

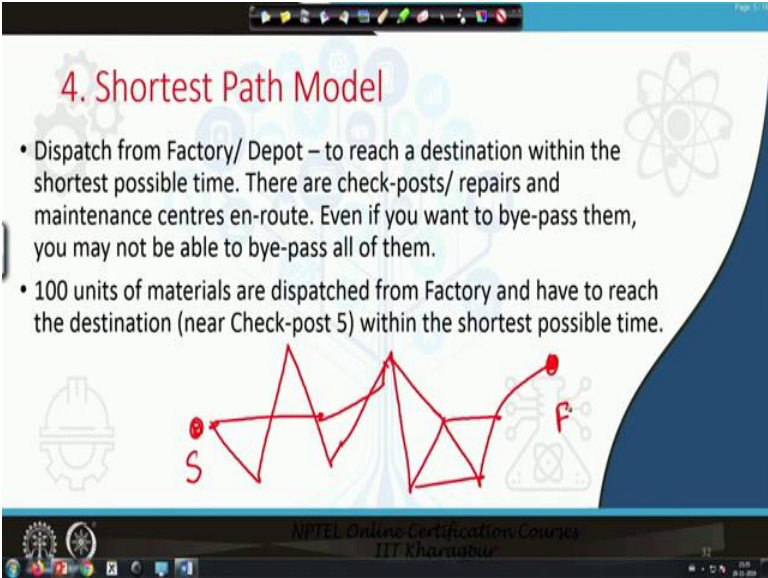
Modelling and Analytics for Supply Chain Management
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Lecture 15

Transportation Decisions: How to Take Decisions on Different Issues with Transportation?
(Contd.)

Hello, welcome to Modelling and Analytics for Supply Chain Management. We were in module 3 transportation decisions, right? And we have finished off with discussing on different types of cost models. Some of these we have finished off. We have finished up with the total cost model, we have finished up with having the Min-Max model that is we want certain quantities to be sent to certain routes. And we then decided on the maximum flow model, for example, if you are having a GIT system, what should we do? Now, today we will discuss on the shortest path model.

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4. Shortest Path Model

- Dispatch from Factory/ Depot – to reach a destination within the shortest possible time. There are check-posts/ repairs and maintenance centres en-route. Even if you want to bye-pass them, you may not be able to bye-pass all of them.
- 100 units of materials are dispatched from Factory and have to reach the destination (near Check-post 5) within the shortest possible time.

The slide features a network diagram with a source node 'S' and a destination node 'F'. A red path is highlighted, showing a zigzag route through several intermediate nodes. The diagram is set against a background with faint icons of a hard hat and a gear.

Shortest path model means I have a factory here, I have a factory here and, sorry, I have a supplier source here and my factory is here. Now, there is no direct road from S to F, there is no direct route, rather there is a zigzag of different combinations of routes that are possible. So, some routes will be like this, some routes will be like this. So, we have to find out the shortest possible route from my supplier to my factory and definitely shortest possible route will mean shortest time and shorter or lesser cost.

So, today's discussion is how to find out the shortest possible route or shortest path model. Now, 100 units, 100 units of materials are dispatched from factory and have to reach the destination near check post 5 within the shortest possible time. So, how should we design this model? That is what our job today is.

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The slide contains five tables and a diagram illustrating a network of roads between a factory/depot and five check-posts.

Factory/Depot	To	Distance (km)
Factory/Depot	Check-post1	20
	Check-post2	25

Check-post3	To	Distance (km)
Check-post3	Check-post1	4
	Check-post4	5
	Check-post5	4

Check-post1	To	Distance (km)
Check-post1	Check-post2	2
	Check-post3	3
	Check-post5	10

Check-post4	To	Distance (km)
Check-post4	Check-post2	6
	Check-post3	5
	Check-post5	8

Check-post2	To	Distance (km)
Check-post2	Check-post1	3
	Check-post4	6

The diagram shows a network of roads connecting the points. The factory (F) is connected to CP1 (20 km) and CP2 (25 km). CP1 is connected to CP2 (2 km), CP3 (3 km), and CP5 (10 km). CP2 is connected to CP4 (6 km). CP3 is connected to CP1 (4 km), CP4 (5 km), and CP5 (4 km). CP4 is connected to CP2 (6 km) and CP5 (8 km). CP5 is connected to CP1 (10 km) and CP4 (8 km).

Now, see what is happening, here is my factory. From factory, I am drawing this the top, the top box, this is my factory, from factory there are 2 roads that are going, 1 is check post 1 and the other is check post 2, say factory to check post 1 or to check post 2. What is the distance from the factory to check post 1? Distance is 20 kilometers. What is the distance from factory to check post 2, distance is 25 kilometers.

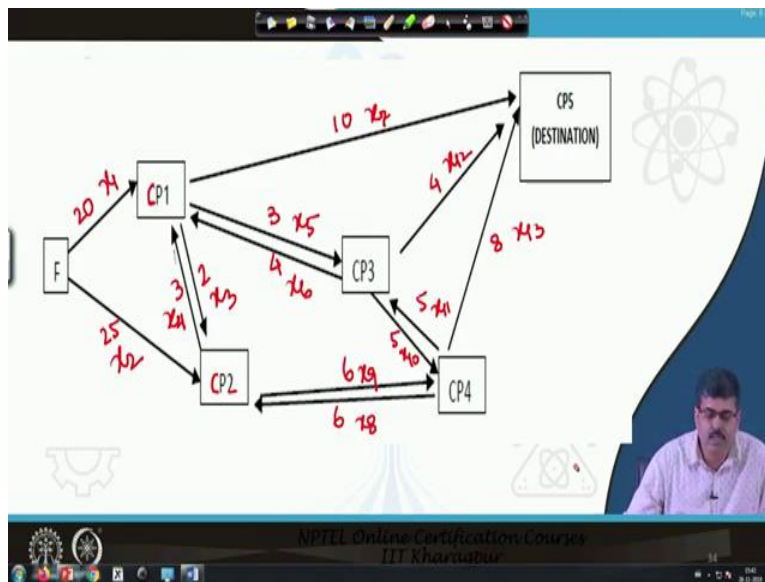
Similarly, from check post 1 see the next, the next box from check post 1 the road takes you to check post 2, there is another road that takes you to check post 3 and there is another road that takes you to check post 5. So, if we can say from check Post 1, there is another road that checks you to check post 2, another road that takes you to check post 2, there is another road that takes you to check post 3 and there is another road that takes you to check post 5.

And then from check post 2 if you see there is another road that takes as we mentioned that there is a road that takes you to check post 1. From check post 2 to check post 1 which I have already

drawn and there is another route that takes you to check post 4. So, in this way, you see what you can do is you can start drawing a network diagram.

You can start drawing a diagram. And then what is the distance from factory to check post 1, 20 kilometers, you see distance from factory to check post 1 is 20, factory to check post 2 is 25. So, these are the things that you can keep on writing down. And what is our job, from factory to destination end point we have to find the shortest possible distance.

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So, what we will do now is, this is if you map the drawing or the diagram from the previous table, if you can draw the network diagram that we were drawing by hand here, we were drawing by hand here, this drawing will lead you to this 1. This drawing will lead you to this 1. Factory to check post 1, factory to check post 2, et cetera. Now, and so if you see, so what you can do, this was my, this was my, this thing 20 units, factory to check post 2 was 25 units, check post 1 to check post 2, check 1 to check post 2 see this arrow, this arrow 2 units check post 1 to check post 3, this arrow is 3 units and check post 1 to check post 5 this arrow is 10 units. Clear.

So, then you see check post 2 to check post 1, see check post 2 to check post 1, what are these 20, 10, 3 distances, road distance, say kilometer. From factory to check post 1 is 20 kilometers, check post 1 to check post 5 is 10 kilometers. Now, from check post 2, if you keep on going back to this problem from check post 2, from check post 2 to check post 1 distance is 3 kilometers,

now see check post 1 to check post 2, CP 1 to CP 2 is 2 kilometers, but CP 2 to CP 1 is 3 kilometers.

Why this discrepancy? Because there may be 1 way restrictions which will make me do a circuit. So, that is why the distances are not same and from CP 1 to CP 5 we have done, that CP 1 and from CP 2 there is a check post 4 it is again 6 kilometers. So, in this way you can keep on putting the numbers. In this way you can keep on putting the numbers.

Then CP 3 to CP 1, let us see, CP 3 to CP 1 is 4 kilometers, CP 3 to CP 4 is 5, CP 3 to CP 5 is 4 kilometers and CP 4 to CP 2, this back arrow, this back arrow, 6, CP 4 to CP 3, this back arrow is 5 and CP 4 to CP 5 is 8. So, you see all our arrows, all our road distances are now marked, all our road distance is are now marked. So, this is the kilometers, road distances in kilometers from 1 point to other.

Remember the road distances in kilometers from 1 point from 1 point to the other. What is our job? 100 units have to move from factory to the destination point check post 5, where my some other unit is there, or where some manufacturing facility or wholesale go-downs or warehouses are there. So, 100 units have to move. This is the kilometers.

So, what is my job? How to move so that my total distance traveled is the lowest. Total distance travelled lowest means time taken is also lowest and cost is also the least. So, we do not know which route it will take. So, x1 quantity is moving, x2 quantity is moving from here, x3, x4, x5, x6, x7, x8, x9, x10, x11, x12, x13. So, these are the possible quantities, these are the, this is the quantity that can move through this route, agreed? So, this is the like the previous day this the same type of a problem.

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• min:
 $20x_1 + 25x_2 + 2x_3 + 3x_4 + 3x_5 + 4x_6 + 10x_7 + 6x_8 + 6x_9 + 5x_{10} + 5x_{11} + 4x_{12} + 8x_{13}$;

• Subject to:

• row1: $x_1 + x_2 = 100$;

• row2: $-x_1 - x_3 - x_4 + x_5 + x_6 + x_7 = 0$;

• row3: $-x_2 + x_4 - x_8 - x_3 + x_9 = 0$;

• row4: $-x_6 - x_{11} - x_{12} + x_5 + x_{10} = 0$;

• row5: $-x_8 - x_{10} - x_{13} + x_{11} + x_9 = 0$;

• row6: $x_7 + x_{12} + x_{13} = 100$;

Net $x_1 + x_4 = x_3 + x_5 + x_7$

CP1 (at CP1)
 CP2 (at CP2)
 CP3 (at CP3)
 CP4 (at CP4)

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So, what is our objective? Our objective is to minimize the total distance traveled. Minimize the total distance travelled, distance traveled is 20, say the first 1, take the first 1, this 20 kilometers distance, 20 kilometers distance how much quantity is being taken, x_1 . 25 kilometers is the distance from 1 check post to the other, how much quantity is being taken, x_2 .

So, in this way we model and the objective is to minimize this total distance that is traveled, minimize the total distance that is traveled. What are the constraints? The constraints are x_1 plus x_2 is equal 100, remember. This was my factory, right, and from factory, it was going to check post 1 and another was going to check post 2. How much quantity and this was x_1 and this was x_2 .

Remember how much quantity is being dispatched from the factory? 100 units. So, x_1 plus x_2 is the total movement from the factory; x_1 plus x_2 . So, x_1 plus x_2 should equal to 100 units, x_1 plus x_2 should equal to 100 units. Now, if you remember this check post 1 from check post 1, 1 road was coming to check post 2 and another was going to check post 3 and another was going to check post 5, from check post 1.

If you remember from check post 1, 1 road was coming to check post 2, another was going to check post 3, another was going to check post 5. The 1 that was coming to check post 3 we have mentioned is x_3 , x_5 , 1 that was coming to check post 2 is x_3 , x_5 and this was x_7 . Now, the

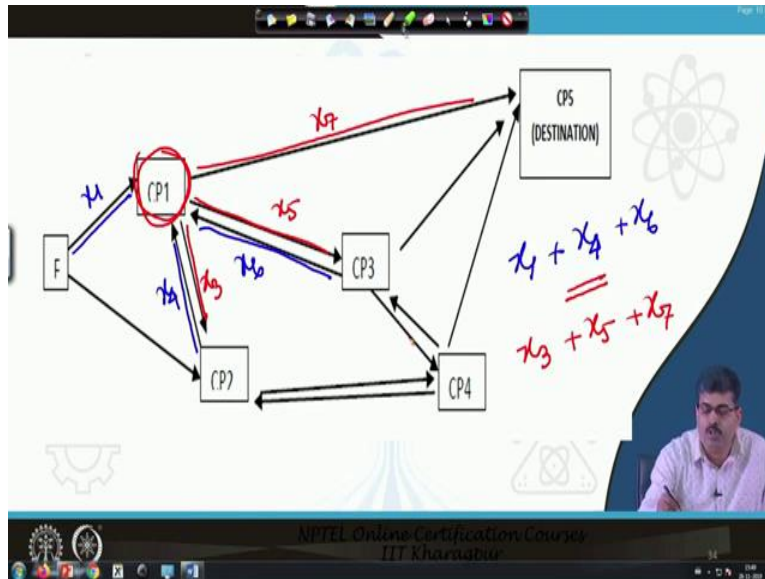
products that are moving from the factory to check post 1 has to move out from check post 1, no unit of the product will stay back at check post by 1.

Whatever is coming into check post 1 is moving out from check post 1, nothing is staying back. So, at check post 1 what is happening, what is coming in, what is coming in, accha another 1, sorry I missed out, that is there was another arrow that is from check post 2 there was a back end road that was there, and we had called this as x4. So, what are the entry points to check post 1, x1 quantity from factory or x4 quantity from check post 2, x1 quantity from factory x4 quantity from check post 2.

So, at check post 1 how much can come in x1 plus x4. At check post 1, how much can come in? How much quantity can come in? x1 plus x4. So, x1 plus x4. How much will have to go out? x3 has to go out from check post 1 plus x5 plus x7. What is going out? x3 quantity plus x5 quantity plus x7 quantity. So, at check post 1 material in is equal to material out, nothing can stay back in check post 1.

So, x1 plus x4, material in x1 plus x4 is equal to material out x3 x5 x7 is equal to x3 x5 x7. This is your second equation, x3 x5 x7 clear, and x1 x5 and there is another 1 that is going out to check post 3, yeah, sorry, another 1 is that is x6. So, we have taken all the move out, moving out on 1 side and we have made them equal to 0. So, basically it is material in is equal to material out, material in is equal to material out. Nothing will stay back at this check post.

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So, just a recap, let us go back to the previous diagram; whatever is coming in, whatever is coming in, whatever is coming in, these are the arrows, the blue arrows are materials that are coming in to check post 4, 3 arrows are coming in that is x_1 x_4 and x_6 that should go out.

What is going out? This one this one and this one. So, x_3 x_5 and x_7 . So, what is happening at check post 1, x_1 plus x_4 plus x_6 the blue colored ones, x_1 plus x_4 plus x_6 should equal to rather let me place it in blue then you will not get going confused, Let me place it in blue. So, what is coming in? x_1 plus x_4 plus x_6 . What is going out? What is going out? In red x_3 x_5 x_7 and these should be equal because nothing can stay back at check post 1 and this applies to all the check posts.

So, my objective is to minimize the total, my objective is to minimize the total distance moved, subject to these conditions that nothing will stay back at the nodes. Nothing will stay back at the notes. Agreed? This is what we wanted to say.

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• min:
 $20x_1 + 25x_2 + 2x_3 + 3x_4 + 3x_5 + 4x_6 + 10x_7 + 6x_8 + 6x_9 + 5x_{10} + 5x_{11} + 4x_{12} + 8x_{13}$;

• Subject to:

- row1: $x_1 + x_2 = 100$;
- row2: $-x_1 + x_3 - x_4 + x_5 - x_6 + x_7 = 0$;
- row3: $-x_2 + x_4 - x_8 - x_3 + x_9 = 0$;
- row4: $-x_6 - x_{11} - x_{12} + x_5 + x_{10} = 0$;
- row5: $-x_8 - x_{10} - x_{13} + x_{11} + x_9 = 0$;
- row6: $x_7 + x_{12} + x_{13} = 100$;

Handwritten annotations: A red bracket groups rows 1-6 with the text "Mat. in = Mat. out". To the right, "at CP1", "at CP2", "at CP3", and "at CP4" are written next to rows 2, 3, 4, and 5 respectively.

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So, this is my, all these constraints are given here, all these constraints we have put equal to on the right hand side just for modelling simplicity purpose, but otherwise it is x_1 plus x_2 plus x_3 is equal to x_4 plus x_5 plus x_6 material in material out, material in is equal to the material out. Remember this material in is equal to material out because we will require this in the subsequent lecture sessions also.

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• LIPS1.11.1 output:

- Optimal solution FOUND :: LIPS 1.11.1
- >> Minimum = 2700
- i.e. 27 kms x 100 units = 2700; therefore,
- shortest distance = 27 kms; equivalent to shortest possible time

Handwritten annotations: "100 units (x)" is written above a box containing "27 kms". Below the box, "= 2700" is written and underlined in red.

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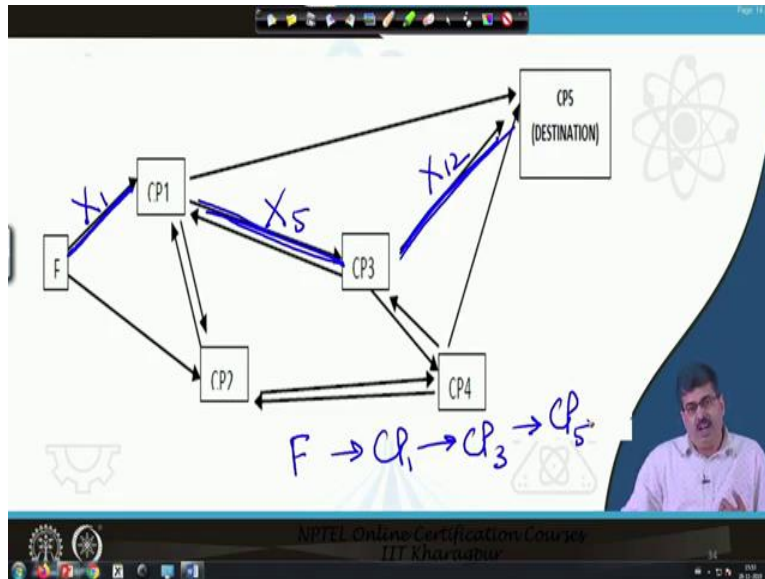
So, once we solve this, once we solve this in LiPS we get A value of 2700. We get a value of 2700. What is this 2700? 100 units multiplied by 27 kilometers is equal to 2700. So, what is the minimum distance travelled? Minimum distance traveled or minimum distance possible is 27 kilometers, agreed? So, 27 kilometers is the shortest distance from factory to my destination point.

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Variable	Value	Obj. Cost	Reduced Cost
x1	100	20	0
x2	0	25	-3
x3	0	2	0
x4	0	3	-5
x5	100	3	0
x6	0	4	-7
x7	0	10	-3
x8	0	6	-3
x9	0	6	-9
x10	0	5	-1
x11	0	5	-9
x12	100	4	0
x13	0	8	0

What is the route, the route is x1 to x5 and then to x12. Remember x1 to x5 to x12, this is my route.

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Let us go back and see what this route was. What was x_1 , x_1 was this, then x_5 was this, and x_{12} was this, all the others are 0, we have seen 100 in these routes. So, x_1 to x_5 to x_{12} . So, my route is factory to check post 1 to check post 3 to check post 5, check post 5 this is my route. I will write it in a fresh manner just give me this space. Factory to check post 1 to check post 3 to check post 5, this is my shortest possible route.

So, when is this model and when is the earlier model, when this model is when you can move in and we do not want to determine what is the maximum number of goods that can be travelled, it is not a JIT system, this is a minimal cost or minimal time system. When there are network of roads and the up-road may have a particular distance and the down-road may have another type of a distance because of no entry restrictions. Now, this much is fine, there is no problems. Now, we have another type of a question in our mind.

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The slide is titled "5. Minimal Spanning Tree Model" in red text. Below the title is a bullet point: "Minimal Spanning Tree is a transportation issue from a materials management perspective". The central part of the slide features a network diagram with six red nodes connected by blue lines. A stick figure is positioned at the bottom-left node. The slide is part of an NPTEL Online Certification Course from IIT Kharagpur, as indicated by the footer. A video feed of a presenter is visible in the bottom right corner of the slide area.

That is, let us say, that is, let us say we are having and we are having a situation when I have, sorry, when I have, when I have 1 warehouse here, 1 warehouse here, 1 here, 1 here, and I have to pick up and these warehouses have to be linked, these warehouses have to be linked.

They are linked by a, so I want to find out... So, you see, so all the roads are to be linked, all the roads are to be linked. I want to find out, what is the minimal number of, minimal time required to move across all these points. What is the minimal time, not the route, remember I am not asking for, there is a basic difference between route planning and minimal time.

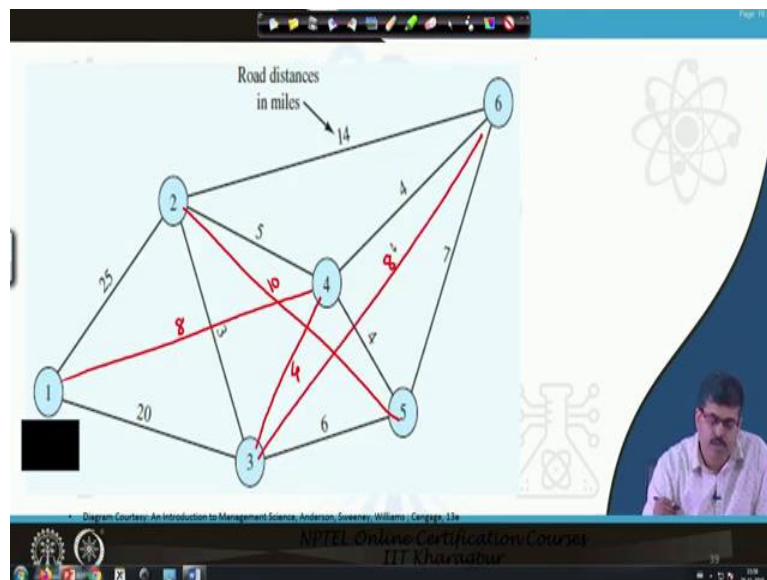
I am not asking that if a person starts moving from here, which route, which pattern should he follow. What I am asking is, which route or which pattern he should he follow that I am not asking, then we may say that he will start from here go there, go there, come there, come here and come back here, that is which route, the sequence. What I am asking is that what is the minimum distance that is to be traveled to cover all the routes, not the sequence, the routes have to be covered.

We are not looking at that we are not asking that, what is the sequence we are not asking, how should he cover, which city should come in first that we are not asking. We are asking he has to cover all the places in 1 day, how much distance he has to travel. Because the distance travelled

among warehouses that will determine your overtime costs and that will increase your warehousing cost.

So, what is the idea, minimize the travel distance. So, we want to know what is the travel distance, what is the total travel distance. Have I been able to explain, what were we doing? We are saying that the person has, it is not the sequence by which he has to go and cover all these, we are saying what is the distance that he has to cover if he wants to cover all these places. This is the problem.

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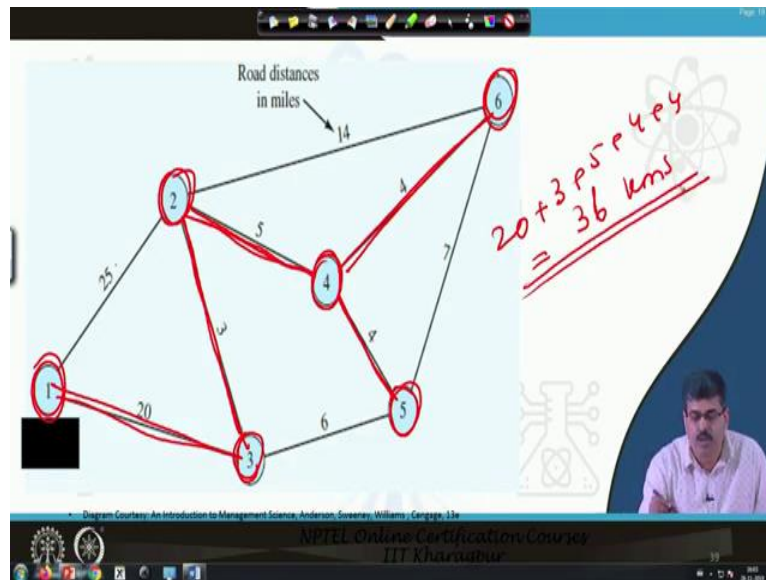


So, let us see, this is, this is a, this thing, this is a diagram or schematic layout diagram we can see that there is a workshop 1 or factory 1 (whatev) or warehouse 1, whatever you want to say. This is a workshop 1, workshop 2, workshop 3, workshop 4, workshop 5, workshop 6. Within a big factory there may be (simps) many workshops like machine shop, lathe shop, heat chamber, cooling chamber, so many types of workshops are there.

So, the issue is that these are the locations of this workshops. If a product has to travel all these workshops, then what is the minimal travel time taken or minimal distance taken? You can say these are the travel time or the travel distance. Not the sequence, in which sequence the product will go from 1 workshop to other that we are not asking, we are asking the minimal travel time.

Say for example, you have a vehicle in the morning that leaves your factory, transportation, your vehicle in a, in the factory it leaves from the factory, factory is here and it has to go to all these warehouses or all these distributors, it has to go to all these warehouses and all these distributors, alright. It has to, there is one arrow there, there will be one arrow here. It has to let us say this is 4 and let us say this is 8. So, it has to, it has to travel to all these spaces. Let us say 10, 8.

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Anyway, anyway let us forget this. Let us stick to this diagram only. So, what we, we are not asking for the sequence, we are saying what is the distance that the work that the that the product has to travel across. Similarly, one vehicle is moving from your factory at one here, one vehicle is moving from your factory.

It has to cover all these wholesalers or distributors or warehouses or retailers whatever they are, but it has to cover all of them. What is the minimal travel distance? Why is it necessary? Because, if the minimal travel distance is such that it cannot be covered in one day. Then what will I do? I will make this as one segment and this is another market segment, this as another market segment.

This part is one segment which can be covered in one day, this part is another region which has to be covered in another day, clear. So, that is why this is required. So, we want to know what is the travel distance. This is helpful for market segmentation, vehicle planning also. If I know that

one vehicle can cover only this much and in one day because of the time, this may be distance, this may be time same thing.

If I know that one vehicle can cover only this much of a distance in one day, so one center remains pending. So, I will put them in another, another area or another location. So, how to do this? Let us see how to do this. Idea is start from here. What are the 2 options 25, 20. Go for the lowest distance or lowest time. So, 1 is covered, then 3 is covered; I am not saying it should go from 1 to 3, their sequence I am not saying, I am saying 1 and 3, this is 1 road, 1 distance.

Now, we will have to take these 2 roads which are circled means which have been covered, which have been touched. From these 2 circles, you see what are the areas where you can go, which have not been circled, these 2 is circled means these 2 have been connected by a road, connected by distance. So, now from here it is 25, from 3 it is 3, from 3 it is 6.

So, which one should we take, we should take the one with the lowest time or lowest distance. So, you see now, wholesaler or distributor 2 is also connected. So, wholesale factory, wholesaler 3 is connected, wholesaler 3 to 2, 2 is also connected. Now, now remember we start, we have, we were looking at 1 and 3 and we said this is 25 this is 3, this is 6, so we went for the lowest 1, assume this was also 3.

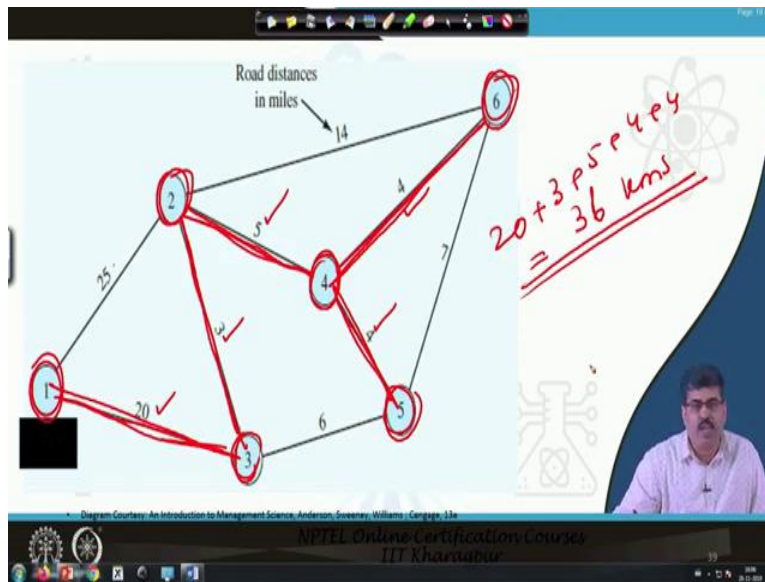
So, 1 could we, could have been connected to 2 or 3 is all, here also it is 3 to 3 could have been also connected. The choice is yours, you can take anyone. So, long as you go for the lowest cost you can take any route. So, 2 is connected. Now, you see 1 has been connected, 2 has been connected, 3 has been connected. So, forget the ones that are, forget the ones that are this side, no need now, move forward, from 3 you can connect 5, 6 kilometers or 6 minutes.

From 2, 5 kilometers to unit 4, 14 kilometers to unit 6. So, 6 kilometers, 5 kilometers 14 kilometer which is the lowest? 5 kilometers. Done. Now, Next one, which one will you consider the outer ones? So, 2, 4, 3 which ones the left? 14 kilometers? 4 kilometers, 4 kilometers, 6 kilometers. 14 is ruled out. 6 is ruled out so, you are left with 4 and 4.

Which one will you take? Let us take this one, let us take this one. Now what are left 2, 4, 5, what are left 2, 4 and 5. 2, 4, 5, 14 kilometers, 14, 4, 7, which one is lowest 4. So, you see one circle it, so you see all the wholesalers are covered and what is the total time taken or total

distance that has to be covered 20, 3, 5, 4, 4; 20, 3, 5, 4, 4. 28, 32, 36 kilometers, is the distance that had to be covered.

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Now, take a situation here there was an arbitrary, take a situation when we were, we were here, we were here. So, we had option 2, 5 and 3, sorry this 2, 4 and 3, how much is moving out 14, 4, 4, 6. So, 14 and 6 are ruled out, we have the option to take this route, we have the option to take this route. What option did we take earlier? This route.

It is arbitrary you take anyone. We took this route now let us take this one. So, this is my route. This is my route, this is not covered. This is not covered. So, what will happen? I have to cover either through this way or I have to cover to this way. Now, since this is the lowest or the cheapest I will not take this route I will also have to take this route.

So, what is the distance now? 20 plus 3, 23, 23 plus 5, 28, 32, 36. 20, 23, 28, 32, 36, same 23, 5, 4, 4. So, it asked me how much time or how much distance I have to cover if I am to cover all the places. Now, you will ask me but this worker coming here has to go back. So, if you are really taking the time you will have to double it.

Now, when is the single route model possible, say your laying a road or laying a cable you know exactly how much of cable is required to cover all the places, how much a cable is required to cover all the places. So, this is what we call as the minimal spanning tree problem. Now, with

this, with this we basically finish off with the transportation modelling and analytics part of Modelling and Analytics for Supply Chain Management.

In the next few lectures we will take you through the other dimensions of modelling. So, what have we covered till now, we have gone into the basics then we looked into supplier selection. Once the suppliers have been selected, the products are being transported. So, supplier selection modelling, transportation modelling and analytics. In the next sessions onwards we will take you through warehouse location decisions and warehouse location analytics.

So, once you have forecasted, how much is required, supply selection, transporting them to my warehouse, and then after transporting them it is where to locate warehousing location decisions. So, supplier selection, transportation, warehousing. So, next module in then next, next topic will be warehousing and warehousing related modelling and analytics. Thank you!