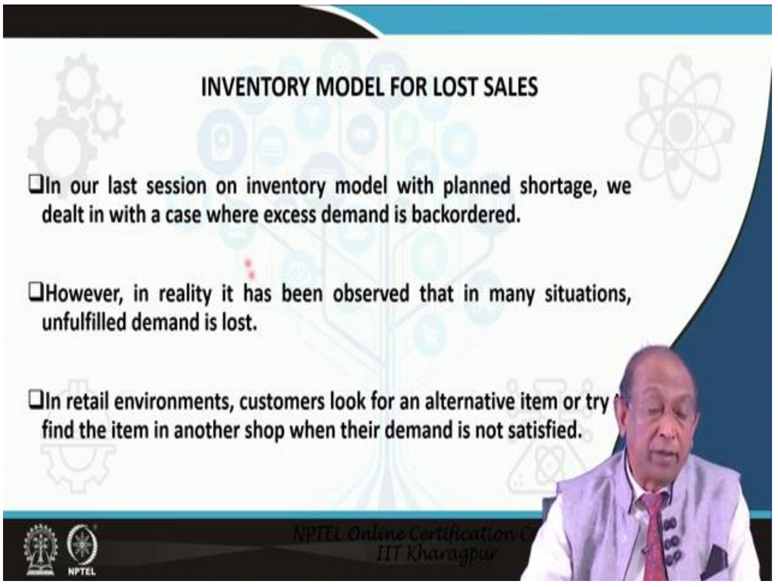
Modelling and Analytics for Supply Chain Management
Professor Kunal Kanti Ghosh
Vinod Gupta School of Management
Indian Institute of Technology, Kharagpur
Lecture 27
Lost Sales

Hi, welcome to our course on modeling and analytics for supply chain management. Today we will be dealing with a very important topic inventory models for lost sales.

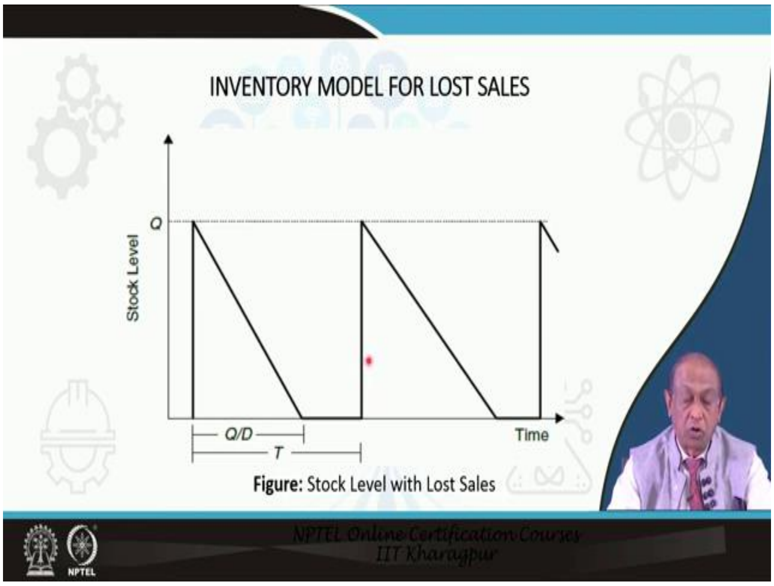
(Refer Slide Time: 00:30)



In our last session on inventory model with planned shortage, we dealt in with a case where excess demand is backordered. However, in reality, it has been observed that in many situations unfulfilled demand is lost. In retail environments, customers look for an alternative item or try to find the item in another shop when their demand is not satisfied.

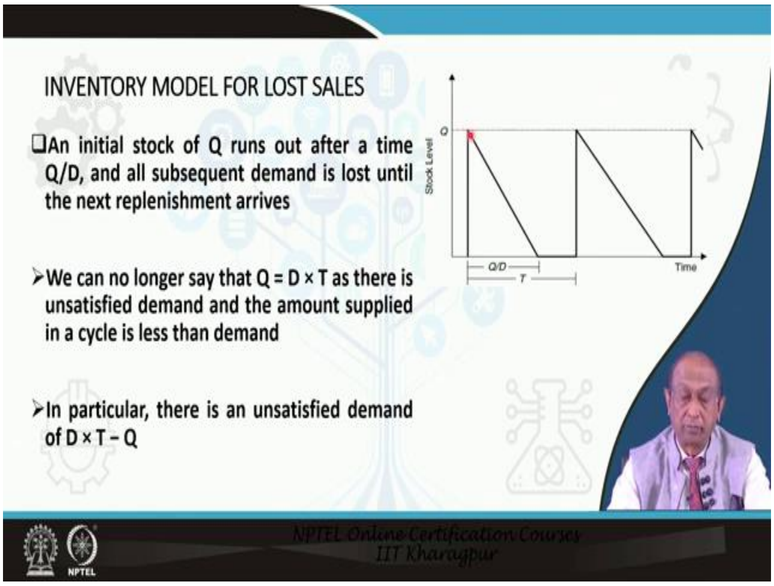
For a example, when you do not find a particular magazine of our choice with a news vendor, we normally do not wait for the next delivery, but simply go to another news agent down the road. So, there is a need to discuss inventory model which deals with the characteristics of lost sales. Inventory systems with lost sales require a different approach in terms of maximizing net revenue rather than minimizing costs.

(Refer Slide Time: 02:13)



This is the inventory model for lost sales, we have plotted stock level along the y axis, x axis as usual represents time, and initials ordered quantity Q is this line. Consumption takes place with a demand read D along this line at this point, there is no stock. So, lost sales occurs over the period from here to this particular point again a fresh stock of Q unit comes in here and the cycle repeats.

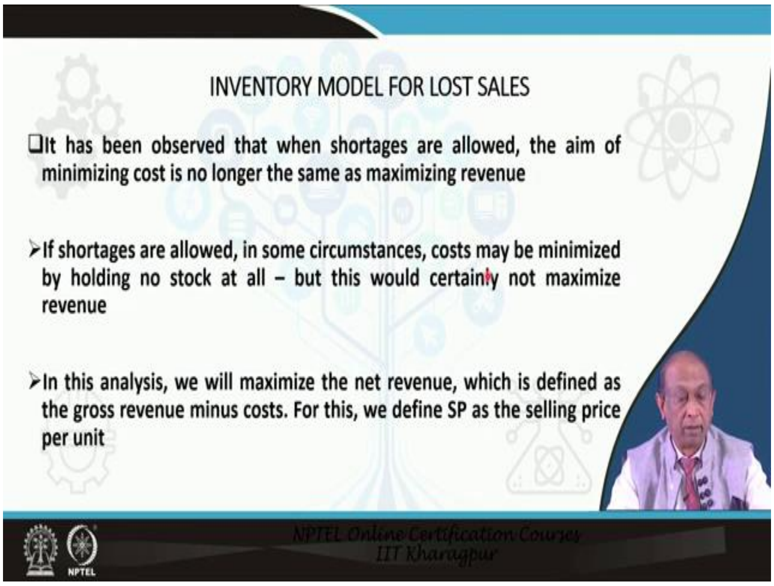
(Refer Slide Time: 03:07)



An initial stock of Q unit runs out after a time Q by D and all subsequent demand its lost until the next replenishment arrives at this point. So, from this to this point in time, there is no stock available. So, if there is any demand during this period of time, it is not made and thereby results in lost sales.

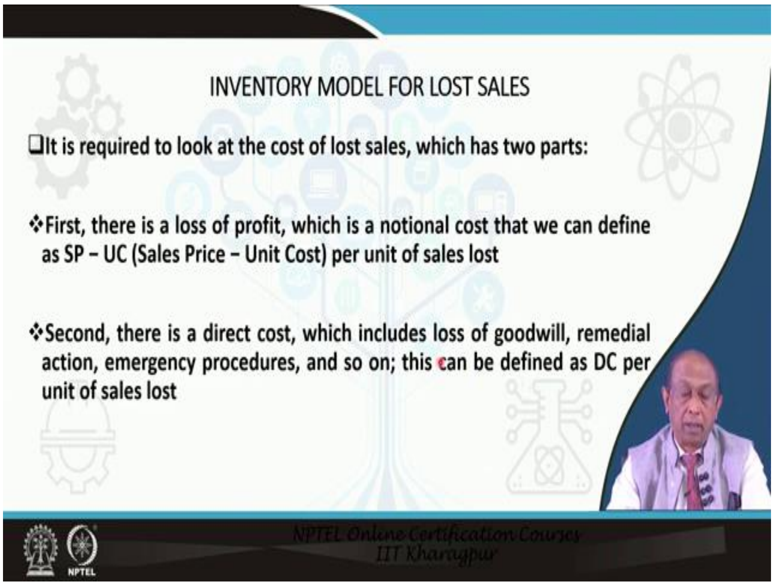
So, T is the cycle length, capital T so, under such situation we can no longer say that Q is demand rate D multiplied by the time period T. Since, there is unsatisfied demand over this period as a result, the amount supplied in a cycle is less than the demand. In particular, there is an unsatisfied demand of D multiplied by T minus Q, D multiplied by T is the total demand over this period T. And we have a stock level of Q. So, D into T minus Q is the amount of unsatisfied demand.

(Refer Slide Time: 05:00)



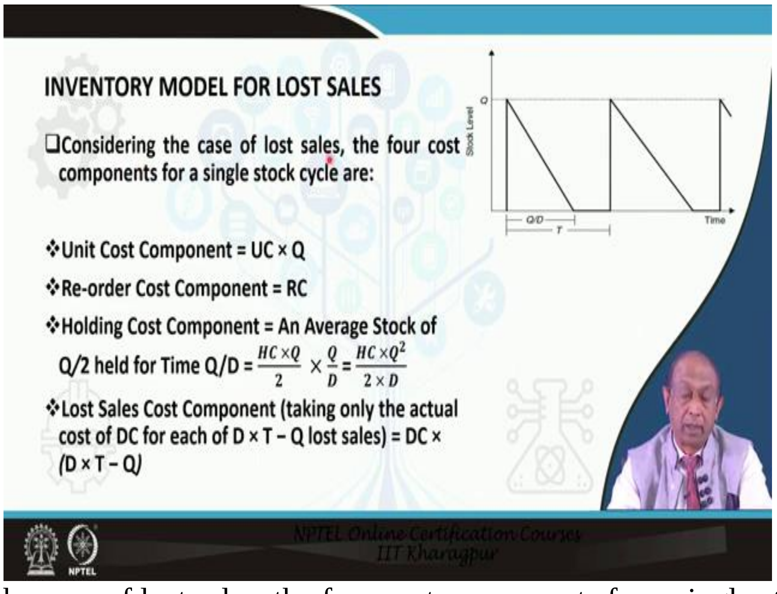
It has been observed that when shortages are allowed, the aim of minimizing cost is no longer the same as maximizing revenue. So, if shortages are allowed, in some circumstances, costs may be minimized by holding no stock at all, but they should certainly not maximize revenue. So, in this analysis we will maximize the net renew, which is defined as the gross revenue minus costs. For this analysis will define SP as a selling price per unit.

(Refer Slide Time: 06:28)



Now, it is required to look at the cost of lost sales which has two parts. First, there is a loss of profit and this loss of profit is a notional cost that we can define as the difference between SP and UC that is loss of profit equals sales price minus unit cost, per unit of sales lost. In the second part there is a direct cost, which is of course very difficult to measure and this direct cost includes loss of goodwill, remedial action, cost of emergency procedures and so on. For the sake of analysis, we define DC per unit of sales lost.

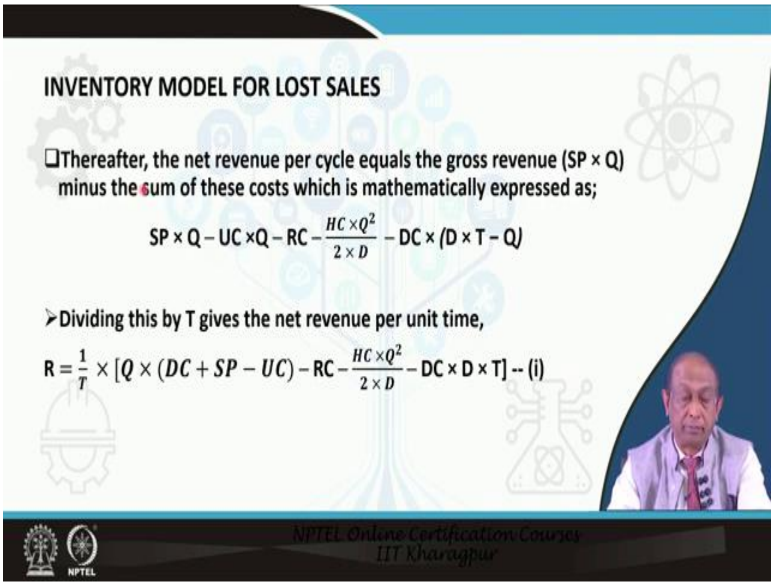
(Refer Slide Time: 08:03)



So, considering the case of lost sales, the four cost components for a single stock cycle that is over the cycle length T, we have one as a unit cost component which is nothing but UC, multiplied by Q. The reorder cost component as usual is RC. The holding cost component equals an average stock of Q by 2 held over a time period Q by D. So, the holding cost components becomes Q by 2 held over a time period Q by D. So, Q by 2 into Q by D multiplied by HC, the holding cost component. So, this becomes HC into Q square divided by 2 into D.

And the fourth one, which is the lost sales cost component. Here one thing please note that we are taking only the actual cost of DC for each D into T minus Q lost sales, the direct cost the actual cost which is equals DC multiplied by D into T minus Q. So, these are the four cost components.

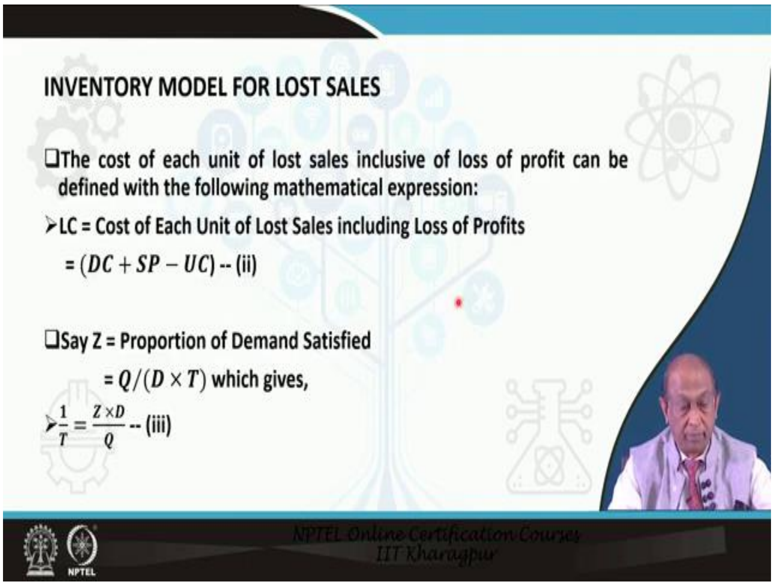
(Refer Slide Time: 10:19)



Thereafter, that net revenue per cycle equals the gross revenue SP into Q minus the four cost components, that is UC into Q minus RC minus HC into Q square by 2 D minus DC multiplied by D into T minus Q. We can divide this expression for net revenue per cycle by the total time period T. So, dividing this by T gives the net revenue per unit time.

So, net revenue per unit time becomes 1 upon T multiplied by this expression Q into DC plus SP minus UC minus RC minus HC into Q square by 2 D minus DC into D into T. We have done some manipulation here.

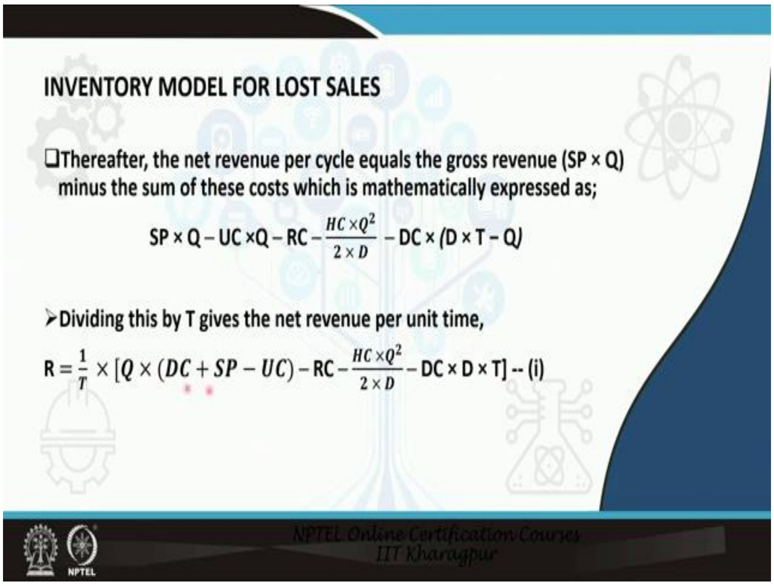
(Refer Slide Time: 11:42)

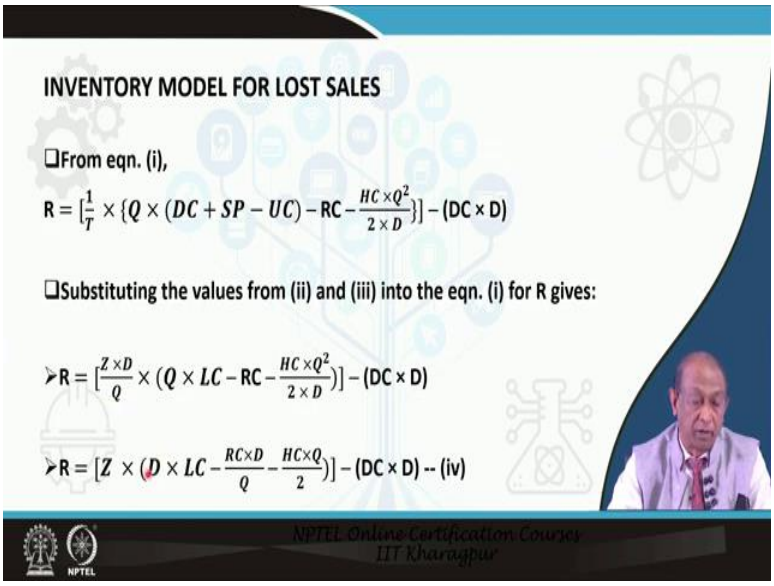


Now, the cost of each unit of lost sales inclusive of loss of profit can be defined with the following mathematical expression. LC equal to cost of each unit of lost sales including loss of profits equals DC plus SP minus UC. Now, we define Z, a variable which is the proportion of demand satisfied.

Now, what is the proportion of demand satisfied? The total demand is D into T and we had a stock level of Q right at the beginning. So, Q divided by D into T gives the proportion of demand satisfied, which we define as Z. So, from this we can derive an expression as 1 upon T equals Z into D divided by Q.

(Refer Slide Time: 13:09)

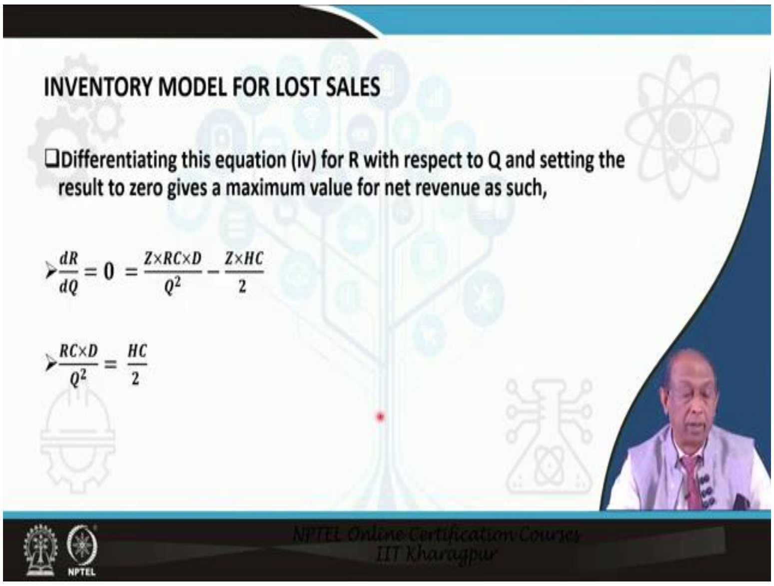


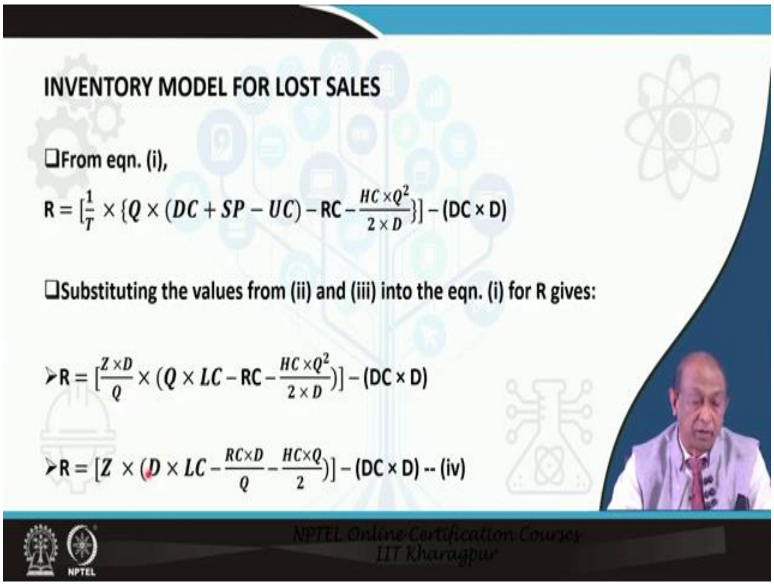


Now, recollect that we had the expression for R as this, this is equation 1. Now, we can substitute that expression for 1 upon T in this equation. So, that will give us R equals Z into D by Q multiplied by Q into LC minus RC minus HC into Q square by 2 D minus DC into D. So, this expression DC plus SP minus UC is nothing but LC. So, instead of DC plus SP minus UC we have written LC.

Thereafter, the expression for R can be written as Z into D where this Q and this Q gets struck off Z into D into LC minus RC into D by Q minus HC into Q by 2, because this D by Q, if you multiply it with this, you get this expression a minus DC by D this is in a separate bracket.

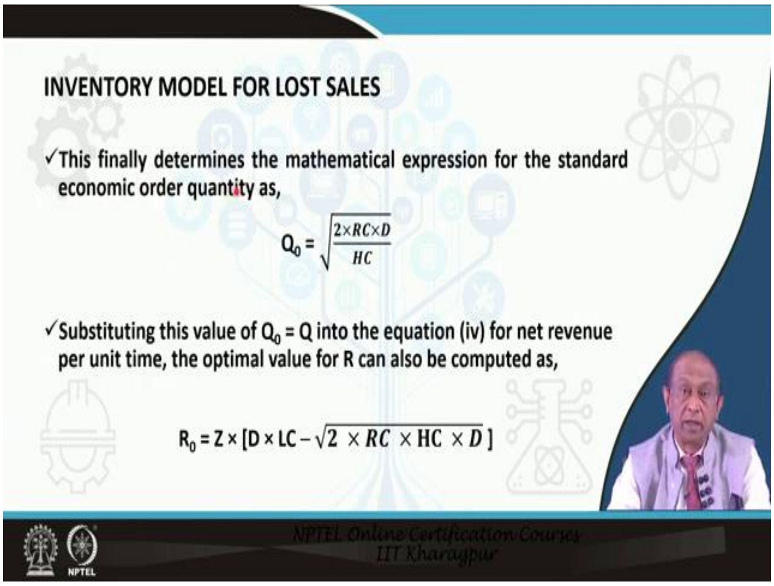
(Refer Slide Time: 14:58)





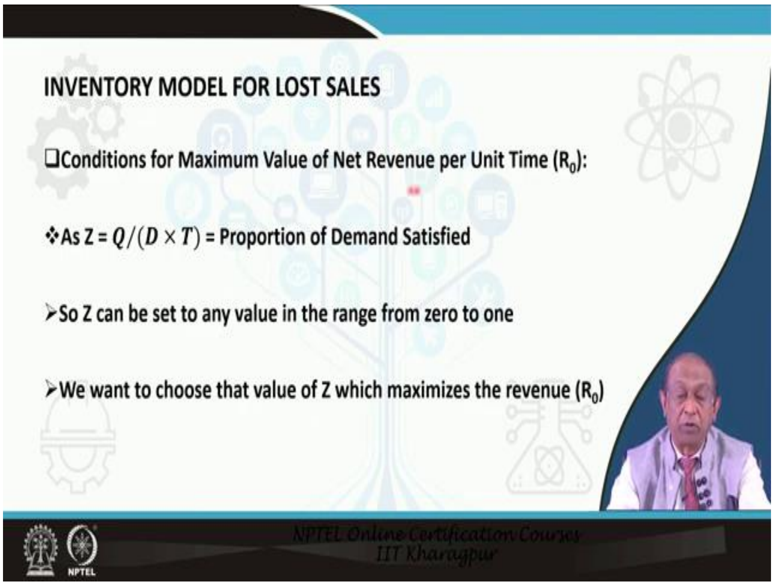
Now, say this equation in order to maximize the revenue net revenue per unit cycle, we have to differentiate it with respect to Q and equate it to 0. If we do that, then we get Z into RC into D by Q square minus, Z into HC by 2 equals 0. So, after side transposing, we get RC into D by Q square equals HC by 2.

(Refer Slide Time: 15:49)



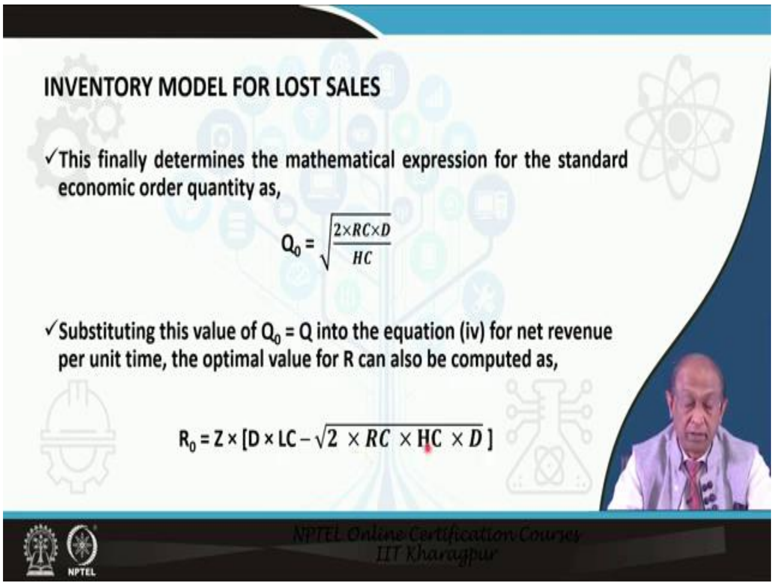
So, this finally determines the mathematical expression for the standard economic order, quantity as Q 0 equals root over of twice RC into D by HC. Substituting this value of Q 0 equal to Q, in the equation four, for net revenue per unit time, the optimal value of R can also be computed as R0 equals Z multiplied by an expression within bracket, which is D into LC minus root over 2 into RC into HC into D. Now, we have to play with this expression in order to determine whether we hold any stock or not, whether we order or not and different type of things.

(Refer Slide Time: 16:59)



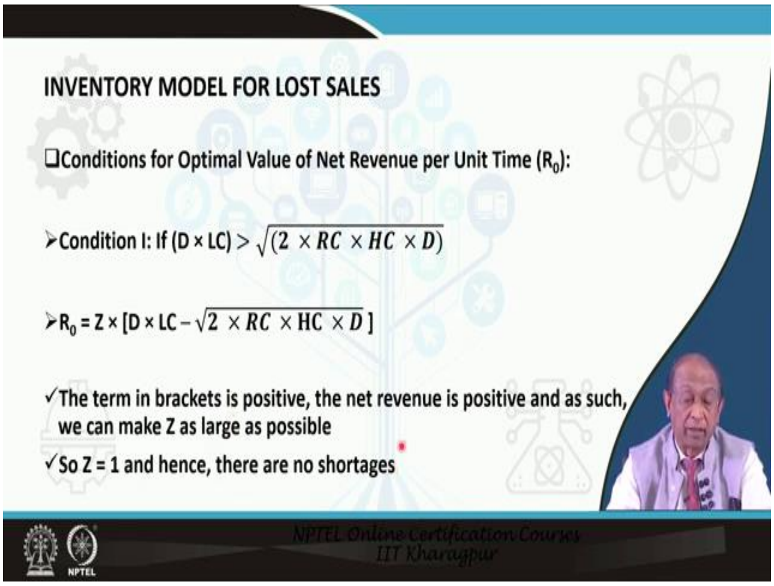
So, that conditions for maximum value of net revenue per unit time we have to determine what are the conditions? Now, recollect that we had defined the variable Z as Q divided by D into T, which is nothing but the proportion of demand satisfied. Now, this Z can be set to any value in the range from 0 to 1. Maximum value is 1, minimum value is 0. Now, we want to choose that value of Z which maximizes the revenue R0.

(Refer Slide Time: 18:01)



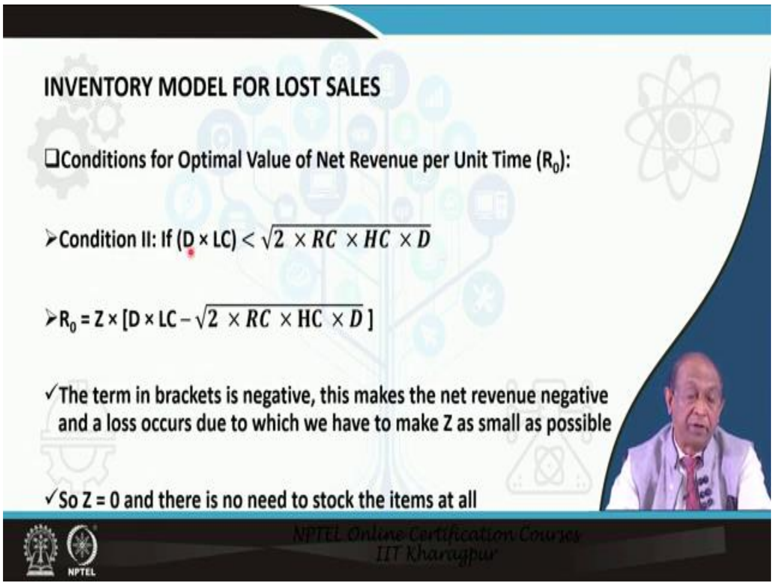
You see in this expression for R0 we have this variable Z. The question is what should be this value of Z? That will depend upon the nature of sign within this bracketed term.

(Refer Slide Time: 18:26)



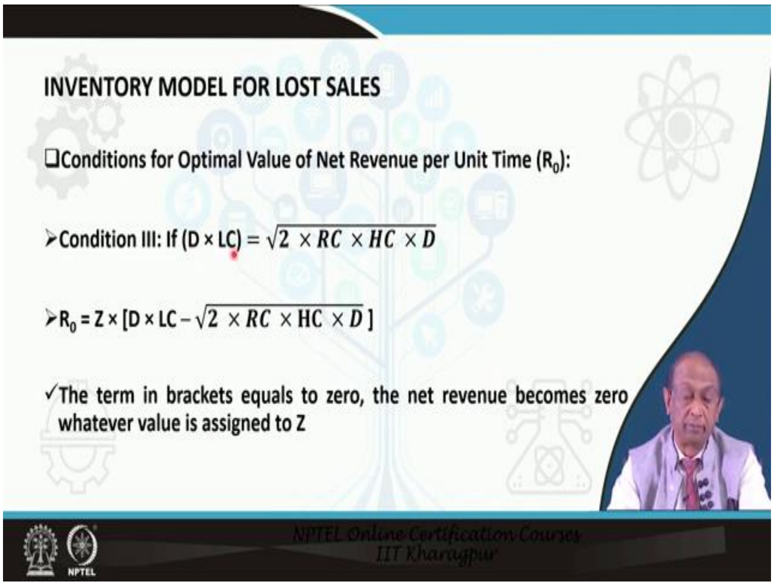
Condition one look at this bracketed term this is expression for R0 net revenue per unit of time optimal value. Now, in this if D into LC is greater than root over of 2 into RC into HC into D then the term within this bracket is positive thereby, the net revenue per unit time is positive and as such, under this circumstances, in order to maximize the revenue per unit of time, we can make Z as large as possible. So, Z equal to 1 can be set and hence under that condition, Z equal to 1 means, there are no shortages. So, that is possible if D multiplied by LC is greater than this term.

(Refer Slide Time: 19:48)



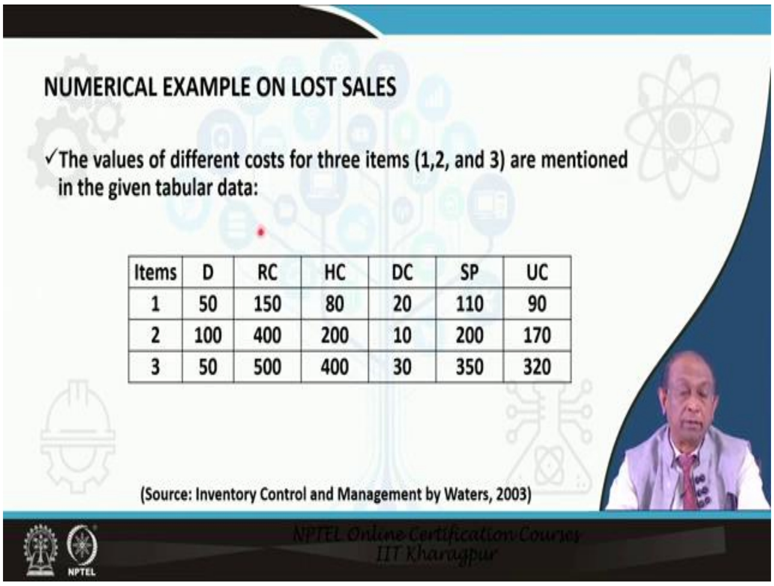
Condition 2 if D multiplied by LC is less than this term then the term in the bracket is negative. This makes the net revenue per unit of time as negative and thereby, a loss occurs due to which we should necessarily make Z as small as possible. If we make Z equal to 0 there is no need to stock the item at all.

(Refer Slide Time: 20:45)



If D into LC equals this term this term inside the bracket will be equal to 0 and hence the net revenue also will be 0 whatever value you assigned to Z.

(Refer Slide Time: 21:23)



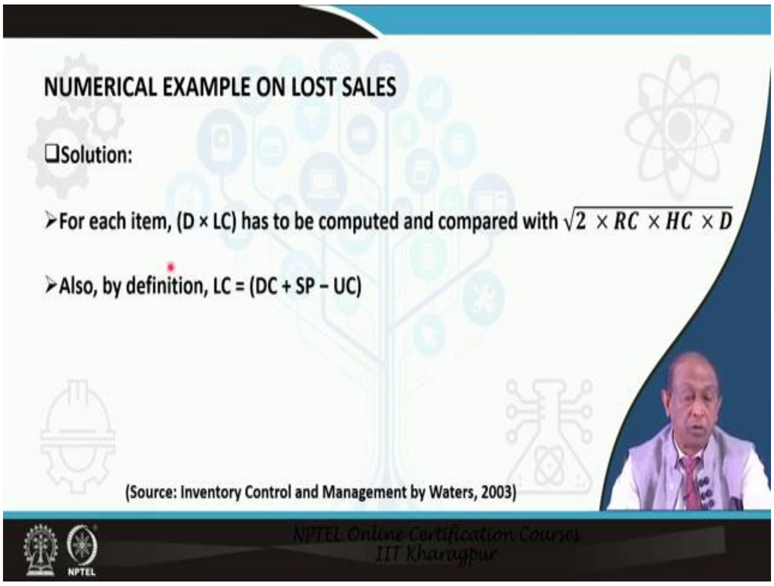
Items	D	RC	HC	DC	SP	UC
1	50	150	80	20	110	90
2	100	400	200	10	200	170
3	50	500	400	30	350	320

Now, let us look at one numerical example, though in real life, estimation of this disease difficult certain notional costs are involved. But suppose we have somehow or other been able to find out this costs DC. We have three items, items 1, 2 and 3. This is the demand rate. This reordering cost for each of these items holding costs, the DC, the selling prices and the unit costs are all given.

(Refer Slide Time: 22:12)

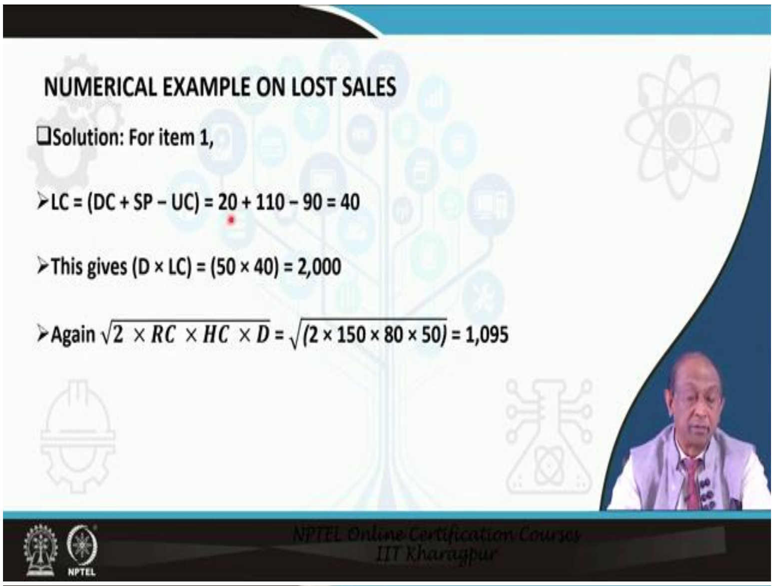
The question to be answered is, if this is the case, determine the best ordering policy for these three items using the given data for different costs.

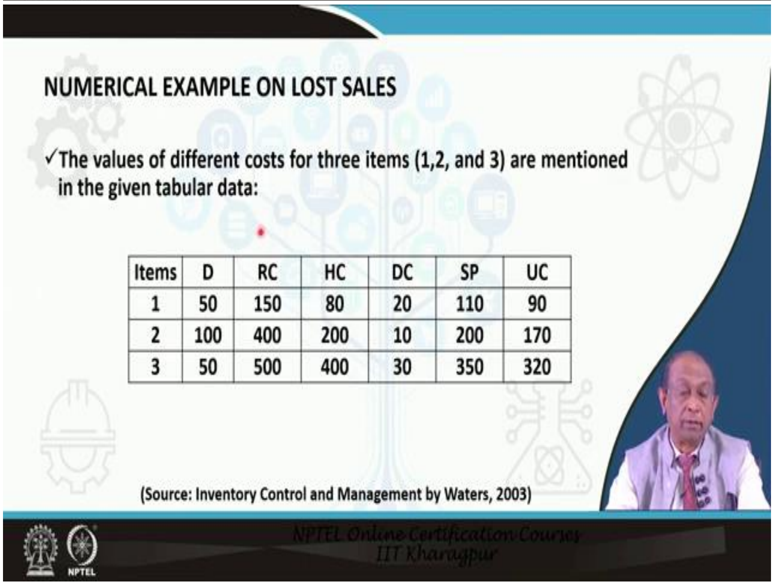
(Refer Slide Time: 22:33)



So, your solution approach should be that for each item, you first compute D into LC and compare that value with root over 2 into RC into HC into D, these are the two expressions within the bracket in the formula for optimal net revenue per unit of time and also by definition, LC is DC plus SP minus UC. This you have to compute.

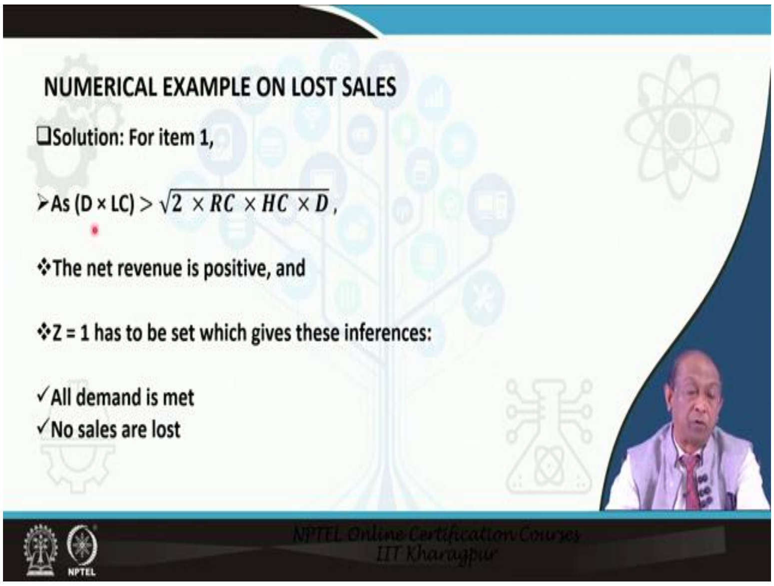
(Refer Slide Time: 23:17)





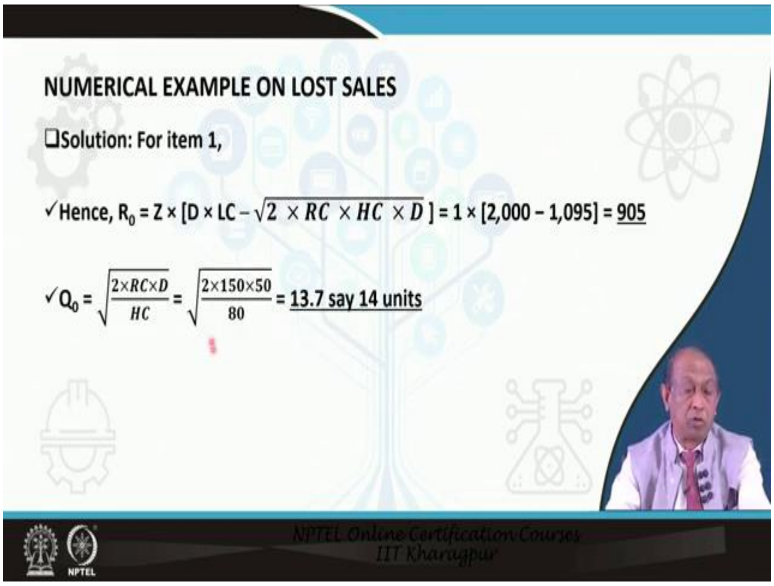
So, for item one, first we compute LC which is 20 plus 110 minus 90 equal to 40. If you recollect, for item 1, DC is 20 plus, SP 110 minus UC is 90, so 130 minus 90 equal to 40 and D into LC equals 50 into 40, which is 2000. And again compute 2 into RC into HC into D, root over of that, which comes out to be 1095.

(Refer Slide Time: 24:22)



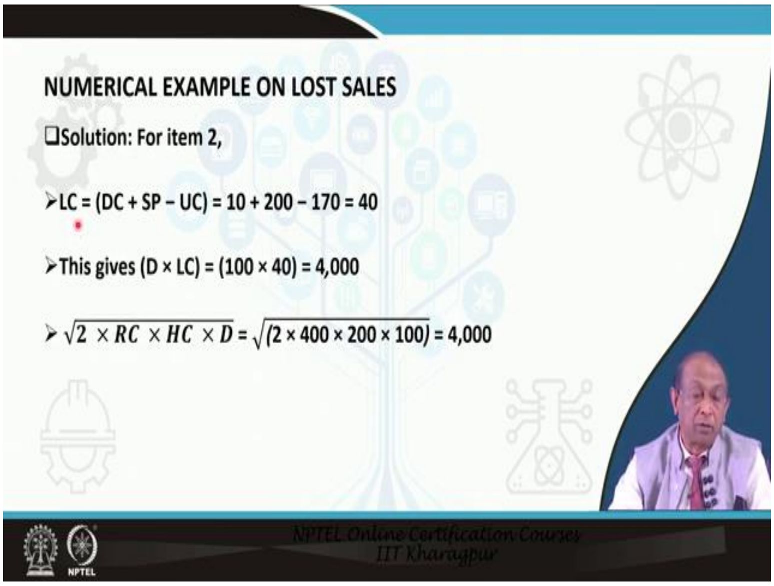
So, now what do you find, for item 1, D into LC is greater than root over up 2 into RC into HC into D. Hence, the net revenue is positive, net revenue per unit time is positive. And thereby, we can set Z equal to 1, leading to the fact that all demand will be made under that circumstance and no sales will be lost.

(Refer Slide Time: 25:05)



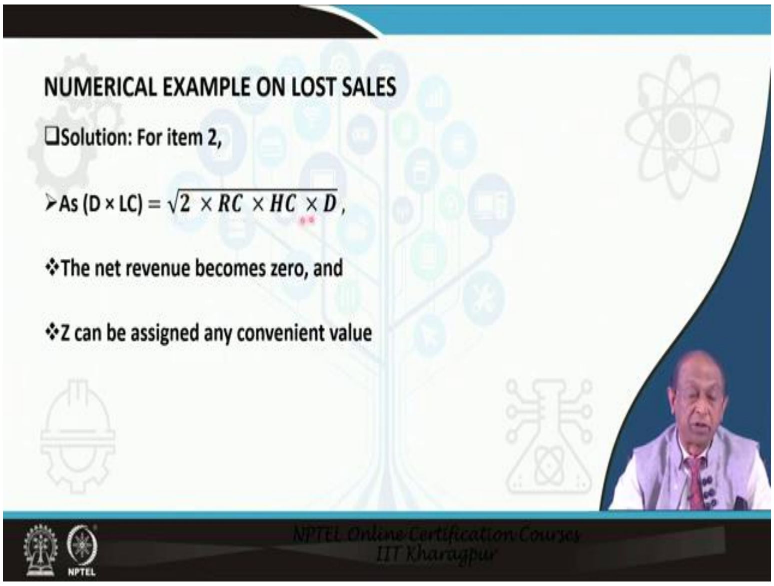
So, the solution for item 1 basically comes down to R0 as 905 units and Q0 in that case comes out to be 14 units because the bracketed term has come out to be positive, we have said Z equal to 1 and the corresponding values of R0 and Q0 have been arrived at.

(Refer Slide Time: 25:40)



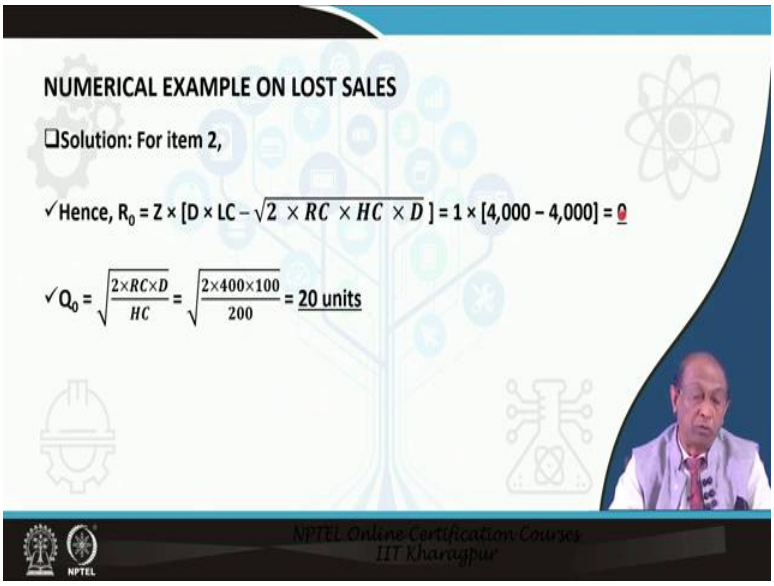
For item 2 similarly, we first work out the value of LC which is again 40 and D into LC in case of item 2 works out to be 4000 and here also it is equal to 4000 this expression, root over of 2 into RC into HC into D.

(Refer Slide Time: 26:17)



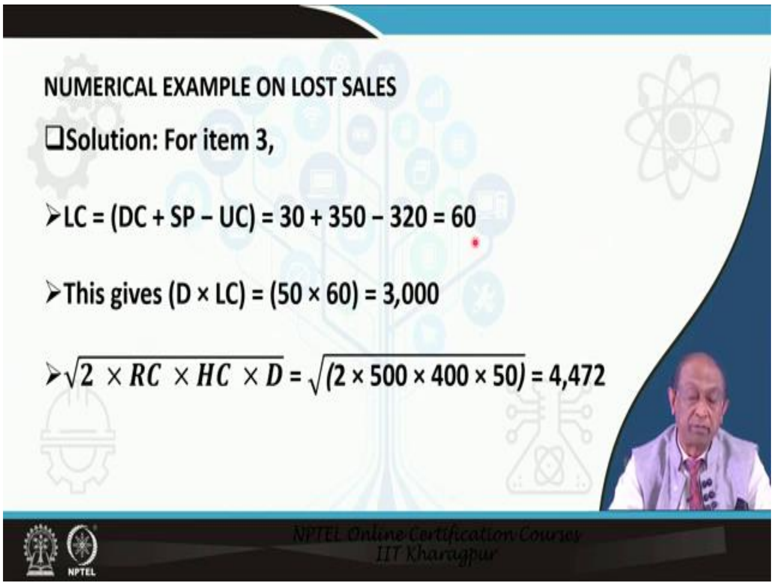
So, what do you find? In this case, the D into LC equals this expression. So, the net revenue per unit time, just look at the expression for R0 that becomes 0. So, you can assign any value to Z.

(Refer Slide Time: 26:46)



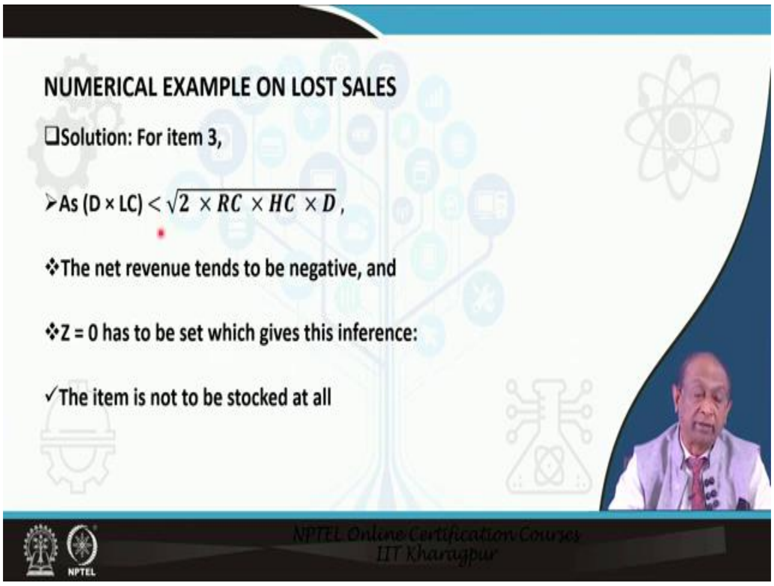
You will get the net revenue per unit time equals 0 and Q0 works out to be 20 units, but does not matter you really get net revenues.

(Refer Slide Time: 27:08)



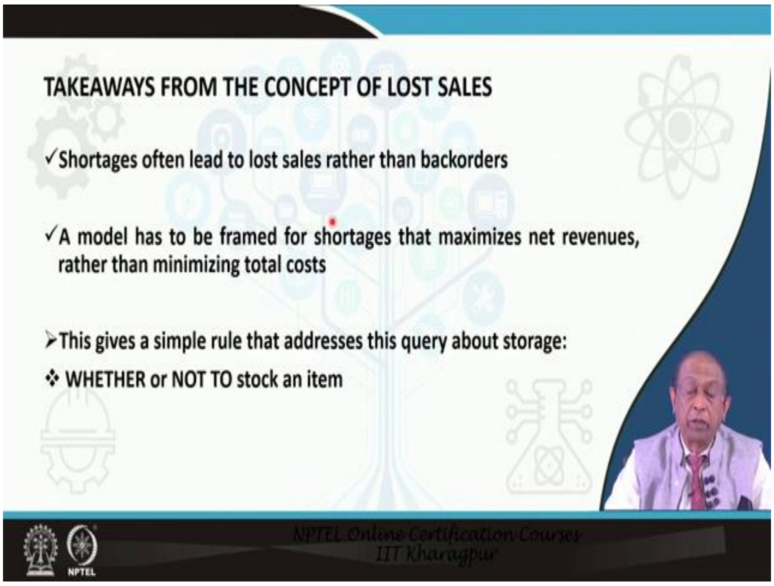
For item 3 again LC works out to be 60 units, D into LC works out to be 3000 unit and the expression under root 2 into RC into HC into D works out to be 4472 unit from the given data.

(Refer Slide Time: 27:31)



So, what do you find? That D into LC is less than root over of 2 into RC into HC into D. So, in this case if you look at the expression for R0 that is the net revenue per unit of time you will find that it will work out to be negative. So, it is better that we set Z equal to 0, meaning thereby that there is no need to stocked this item at all.

(Refer Slide Time: 28:10)



So, the takeaway was from the concept of lost sales is that the shortages often lead to lost sales rather than backorders. Hence, there is a need to frame a particular inventory model for shortages that maximizes net revenues rather than minimizing total costs. And this basically, model tells us under the condition of lost sales, whether or not we should stock the item at all.

Thank you.

(Refer Slide Time: 28:55)

We have used references mainly from this book, Waters, D Inventory Control and Management. Thank you