**Modelling and Analytics for Supply Chain Management**
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**Lecture 26**
**Inventory Analytics – I (Contd.)**

Good afternoon and welcome to module 2 of inventory analytics.

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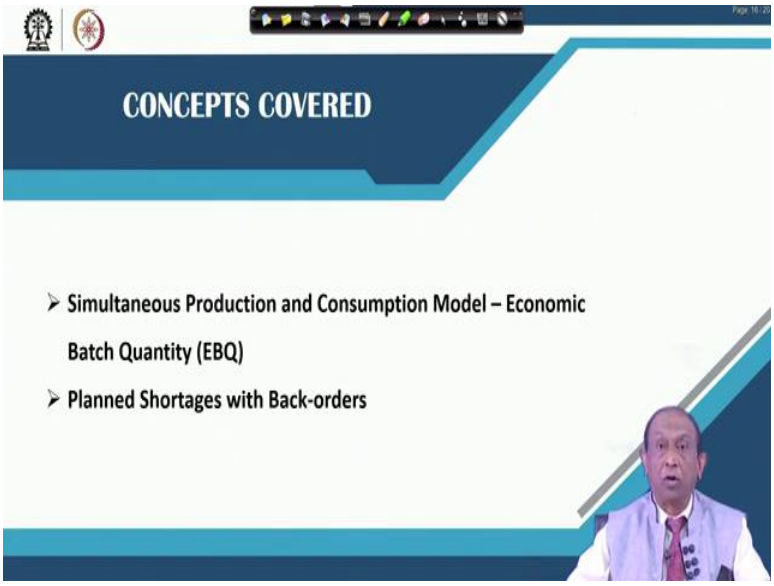


Today, we are going to discuss the concepts related to a situation where there is simultaneous production and consumption is happening and what should be the economic batch quantity under such a situation. And we will be next dealing with planned shortages with backorders, how to model these 2 kind of situations.

In the last session, we had discussed the fundamentals of economic order quantity model, wherein there were 2 important assumption that the demand rate is constant and known with certainty. Another important assumption was that the lead time is constant or it may be even 0. So, the order replenishment under such situation is instantaneous. But, in real life whenever there is a production situation then the production is going on simultaneously, consumption is taking place. Under such a situation what should be the economic batch quantity?

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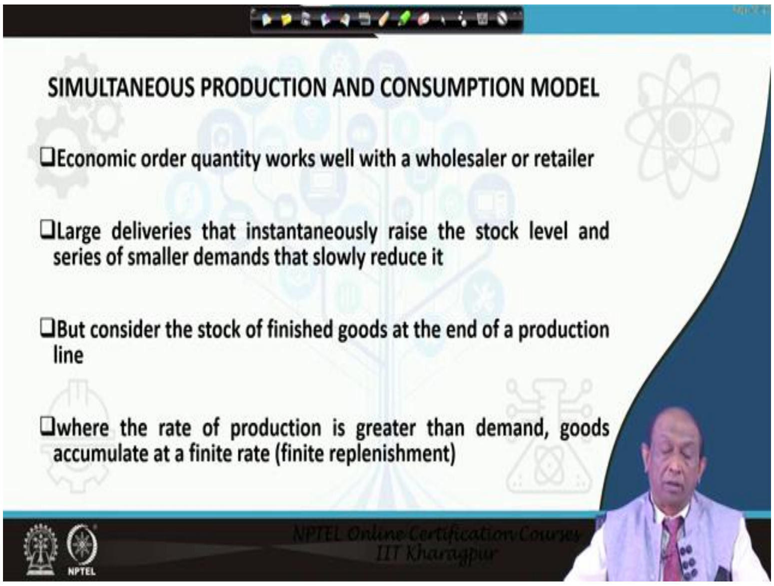


So this economic order quantity works well with a wholesaler or a retailer. And in here, we find that large deliveries instantaneously raise the stock level and series of smaller demands slowly reduces the stock level. But consider a situation wherein there is production and consumption happening at the same time. For example, how do the stock of finished goods at the end of a production line accumulate where the rate of production must be greater than the demand and under such situation goods accumulate at a finite rate. So the replenishment is finite but not instantaneous.

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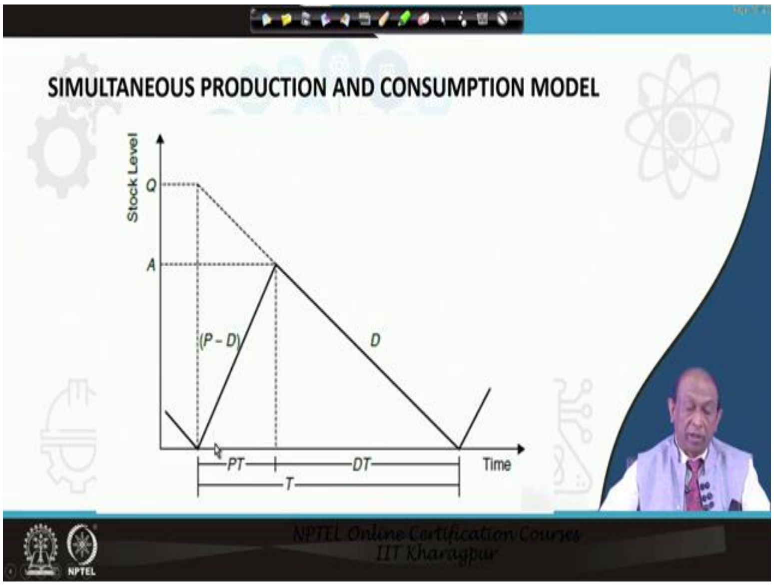


Look at this typical model. Again in here the stock level is represented along the y axis and x axis represents the time. This is the rate at which the stock is getting accumulated, which is nothing but the difference between the production rate P and the demand rate D. This slope represents the difference in rate between production and demand. PT is the time over which the production is taking place and the production is stopped at this point in time.

For the maximum stock level that can get accumulated is A. The demand takes place over this time also when there is no production. So DT represents the time interval over which only when consumption is there and no production and T is the total time which is sum of this PT and DT representing total time over which this cycle of production and consumption is taking place and this pattern is going to get repeated.

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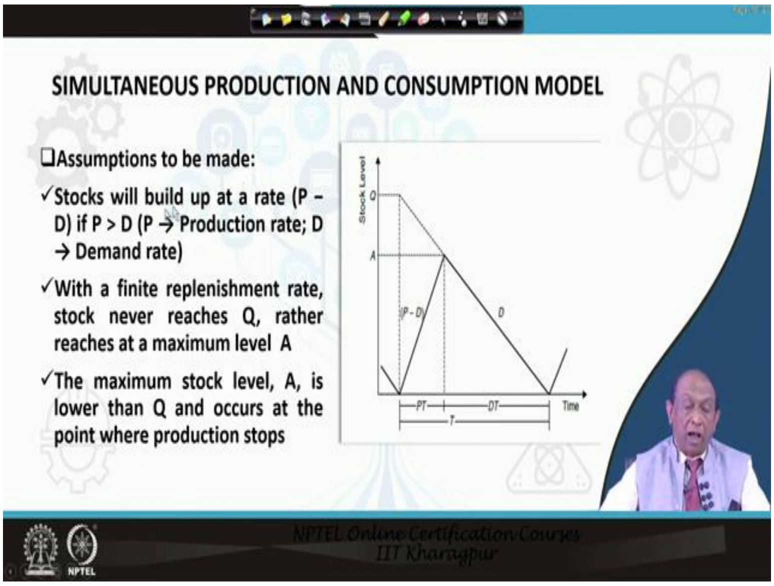


Here, the assumptions to be made are that the stocks will build up at a rate which is equal to P minus D provided the production rate P is greater than the rate of demand D. You see in this situation with a finite replenishment rate, stock will never reach Q rather it reaches a maximum level A stock the maximum stock level A is lower than Q and it occurs at that particular point in time when production stops.

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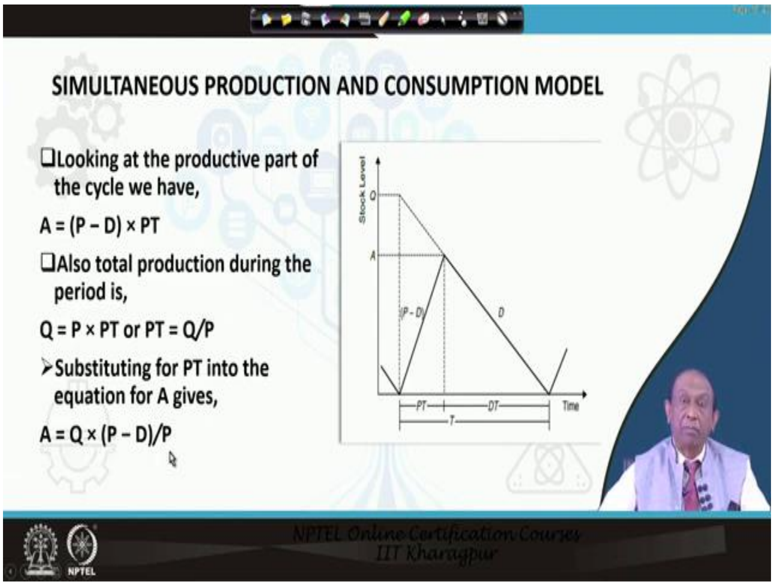


So if we look at the productive part of the cycle that is over time period PT, what we find, that this maximum stock Level A is nothing but P minus D multiplied by PT. Also the total production that is taking place during this time period PT is nothing but Q which is equal to the production rate P multiplied by PT. So Q is nothing but P times PT or in other words, PT is Q by P. So if we substitute the expression of PT into the equation for A, we get A equals Q times P minus D divided by P.

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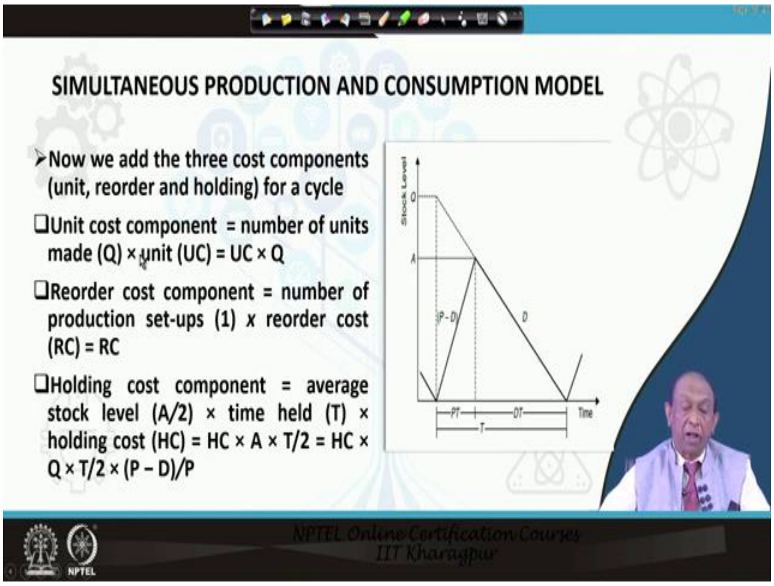


Now, let us consider this particular cycle where the total time is t. Here we add the three cost components. For example, unit cost plus the reordering cost and the holding cost for a cycle. The unit cost component is nothing but the number of units produced, which is Q multiplied by unit cost of the item, which is equal to UC multiplied by Q. The reorder cost component equals the number of production setups, which is in this case is only 1 multiplied by reorder cost, RC because in any production situation whenever you change from one product type to the other, the setup cost is associated with preparing the equipment for the next production type or product type in terms of changing the dyes and tools related to the first product.

This takes time over which no production might take place. So there is an opportunity cost of not having production over which the setup is taking place. And hence, this cost is also known as setup cost which is nothing but (()) (11:14) expression as reorder cost. Inventory holding cost is the average stock level which is A by 2 into the time over which this particular inventory is held which is T multiplied by the holding cost. So the holding cost component becomes HC multiplied by A multiplied by T by 2 which is HC into Q into T by 2 into P minus D by P because we are substituting the expressions for this.

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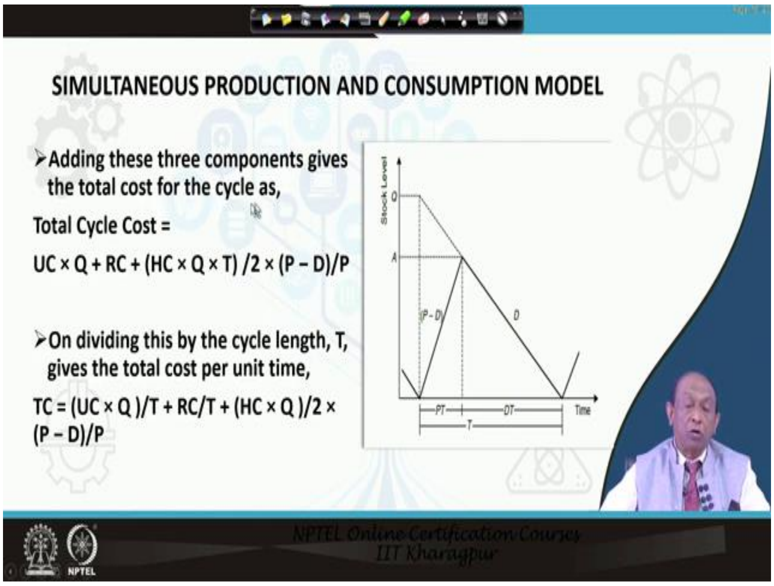

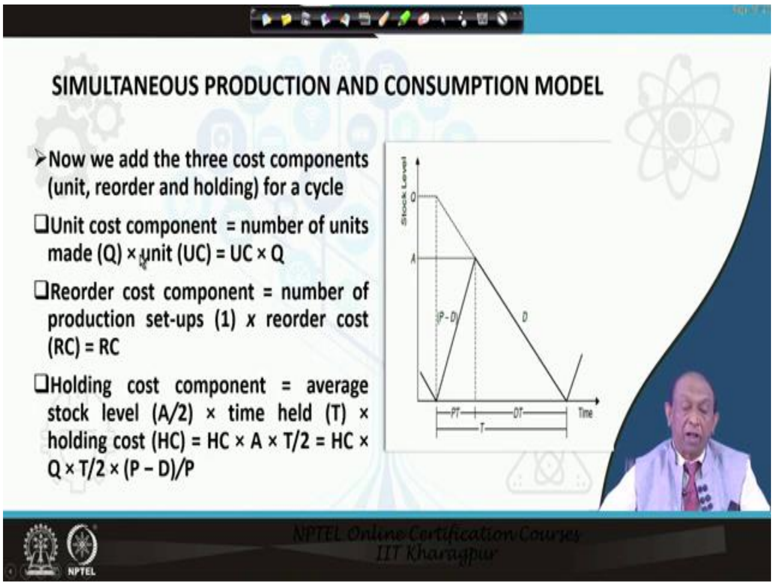


Adding these three components, these are the total cost for the cycle T as total cost equals UC multiplied by Q plus RC plus HC into Q into T divided by 2 multiplied by P minus D divided by P. If you look at this see, we have just rearranged this equation. These three components we have added together and then we have found this expression.

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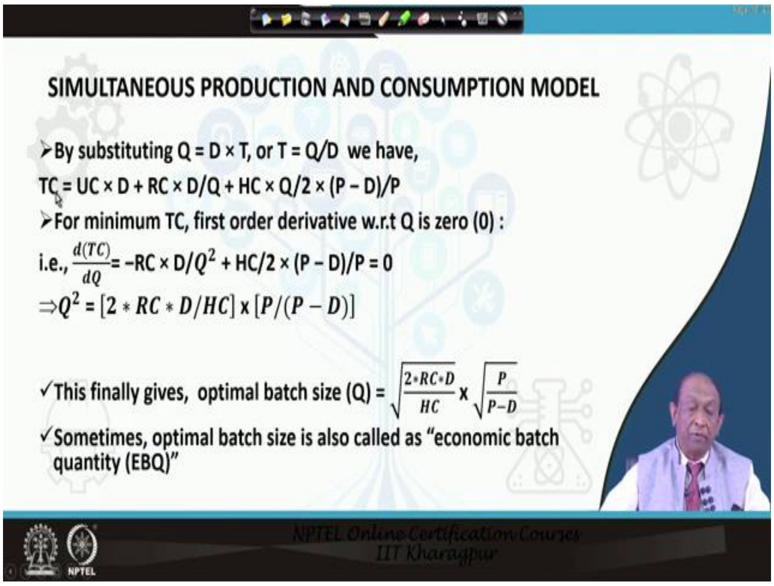


So by substituting Q which is equal to D into T, we get the expression for T as Q by D. So we have the expression for total cost as UC into D plus RC into D by Q plus HC into Q by 2 into P minus D by P. We have to minimize this cost, total cost. So, and it is the function of Q because we want to determine that particular batch size Q, which will minimize this total cost. So for minimum TC, the first order derivative with respect to Q must be 0. That is ddQ of TC equals 0, which in turn gives us, this is not a function of Q.

So this portion goes out minus RC into D divided by Q square plus HC by 2 multiplied by P minus D divided by P, this is equated to 0. Then if we transfer this to this side, we get the expression for Q square as 2 times RC into D by HC multiplied by P divided by P minus D. And this finally gives the expression for optimal batch size Q as root over 2 times RC into D divided by HC multiplied by root over of P by P minus D. Sometimes this optimal batch size is also known as economic batch quantity, this RC here is nothing but reordering cost, which is the same as setup cost, D is the annual demand, HC is the holding cost. Now, let us look at one numerical example, wherein we have illustrated how to compute the optimal batch size.

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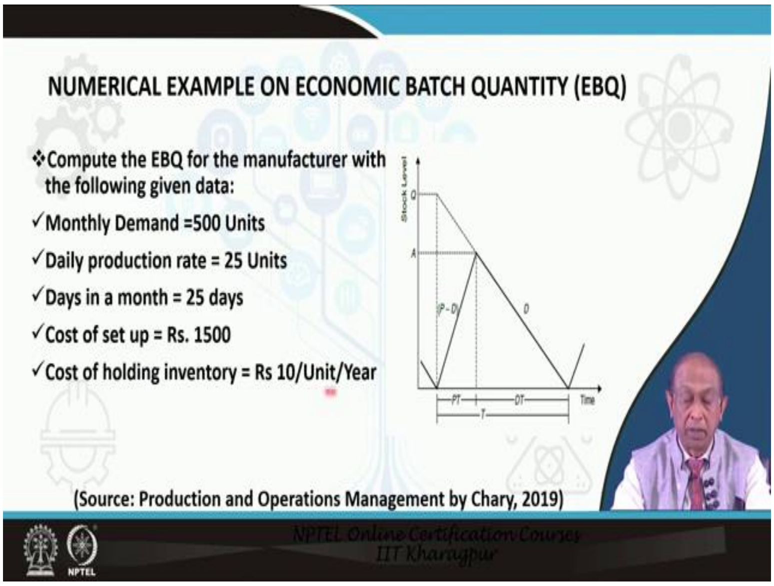


The problem is that for a manufacturer the following data has been given. Monthly demand 500 units, daily production rate 25 units, days in a month 25, cost of setup 1500 rupees and cost of holding inventory rupees 10 per unit per year.

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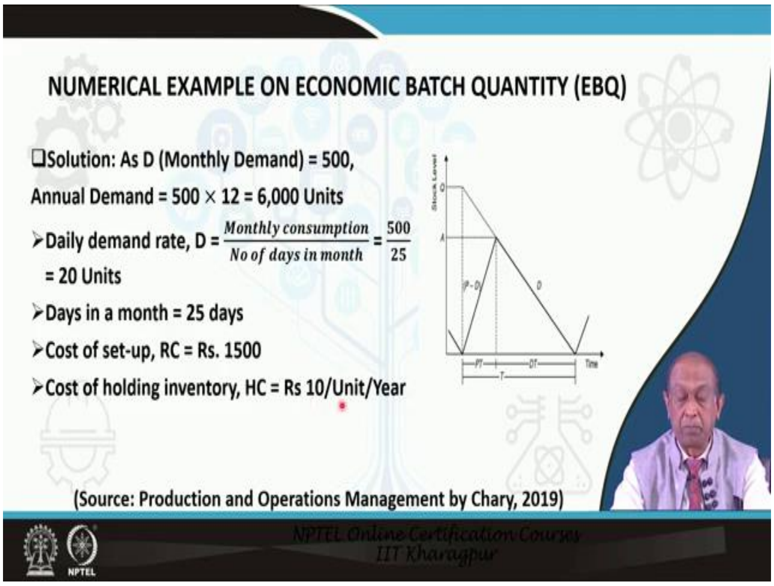


So in order to determine the economic batch quantity, we have to first compute the annual demand which is nothing but 500 multiplied by 12 equals 6000 units. In this case the demand rate, the monthly consumption of 500 units by number of days in a month 25 which works out to be 20 units. Days in a month 25, the cost of setup is 1500 rupees given and cost of holding inventory is rupees 10 per unit per year.

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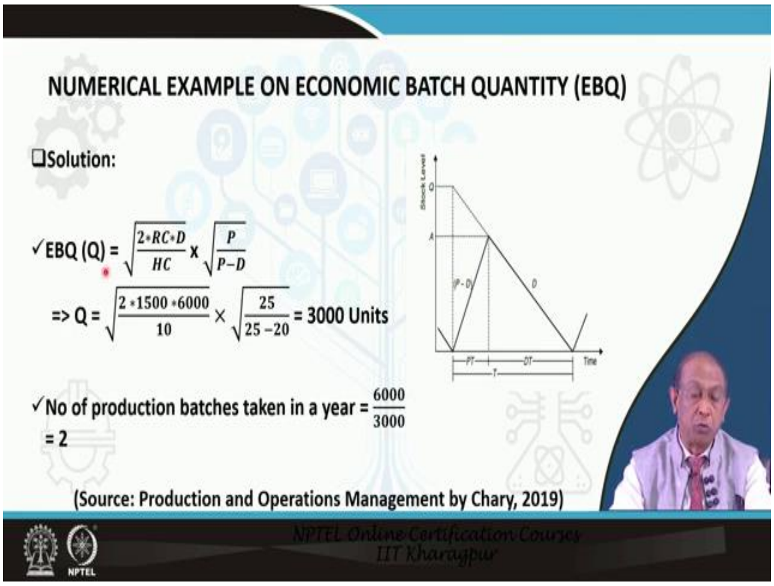


So if the EBQ as per our derivation is root over of twice RC into D divided by HC into root over of P divided by P minus D. So EBQ works out to be 2 into 1500 into 6000 divided by 10 multiplied by this particular factor of P by P minus D which is 25 by 25 minus 20. And this entire thing works out to be 3000 units. This is the optimal batch size under a production situation, simultaneous production and consumption model. And the number of production batches with this optimal batch size of 3000 units to be taken in a year is equal to 6000 divided by 3000 which is equal to 2.

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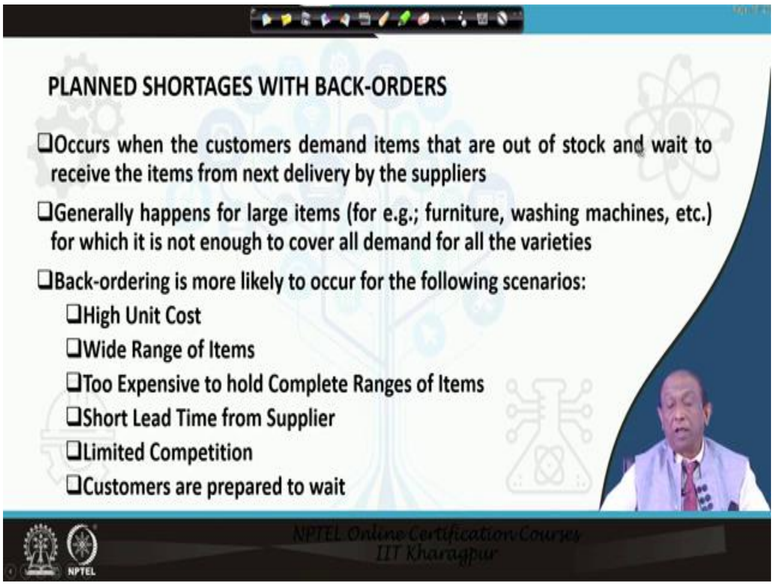


Now come, let us discuss about the planned shortages with backorders. This kind of situation occurs when the customers demand items that are out of stock but they are prepared to wait, to receive these items from next delivery by the suppliers. Generally, this kind of situation happens for large items, for example furniture, washing machines and so on, for which it is not enough to cover all demand for all the varieties.

Back ordering is more likely to occur for the following scenarios. When the unit cost of the item is very high. Because in that situation if we are carrying large number of items in inventory then the inventory carrying cost will be high and so it will not be economical. When there is a wide range of items to be covered, we cannot or is not economical to cover or to stock all the different kinds of items.

Back ordering is more likely to occur when it is too expensive to hold complete range of items. And back ordering is more likely to occur when the lead time from the supplier is very short so that we can quickly get the items from the supplier when we place the order. Back ordering is more likely to occur when there is limited competition and above all, the customers are prepared to wait under that because they value those items and they are prepared to give them some more time.

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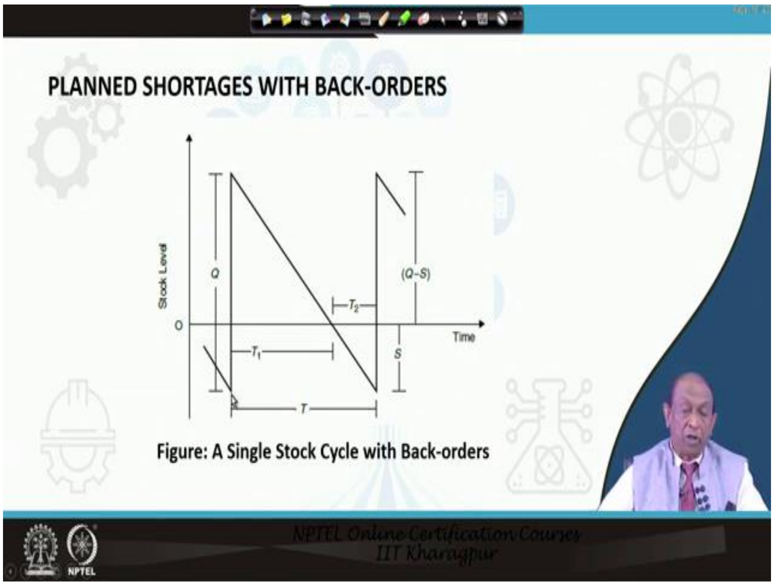


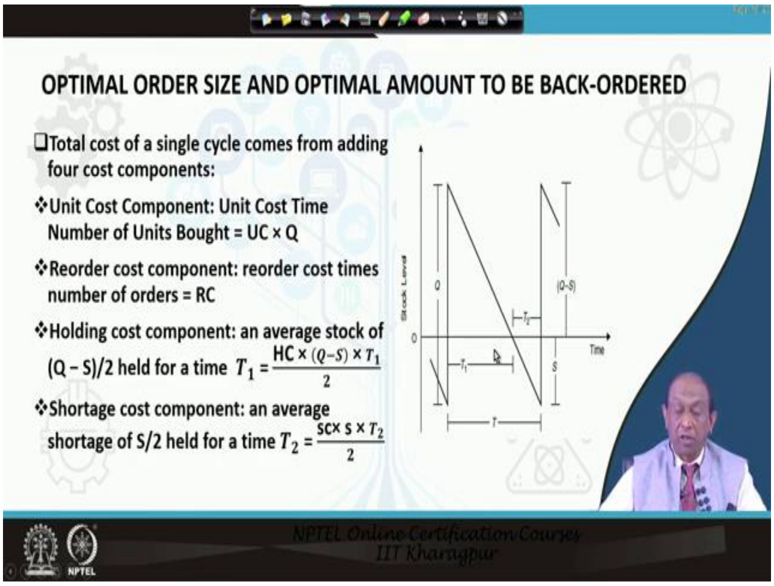


So a single stock cycle with backorders, this is the model that we will be dealing with. Again here, the stock level is reflected along the y axis, x axis represents the time and the total cost of a single cycle comes from adding 4 cost components. One is unit cost times number of units bought, that is the unit cost component, which is UC multiplied by Q. This is the item that is in order.

Holding cost component. Here, what do you see? That over a period T1, we have an average stock of Q minus S held for a time T1, because S is the amount in backorder. The stock level has gone down to 0 at this point in time, but still there is demand which is made from the next replenishment. From here to here, there is demand but no stock. So the total amount backorder amount is S.

So from this portion to this portion, the stock level is Q minus S and the backorder cost is, backorder amount is S. So the cost associated with this backorder amount, that is a shortage cost component equals average shortage of S by 2 held over a period T2. So holding cost component equals average stock of Q minus S held over a period T1 which is nothing but Q minus S by 2 is average stock multiplied by the holding cost HC into the time T1 over which this cost is incurred. And the shortage cost component is nothing but shortage cost SC multiplied by S by 2 into T2.

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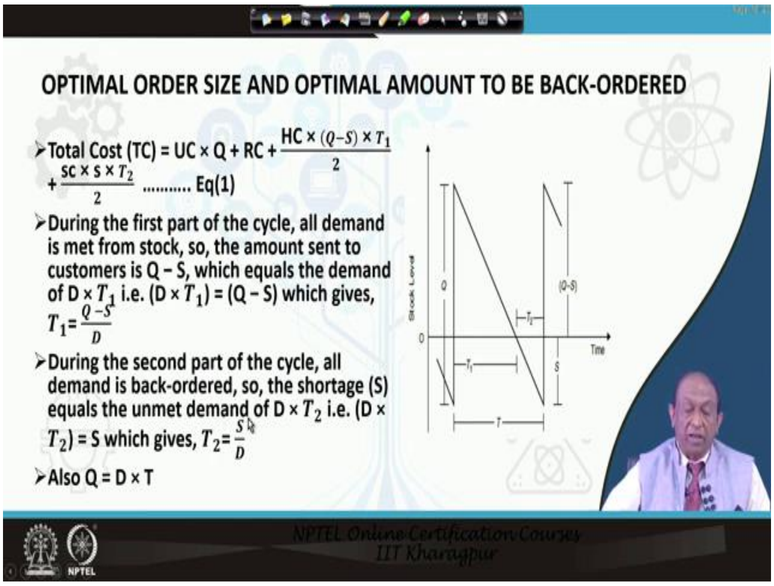


So the total cost TC equals unit cost UC multiplied by Q plus RC that is the reordering cost plus HC into Q minus S by 2 into T1 plus HC into S into T2 by 2 where T is nothing but T1 plus T2. So what we see is that during the first part of the cycle, all demand is made from stock. So the amount sent to customers is nothing but Q minus S and this equals the demand of D into T1, the demand rate is D.

So what we find here? D multiplied by T1 equals Q minus S which gives T1 equals Q minus S divided by D. During the second part of the cycle in here over T2, all demand is backordered. So the shortage amount S equals the unmet demand which is equal to D multiplied by T2. So D multiplied by T2 equals S, which gives an expression for T2 equal to S by D and we have also Q equals D by T.

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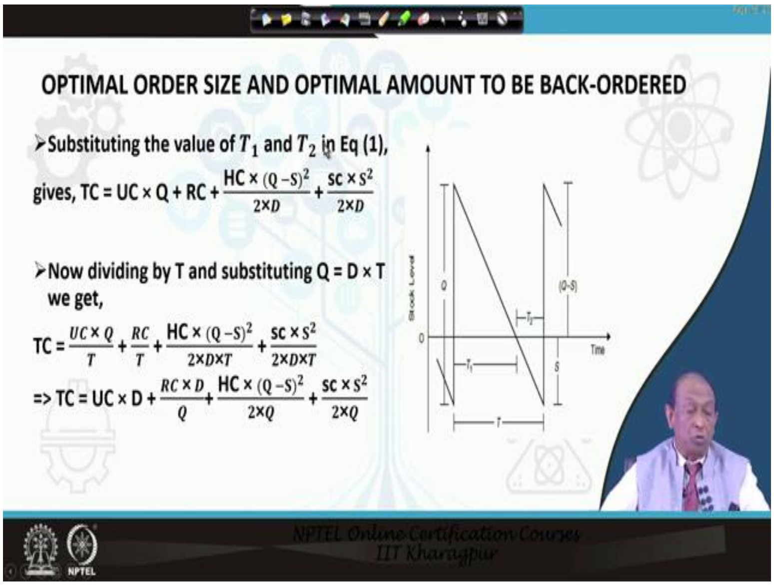


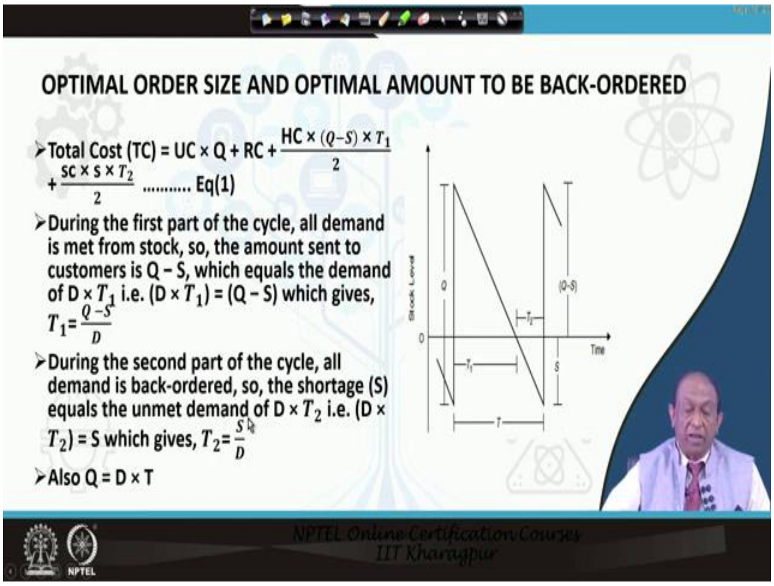


With this when we substitute the value of T1 and T2 in the previous equation, that is the equation 1 for the expression for total cost TC, we get TC equals UC into Q plus RC plus HC into Q minus S whole square by 2D plus HC into S square by 2D. Now, if we divide by T and substitute Q equals D into T, we get total cost per cycle equals UC into Q by T plus RC by T plus HC into Q minus S whole square by 2 into D into T plus HC into S square divided by 2 into D into T. The total costs TC equals UC into D plus this expression.

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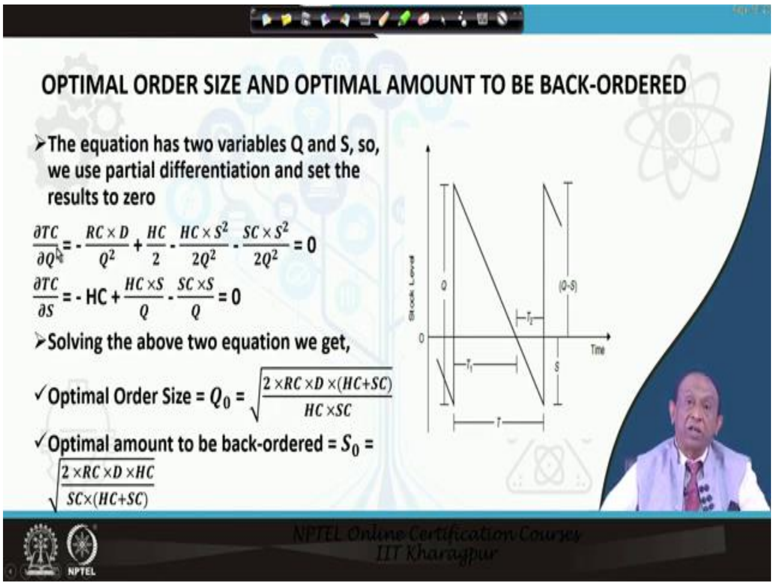


Now, if we partially differentiate this expression for total cost with respect to Q and S, first with respect to Q, we get del Q of TC equals minus RC into D by Q square plus HC by 2 minus HC into S square by 2Q square minus this equals 0. And the second expression del del S of TC equals minus HC plus HC into S by Q minus SC into S by Q equals 0. Solving the above 2 equation, we get optimal order size Q2 equals root over of 2 times RC into D into HC plus SC divided by HC into SC and substituting this expression of Q0, we get optimal amount to be backordered, S0 equals this.

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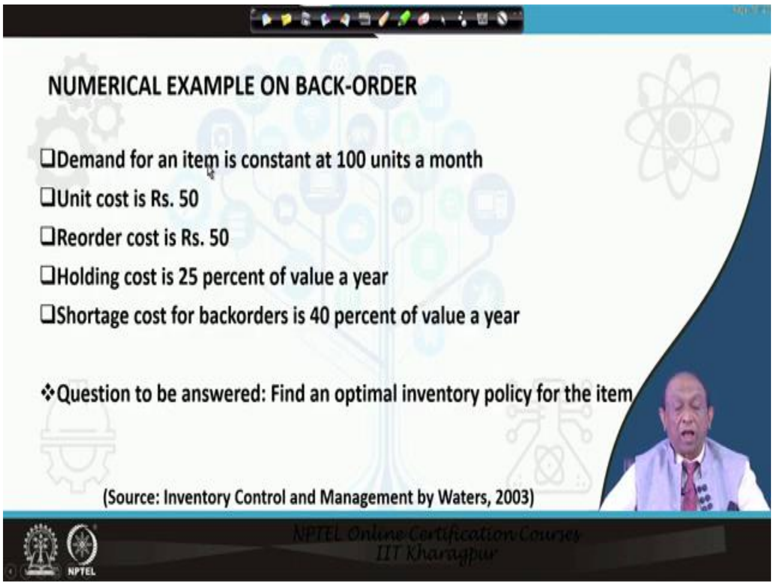


Let us take an example. Demand for an item is constant at 100 units a month, unit cost is rupees 50, reordering cost is rupees 50, holding cost is 2 percent of value a year, shortage cost for backorders is 40 percent of value a year. The question to be answered is, find an optimal inventory policy for the item.

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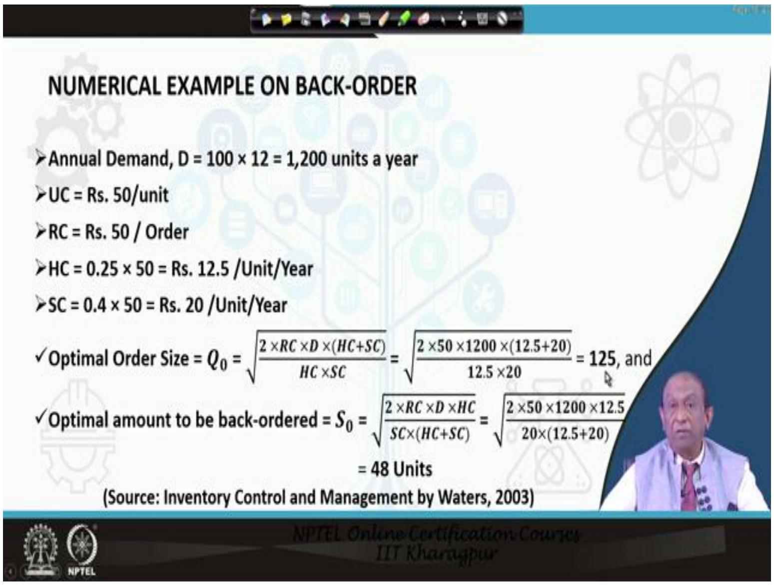


So it is very simple. You compute the annual demand which is nothing but monthly demand into 12 which is 1200 units a year. Unit cost is rupees 50 per unit, RC is also the same per order. So holding cost is nothing what 0.25 five into 50 which is rupees 12.5 per unit per year. SC or the shortage cost from the given data is 0.4 times 50 which is rupees 20 per unit per year. Hence, optimal order size works out to be 125 and the optimal amount to be backordered is 48 units. Thank you.

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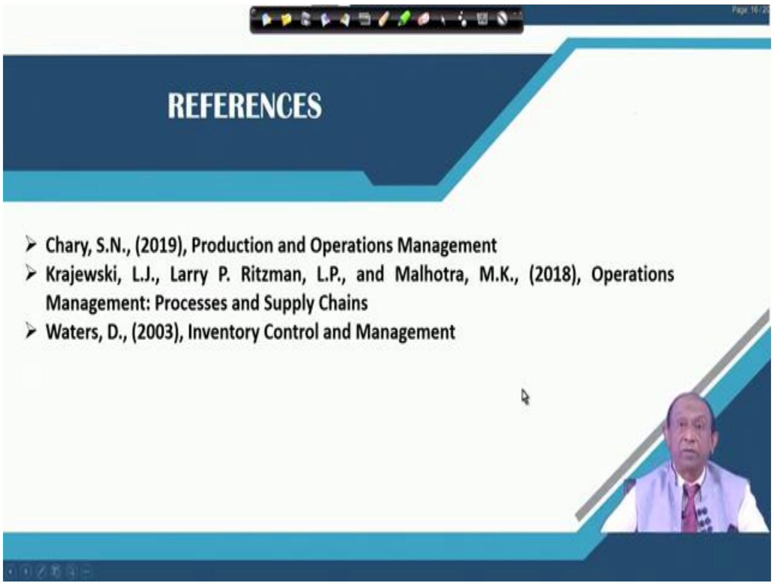

Next class we will be dealing with safety stock and how to determine it and the two popular inventory systems used in practice, which is the P system and the Q system. Thank you.