Modelling and Analytics for Supply Chain Management
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Lecture-19
Warehouse location models - IV

Welcome to module 4, week 4. For the course modelling and analytics for the supply chain management. Now so long we had touched up on different models for warehouse location. We started off with the model were the company is not willing to share cause data with you. That was the factor rating model and this module was applicable. when your business is with in very very not very very but when your business is within a smaller geographical region and the number of warehouses are also not that many. Then we came up with another model were in only the demands was given, but as again the cause data was not shared.

Then the centre of gravity model was used. And again then we came up with another model were the demand was given and, you were calculating the cost putting in distances as a proxy. And there the decision was we are not setting up 2, 3 warehouses at the same time.

We were setting up 1 warehouse, 1 after the other. That means we will set up warehouse 1 then 2 then 3 then 4, that was a heuristic, which is Ardalan heuristic. Then we came up with the model which is total cost model. When all the cost data is given to you and you are calculating which warehouse you should keep open and which how much quantity of product should remove from which warehouse to which market?

Today, we will deal with a model were we have different warehouse, choices and we have the option to set up a small warehouse and a large warehouse. Let us understand we have different markets, let us say Delhi, Mumbai, Kolkata, Madras or Chennai and we have 3, 4 probable warehouse location.

At each of this locations we have the option to have a small warehouse and slash or you can have both, and or a big warehouse. Our job is to decide on 1st whether to have a warehouse. 2nd whether to have a small warehouse, 3rd whether to have a large warehouse. 4th whether to have small or a large warehouse.

Now you will ask me, when I have the option to have a large warehouse, why shall I have a small warehouse. Well sometimes spaces may be concrete. So you might not get enough space to have as bigger warehouse that you are looking for. Sometime distance to the market is also a constraint. . So you will love to have to big warehouse outside the city.

And 1 or 2 small warehouses, right inside the city so that you can very quickly cater to the demand of the market. So large warehouse and small warehouse, simultaneously in the same location is not an astonishing fact. We can have large as well as small warehouse. So coming back what decision are we taking?

We have entire cost information and we have to take decision whether to have a warehouse, whether to have small warehouse, whether to have large warehouse, whether to have both. This is our agenda today.

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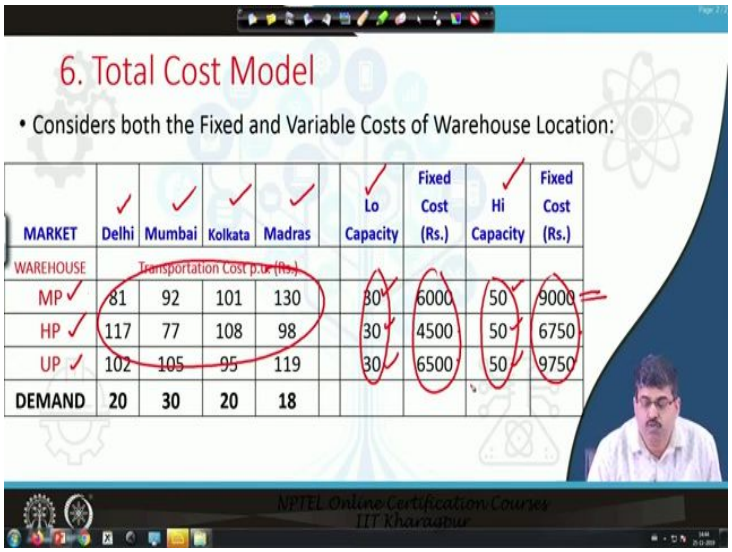

If you look at the issue that we were just explaining. This is the cost matrix that we were talking about. I have taken a very small 1 as usual just to fit just to fit your screen, you can keep on expanding the rows and columns. The basic solving methodology will remain the same. Let us understand this problem you have 3 probable locations of warehouses.

Madhya Pradesh, Himachal Pradesh and Uttar Pradesh and you have 4 locations for 4 probable markets. Delhi, Mumbai, Kolkata and Chennai. Now the transportation cost of transporting 1 unit of a commodity from a Madhya Pradesh warehouse to the Delhi market is rupees 81. 1 unit of product to transport from Madhya Pradesh warehouse to Mumbai market is rupees 92.

In this way. This entire cost matric is given to you. The transportation cost of 1 unit of product from Uttar Pradesh UP to Chennai is given as rupees 119. Now the demand at all this location is also given. 20, 30, 20 and 1 8 18. This may be 20 units this may be 20000 units, 200000 units, it is just a unit.

Now at Madhya Pradesh you have the option to have a small warehouse that is a low capacity warehouse and you have the option to have a high capacity warehouse. What is the capacity? What is a storing capacity of the low capacity warehouse? It is 30 units. 30 matric terms, 30000 units or it is 30.

What is the storing capacity of a high capacity warehouse? 50. Now definitely the rental of a high capacity warehouse will be more than the low capacity warehouse. So the fixed cost of a low capacity warehouse is rupees 6000 and the fixed cost of a high capacity warehouse rupees is 9000.

This same data or the same situation is replicated for warehouses for Himachal Pradesh and Uttar Pradesh. Low capacity warehouse high capacity warehouse. So what do you have? We have the per unit transportation cost given to you. You have the capacity given to you and you have the fixed cost given to you.

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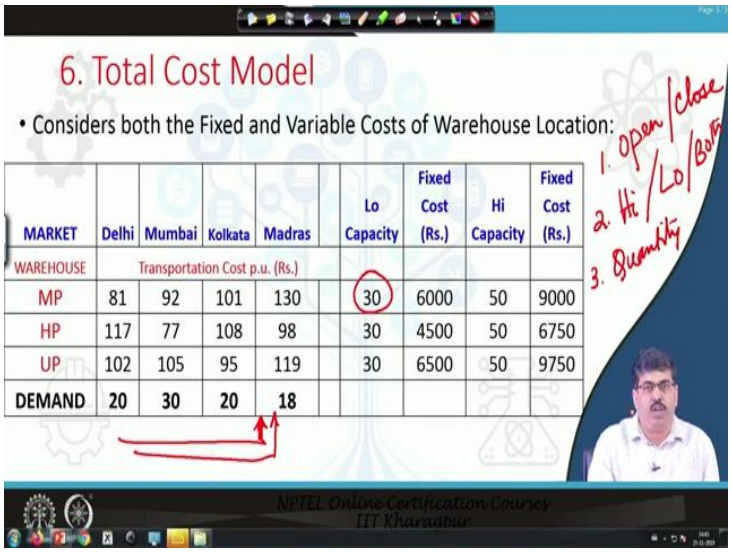

MARKET	Delhi	Mumbai	Kolkata	Madras	Lo Capacity	Fixed Cost (Rs.)	Hi Capacity	Fixed Cost (Rs.)
WAREHOUSE	Transportation Cost p.u. (Rs.)							
MP	81	92	101	130	30	6000	50	9000
HP	117	77	108	98	30	4500	50	6750
UP	102	105	95	119	30	6500	50	9750
DEMAND	20	30	20	18				

Our purpose is to find out what? Our purpose is to find out 1 whether this particular warehouse of Madhya Pradesh should be there or not. Should be open or should be closed. Next whether it should be a high capacity or a low capacity or both. And number 3 as usual how much quantity should be transported from these warehouses to these markets. How much quantity should be transported from this warehouse to these markets?

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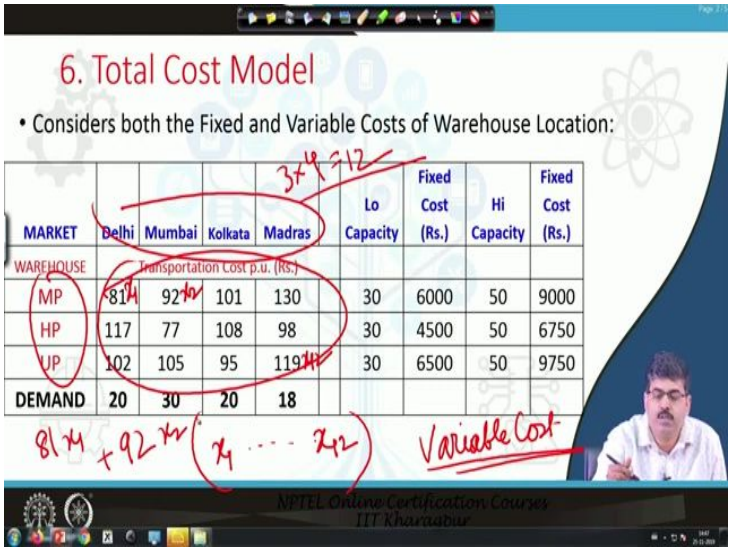

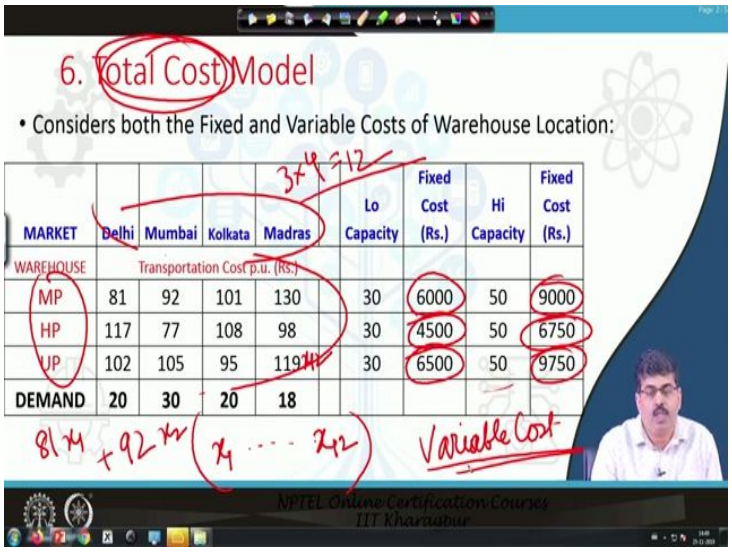

So how will you solve this problem? Let us see if we go back to our simple cost model. How much quantity are we transporting from Madhya Pradesh to Delhi? Cost is 81 rupees per unit, cost. But how much quantity are we transporting? We do not know? So this is x 1. How much quantity are we transporting from Madhya Pradesh to Mumbai? That is x 2. We do not know? It may be 0 also, x 2 may be 0 also. So how much quantity we are transporting is x 1 to 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12? How much quantity we are transporting is x 1 to x 12?

Why? 3 origins 4 destinations with the metrix is 3 by 4 that is equal to 12. So x 1 to x 12, so x1 quantity from Madhya Pradesh to Delhi and x 12 quantity from Uttar Pradesh to Chennai. So this is the quantity that can be transported, agreed. What is the cost?

Now this is what cost? This transportation cost what cost? This transportation cost is my variable cost. so what is the transportation cost? 81 x 1 + 92 x 2, you see this is right agreed. This is 81 x 1 + 92 x 2 + 100 1 x 3 + 130 x 4 in this way we will continue up to 119 x 12. That is my total variable cost. What is my fixed cost?

Is it I am stopping for a second just for you to gauge. It is 81, this is my variable cost, 81 x 1, if I do not transport any goods here. My x 1 becomes 0, so this cost also become 0. That is why it is variable cost. So 81 x1 + 92 x 2 + 101 x 3 130 x 4 et cetera up to 119 x 12. What is my fixed cost?

Fixed cost possible we are not saying that all the warehouses will be kept open. Or we will start off with all the warehouses. We are saying possible fixed cost, if I have all the warehouses small big et cetera. At least 1 warehouse in every location if I have the fixes cost possible is 6000 + 9000 + 4500 rupees + 6750 rupees + 6500 rupees + 9750 rupees.

So this is the total fixed cost that is possible. Possible we are not if this warehouse I do not open, this fixed cost will become how much? 0 So by total cost that is possible that is what the model is all about total cost model. By total cost that is possible is variable cost. Variable cost + fixed cost agreed? Now here we want to bring in something.

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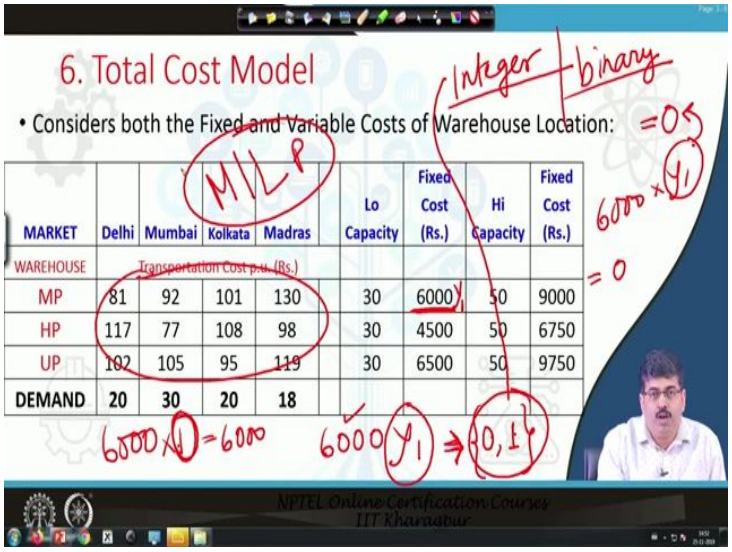

Here we want to bring in something this 6000 is the fixed cost for a low capacity warehouse in Madhya Pradesh, fixed cost for a low capacity warehouse in Madhya Pradesh. I will incurred this fixed cost only when I open the warehouse. Only when I open the warehouse. So just like we had x 1 here. X 1 was what the quantity that is transported. If no quantity is

transported this value will be will become 0. Similarly I have similarly this 6000, beside 6000 I will put a y 1, so it will become 6000 y 1. What is this y 1? Y 1 can be either 0 or 1.

Let us try to understand y 1 can be either 0 or 1. If this warehouse is open if I decide to keep the low capacity warehouse open, then what is the cost of the warehouse operation? If I keep the low capacity of this low capacity warehouse open for Madhya Pradesh, what is the cost? The cost is rupees 6000. Then what should be the value of y 1?

Y 1 should be 6000 into 1 equal to 6000. So y 1 value should be 1 agreed. Y 1 value should be 1, if I do not open this warehouse then what will my cost for low capacity warehouse to Madhya Pradesh? If I do not open? This cost will be I am writing here will be 0. I already have 6000 into y 1. So to make it 0 what value should y 1 18, y 1 should be 0.

So 6000 into 0 is equal to 0, if I do not open the warehouse the cost is 0. So y 1 value might be only 1 if the warehouse is open, repeating 1 if the warehouse is open, and 0 if the warehouse I do not open close, So y 1 can be either 0 or 1. This is this 0 or 1 is what we called at integer. right okay agreed. This 0 and 1 is called integers for programming some people call it as binary, binary also means 0 and 1.

So y 1 can be either 0 or 1, either not both. Y 1 can be either 0 or 1. That is why this type of model this is variable cost this type of a model is called as mixed integer linear programming.

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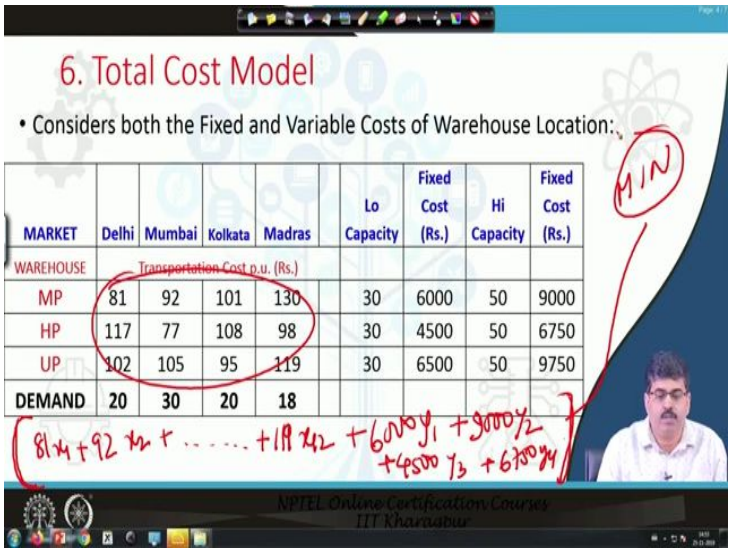

So now okay so now let us try to see what we have got? We have got all the variables cost. 81 x 1 + 92 x 2 + dot dot +119 x 12 + 6000 this is your fixed cost y 1 + 9000 y 2 + 4500 y 3 +

6750 y 4 in this way you continue. What is the value of y 1? We again repeat it is either 0 or 1. So this is your cost. What is the objective? You have to minimize this cost, agreed?

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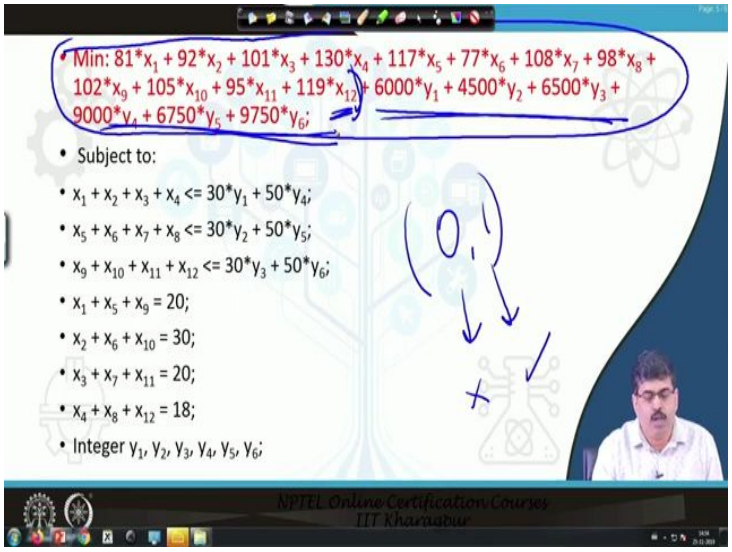

This is what we were talking about this is my cost model. 1st up to this as we mention x 12 is your variable cost. After that all your fixed cost. This is the possibility you may not have warehouses in all this location. This is the possibility of the total number of warehouses to be build. We should keep all the option open. Then my 0 1 issue will tell us where to open the warehouse? And where not to open the warehouse? So we should but we should keep all the possibilities open. What are the other kind… this is the objective. What are the other constraints?

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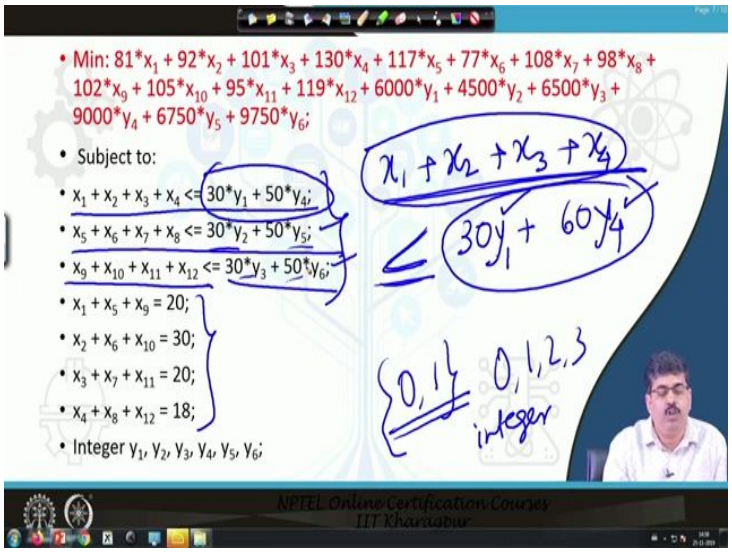

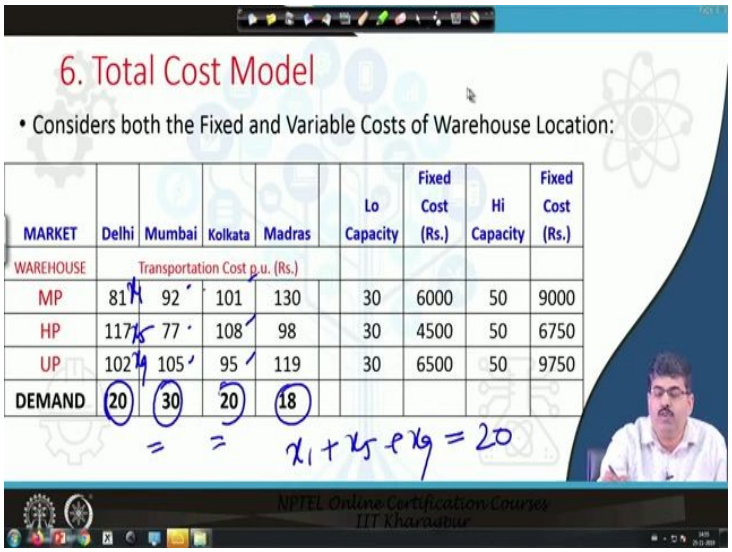

Other constraints are my demand constraints, these are my demand constraints. If you go back to the previous slide, yup sorry, if you go back to the previous slide you will see that this was my demand constraints, this 20, 30, 20 and 18. This was my demand constraints you agreed. So where were we? We were here x 1 + x, x 1, x 2, x 3, x 4, x 5 up to x 8, x 9, so we were here x 1 + x 5 + x 9 is equal to 20. X 2 + x 8 + x 10 is equal to 30. x 7, x 3, x 7, x 11 is equal to 20. So these are my demand equations. As we mentioned this are my demand equations. We just now showed it to you. What are my supply constraints?

Remember this was the supply from Madhya Pradesh. x 1 am righting it bigger x 1, x 2, x 3 + x 4 is the total quantity that Madhya Pradesh can supply. How many warehouses? What is the capacity? 30 and 60, y 1 and from the matrix we wrote y 1, y 2, y 3, y 4, y 5, y 6, just 1 below the other. So y 1 + y 4 this is the total supply capacity, y 1 can be 1, 2, 3 4 anything. In the previous one we showed y 1 can be 0 or 1. That means y 1 can be 0 or 1 means either the warehouse is open or it is closed. The moment we write y 1 can be 0, 1, 2, 3 and integer that means y 1 can be 1, 2, 3 anything, that means we can open as many warehouses as possible 1, 2, 3 anything. So y 1 y 2 can be anything 0, 1, 2, 3 anything.

And this total supply is less than equal to this possible, Agreed? x 1 + x 2 + x 3 + x 4 is less than equal to 30 y 1 + 50 y 4. That is the total supply from Madhya Pradesh should be less than equal to warehouse capacity small warehouse, warehouse capacity large warehouse.

Similarly total capacity of the other warehouses this 1 again small warehouse, large warehouse total capacity of Uttar Pradesh warehouse again small warehouse large warehouse.

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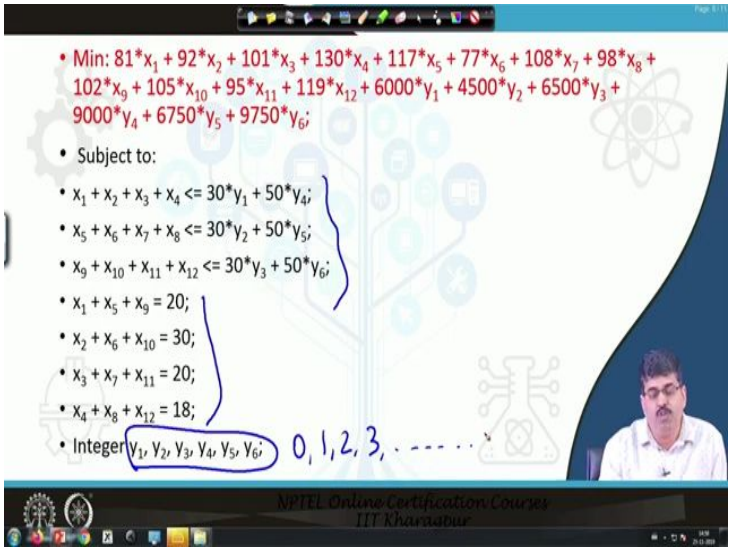

So my demand is given by this and my demand is given by this and this is my supply possible. And y 1, y 2, y 3, y 4, y 5, y 6 are integers. That is means it can be 0, 1, 2, 3 anything. So we can have as many warehouses possible. Integers mean it cannot be a fraction, it is a whole number. So this is the total cost model were we have both the fixed cost and the variable costs.

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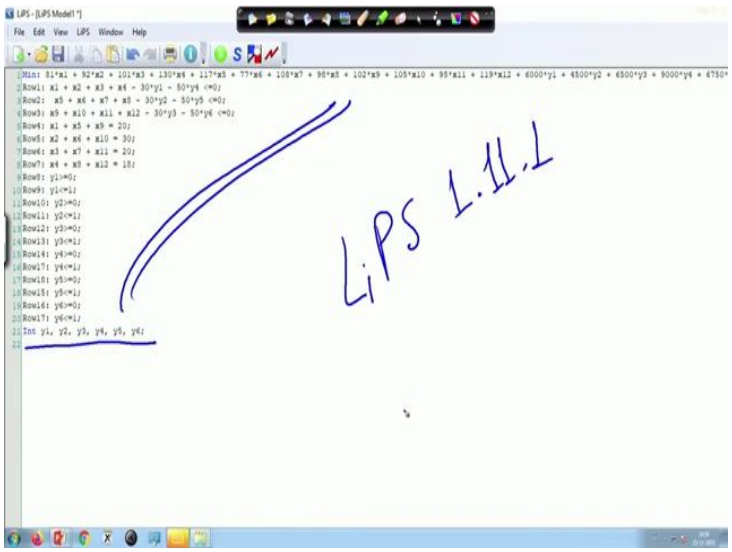

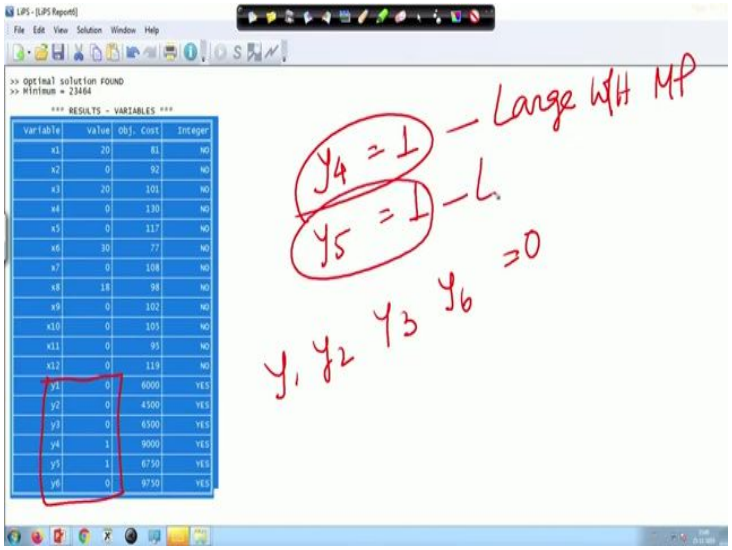

Now so what we do this entire equation we put it in a in a software which is again freely downloadable. And it is very easy whatever we wrote down in generally equation form, we have just written the same thing in this equation form nothing else. See at the bottom integers y 1, y 2, y 3, y 4, y 5, y 6 and we can solve it using the text model where you just write this equation.

We can also solve it using the column model And this is the results, it will be bit difficult for you to visualize the result from there but what I will tell you is here is your y 1, to y 6. There it says y 4 is equal to 1, y 5 is equal to 1, All the others are y 1, y 2, y3, y 6 are 0. That means we will have 1 large warehouse at Madhya Pradesh.

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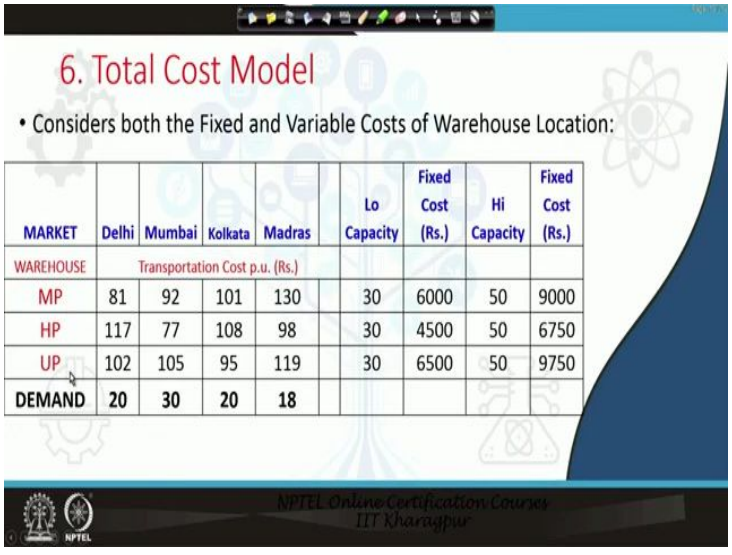

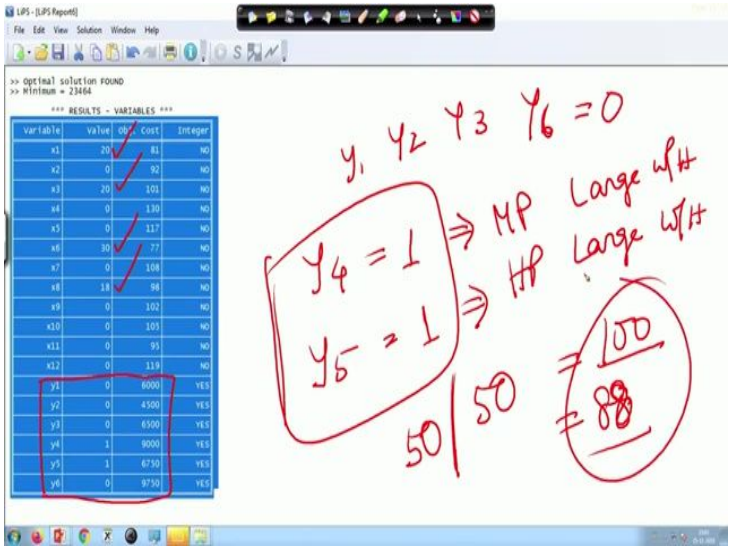

And again if you see we will just quickly go back Madhya Pradesh, Himachal Pradesh and Uttar Pradesh. y 1, y 2, y 4, y 5, y 6, y 1, y 2, y 3, y 4, y 5, y 6. Madhya Pradesh, Himachal Pradesh and Uttar Pradesh. So now if you go back we were having this Y 1, y 2, y 3, y 6 is equal to 0, and y 4 is equal to 1. That is basically Madhya Pradesh with a large warehouse and y 5 is equal to 1, that is your Himachal Pradesh with a large warehouse. So this is your solution. Now what was the warehouse capacity? Large warehouse 50 and 50. So total capacity 100.

What was your total demand? If you see 20, 20, 30, 18, this was your total demands. So is your total demand satisfied? 20 + 20 is 40, 40 + 30, 70, 70 + 18, 88. Your total demand is 88 and your warehouse supply capacity is 100. So this problem is met this problem is solved.

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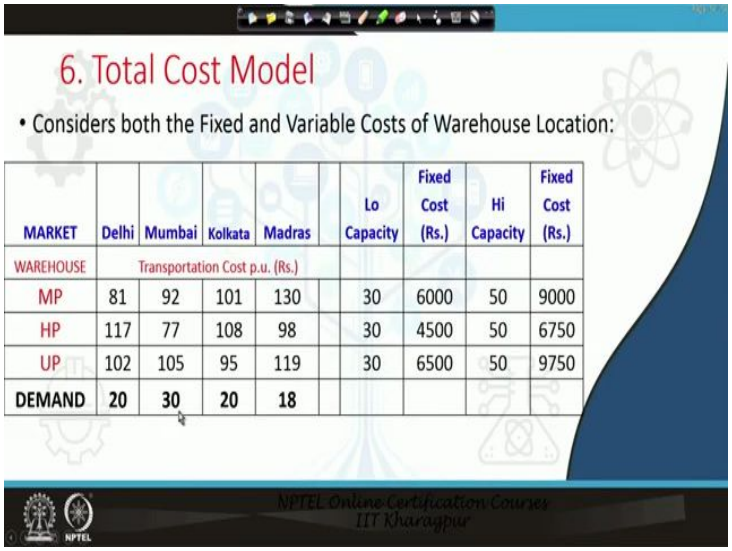

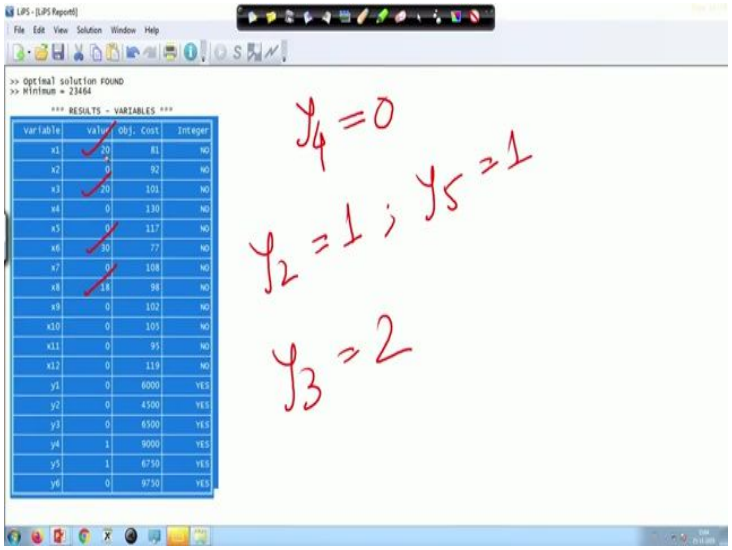

Now I also have something else to mention. This was your total demand if you mention 20, 30, 20, 20 Now I have a question and the question is all along this y 1 we were integers, now suppose we do not want a warehouse at do not want a large warehouse at Madhya Pradesh. We do not want a large warehouse at Madhya Pradesh.

So we do not want a large warehouse Madhya Pradesh. Now Madhya Pradesh is what? Large warehouse Madhya Pradesh is y 4 we do not want a large warehouse at Madhya Pradesh. So in the equations that we are running the modelling the problem the the problem formulation. What should we write? We do not want a warehouse at Madhya Pradesh.

Y 4 is equal to 0. We want both the small warehouse and a large warehouse at Himachal Pradesh. We want 2 small warehouses at Uttar Pradesh, definitely so let others happens as a as the mathematical problem will take over. So we can keep on adding the constraints to the main problem. We can keep on adding the constraints to the main problem.

And then you can see how the order quantities are behaving. The solution will give you how much quantity to be transported from which origin to which destination? How much quantity to be transported from which origin to which destination. Here for this you see x 1 is 20, x 3 is 20, 30, 40, okay yeah.

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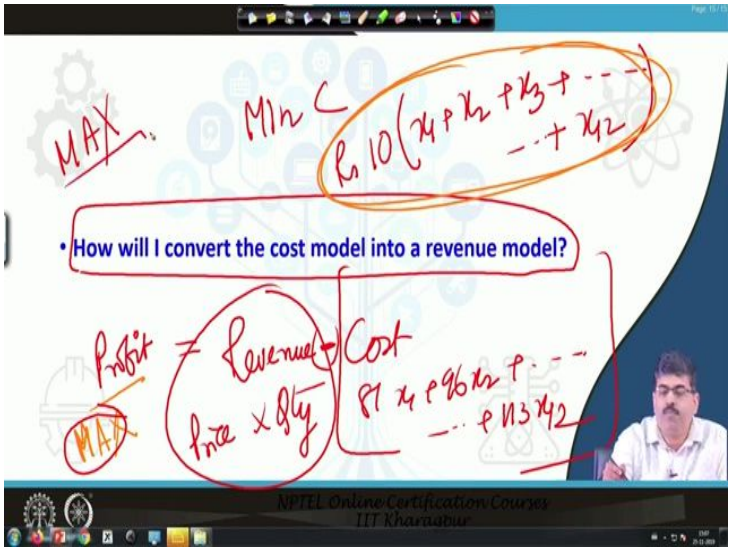

Now I have a trick question, and that question is, how will I convert this was the cost model. minimized cost. How will I convert the cost model into a revenue model? Because companies are not interested in cost. Companies are interested in how much profit they will generate? So next question is how will I convert the cost model into a revenue model? See all along we had this cost structure. Your profit your profit is what?

Profit is equal to revenue - cost. This cost data you got 81 x 1 + 96 x 2 + dot dot + 113 x 12. This is your cost data. right clear. What is your revenue? Your revenue is basically the market price into quantity. Revenue is price into quantity agreed What quantity what quantity? What will be the value of quantity? Say mrp price is mrp that is printed on the packet.

So price is basically let us say 10 rupees for a product rupees 10. What is the quantity? X 1 + x 2 + x 3 + x 12. That is your revenue. That is your revenue rupees 10 into yeah this is your revenue rupees 10 into x 1 + x 2 + x 3 + x 4 up to x 12. That is your revenue - this is your cost and profit. So you will have to change it and make it a maximization problem. That is the profit you have to change it and make it a maximization problem.

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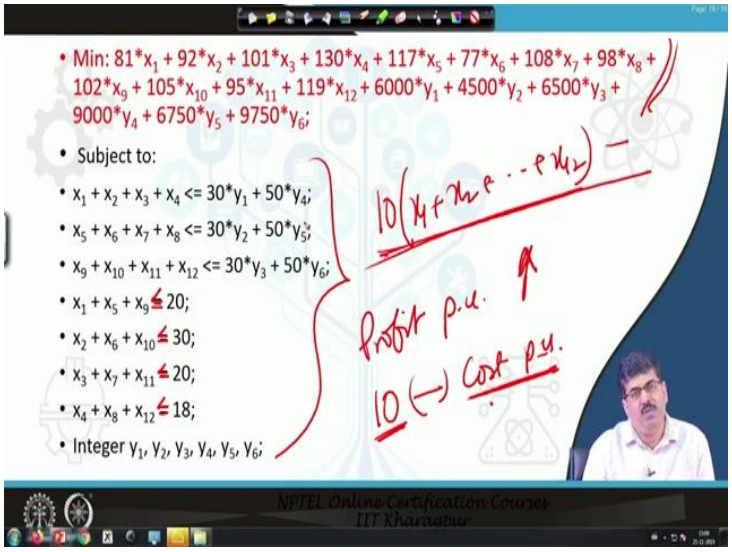

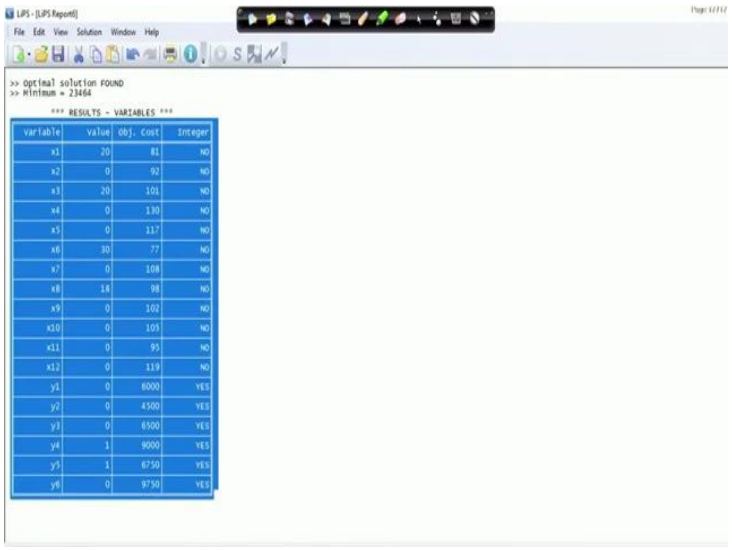

I have another question and that question is this was my original model. Now we said that this cost will remain this cost will remain and we will have a price of 10 into x 1 + x 2 dot dot dot up to x 12 - this cost will any of this constraint change? This is my revenue model, profit modelled, selling price - this cost that has to be maximized just now we showed you that. Will any of this cost… other this constraints changed. Answer is yes, constraints will change. How? See because it is a maximization problem. I will sell only in those areas where my profit per unit it is higher and profit per unit is what? 10 rupees - cost per unit.

10 rupees selling price is fixed. So that is profit is selling price - cost per unit. Sees my selling price is fixed, where were which which ever destination cost is less, cost of transportation is

less. I will send more and more products there. and try to sell it. So demand now cannot be equal if I put equal to sign then I am not maximizing the profit.

I will send more goods to those locations where my margin is more. That is revenue - cost. So in that way all demand from all the centers individual demand will not be satisfied. Total demand will be satisfied total production will be sold out. But individual city wise demand may not be satisfied. Some cities I will send more some cities I will send less based on my transportation cost, based on my variable cost.

So my demand equation will now be less than equal to, my demand equation will now be less than equal to supply any anyway was less than equal to my demand and equation will be less than equal to. That will convert the cost model into profit model. That will convert cost model into profit model. Clear? So with these we will our cost this cost models we are taking care of okay.

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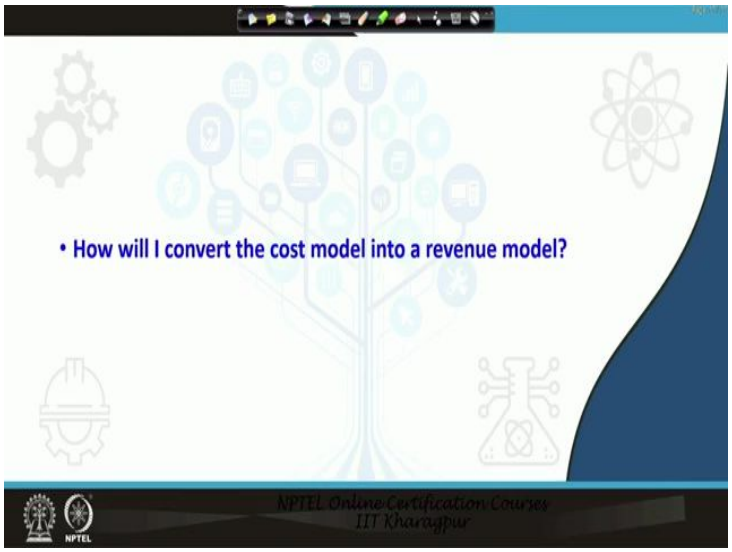

Now in the next session we will take up how will I convert the cost… how will I take care of the issue of goods and service tax that is gst in building this warehouse location models. Thank you.