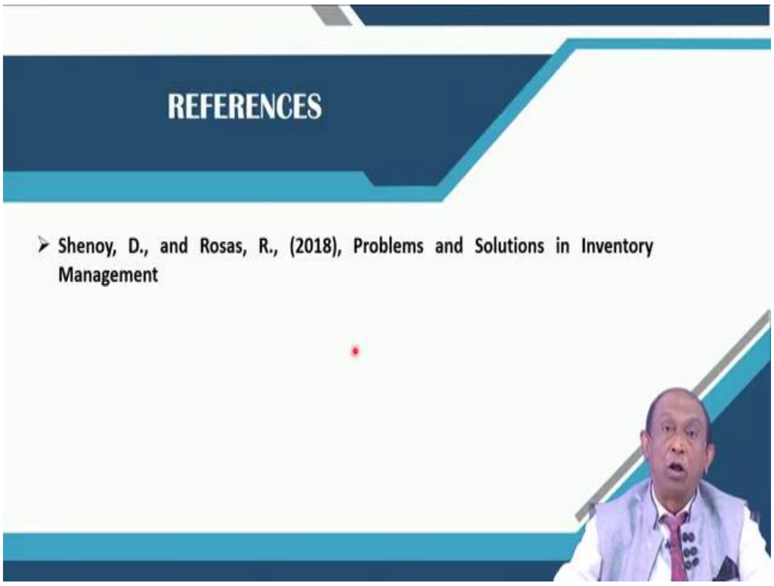
(Refer Slide Time: 29:41)





m is B by EOQ j into Cj, this particular expression in this problem the budgetary constraint is B is dollar 10,000 and this comes out to be this one. So, we get m equals 0.899. Multiplying the initial EOQs with m, we get QA QB and QC as 62, 58, and 74 units, which tallies with what we had worked out earlier, but in this case, the solution is much simpler. Thank you all.

(Refer Slide Time: 30:46)



This is the reference that I have used for explaining you this particular situation that is constraint inventory problem where there is a restriction on the total amount of investment in inventory. Thank you