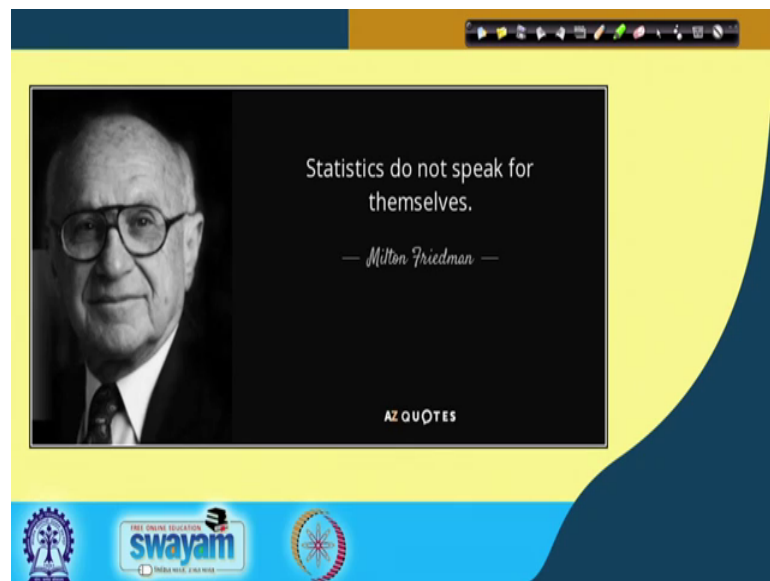


Six Sigma
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Lecture – 57
Design of Acceptance Sampling Plans for Variables

Hello friends, I welcome you to our ongoing journey on Six Sigma, and to remind you we are in the final phase of DMAIC cycle and we are discussing the control phase. So, as a part of control phase, we have talked about statistical process control, acceptance sampling. We have covered in detail the acceptance sampling plan for attributes and now as a part of lecture 57 we will focus on Design of Acceptance Sampling Plans for Variables.

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So statistics do not speak for themselves, we have to interpret it, we have to select the right strategy, so that we can infer the appropriate conclusions and set the necessary actions.

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Recap

- ❑ Lot-by-Lot Attribute Sampling Plans
 - Standard Sampling Plans
 - ANSI/ASQ Z1.4/ MIL –STD-105D
 - Dodge-Romig
- ❑ Deming's kp Rule

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So, if you see the recap, then the last lecture which was on attribute, acceptance sampling for attributes part 2, we talked about standard sampling plans and broadly. There were two; one is ANSI ASQ MIL STD military standard and another one was, say Dodge Romig acceptance sampling plan. We had also seen a very easy to use rule that is Demings kp rule and it considers the cost of rework, as well as the cost associated with finding of the non conforming items from the extra supply provided by the vendor.

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CONCEPTS COVERED

Concepts Covered:

- ❑ Advantages of Variable Plan
- ❑ Disadvantages of Variable Plan
- ❑ Variable Sampling Plans for a Process Parameter

Parameter

- ✓ Case 1:
Estimating Process Average: Single Specification Limit and Known Process Standard Deviation
- ✓ Case 2:
Estimating Process Average: Double Specification Limits and Known Process Standard Deviation
- ✓ Case 3:
Estimating Process Average: Single Specification Limit and Unknown Process Standard Deviation

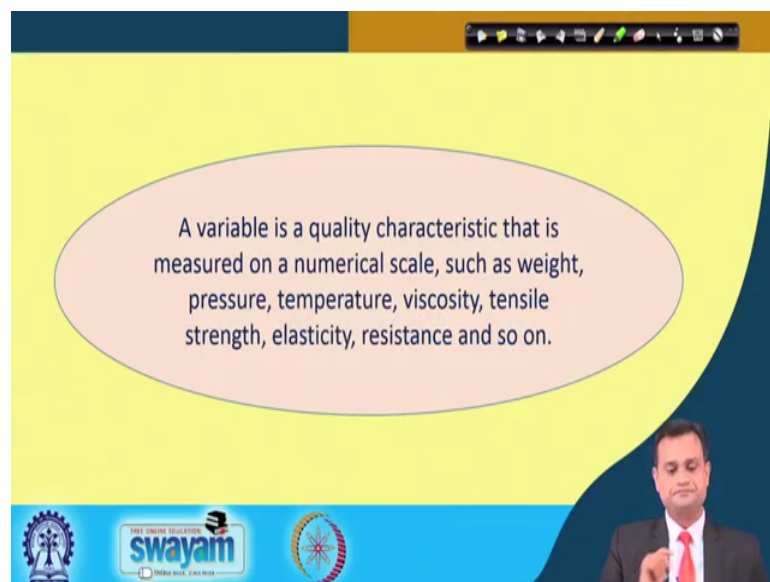
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So, we have appreciated this concepts, now as a part of this lecture on acceptance sampling plan for variables, we will focus on advantages of variable sampling plan, disadvantages of variable plan.

And we will discuss three cases as a part of variable sampling plan for process parameter. We have case 1; estimating process average, single specification limit. Suppose you are only interested in one specification limit; upper specification or lower specification and process standard deviation is known. Case 2, again estimating process average, double specification limits and known process standard deviation. So, in this case you are interested to deal with two specification limit; upper and lower both. If you see the case 3, then estimating process average, single specification limit and unknown process standard deviation.

So, here I am not very much clear or sure about the process standard deviation, my population parameter and it is unknown. So, we will deal with this three cases and see that how an appropriate acceptance sampling plan for variable quality characteristic can be developed.

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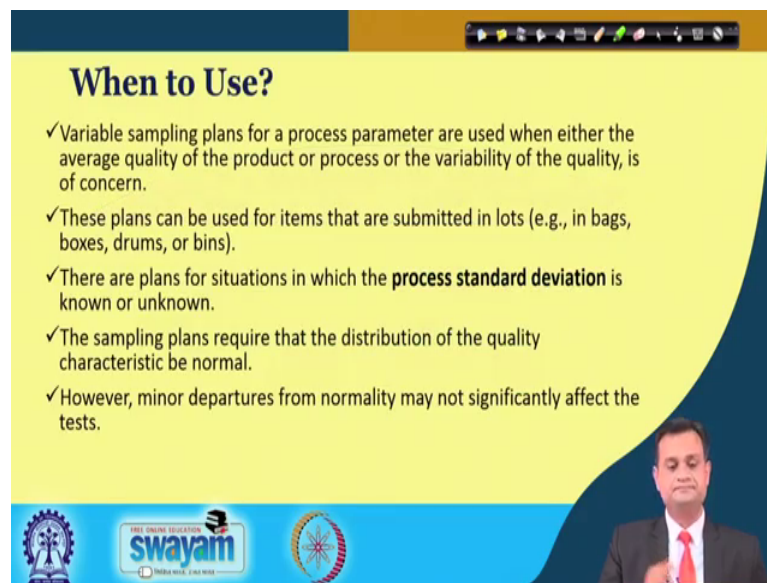


So, we know that variable quality characteristic is basically something which is measured on a numerical scale, and such as height, weight, pressure, temperature, viscosity, tensile strength and so on. So, we had a detailed discussion even on the

statistical process control, control charts for variables and we had seen X bar chart, r chart, s chart.

So, the variable quality characteristic is one which is measured on a scale and numerical scale, and there could be the examples like weight, height and others. So, this session is dedicated on development of say variable sampling plans or sampling plans for variable quality characteristic.

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The slide is titled "When to Use?" and lists five conditions for using variable sampling plans. It features a yellow background with a blue border on the right and bottom. A small video inset of a man in a suit is visible in the bottom right corner. The slide includes logos for IIT Bombay, Swayam, and the Ministry of Education, India.

- ✓ Variable sampling plans for a process parameter are used when either the average quality of the product or process or the variability of the quality, is of concern.
- ✓ These plans can be used for items that are submitted in lots (e.g., in bags, boxes, drums, or bins).
- ✓ There are plans for situations in which the **process standard deviation** is known or unknown.
- ✓ The sampling plans require that the distribution of the quality characteristic be normal.
- ✓ However, minor departures from normality may not significantly affect the tests.

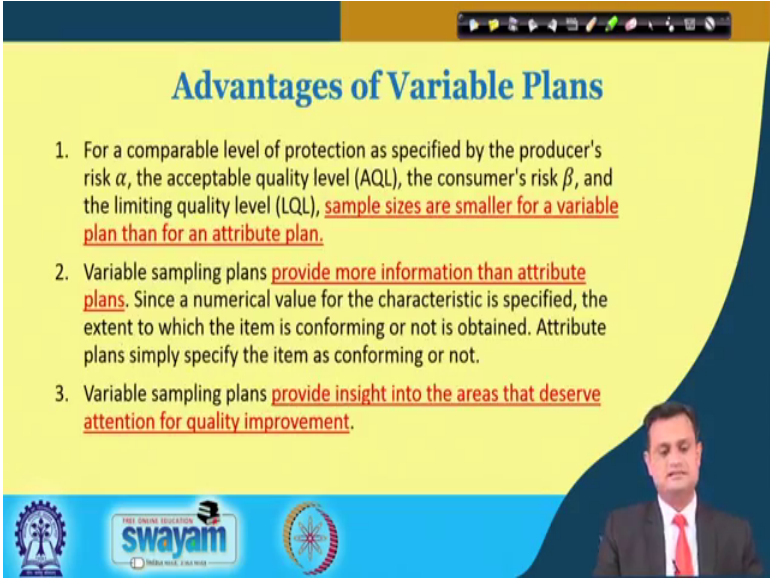
So, when to use? Before we start discussing the various cases and development of sampling plan, let us try to appreciate when would I like to go for this kind of sampling plan. So, typically when a process parameter used, when either the average quality of the product or process or the variability of the quality is of concern. So, I am concerned about the variability in the process or product quality, then there is a point that I would like to go for, say variable acceptance sampling plan.

Second is, this plans can be used for items that are submitted in lots; example in bags, boxes, drums and bins and I can effectively use the variable sampling plan. There are plans for situation in which process standard deviation is known or unknown. So, I may encounter both the situation and I can effectively use variable sampling plan for any situation where process standard deviation is known or it is unknown. The sampling plan requires that distribution of quality characteristic be normal. So, if this is the requirement then variable sampling plan could be a fit case, but if there is a little departure or

deviation from the normality, then this may not affect the sampling plan in its effectiveness significantly.

So, these are the cases which helps me to appreciate that what could be the appropriate case for using the variable Sampling plan.

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Advantages of Variable Plans

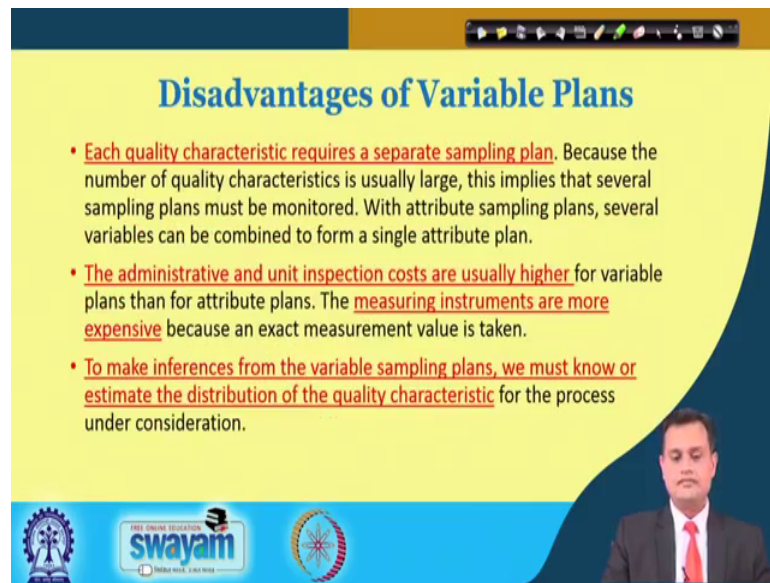
1. For a comparable level of protection as specified by the producer's risk α , the acceptable quality level (AQL), the consumer's risk β , and the limiting quality level (LQL), sample sizes are smaller for a variable plan than for an attribute plan.
2. Variable sampling plans provide more information than attribute plans. Since a numerical value for the characteristic is specified, the extent to which the item is conforming or not is obtained. Attribute plans simply specify the item as conforming or not.
3. Variable sampling plans provide insight into the areas that deserve attention for quality improvement.

There are couple of advantages of variable sampling plan that sample size you require for a variable sampling plan, is less than the which is required for the attribute sampling plan in order to have the same level of protection in terms of alpha, producers risk, beta consumers risk, acceptable quality level A Q L and limiting quality level L Q L. Second this plan provide more information, because actually you are measuring the quantity. You are not just saying acceptable or not acceptable good or bad, fair or poor. So, compare to attribute plans they provide more information about the process and we do not simply end up with the decision like confirming or not confirming.

Third variable sampling plans provide insight into the areas that deserved attention for quality improvement, because you are measuring the numerical value. And when I go with the acceptance sampling plan for variables, I develop greater insight into the improvement areas.

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Disadvantages of Variable Plans

- Each quality characteristic requires a separate sampling plan. Because the number of quality characteristics is usually large, this implies that several sampling plans must be monitored. With attribute sampling plans, several variables can be combined to form a single attribute plan.
- The administrative and unit inspection costs are usually higher for variable plans than for attribute plans. The measuring instruments are more expensive because an exact measurement value is taken.
- To make inferences from the variable sampling plans, we must know or estimate the distribution of the quality characteristic for the process under consideration.

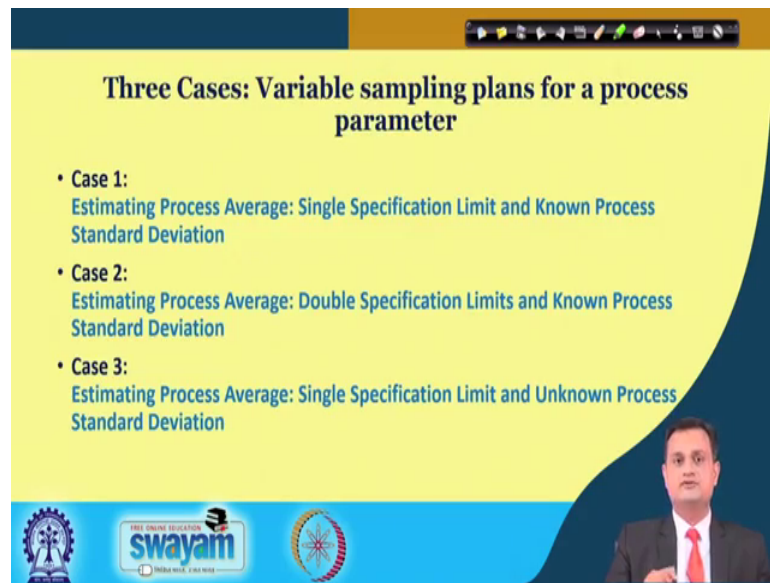
Logos for Swamyam and other institutions are visible in the footer.

Now, there are couple of disadvantages that for each quality characteristic you need to have a separate variable sampling plan. So, there could be many quality characteristic. For example, if you take bearing then you might be interested in surface finish, outer diameter, inner diameter and thickness, width. So, there could be many quality characteristic and each one of these might be critical. So, you may have to say develop the separate variable acceptance sampling plan for each quality characteristic.

In case of attribute you just say, whether it is good or bad, and there is no need to develop a separate sampling plan for the different quality characteristic, but here this administrative cost of maintaining the separate sampling plan we have to accept. Then the administrative and unit inspection costs; obviously, are high, because you may have to maintain more number of variable sampling plan and also you need very precise and accurate measuring instrument. And finally, to make inferences from variable sampling plan, we must know or estimate the distribution of the quality characteristic for a particular process which is under consideration.

So, this information we need if we have to execute with the variable sampling plan and this can be seen as couple of disadvantages. Now we have three different cases as I mentioned right in the concepts covered.

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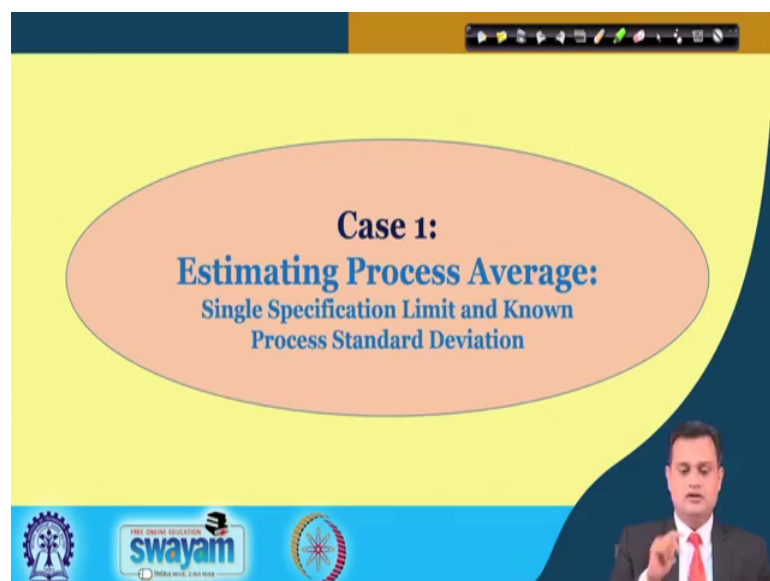
Three Cases: Variable sampling plans for a process parameter

- **Case 1:**
Estimating Process Average: Single Specification Limit and Known Process Standard Deviation
- **Case 2:**
Estimating Process Average: Double Specification Limits and Known Process Standard Deviation
- **Case 3:**
Estimating Process Average: Single Specification Limit and Unknown Process Standard Deviation

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So, variable sampling plan, we have three cases. Case 1 estimating process average; single specification limit and known process standard deviation. Case 2 estimating process average, double specification limit, known process standard deviation. So, first case is about single specification limit, second is about double specification limit and in third case I am focusing on single specification limit, but my process standard deviation is unknown.

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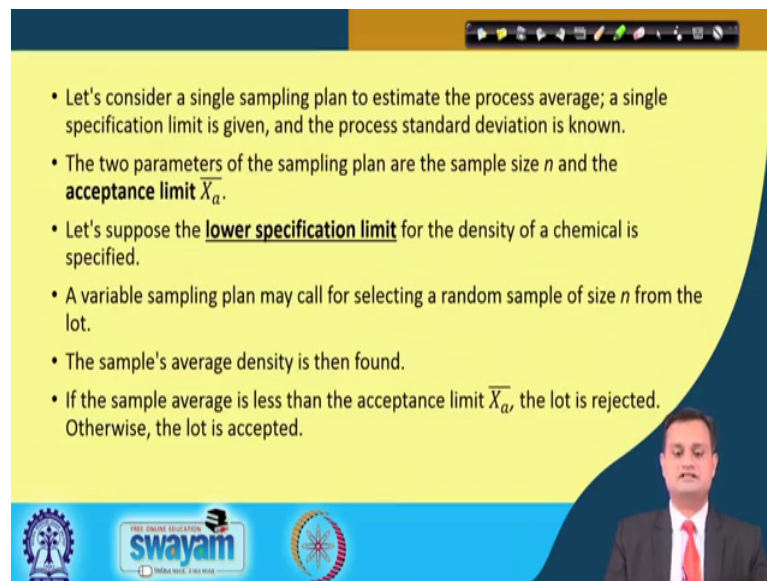
Case 1:
Estimating Process Average:
Single Specification Limit and Known
Process Standard Deviation

This slide is a zoomed-in view of the first case from the previous slide. It has a yellow background with a dark blue wavy border on the right. The text is centered in a light orange oval. At the bottom, there are logos for 'swayam' and other educational institutions, along with a small video inset of a man in a suit.

So, now, let us try to discuss each case and appreciate that how can I develop a variable sampling plan for making a decision of accept or reject for a particular lot and how it can be executed.

. So, case 1 estimating process average, it is a case of single specification limit and known process standard deviation.

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- Let's consider a single sampling plan to estimate the process average; a single specification limit is given, and the process standard deviation is known.
- The two parameters of the sampling plan are the sample size n and the acceptance limit $\bar{X}_a$.
- Let's suppose the lower specification limit for the density of a chemical is specified.
- A variable sampling plan may call for selecting a random sample of size n from the lot.
- The sample's average density is then found.
- If the sample average is less than the acceptance limit $\bar{X}_a$, the lot is rejected. Otherwise, the lot is accepted.

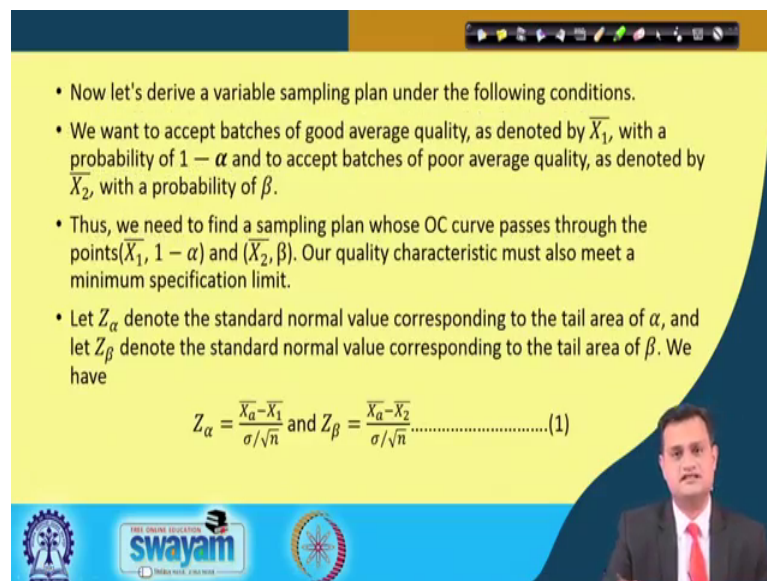
So, let us see that what we are exactly trying to do, but I would just like to remind you that in case of attribute sampling plan we had the acceptance number, rejection number. And based on that we were trying to say take the decision and also the discriminatory power, means the slope of the O C curve had lot of influence on my producers risk, consumers risk or ability to detect. Here also more or less I need to follow the same procedure, but because it is a variable quality characteristic I have to set the decision rule, I have to fix or decide about the acceptance criteria, and for that we just follow a simple procedure.

So, let us consider a single sampling plan to estimate the process average and a single specification limit is of interest or given and the process standard deviation is known. NOW two parameters of the sampling plan are sample size n and acceptance number $\bar{X}_a$, you need to have in order to execute your variable sampling plan. So, you need to have n and you need to have $\bar{X}_a$. So, n is your sample size and $\bar{X}_a$ is basically the decision rule, that whether you should accept a particular sample or you should

reject, so its a critical value $\bar{X}$ a bar acceptance number and this I need to determine. So, when I have the case of lower specification limit, for let us say density of chemical is specified and a variable sampling plan may call for selecting a random sample of n from the lot.

And I will try to say find out the sample average for each particular sample. And if it is less than the acceptable limit which is $\bar{X}$ a bar then the lot is rejected; otherwise lot is accepted. I am referring the case of lower specification limit, and hence my decision rule goes like this. So, lower specification limit means, anything which is falling below lower specification is of no use, it will simply be scraped, it will be rejected. And that is why I am setting the threshold value critical value in terms of $\bar{X}$ a bar. So, anything which is a sample average, which is less than the $\bar{X}$ a bar, you reject the particular sample or the lot and this is the case. If you assume upper specification limit it will just be reverse, because in that case anything which is falling above the upper specification limit is a problem and I have to reject it.

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- Now let's derive a variable sampling plan under the following conditions.
- We want to accept batches of good average quality, as denoted by $\bar{X}_1$, with a probability of $1 - \alpha$ and to accept batches of poor average quality, as denoted by $\bar{X}_2$, with a probability of β .
- Thus, we need to find a sampling plan whose OC curve passes through the points $(\bar{X}_1, 1 - \alpha)$ and $(\bar{X}_2, \beta)$. Our quality characteristic must also meet a minimum specification limit.
- Let Z_α denote the standard normal value corresponding to the tail area of α , and let Z_β denote the standard normal value corresponding to the tail area of β . We have

$$Z_\alpha = \frac{\bar{X}_a - \bar{X}_1}{\sigma/\sqrt{n}} \text{ and } Z_\beta = \frac{\bar{X}_a - \bar{X}_2}{\sigma/\sqrt{n}} \dots \dots \dots (1)$$

So, this is all you can set the rule for single specification, rather it is lower or whether it is upper that is the interest for a particular process. So, now, let us try to appreciate couple of things, which can help us to derive a sampling plan for variable. So, what we want? We want to accept the batches of good average quality, as denoted by $\bar{X}_1$ bar. So, please try to understand couple of things that I have a process and this process is

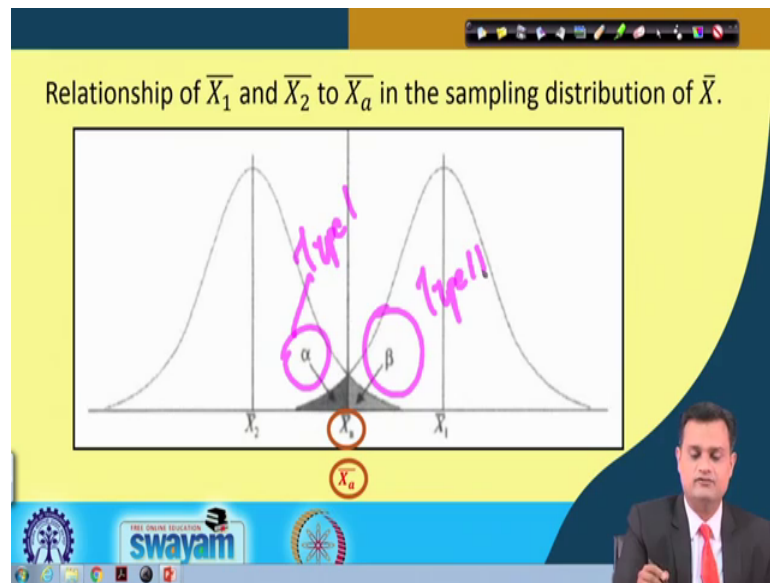
producing some component or some product and I want to say have the average of good batches. So, I want to accept the batches of good average quality and this is denoted by $\bar{X}_1$ with a probability $1 - \alpha$.

Now you can understand $1 - \alpha$ why it is α is producers risk. So, if it is 5 percent then 95 percent time I should be able to accept the batches which is of good quality. And accept the batches of poor average quality as denoted by $\bar{X}_2$ with a probability β . So, I hope it is clear $\bar{X}_1$ is specific to the average quality, specific to good batches. $\bar{X}_2$ is basically average quality of poor batches and these are the two notations we will use. NOW we need to find a sampling plan whose O C curve will pass through $\bar{X}_1$ $1 - \alpha$ and $\bar{X}_2$ β . If you recall we said the O C curve similarly for attribute where we use the A Q L and L Q L as the criteria.

In this case $\bar{X}_1$ $1 - \alpha$ and $\bar{X}_2$ β are the points, and I want my O C curve to pass through these points in order to be acceptable to both my consumer as well as producer. So, now, Z_α denote the standard normal value corresponding to the tail area of say α . So, α is your basically rejection region, and it is called producers risk. And Z_β denote the standard normal value corresponding to the tail, area of β , and we can have Z_α is equal to $\bar{X} - \bar{X}_1$ divided by $\sigma \sqrt{n}$. So, it is a very simple expression, usually you write like this $\bar{X} - \mu$ by $\sigma \sqrt{n}$. Here this is replaced by $\bar{X} - \mu$.

This μ is basically replaced by $\bar{X}_1$ and $\bar{X}_1$ is basically the average quality of good batches. And σ is replaced by $\sigma / \sqrt{n}$, because I am going by central limit theorem. Same way you can have the expression for Z_β . So, $\bar{X} - \bar{X}_2$ divided by $\sigma \sqrt{n}$.

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So, once you have appreciated this then you can very well see that what exactly I want to say. Now here what do you see? You see $\bar{X}_1$. And I would like to remind you that my interest is in developing a sampling plan, variable sampling plan with one specification limit and here I am bothered about the lower specification limit.

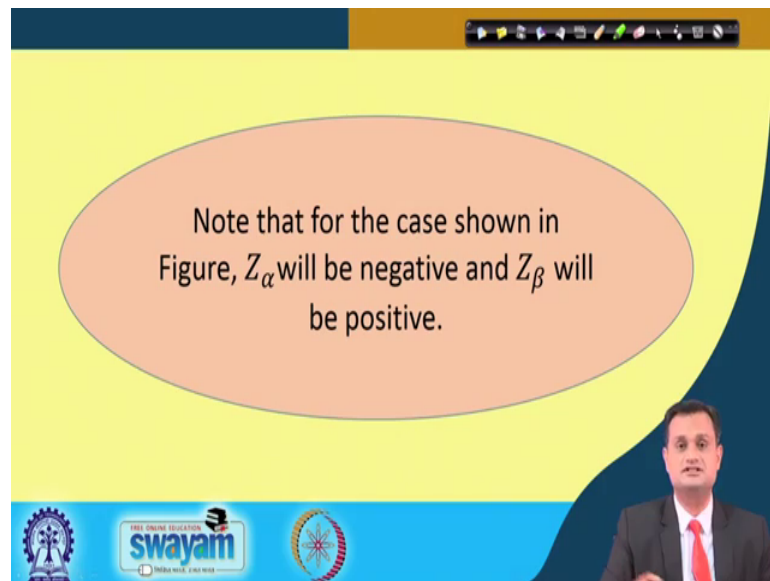
So, now just see that what is happening. Forget about this second this particular curve, just forget. Focus on these and this is my average quality for good batches or good lots. And in this case I have this much alpha, so if it will fall below this then it would be reject. Now just see I have set here $\bar{X}_a$. This is basically $\bar{X}_a$, so $\bar{X}_a$ is my threshold limit, anything which is going below for a lower specification case is of reject. So, this is my alpha, this is basically my $\bar{X}_a$. Now when I look at the other process this one, now what you see here this is $\bar{X}_1$ bar, this is $\bar{X}_2$ bar. So, what is $\bar{X}_2$ bar? It is a same process, no change in process, but a process can produce the good quality lots, as well as a process can produce bad quality lots.

So, $\bar{X}_2$ bar is the average quality specific to the bad quality lots, poor quality lots and you can see that there is some shift in my process. So, let us say with respect to lower specification limit, I am finding that this is the $\bar{X}_2$ bar which has got shifted towards the lower specification limit and $\bar{X}_2$ bar is the average quality for the poor quality batches, and when you see this you will find that this is my beta. When you see both the curve when there is a shift in the process, you will say that this is beta. It means when there is a

shift, we have already discuss this concept, when there is a shift in the mean value of the process then the area which falls in a particular acceptance zone, I will consider that as the beta, this is type 2 error.

So, this is what is type 2 error here, and I am considering both producers risk that is alpha, consumers risk that is beta, type 1 error. This is type 1 error, this is type 2 error and I would like to develop my acceptance sampling plan considering both.

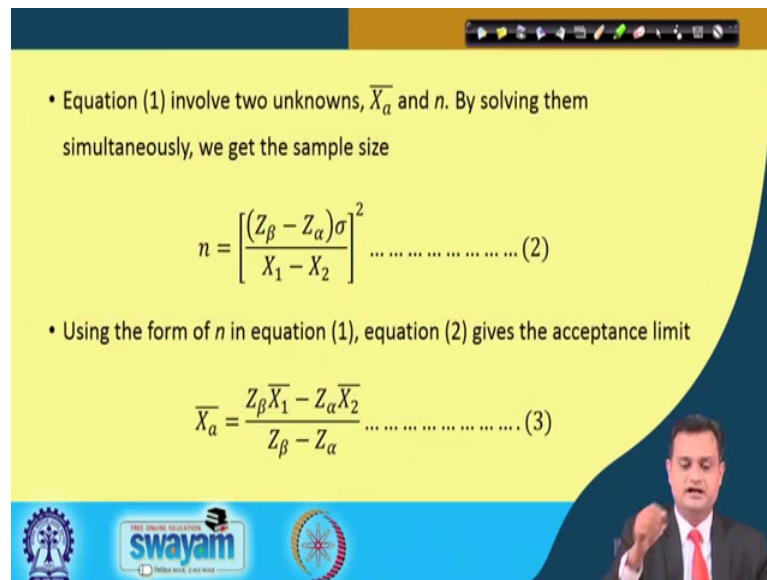
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Note that for the case shown in Figure, Z_{α} will be negative and Z_{β} will be positive.

So, we have the case in figure, where Z_{α} will be negative and Z_{β} be positive as you have seen from the figure.

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• Equation (1) involve two unknowns, $\bar{X}_a$ and n . By solving them simultaneously, we get the sample size

$$n = \left[\frac{(Z_\beta - Z_\alpha)\sigma}{\bar{X}_1 - \bar{X}_2} \right]^2 \dots \dots \dots (2)$$

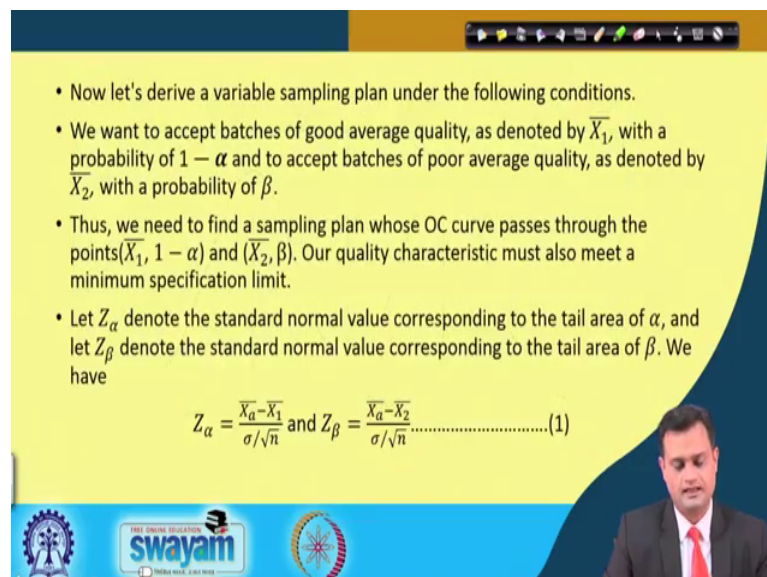
• Using the form of n in equation (1), equation (2) gives the acceptance limit

$$\bar{X}_a = \frac{Z_\beta \bar{X}_1 - Z_\alpha \bar{X}_2}{Z_\beta - Z_\alpha} \dots \dots \dots (3)$$

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Now I would just try to rearrange the equations or equate them, and equation 1 basically involve two unknowns. So, what is my equation 1? If I just go back then I have this equation 1, this is equation 1 and when I see my equation 1 it has two unknown $\bar{X}_a$ and n .

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• Now let's derive a variable sampling plan under the following conditions.

- We want to accept batches of good average quality, as denoted by $\bar{X}_1$, with a probability of $1 - \alpha$ and to accept batches of poor average quality, as denoted by $\bar{X}_2$, with a probability of β .
- Thus, we need to find a sampling plan whose OC curve passes through the points $(\bar{X}_1, 1 - \alpha)$ and $(\bar{X}_2, \beta)$. Our quality characteristic must also meet a minimum specification limit.
- Let Z_α denote the standard normal value corresponding to the tail area of α , and let Z_β denote the standard normal value corresponding to the tail area of β . We have

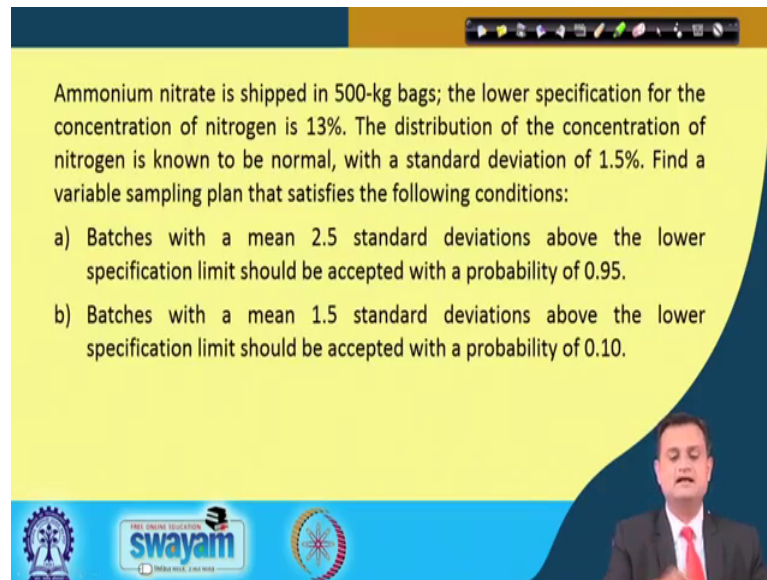
$$Z_\alpha = \frac{\bar{X}_a - \bar{X}_1}{\sigma/\sqrt{n}} \text{ and } Z_\beta = \frac{\bar{X}_a - \bar{X}_2}{\sigma/\sqrt{n}} \dots \dots \dots (1)$$

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So, if I solve this two equations then I can easily get the expression for n , which is Z_β minus Z_α into sigma divided by $\bar{X}_1$ minus $\bar{X}_2$ whole square. I can use the form of n in equation 1 and equation two which gives me the acceptance limit, which is the

threshold point critical point, and I need to compare these average of each sample with respect to this decision criteria $\bar{X}$. So, I will get the expression like $Z_{\beta} \times \bar{1}$, $Z_{\alpha} \times \bar{2}$, $Z_{\beta} - Z_{\alpha}$. So, once you have this expressions you can very easily design the plan.

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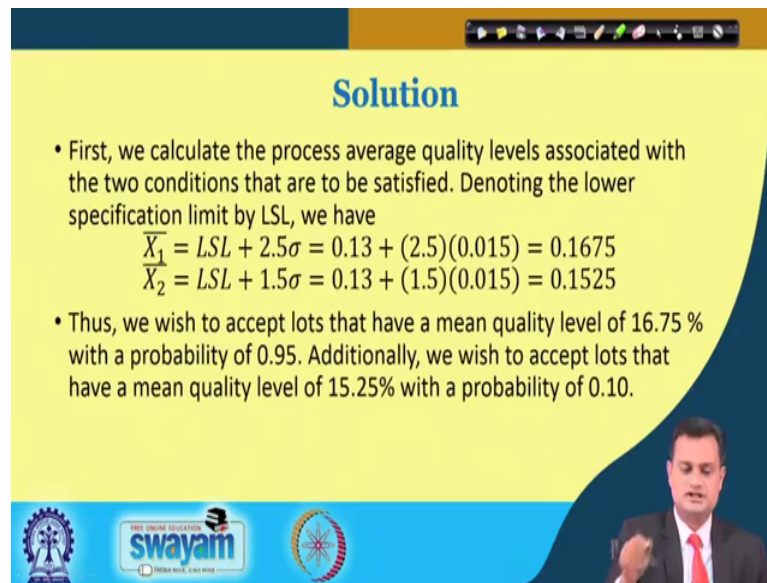
Ammonium nitrate is shipped in 500-kg bags; the lower specification for the concentration of nitrogen is 13%. The distribution of the concentration of nitrogen is known to be normal, with a standard deviation of 1.5%. Find a variable sampling plan that satisfies the following conditions:

- a) Batches with a mean 2.5 standard deviations above the lower specification limit should be accepted with a probability of 0.95.
- b) Batches with a mean 1.5 standard deviations above the lower specification limit should be accepted with a probability of 0.10.

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Let us see an illustrative example, say ammonia nitrate is shipped in 500 kg bags. The lower specification for the concentration of nitrogen is 13 percent. The distribution of the concentration of nitrogen is known to be normal with a standard deviation of 1.5 percent. Find a variable sampling plan that satisfies the conditions; a, batches with a mean 2.5 standard deviations above the lower specification limit should be accepted with a probability $1 - \alpha$ or 0.95 or $\alpha = 0.05$. Batches b, batches with a mean 1.5 standard deviation, above the lower specification limit should be accepted with a probability of 10 percent.

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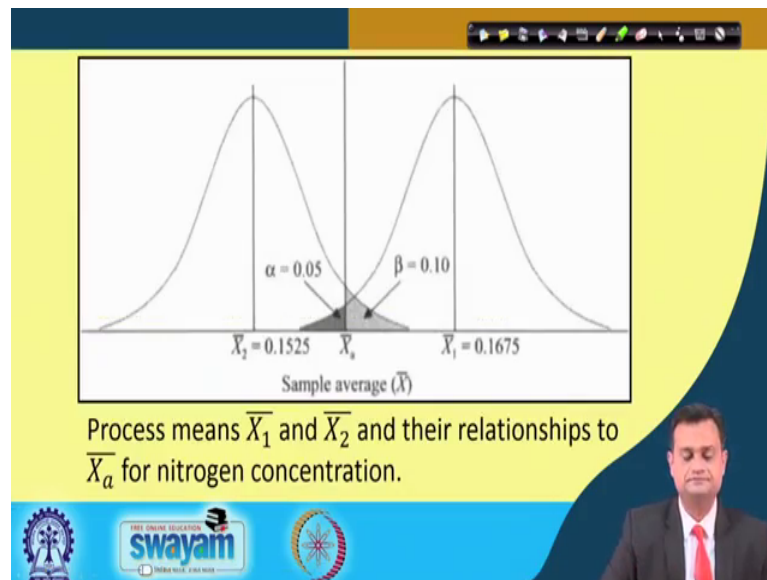
Solution

- First, we calculate the process average quality levels associated with the two conditions that are to be satisfied. Denoting the lower specification limit by LSL, we have
$$\bar{X}_1 = LSL + 2.5\sigma = 0.13 + (2.5)(0.015) = 0.1675$$
$$\bar{X}_2 = LSL + 1.5\sigma = 0.13 + (1.5)(0.015) = 0.1525$$
- Thus, we wish to accept lots that have a mean quality level of 16.75 % with a probability of 0.95. Additionally, we wish to accept lots that have a mean quality level of 15.25% with a probability of 0.10.

. So, this is what, so this is what is being given to me. Now I can easily find my $\bar{X}_1$ and $\bar{X}_2$. So, $\bar{X}_1$ is basically L S L plus 2.5 sigma and this is $\bar{X}_2$ is L S L plus 1.5 sigma, because I am referring the lower specification limit case. So, L S L is 0.13, you have 2.5, you have 1.5 and you are putting this sigma. So, if you just go back, then you have this 1.5 percent, this is your 2.5, this is your 1.5 and you are just trying to use this data for calculating this. So, L S L is equal to 2.5 sigma and L S L plus 1.5 sigma. So, you get the values of, values like 0.1675 and 0.1525.

So, we wish to accept lots that have a mean quality level 16.75 percent with a probability 0.95, and we wish to accept lots that have mean quality level 15.25 percent with a probability of 0.1. So, this is what we are trying to do with the given data.

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Now, the picture will be extremely clear that you have the $\bar{X}_1$ bar. This is my good quality, average of good quality batches. This is my average of poor quality batches. So, this is process when it is performing and delivering the good quality batches, this is the process when it is performing and delivering the bad quality batches, because it is a lower specification. So, this is your alpha and this is typically your beta and I have already completed $\bar{X}_1$ bar and $\bar{X}_2$ bar.

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- We have $1 - \alpha = 0.95$, $\bar{X}_1 = 0.1675$, $\beta = 0.10$, $\bar{X}_2 = 0.1525$. From Appendix A-3, we have $Z_\alpha = Z_{0.05} = -1.645$ and $Z_\beta = Z_{0.10} = 1.282$. From equation (2), the sample size is

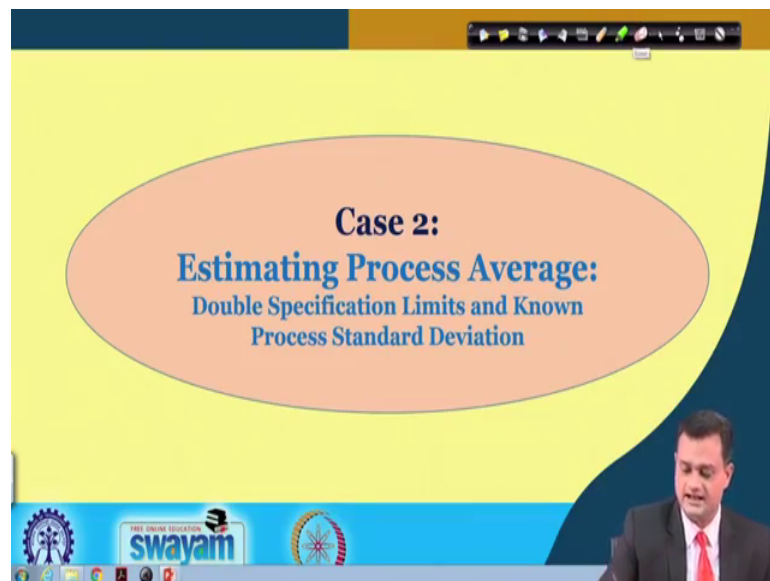
$$n = \left\{ \frac{[1.282 - (-1.645)](0.015)}{0.1675 - 0.1525} \right\}^2 = 8.57 \approx 9$$
- From Equation (3), the acceptance limit is

$$\bar{X}_a = \frac{(1.282)(0.1675) - (-1.645)(0.1525)}{1.282 - (-1.645)} = 0.1591$$
- A random sample of size 9 is chosen from the lot, and the sample average concentration of nitrogen is found. If the sample average is less than 15.91%, the lot is rejected; otherwise, it is accepted.

So, this is the $\bar{X}_1$ and $\bar{X}_2$. Now $\bar{X}_a$ is the deciding criteria that will help me to analyze. So, what you do, we have $1 - \alpha = 0.95$ $\bar{X}_1$ β $\bar{X}_2$, Z_α you can refer the normal table and get it, Z_β you can get it and then you can put all the values in n , so you will get n is equal to 9. Now when you substitute the values in the expression of $\bar{X}_a$, you will get 0.1591, and here this sets the decision rule. So, what it says that you have collected the data let us say. Now just take the first nine and consider that as a sample, because you have sample size 9. Take the average and compare it with your $\bar{X}_a$.

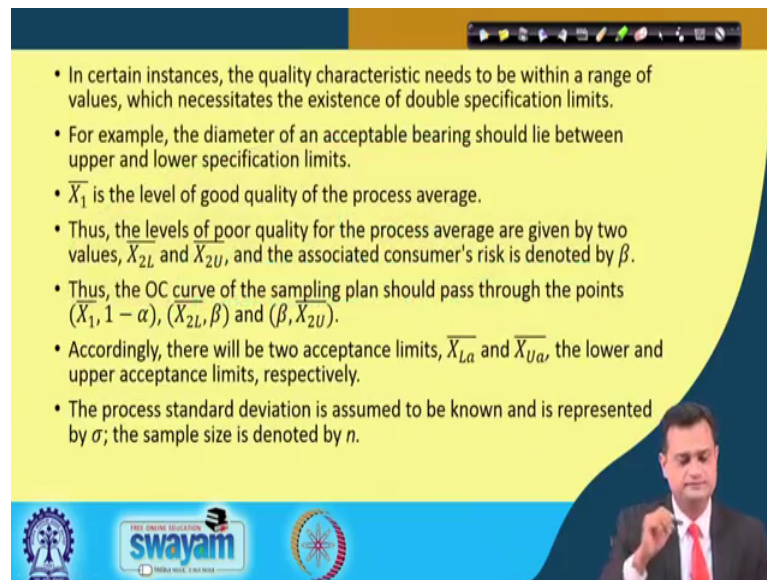
So, if let us say, the sample average concentration of nitrogen is found and if the sample average is less than 0.1591 or 15.91 percent it is rejected. I am referring the lower specification, anything which is below less than this limiting criteria, my critical value; that is 0.1591, I should reject. So, this is a very simple system by which you can design the variable sampling plan with a single specification limit.

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Now let us see the case 2. Here my focus would be on considering the double specification limits, but the remaining procedure will remain as it is, so I hope there would not be much difficulty.

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- In certain instances, the quality characteristic needs to be within a range of values, which necessitates the existence of double specification limits.
- For example, the diameter of an acceptable bearing should lie between upper and lower specification limits.
- $\bar{X}_1$ is the level of good quality of the process average.
- Thus, the levels of poor quality for the process average are given by two values, $\bar{X}_{2L}$ and $\bar{X}_{2U}$, and the associated consumer's risk is denoted by β .
- Thus, the OC curve of the sampling plan should pass through the points $(\bar{X}_1, 1 - \alpha)$, $(\bar{X}_{2L}, \beta)$ and $(\beta, \bar{X}_{2U})$.
- Accordingly, there will be two acceptance limits, $\bar{X}_{La}$ and $\bar{X}_{Ua}$, the lower and upper acceptance limits, respectively.
- The process standard deviation is assumed to be known and is represented by σ ; the sample size is denoted by n .

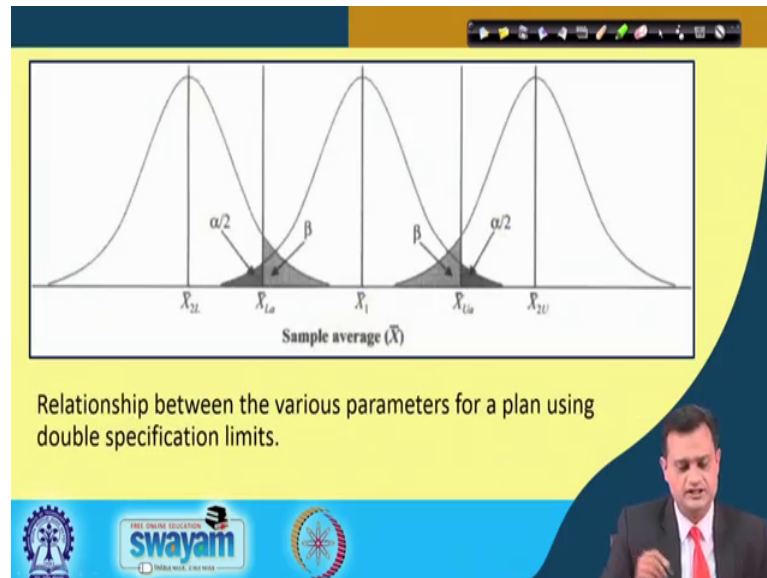
So, let us try to see that now here I have $\bar{X}_1$ bar is the level of good quality of the process average. I am considering both upper and lower specification limit. Now the level of poor quality or the process average is given by two values; $\bar{X}_{2L}$ bar and $\bar{X}_{2U}$ bar and the associated consumer risk is beta. So, when it is a poor quality, you always talk about the consumer risk, because my consumer is at risk if the poor quality lot is accepted. Now what I have done here, that I have separated out my entire say reading my data into $\bar{X}_1$ bar and $\bar{X}_2$ bar.

So, $\bar{X}_1$ bar is the level of good quality of process average and $\bar{X}_2$ bar is basically divided into lower and upper, so I have the two levels of poor quality for the process average, so these are the two values. Now I want my O C curve to pass through the points like $\bar{X}_1$ bar $1 - \alpha$, $\bar{X}_{2L}$ bar beta and beta $\bar{X}_{2U}$ bar.

So, I must understand that what are the points from which my O C curve will pass. In the previous case when it was a single specification limit I wanted my O C curve to pass through $\bar{X}_1$ bar and your $1 - \alpha$ $\bar{X}_2$ bar and beta. In this case it is $\bar{X}_1$ bar $1 - \alpha$ $\bar{X}_{2L}$ bar lower part of the bad quality beta and similar way beta $\bar{X}_{2U}$ bar. So, we need to have two acceptance limit; obviously, because I am referring the case of two specification limit. So, I need to have $\bar{X}_{La}$ bar and $\bar{X}_{Ua}$ bar. In the previous case I had only one deciding criteria that was $\bar{X}_a$ bar. Here I need two in order to see that what

is that range within which my particular quality characteristic reading should fall, if it is falling below it or above it, because it is a two specification limit case it will be rejected.

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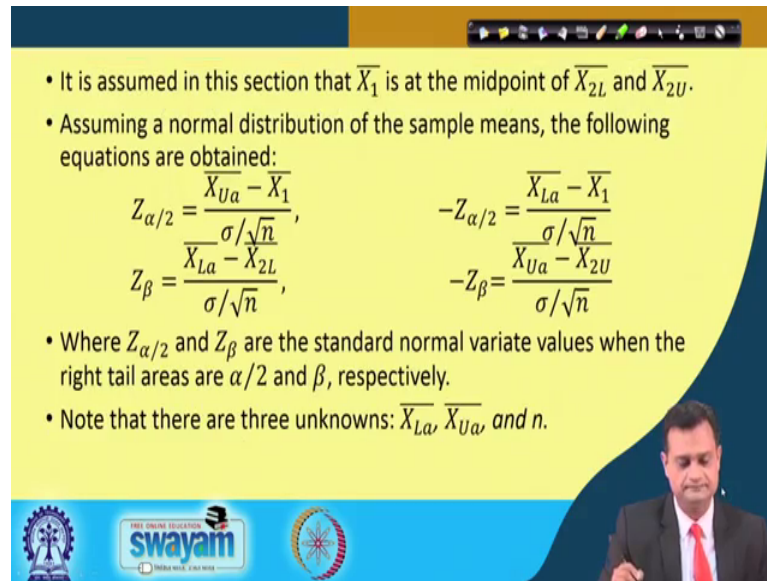


So, now with this we need to understand that let us say this is $\bar{X}_1$, actually what is happening and this will make the picture clear. Now this is the process which is operating at a level $\bar{X}_1$ and it is giving me the good average of good quality lots. There could be possibility that process has shifted to this. And on the upper side it is basically bad quality, average of bad quality lot and it is possible that it can shift one the lower side and it can give me the average of lower quality lot.

So, now, it is very simple that I need to have $\bar{X}_{2U}$, $\bar{X}_{2L}$, $\bar{X}_1$ and I need to have $\bar{X}_{La}$, $\bar{X}_{Ua}$, this is most important it is my deciding criteria. And once I have this I will only accept the product which falls within this range; otherwise the other product, the products falling beyond this below or above is not of acceptable quality.

So, it is all about finding a critical value of $\bar{X}_a$. In case of double specification it is $\bar{X}_{La}$ and $\bar{X}_{Ua}$.

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- It is assumed in this section that $\bar{X}_1$ is at the midpoint of $\bar{X}_{2L}$ and $\bar{X}_{2U}$.
- Assuming a normal distribution of the sample means, the following equations are obtained:

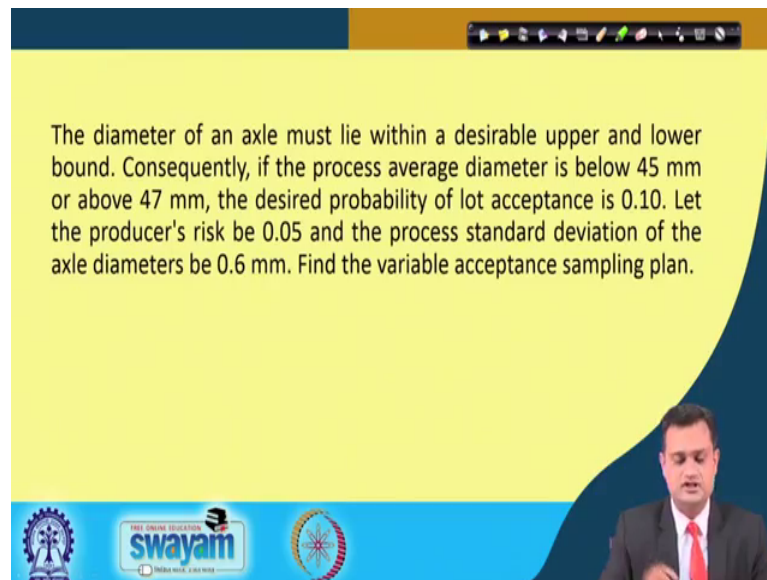
$$Z_{\alpha/2} = \frac{\bar{X}_{Ua} - \bar{X}_1}{\sigma/\sqrt{n}}, \quad -Z_{\alpha/2} = \frac{\bar{X}_{La} - \bar{X}_1}{\sigma/\sqrt{n}}$$

$$Z_{\beta} = \frac{\bar{X}_{La} - \bar{X}_{2L}}{\sigma/\sqrt{n}}, \quad -Z_{\beta} = \frac{\bar{X}_{Ua} - \bar{X}_{2U}}{\sigma/\sqrt{n}}$$
- Where $Z_{\alpha/2}$ and Z_{β} are the standard normal variate values when the right tail areas are $\alpha/2$ and β , respectively.
- Note that there are three unknowns: $\bar{X}_{La}$, $\bar{X}_{Ua}$, and n .

So, with this understanding what you can do? You can say have $\bar{X}_1$ that you can compute from the data collected, $\bar{X}_{2L}$ and $\bar{X}_{2U}$ and assuming the distribution of sample mean following normal, you can have $Z_{\alpha/2} \times \sigma - \mu$ by sigma, same expression, but just here because I have $\bar{X}_{La}$ $\bar{X}_{Ua}$, I need to find out this, I need to find out this. Also I would like to find out the minus side, so minus $Z_{\alpha/2}$ and minus Z_{β} . So, this would give me the expressions for a case where two specification limits are considered.

So, $Z_{\alpha/2}$ and Z_{β} are the standard normal variate that you can refer from the normal table for $\alpha/2$ and β , and these are three unknowns. So, in the previous case I had sample size and $\bar{X}_{Ua}$ unknown here, I have $\bar{X}_{La}$ and $\bar{X}_{Ua}$ and n three unknowns.

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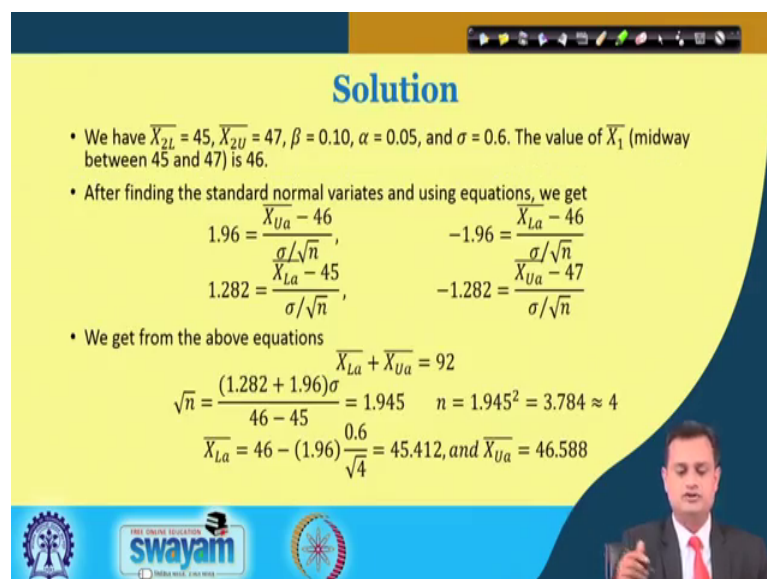


The diameter of an axle must lie within a desirable upper and lower bound. Consequently, if the process average diameter is below 45 mm or above 47 mm, the desired probability of lot acceptance is 0.10. Let the producer's risk be 0.05 and the process standard deviation of the axle diameters be 0.6 mm. Find the variable acceptance sampling plan.

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So, let us see the illustrative example, diameter of an axle must lie within a desirable upper and lower specification. And consecutively if the process average diameter is below 45 millimeter or above 47, you are given the two the desired probability of lot acceptance is 0.1. Let the producers risk be 0.05 and the process standard deviation of the axle is known; that is 0.6. Find the variable acceptance sampling plan. It means you find the value of n sample size, you find $\bar{X}_L$ a bar $\bar{X}_U$ a bar.

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Solution

- We have $\bar{X}_{2L} = 45$, $\bar{X}_{2U} = 47$, $\beta = 0.10$, $\alpha = 0.05$, and $\sigma = 0.6$. The value of $\bar{X}_1$ (midway between 45 and 47) is 46.
- After finding the standard normal variates and using equations, we get

$$1.96 = \frac{\bar{X}_{Ua} - 46}{\sigma/\sqrt{n}}, \quad -1.96 = \frac{\bar{X}_{La} - 46}{\sigma/\sqrt{n}}$$

$$1.282 = \frac{\bar{X}_{La} - 45}{\sigma/\sqrt{n}}, \quad -1.282 = \frac{\bar{X}_{Ua} - 47}{\sigma/\sqrt{n}}$$
- We get from the above equations

$$\bar{X}_{La} + \bar{X}_{Ua} = 92$$

$$\sqrt{n} = \frac{(1.282 + 1.96)\sigma}{46 - 45} = 1.945 \quad n = 1.945^2 = 3.784 \approx 4$$

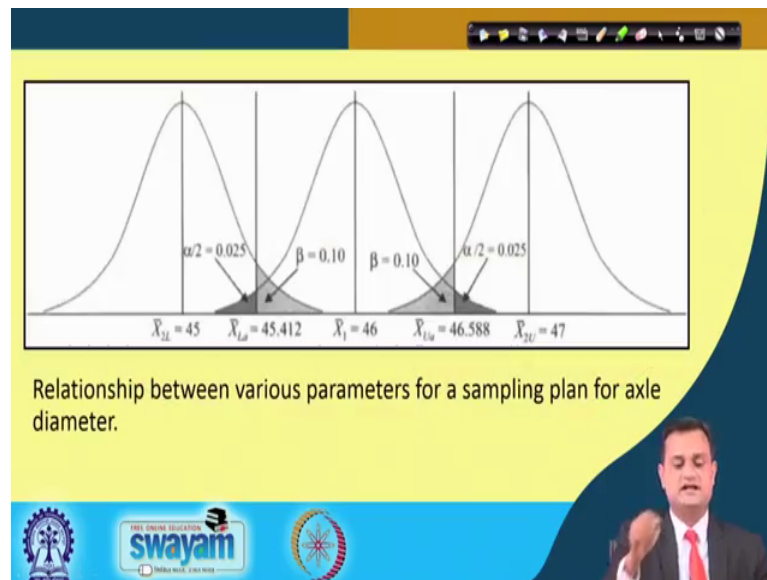
$$\bar{X}_{La} = 46 - (1.96)\frac{0.6}{\sqrt{4}} = 45.412, \text{ and } \bar{X}_{Ua} = 46.588$$

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So, if you can do this, it is done. So, I am just substituting the values I have $X_L = 45$ and $X_U = 47$, so I will take it 46. So, you can just put the values here, and you get the equations by equating this appropriately, and you have $X_L + X_U$ is equal to 92. So, you can easily find square root of n and this will give you n is equal to 4. Now if I put it in this expression of X_L then I will get X_L that is 45.412 and X_U 46.588. So, I would advise you to rewrite this equation, substitute the value and do the calculation, so that you really get the feel of entire process.

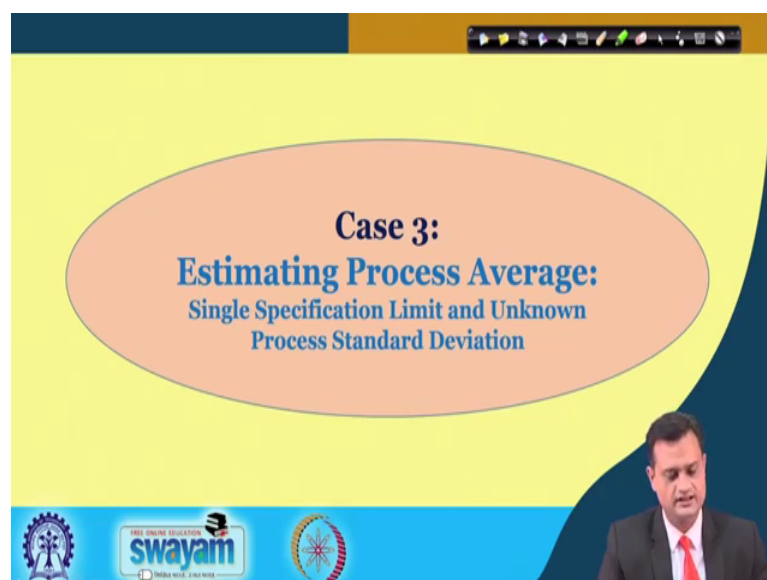
- ✓ The sampling plan operates as follows.
- ✓ A random sample of size 4 is chosen from the lot, and the average diameter of the axles is computed.
- ✓ If the sample average is less than 45.412 mm or greater than 46.588 mm, the lot is rejected.
- ✓ Otherwise, the lot is accepted.

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So, you can see the complete picture here which is very easy to interpret. Just see it here, you have $\bar{X}_1$ you have $\bar{X}_2$ L bar you have $\bar{X}_2$ U bar. These are given, you find the average of this is $\bar{X}_1$ bar, you computed this, you computed this and then you set the decision rule that if it is within this for n is equal to 4 you computed, you accept this sample otherwise you reject the sample. So, I think it is really interesting.

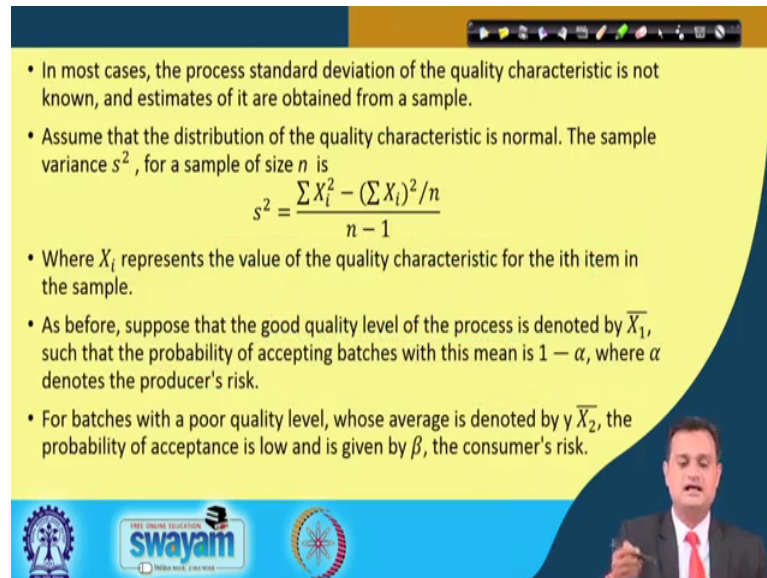
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Now let us see the final case that is case 3. In this case I want to again estimate the process average for single specification, but my unknowns are process standard

deviation. I am not aware of the process standard deviation and in this situation I need to develop a plan.

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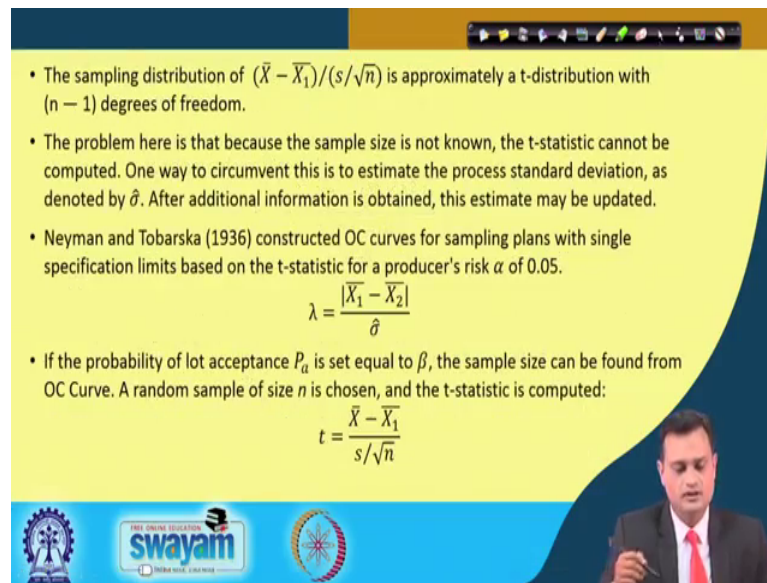


- In most cases, the process standard deviation of the quality characteristic is not known, and estimates of it are obtained from a sample.
- Assume that the distribution of the quality characteristic is normal. The sample variance s^2 , for a sample of size n is
$$s^2 = \frac{\sum X_i^2 - (\sum X_i)^2/n}{n - 1}$$
- Where X_i represents the value of the quality characteristic for the i th item in the sample.
- As before, suppose that the good quality level of the process is denoted by $\bar{X}_1$, such that the probability of accepting batches with this mean is $1 - \alpha$, where α denotes the producer's risk.
- For batches with a poor quality level, whose average is denoted by $\bar{X}_2$, the probability of acceptance is low and is given by β , the consumer's risk.

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So, you need to find the distribution of the characteristic which is normal and the sample variance can be computed like this; $\sum X_i^2 - (\sum X_i)^2/n$ divided by $n - 1$ this is the standard expression for finding, the standard deviation or variance of the sample. Now X_i represents the value of the quality characteristic from the i th item, and you want that good quality level of the process is $\bar{X}_1$ I am consistent in referring the notations. And the probability of accepting batches is $1 - \alpha$, where α is producers risk. Same way the average of poor quality batch is this $\bar{X}_2$ and the associated risk is consumer risk; that is β .

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- The sampling distribution of $(\bar{X} - \bar{X}_1)/(s/\sqrt{n})$ is approximately a t-distribution with $(n - 1)$ degrees of freedom.
- The problem here is that because the sample size is not known, the t-statistic cannot be computed. One way to circumvent this is to estimate the process standard deviation, as denoted by $\hat{\sigma}$. After additional information is obtained, this estimate may be updated.
- Neyman and Tobarska (1936) constructed OC curves for sampling plans with single specification limits based on the t-statistic for a producer's risk α of 0.05.

$$\lambda = \frac{|\bar{X}_1 - \bar{X}_2|}{\hat{\sigma}}$$

- If the probability of lot acceptance P_a is set equal to β , the sample size can be found from OC Curve. A random sample of size n is chosen, and the t-statistic is computed:

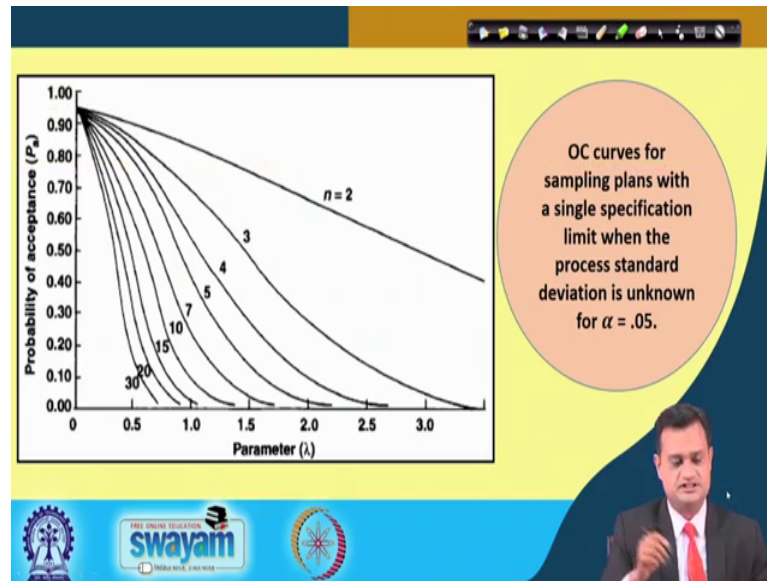
$$t = \frac{\bar{X} - \bar{X}_1}{s/\sqrt{n}}$$

So, now, here we will do little bit something like this, that the sampling distribution of $\bar{X}$ bar minus $\bar{X}_1$ bar divided by s divided by square root n is approximately following the t distribution. So, I am deriving a quantity for the sampling distribution and this follows the t distribution. So, I would use the t distribution to finally, take a decision that whether this is an acceptable sample or a disrejectable sample.

So, because you have unknown standard deviation about the process, you here try to estimate this σ hat and take λ $\bar{X}_1$ bar minus $\bar{X}_2$ bar. So, this procedure is recommended by Neyman and Tobarska 1936 for constructing the O C curve for variables when your standard deviation is unknown, variance is unknown. And for α is equal to 0.05 you can compute a value λ , we will use this λ in the development of O C curve.

And this is the well-known that you can find the t statistics to take a final decision about the acceptance or rejection of the lot and we will make use of this expression.

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So, just see this, you computed the parameter lambda and this is the probability of acceptance. And what you find here, you find here n number of say O C curves, each one for a particular sample size. So, as you increase the sample size; obviously, the discriminatory power slope is increasing, this is very obvious, we have seen it number of times. Here the quantity which I am computing is lambda. So, this is a standard say developed recommended O C curve which can help to find a particular sample size for a given level of lambda as well as the probability of acceptance.

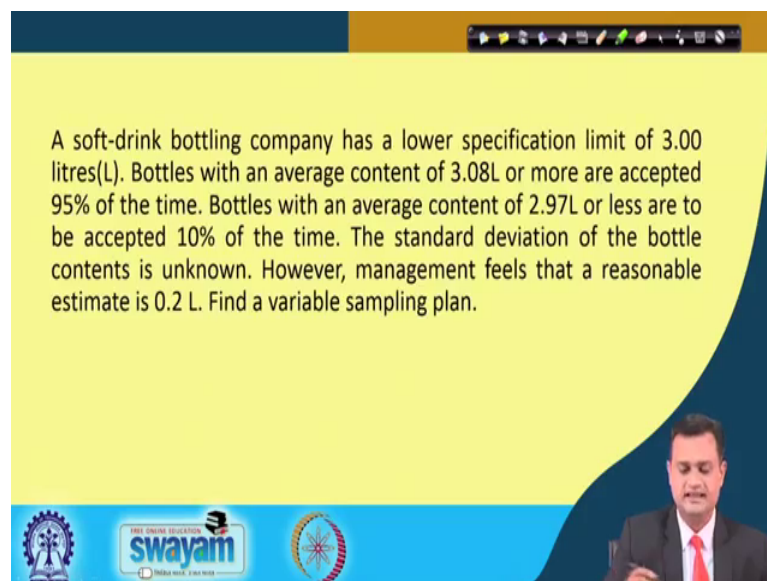
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- ✓ If a lower specification limit is given, the lot is rejected if the calculated value of $t < t_{\alpha, n-1}$, where $t_{\alpha, n-1}$ is the 100α percentile point of the t-distribution with $(n - 1)$ degrees of freedom.
- ✓ If an upper specification limit is given, the lot is rejected if $t > t_{1-\alpha, n-1}$ where $t_{1-\alpha, n-1}$ is the $100(1 - \alpha)$ percentile point of the t-distribution within $(n - 1)$ degrees of freedom.

Logos for Swamyam and other educational institutions are visible at the bottom of the slide.

So, once I have this the rule is like this if a lower specification limit is given. The lot is rejected if the calculated value of t , we have done such kind of thing in hypothesis testing. So, the calculated value of t is less than $t_{\alpha, n-1}$, so you need value of α in degree of freedom if we have to do refer the t table. So, where $t_{\alpha, n-1}$ is the 100 α percent point of the t distribution, and if upper specification limit is given, lot is rejected if the calculated value t is greater than $t_{1-\alpha, n-1}$. So, this is my decision rule.

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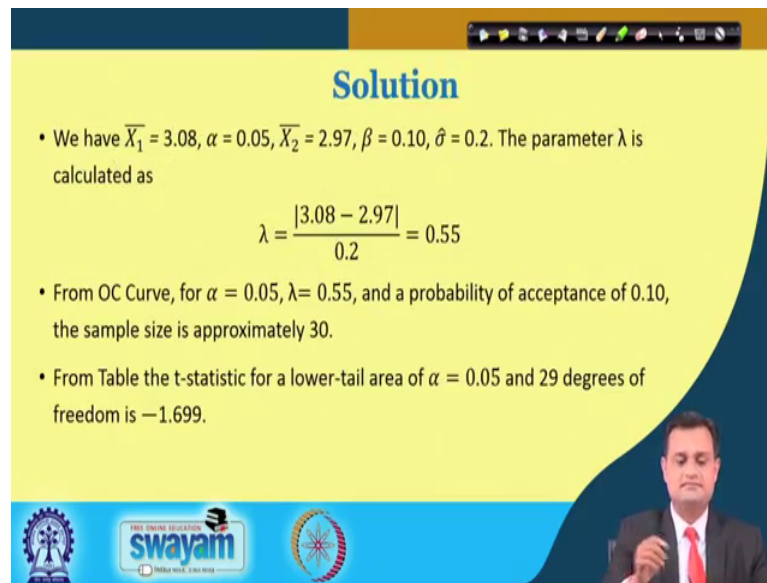
A soft-drink bottling company has a lower specification limit of 3.00 litres(L). Bottles with an average content of 3.08L or more are accepted 95% of the time. Bottles with an average content of 2.97L or less are to be accepted 10% of the time. The standard deviation of the bottle contents is unknown. However, management feels that a reasonable estimate is 0.2 L. Find a variable sampling plan.

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Let us quickly see the illustrative example; a soft drink bottling company has a lower specification limit of 3.0 liters, we all purchase usually 1 liter bottle.

Bottles with an average content of 3.08 liter or more are accepted 95 percent of the time; that is what I want through my sampling plan. Bottle with an average content of 2.97 liter or less are to be accepted only 10 percent, because this is where my consumer is at risk. You are purchasing a bottle of 1 liter and you only get 700 ml, so you are never happy, so consumer is at risk. So, I want this thing to happen only 10 percent of the time. The standard deviation of the bottle content is unknown and; however, management feels that a reasonable estimate may be based on past data or may be based on say some experiences of similar kind of process, let us say it is 0.2 L. Find the variable sampling plan.

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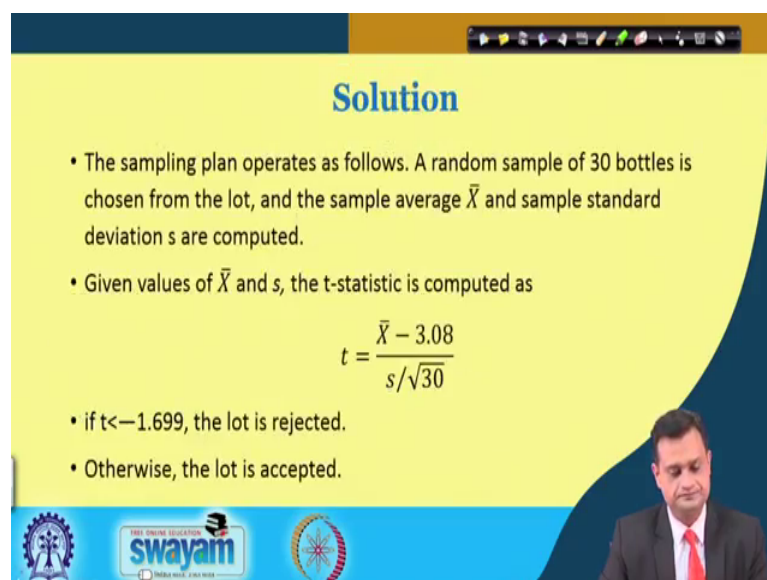
Solution

- We have $\bar{X}_1 = 3.08$, $\alpha = 0.05$, $\bar{X}_2 = 2.97$, $\beta = 0.10$, $\hat{\sigma} = 0.2$. The parameter λ is calculated as
$$\lambda = \frac{|3.08 - 2.97|}{0.2} = 0.55$$
- From OC Curve, for $\alpha = 0.05$, $\lambda = 0.55$, and a probability of acceptance of 0.10, the sample size is approximately 30.
- From Table the t-statistic for a lower-tail area of $\alpha = 0.05$ and 29 degrees of freedom is -1.699 .

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So, I have the data $\bar{X}_1$, I have the data $\bar{X}_2$, α , β , $\hat{\sigma}$ and I can put it into the expression of λ which is $\bar{X}_1 - \bar{X}_2$ divided by $\hat{\sigma}$. So, what I get it is 0.55. So, for $\alpha = 0.05$ and $\lambda = 0.55$ and a probability of acceptance 0.1; that is β , I can have a sample size 30. So, let us see how this is done, so I have $\lambda = 0.55$ and $\beta = 0.10$.

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Solution

- The sampling plan operates as follows. A random sample of 30 bottles is chosen from the lot, and the sample average $\bar{X}$ and sample standard deviation s are computed.
- Given values of $\bar{X}$ and s , the t-statistic is computed as
$$t = \frac{\bar{X} - 3.08}{s/\sqrt{30}}$$
- if $t < -1.699$, the lot is rejected.
- Otherwise, the lot is accepted.

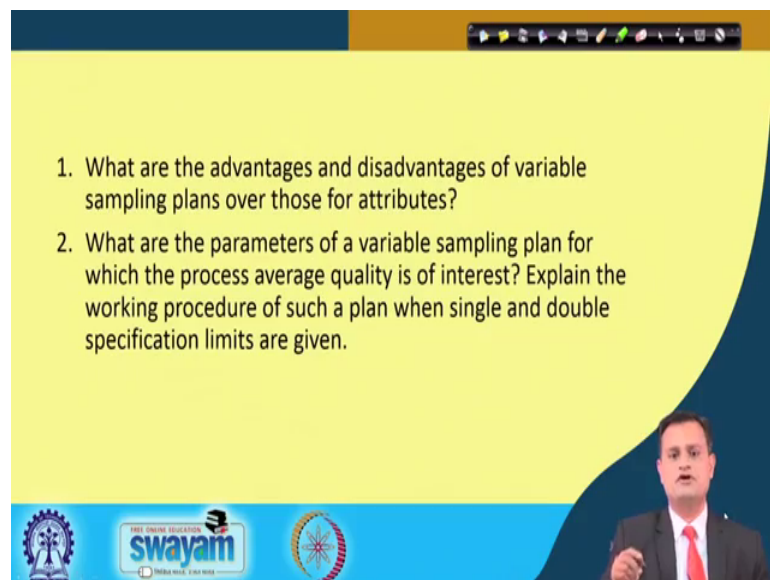
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So, for $\lambda = 0.55$, and just see this λ is 0.55, so somewhere here and my acceptance number is 0.1. So, exactly you are falling in this. So, your n is equal to 30.

So, I hope this is clear that how we find the sample size. So, n is equal to 30 you have found from the standard operating characteristic curve, developed by normal end for conducting the t stage you will say that lower area of α is 0.05; obviously, $n - 1$ that is 30 minus 1 is the degree of freedom, and the table of t statistic the standardized value is minus 1.699.

So, here I am leaving it to you, you make the decision, but finally, your decision should fall like this that if t is less than minus point 1.699, lot is rejected otherwise the lot is accepted. So, you need to calculate the value of t , compare this with the tabulated value and you take the decision whether lot should be accepted or rejected.

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1. What are the advantages and disadvantages of variable sampling plans over those for attributes?

2. What are the parameters of a variable sampling plan for which the process average quality is of interest? Explain the working procedure of such a plan when single and double specification limits are given.

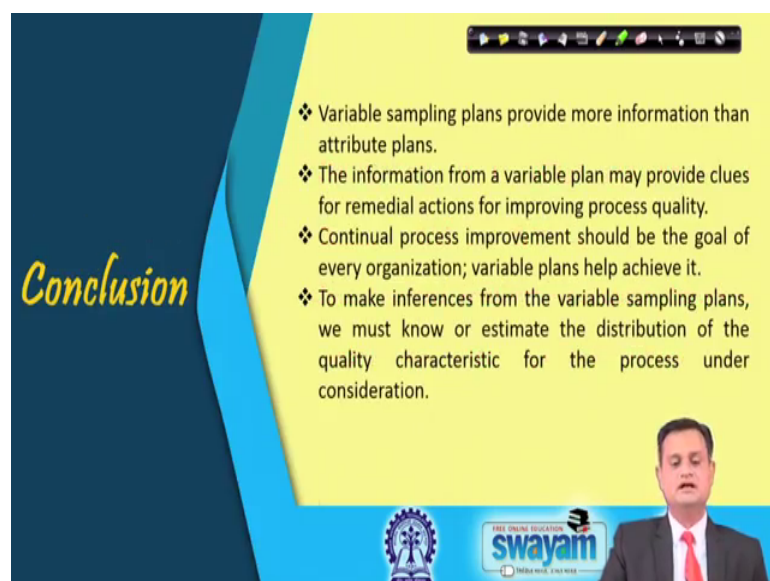
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So, before we end I would just like to float couple of think it question, what are the advantages and disadvantages of variable sampling plan over those for attributes. What are the parameters of a variable sampling plan for which the process average quality of interest, and explain the working procedure of such plan when single and double specification limits are given.

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Mainly, I have used the Mitra fundamentals of quality for this particular lecture. And we can say that variable sampling plan provide more information insights into the corrective steps to be taken for the improvement of the process, and we can have better inferences. Only the administration cost desire, because you need to maintain variable sampling plan for each of the quality characteristic, not like your attribute.

So, thank you very much for your interest in learning the variable sampling plan or the sampling plan for variables. I advise you to revise the concepts from the lecture as well

as refer the book and try to internalize the concept by solving couple of examples or taking some hypothetical data or real life data, and feel more confident about a particular concept. So, we have advanced a lot in our six sigma journey. We are also about to finish our control phase, and then we will have finally, some topics related to six sigma implementation challenges so.

Thank you very much be with me enjoy.