

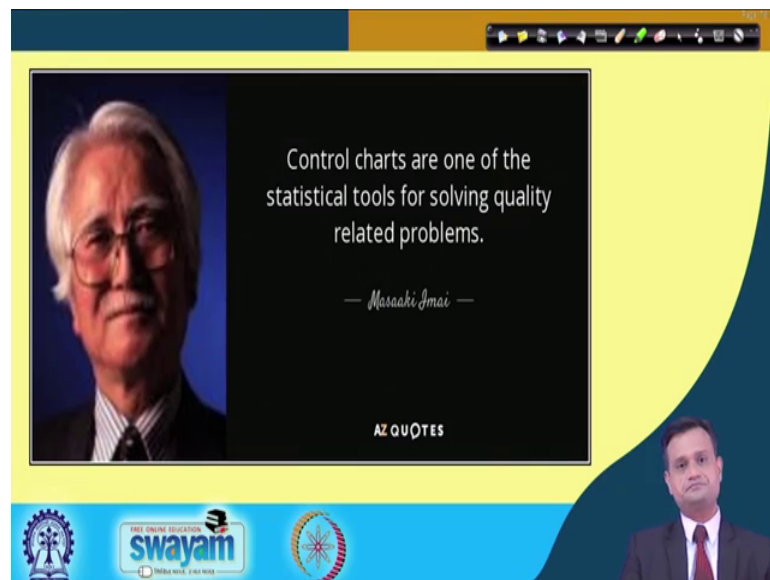
Six Sigma
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Lecture - 55
Design for Acceptance Sampling Plans for Attribute (Part-1)

Hello friends, once again I welcome you to our ongoing journey on Six Sigma. And I would like to remind you that we are in the final phase of our DMAIC cycle. And we are discussing the various issues in the control phase; mainly we are focusing on statistical quality control, as a part of that we have talked about statistical process control, control charts for variables and control charts for attributes. Now we are talking about acceptance sampling. We have seen the key concepts like producer's risk, consumer's risk, AQL, AOQL and couple of other things in the last lecture. Now this particular lecture 55, we will focus on design of acceptance sampling plan for attributes.

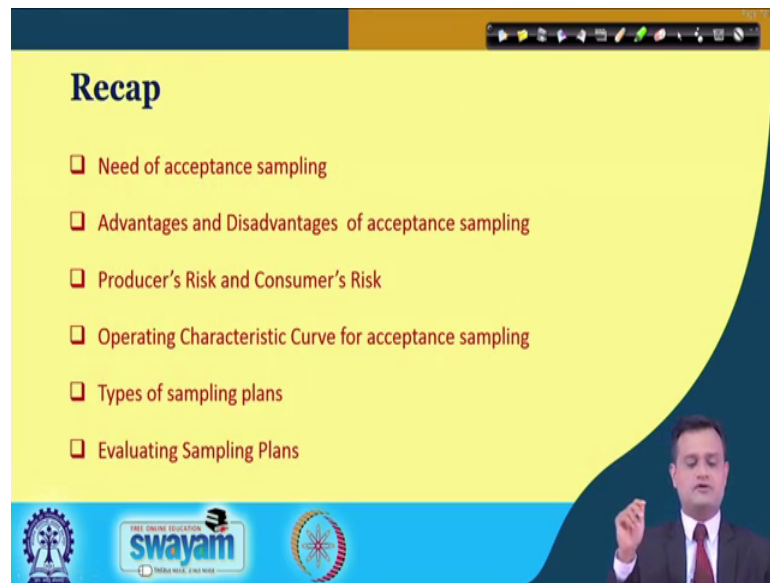
As we need to discuss in detail and there are lot many things to cover, this particular lecture is divided into part 1 and part 2.

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So let us begin with lecture 55. Once again I would emphasize that, there is no substitute of control chart and statistical process control and acceptance sampling are must in order to have fact based management in the organisation.

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The slide is titled "Recap" in a bold, dark blue font. It features a list of six topics, each preceded by a red square icon. The topics are: "Need of acceptance sampling", "Advantages and Disadvantages of acceptance sampling", "Producer's Risk and Consumer's Risk", "Operating Characteristic Curve for acceptance sampling", "Types of sampling plans", and "Evaluating Sampling Plans". The slide has a yellow background with a dark blue curved border on the right side. At the bottom, there is a blue banner with logos for "swayam" and "INDIA WISE, FUTURE WISE". A small video inset of a man in a suit is visible in the bottom right corner.

Recap

- ❑ Need of acceptance sampling
- ❑ Advantages and Disadvantages of acceptance sampling
- ❑ Producer's Risk and Consumer's Risk
- ❑ Operating Characteristic Curve for acceptance sampling
- ❑ Types of sampling plans
- ❑ Evaluating Sampling Plans

So, this is what we have covered, and we also had little bit glimpses on types of sampling plan and their key characteristics and we had seen the sum of the evaluating criteria for sampling plan.

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The slide is titled "CONCEPTS COVERED" in a bold, yellow font. It features a list of concepts, each preceded by a red square icon. The concepts are: "Lot-by-Lot Attribute Sampling Plans", "Single Sampling Plans", "Double Sampling Plans", and "Multiple Sampling Plans". The slide has a yellow background with a dark blue curved border on the left side. At the bottom, there is a blue banner with logos for "swayam" and "INDIA WISE, FUTURE WISE". A small video inset of a man in a suit is visible in the bottom right corner.

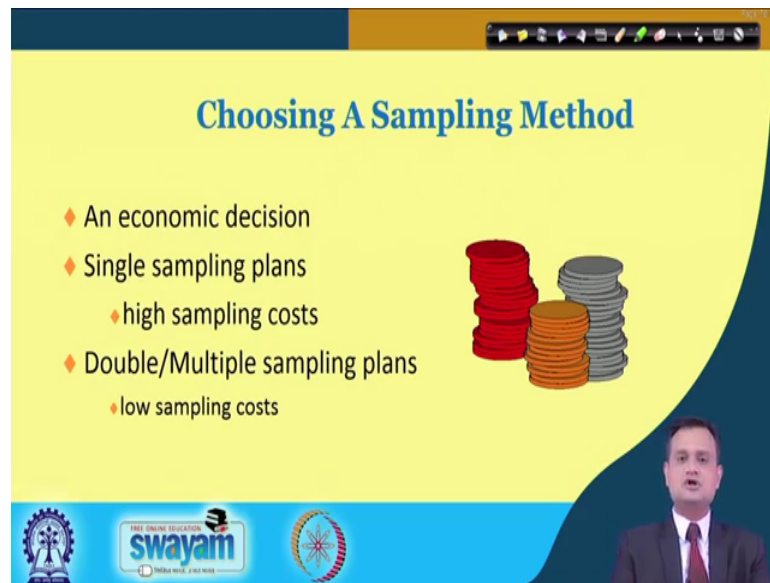
CONCEPTS COVERED

Concepts Covered:

- ❑ Lot-by-Lot Attribute Sampling Plans
 - Single Sampling Plans
 - Double Sampling Plans
 - Multiple Sampling Plans

So, this lecture we will basically focus on lot by lot attribute sampling plan; and we will talk about single sampling plan, double sampling plan and multiple sampling plan. Our focus mainly is on the design of the sampling plan typically under the category of lot by lot attributes sampling plans.

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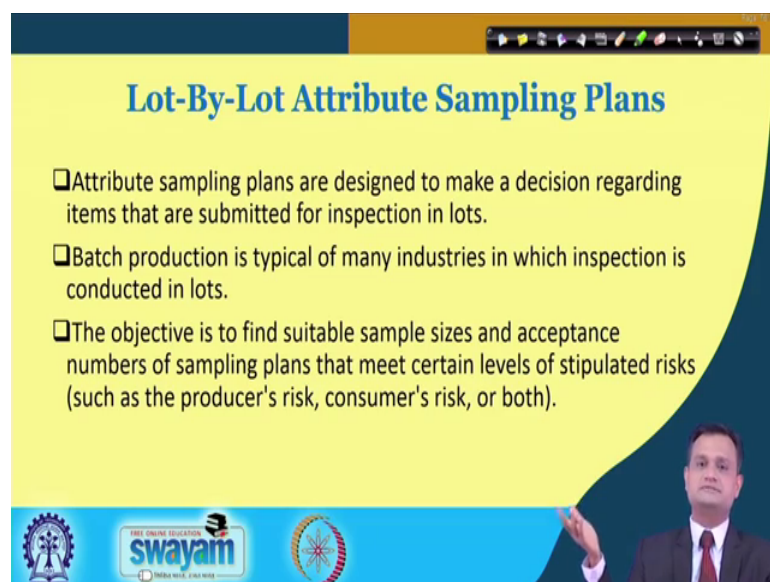
Choosing A Sampling Method

- ♦ An economic decision
- ♦ Single sampling plans
 - ♦ high sampling costs
- ♦ Double/Multiple sampling plans
 - ♦ low sampling costs

The slide features an illustration of three stacks of coins: a tall red stack, a medium orange stack, and a shorter grey stack. At the bottom, there is a Swayam logo and a small video feed of a man in a suit.

So, choosing a sampling method is an economic decision I cannot go with 100 percent inspection. But obviously, say I have to see the trade off between producers risk and consumers risk; and single sampling plan, there is a high sampling cost; double multiple sampling plan, I may take a decision on a very first sample and my sample size would be less.

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Lot-By-Lot Attribute Sampling Plans

- ❑ Attribute sampling plans are designed to make a decision regarding items that are submitted for inspection in lots.
- ❑ Batch production is typical of many industries in which inspection is conducted in lots.
- ❑ The objective is to find suitable sample sizes and acceptance numbers of sampling plans that meet certain levels of stipulated risks (such as the producer's risk, consumer's risk, or both).

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So, my sampling cost is little bit lower compared to the single sampling plan.

Now, if you see the lot by lot attributes sampling plan, then this plans are designed to make a decision regarding items that are submitted for the inspections in lot. So typically if you see, the batch products and then, items are produced in lot and then they are submitted for the sampling and inspection decision making in a particular lot. And the objective is to find here, sample size and acceptance number of a sampling plan which is acceptable to both producer and consumer. Once again I would like to remind you that producers risk is type one error denoted as α and producer would not like his good quality lot to be rejected. Consumers risk is beta type two error and consumer would not like the bad quality lot to be accepted.

So, we have to see the trade off between producers risk and consumer's risk; and unless you consider the judicious trade off between these two you cannot really design a sampling plan with sample size, an acceptance number and rejection number and other criteria. So, the whole exercise, we will do in this particular lecture is all about to see that how producers stipulated risk and consumers stipulated risk can be appropriately accommodated, so that I can devise a judicious, logical and mutually acceptable sampling plan to both the parties.

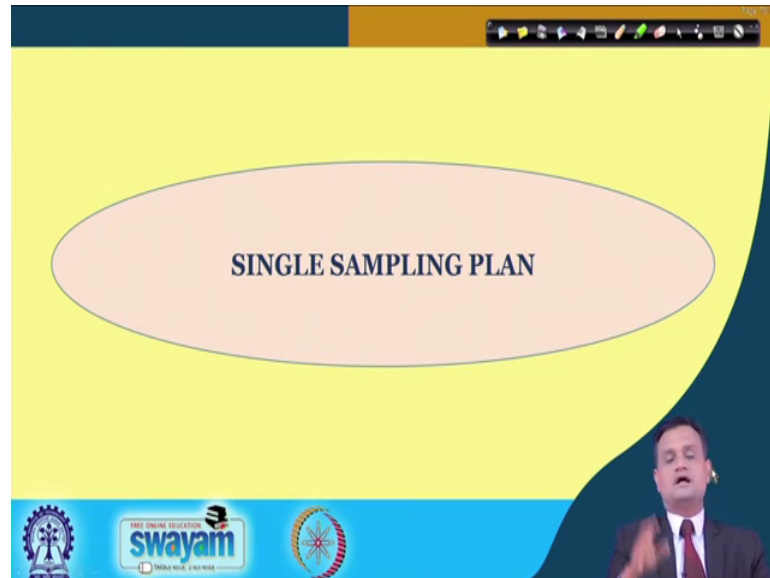
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So, this is exactly we will try to do. So, we have broadly say four different categories of lot by lot attributes sampling plan single sampling plan, double sampling plan, multiple sampling plan and standard sampling plans. So, there are totally four. In this particular

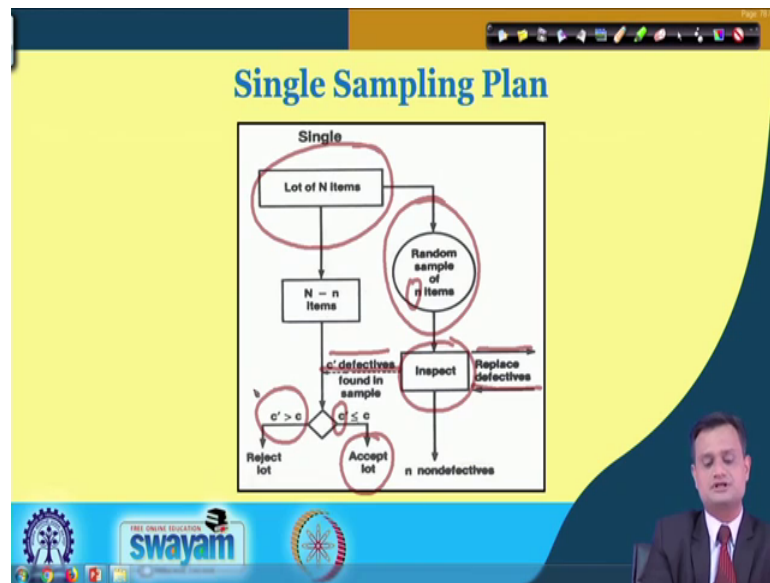
lecture, we will focus on these three that is single sampling plan, double sampling plan, single, double and multiple and we will focus on standard sampling plan in the next lecture.

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So, this is what we want to do. Now let us see the first the design of the single sampling plan, which is easiest to employee but because we only rely on single sample and sample size may be larger so, the overall cost of inspection maybe more compared to the double sampling and the multiple sampling. But so for the administration is concerned single sampling plan is the best to use and implement.

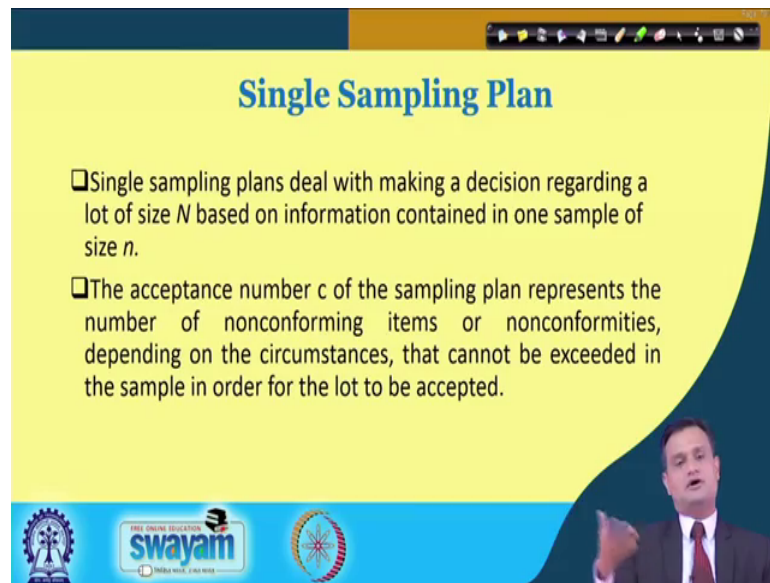
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So, the overall picture goes like this in single sampling, I have the lot of N items and then I have random sample of n item that is the small n . Now I inspect it, after inspection I will find that there are c' defectives found in the sample. I can also replace the defectives as a part of my rectification process, but here just try to focus on these that c' is the defectives found in a sample. Now if this c' is less than or equal to c which is the acceptance number, I will accept the lot, because it is less than the acceptance number and I will accept the lot. If it is greater than c means number of defectives, then I will reject the lot and you can have n non defectives from a lot of say capital N in a sample of n .

So, this is what exactly I do in a single sampling plan and it is called single sampling because I just draw collect one sample can then based on that, I try to compare the number of defectives in the sample with the acceptance number and make my decision.

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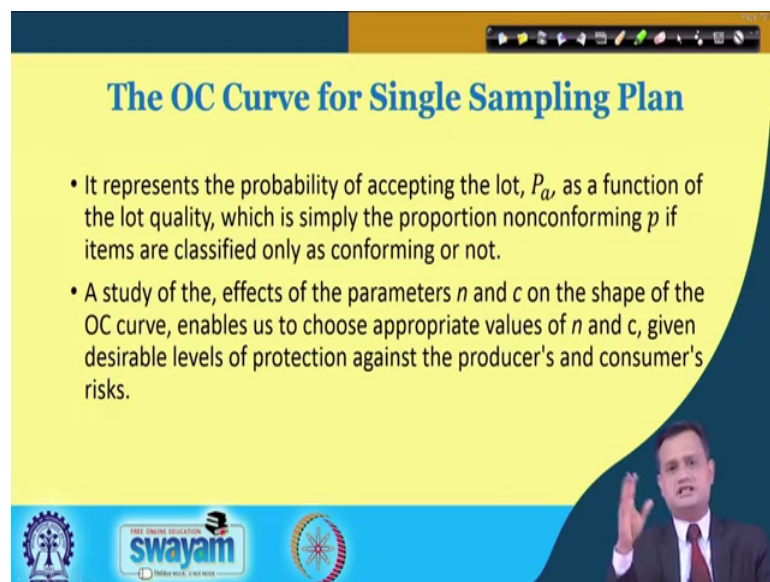
Single Sampling Plan

- ❑ Single sampling plans deal with making a decision regarding a lot of size N based on information contained in one sample of size n .
- ❑ The acceptance number c of the sampling plan represents the number of nonconforming items or nonconformities, depending on the circumstances, that cannot be exceeded in the sample in order for the lot to be accepted.

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So, single sampling plan basically deals with the decision regarding a lots of size N and sample size n and you have the acceptance number c to decide that, whether this is accepted or this is rejected.

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The OC Curve for Single Sampling Plan

- It represents the probability of accepting the lot, P_a , as a function of the lot quality, which is simply the proportion nonconforming p if items are classified only as conforming or not.
- A study of the, effects of the parameters n and c on the shape of the OC curve, enables us to choose appropriate values of n and c , given desirable levels of protection against the producer's and consumer's risks.

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So, typically if you see the operating characteristic curve which is a measure of performance, either of control chart or the acceptance sampling plan. Basically it provides the discriminatory power I am repeating this statement again and again to my acceptance sampling plan. And how discriminatory it is in terms of say separating the

good lot from the bad lot and this is what we try to appreciate as a part of operating characteristic curve.

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The OC Curve for Single Sampling Plan

- Let's suppose that both the acceptable quality level (AQL), which is the measure of a good lot and is associated with the producer's risk α , and the limiting quality level (LQL), which is the measure of a poor lot and is associated with the consumer's risk β are specified.
- For a sampling plan specified by n and c , lots with a proportion nonconforming level of AQL that come in for inspection should be accepted $100(1 - \alpha)\%$ of the time.
- With an AQL of 0.03 and an α of 0.05, lots that are 3% nonconforming should then be accepted 95% of the time by the given sampling plan.
- Similarly, if the proportion nonconforming of batches coming in for inspection is LQL, they should be accepted $100\beta\%$ of the time.
- Thus, if the LQL is 0.09 and β is 0.10, then lots that are 9% nonconforming should be accepted only 10% of the time.
- A suitable choice of n and c ensures that good lots will be accepted a large percentage of the time and that bad lots will be accepted infrequently.

So, if you just see the couple of issues that we discussed in the last lecture, then one of them was AOQ or AQL that is acceptable quality level. So, acceptable quality level is the measure of a good lot and typically it is associated with the producers risk alpha. So, my producer would not like a good lot to be rejected, an AQL acceptable quality level is typically the one which is associated with producer's risk that is alpha. If you look at the another definition that is LQL this is limiting quality level, then this is the measure of a poor lot and typically it is associated with the consumers risk and consumer would not like a poor quality lot to be accepted.

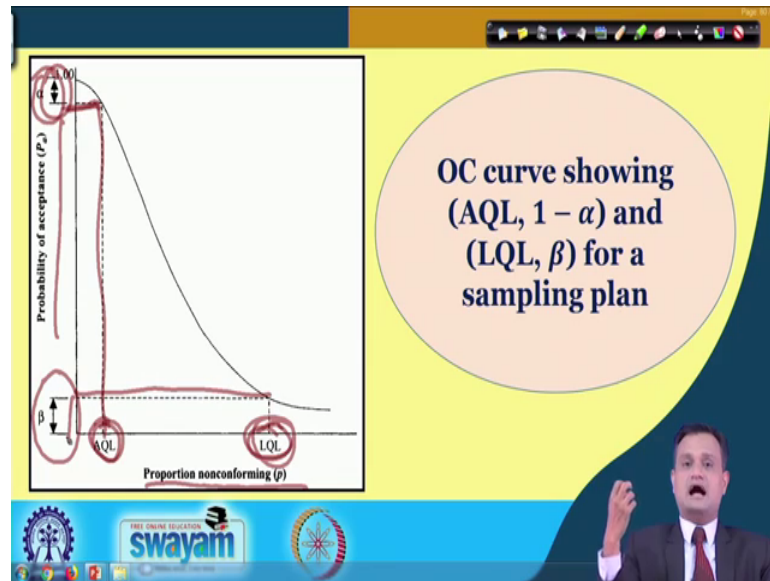
So, in the previous lecture average outgoing quality, average outgoing quality level this concepts we have seen. Now I am introducing two more new terms, one is acceptable quality, other is say your limiting quality levels that is LQL. Now suppose I consider a sampling plan specified by n and c , lots with a proportion nonconforming level of AQL, that come in for inspection should be accepted $100(1 - \alpha)\%$ of the time. It means number of lots coming for the inspection and out of this many lots I will select $100(1 - \alpha)\%$ of the time out of the submitted lots and this is something which refers to my AQL. So, same way I can say that AQL is 0.03 and alpha of 0.05.

So, I would say that lots that are 3 percent nonconforming should than be accepted 95 percent of the time by the given sampling plan. So, you may make a mistake; 3 percent is the level that you need to achieve that is the acceptable quality level. And let us say if it is 0.03, 3 percent and I operate with a particular level alpha, it means the lot is of good quality type one error, but I will reject it. So, I will say that if my alpha is 0.05 that is 5 percent, then the lot which contains 3 percent nonconforming or defective 95 percent of the time, it should be accepted and that is what my operating characteristic curve or my acceptance sampling plan must incorporate.

So, this is the clear cut understanding, similarly if the proportion of non-conforming of the batches in coming in for inspection is LQL. Now let us think from the consumers point of view, suppose I am referring the limiting quality level this is the quality of a poor lot. So, let us say LQL, they should be accepted by 100 beta percent, means the bad quality lot should be accepted by 100 beta percent. Let us try to appreciate LQL is 0.09 and beta is 0.10. So, what would I say that the lot that are 9 percent nonconforming 9 percent nonconforming should be accepted only 10 percent of the time.

So, the idea is very clear, let us say AQL 0.03 that is 3 percent. And if I have the alpha is equal to 0.05, I will say the number of lots submitted out of this number of lots the lots having 3 percent defective or nonconforming (Refer Time: 12:00) 95 percent of the time it should be accepted. It is only 5 percent time producers risk will come in picture and I will check the wrong decision. Similar way if I refer the bad quality lot let us say 9 percent is LQL and beta is my 10 percent 0.10, then I will say that the lots which has 9 percent defective this lot should only be accepted 10 percent of the time. So, that my consumers risk remains at 10 percent and he feels protected. So, it is all about protecting your producers and consumers and I would say that what is the level of protection, your acceptance sampling plan ensures to your consumer and producer and acceptable to them decides the goodness of your acceptance sampling plan.

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So, this is something which is very important and you must appreciate, just see the OC curve for AQL $1 - \alpha$ that is specific to your producer's risk. And LQL β that is specific to your consumer's risk you can see it very well here, AQL is this and this is my α that is producer's risk. It means that the lot which has this much percent of non conforming the lot which has this much percent of non conforming AQL, it will be accepted 95 96 whatever is this, α as per that that many times. But the lots which has LQL quality level that is very poor lot maybe let us say 9 percent 8 percent whatever.

Then it would only be say accepted this much of the time and this is my basically probability of acceptance. So, I hope the idea is crystal clear that a producer would like the lot to be accepted at AQL; and he would not like any lot to be rejected. If it is below AQL or at AQL and similar way a consumer would not like to accept the lot which is of very poor quality LQL with β percent risk.

So, again I would remind you that we are dealing with the probability, we are dealing with the sampling and we always take the decision or test the hypothesis by accepting some level of type one and type two error. We cannot get rid of say making the error, but we can just try to see that how this error can be kept at minimum level? And this is acceptable typical in acceptance sampling plan to my both producer and consumer. So, this is something that I wanted to emphasize.

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Design of Single Sampling Plans

- ✓ Now we will discuss several approaches for designing single sampling plans.
- ✓ Basically, these approaches involve determining the sample size n and acceptance number c of the plan.
- ✓ The criteria selected influences the parameters of the plan. Sometimes, more than one plan will satisfy the criteria.
- ✓ What follows is a series of procedures for determining the parameters of a single sampling plan based on the criteria specified.
 - Stipulated Producer's Risk
 - Stipulated Consumer's Risk
 - Stipulated Producer's and Consumer's Risk

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Now, if you see the design of a single sampling plan, then there are three issues. Number one stipulated producers risk. So, let us say AQL is something related to producers risk alpha we have seen in the figure and consumers risk that is beta specific to your consumer. And you may have stipulated producers and consumers risk. So now when I want to design the single sampling plan, meaning to say I want to decide about sample size and; and the acceptance numbers c , I have to consider three different situation or one of these three either I design it for stipulated producers risk. It means my plan will exactly try to meet the producers risk or I may try for consumers risk or I may try to incorporate both simultaneously.

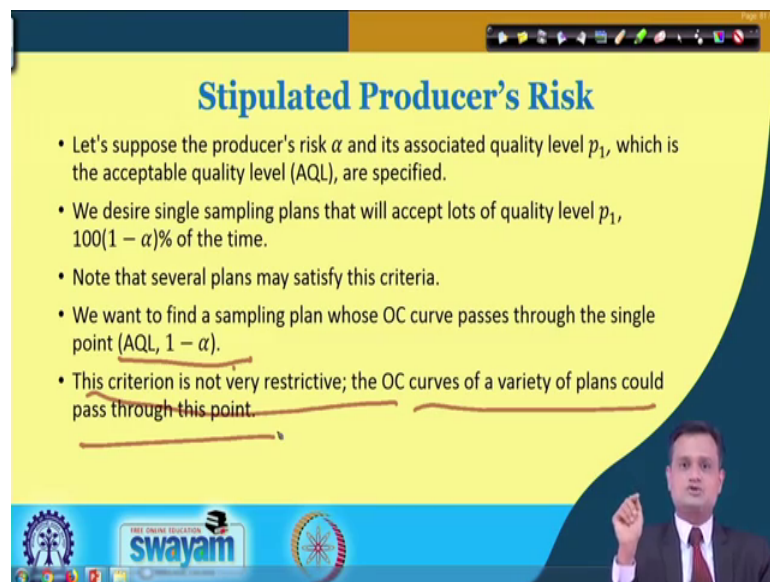
So, let us try to see how we can design this single sampling plan with this three different criteria.

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So, let us see the first one design of single sampling plan based on stipulated producer's risk.

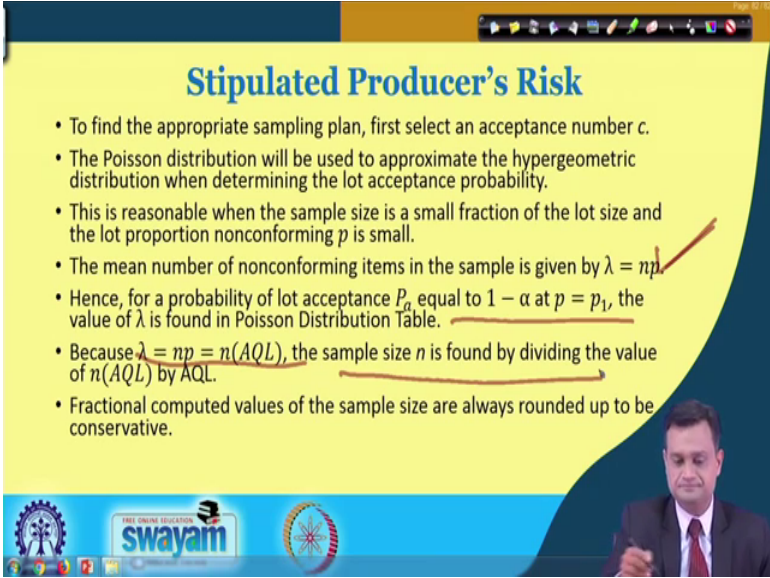
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So, stipulated producers risk it is say given by alpha, associated quality level p_1 which is basically AQL. And I would like the lot containing p_1 non conforming to be accepted $100(1 - \alpha)\%$ of the time in a very simple way. If your type one error alpha producers risk is 5 percent, then I would like the lot having AQL percent defective to be accepted 95 percent of the time.

Now, what would I will accept here that we want to find a sampling plan whose OC curve will basically pass through the point which is tagged as AQL 1 minus alpha. Now remember that this criteria is not very restrictive the OC curves of a variety of plans could pass through this point. So, you may have different sample size you may have different acceptance number. So, you cannot say there would be only one OC curve that will pass through this particular point AQL 1 minus alpha, but there could be multiple plans that you can devise, but my point here is to see that how best I can address the producers stipulated risk, in designing my single sampling plan.

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Stipulated Producer's Risk

- To find the appropriate sampling plan, first select an acceptance number c .
- The Poisson distribution will be used to approximate the hypergeometric distribution when determining the lot acceptance probability.
- This is reasonable when the sample size is a small fraction of the lot size and the lot proportion nonconforming p is small.
- The mean number of nonconforming items in the sample is given by $\lambda = np$.
- Hence, for a probability of lot acceptance P_a equal to $1 - \alpha$ at $p = p_1$, the value of λ is found in Poisson Distribution Table.
- Because $\lambda = np = n(AQL)$, the sample size n is found by dividing the value of $n(AQL)$ by AQL.
- Fractional computed values of the sample size are always rounded up to be conservative.

So, with this let us try to move it and what I would like to do to find the appropriate sampling plan for select an acceptance number that is the c the procedure I am just going through. The Poisson distribution will be used to approximate the hypergeometric distribution, when determining the lot acceptance probability. So, we have done this exercise previously also. So, your Poisson distribution table will have mean as well as the value individual value; by looking at these you can figure out what is the probability of acceptance. And this will give you the cumulative probability from the Poisson distribution table.

Now, mean number you can find for referring the Poisson distribution table, that is lambda is equal to $n p$ and P_a equals to $1 - \alpha$. So, you have a probability of acceptance and p is equal to p_1 , that is the value of lambda found in the Poisson

distribution table. Now here because you have p that is AQL, so I will say λ is equal to $n p$ is equal to n into AQL and this is what you will try to refer when you will see the cumulative Poisson distribution table.

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Example

Find a single sampling plan that satisfies a producer's risk of 5% for lots that are 1.5% nonconforming.

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So, now let us try to see this through example. So, when you see the numbers the idea is better clear. Now find or design a single sampling plan that satisfies the producers risk 5 percent as usual and you have the lots with 1.5 percent non-conforming. So, you are given this two value α 5 percent and the lots with 1.5 percent non conforming, we are supposed to design single sampling plan.

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Standard Table of Values of np for a Producer's Risk of 0.05 and a Consumer's Risk of 0.10 for different values of c

[Source: F.E. Grubbs (1949), "On designing single sampling Plans," Annals of Mathematical Statistics, XX, 256]

Note:

- ✓ np_1 values are used for deriving single sampling plan that satisfies producer's risk
- ✓ np_2 values are used for deriving single sampling plan that satisfies consumer's risk

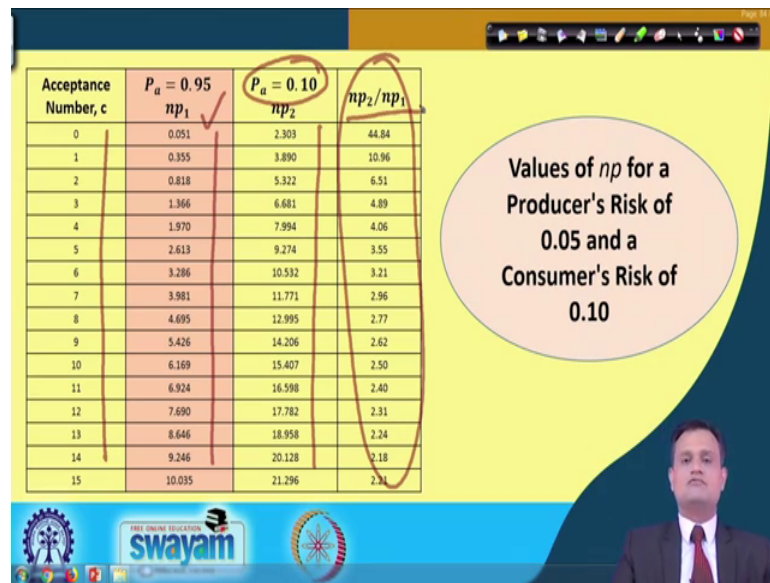
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So, here I would refer a particular standard table, we are not interested in seeing how this table have come? But I will refer a particular standard table of values of np for a producer's risk of 0.05 and consumers risk 0.10 this particular table is developed and published by F.E Grubbs in 1949 long back, on designing single sampling plans and it was published in Annals of Mathematical Statistics.

So, typically I will have two things here one is the np_1 values that are used for deriving single sampling plan, that satisfies the producers risk. And np_2 these are used for deriving single sampling plan that satisfies the consumers risk. So, this researcher F.E Grubb has designed a table, standard table which can help us to find the value of np_1 and np_2 for the producers risk and respective say consumers risk respectively. So, that we can easily figure out our single sampling plan for a stipulated producer risk or a stipulated consumer risk and also, we can use this table to find a plan which meets the stipulated producer and consumer risk both.

So, we have three conditions for which we will try to develop the single sampling plan

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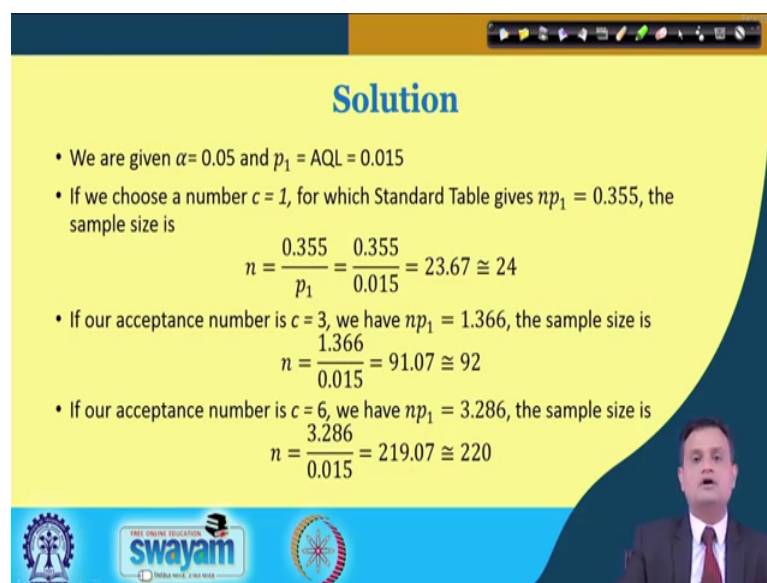
Acceptance Number, c	$P_a = 0.95$ np_1	$P_a = 0.10$ np_2	np_2/np_1
0	0.051	2.303	44.84
1	0.355	3.890	10.96
2	0.818	5.322	6.51
3	1.366	6.681	4.89
4	1.970	7.994	4.06
5	2.613	9.274	3.55
6	3.286	10.532	3.21
7	3.981	11.771	2.96
8	4.695	12.995	2.77
9	5.426	14.206	2.62
10	6.169	15.407	2.50
11	6.924	16.598	2.40
12	7.690	17.782	2.31
13	8.466	18.958	2.24
14	9.246	20.128	2.18
15	10.035	21.296	2.14

Values of np for a Producer's Risk of 0.05 and a Consumer's Risk of 0.10

So, now just see the table and what it tells? You have the acceptance number in the first column and you have np_1 for P_a is equal to 0.95 so 1 minus alpha percentage. It means I am referring to the producers risk and these are the values specific to np_1 it is a standard table. So, we just accept it in order to design our single sampling plan; np_2 it is for 10 percent beta you can see here 10 percent beta.

And this is for used for my stipulated consumer risk based single sampling plan. You will see the third column where you have np_2 divided by np_1 , this we will use for devising or developing the single sampling plan which can accommodate stipulated producers as well as consumers risk.

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Solution

- We are given $\alpha = 0.05$ and $p_1 = \text{AQL} = 0.015$
- If we choose a number $c = 1$, for which Standard Table gives $np_1 = 0.355$, the sample size is
$$n = \frac{0.355}{p_1} = \frac{0.355}{0.015} = 23.67 \cong 24$$
- If our acceptance number is $c = 3$, we have $np_1 = 1.366$, the sample size is
$$n = \frac{1.366}{0.015} = 91.07 \cong 92$$
- If our acceptance number is $c = 6$, we have $np_1 = 3.286$, the sample size is
$$n = \frac{3.286}{0.015} = 219.07 \cong 220$$

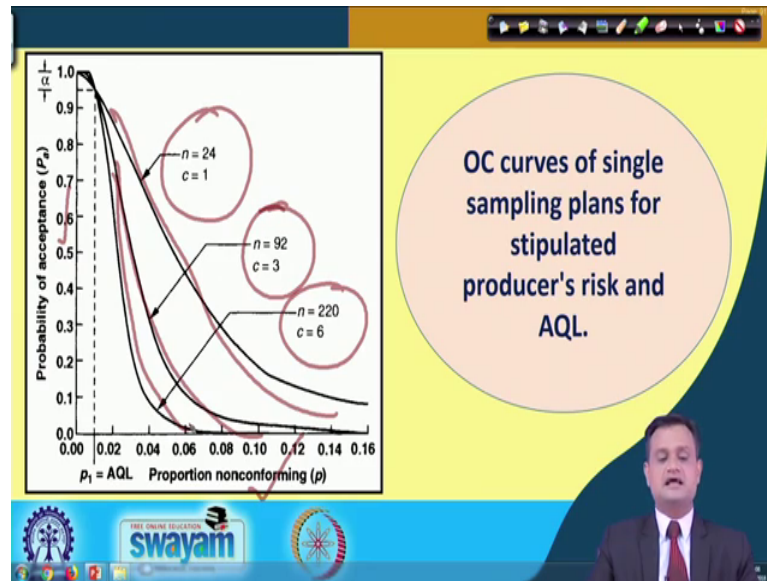
Logos at the bottom: Swayam, and other institutional logos.

So, now, the thing is very simple that we have this table. And now the data given is like this alpha is equal to 0.05 and you have p_1 is equal to AQL 0.015. So, 1.5 percent defective is 0.015. So, this is my p_1 is equal to AQL. And I have alpha is equal to 0.05.

Now just refer the table and for c is equal to 1 what do you find? So, let me go back to help you. So, for c is equal to 1 what you find here is 0.355, we will only refer this particular column which I have highlighted in pink. When we will talk about the next case I will highlight this second column, third case I will highlight third column in pink, so that you can focus on that particular column. So, here you can see that the value of np_1 basically 0.355. So, once you get this value from the table, you just see here that for c is equal to 1 np_1 is 0.355.

So, I can just rewrite it like this and my p_1 is 0.015 so I will have 24 as the sample size. Now let us say, if our acceptance plan is c is equal to 3 acceptance number is c equal to 3, then I will get corresponding np_1 1.366. Let us verify, so that you become crystal clear. So, for c is equal to 3 you can see that, c is equal to three it is 1.366. So, I hope now it is clear that I am checking the value for different c the value of np_1 and this is 92 same way this will be 220 for c is equal to 6.

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Now once I done this calculation, you can easily plot on your operating characteristic curve for single sampling plan for stipulated producers risk and AQL. So, I referred AQL I referred Grubbs table for typically stipulated produces risk and this is where I get the probability of acceptance, probability of non conforming I get n is equal to 24 for c is equal to 1 this is the OC curve, then n is equal to 92 c is equal to 3 this is the OC curve n is equal to 220 c is equal to 3, this is the OC curve. You can see that, as the sample size increases I am going for steeper and steeper operating characteristic curve. It means my discriminatory power increases and this is exactly I told that not necessary that you will have only one OC curve passing through a point AQL and 1 minus alpha percent. But you can have multiple OC curve depending upon sample size n and acceptance number c.

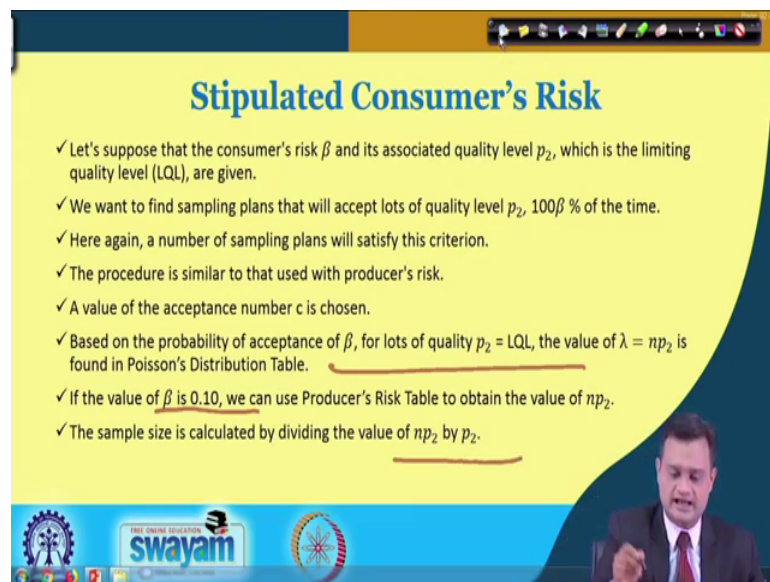
So, now, out of these which one to choose depends on the criticality of the quality characteristic and the consequences that it may have on the final quality of the product. So, depending upon that you can decide or you can choose judiciously the discriminatory power of the single sampling plan.

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So, when you have understood this, let us try to apply the same for the second case and second case is stipulated consumers risk.

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Now, here same way I will try to follow the procedure, but I will refer the beta and p_2 is equal to LQL. So, my limiting quality level typically indicates that the bad quality lot at some beta percent consumers risk and this is what I will try to use. So, my beta is 0.10.

And I will try to find np_2 from the Grubbs table so that I can figure out my particular acceptance sampling plan.

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Example

Find a single sampling plan that will satisfy a consumer's risk of 10% for lots that are 8% nonconforming.

swamy

So, let us see the example and it would be clear, that single sampling plan that will satisfy a consumer's risk of 10 percent for lots that are 8 percent nonconforming. So, I will once again refer this particular table and if you see I have now highlighted the second column in pink. So, I have to refer these because np_2 is specific to consumers risk and I have been given the value of consumer's risk 0.10.

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Solution

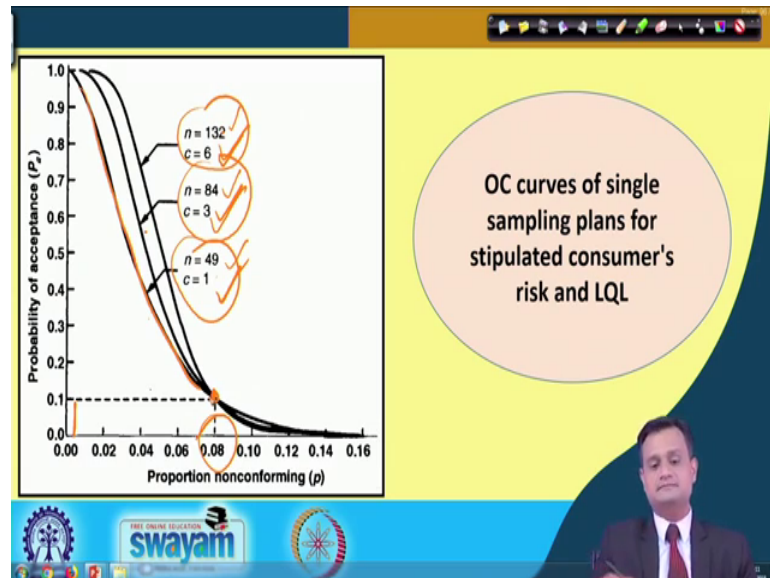
- We are given $\beta = 0.10$ and $p_2 = LQL = 0.08$
- If we select a number $c = 1$, for which Standard Table gives $np_2 = 3.890$, the sample size is
$$n = \frac{3.890}{p_2} = \frac{3.890}{0.08} = 48.62 \cong 49$$
- If our acceptance number is 3, we have $np_2 = 6.681$, and the sample size is
$$n = \frac{6.681}{0.08} = 83.51 \cong 84$$
- If our acceptance number is 6, we have $np_2 = 10.532$, the sample size is
$$n = \frac{10.532}{0.08} = 131.65 \cong 132$$

swamy

So, what I get here? When I refer the table for c is equal to 1 I get the value np_2 3.890.

I get n by putting value in this similar way for c is equal to 3, I get n is equal to 84 6 I get this.

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So, finally, you can figure out an acceptance sampling plan for single sampling with n is equal to 132 and n is equal to 84, n is equal to 49 and what do you see here? My sample size is reducing, but this is 132 this is 84 this is 49, but simultaneously my value of c is also reducing and you will find that the combination of these two is giving me a better discriminatory power. So, here it is not straight forward, what we have seen that you increase the sample size for producer's risk and your curve becomes steeper.

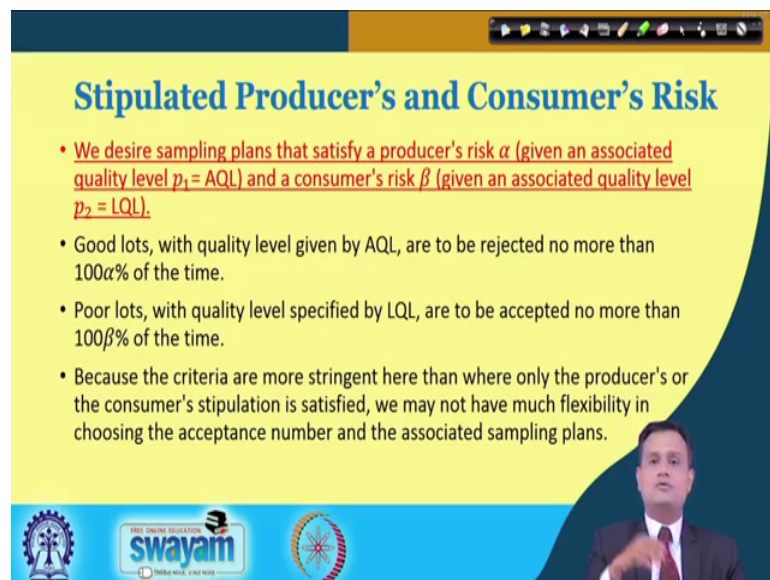
Here even though the sample size is decreasing or if you say increasing, my curve is not behaving like that, but simultaneously you should see the effect of c and you will find that for this beta my consumers risk and this is the limiting quality level, I have three different OC curves which is passing through this LQL point. So, LQL and beta basically say designs my single sampling plan for stipulated producer's consumers risk.

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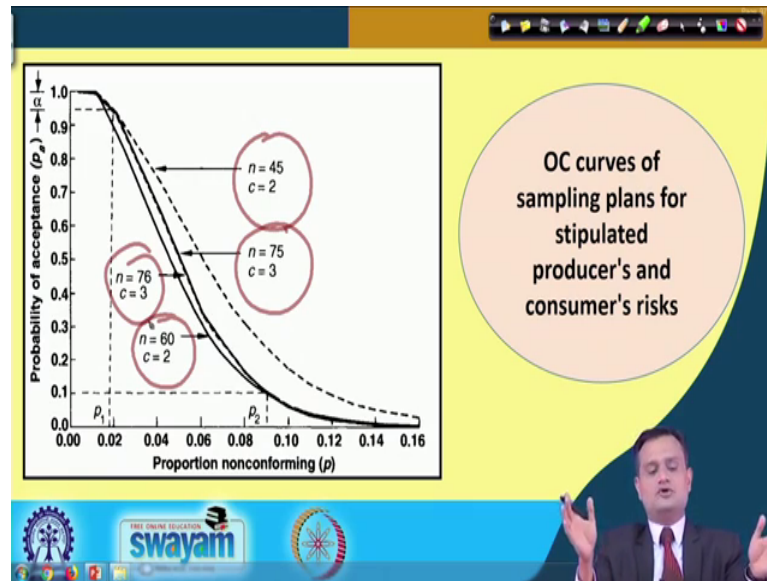
Now, let us see the third case I want to design a plan for stipulated producers and consumers risk both.

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So, now what I want? I want a plan basically which satisfy a producer risk alpha given an appropriate quality that is p_1 is equal to AQL; and also consumers risk beta given an associated p_2 that is LQL. So, now, I want to consider both previously I considered only AQL that is p_1 and alpha in the second one I considered LQL and then, beta here I want to consider both my LQL as well as AQL beta and alpha.

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So, for this I would first try to present the operating characteristic curve and what it indicates that, my sampling plan whatever I have. Let us say n is equal to 45 and c is equal to 2 n is equal to 75 and c is equal to 3 and n is equal to 76 c is equal to 3 n is equal to 60 and c is equal to 2. So, now, here you will see that there are two plans which are indicated in dotted and there are two plans which are indicated in full black line. So, if you see the plans which are indicated in dotted they are passing through this particular that is the AQL, and the plans which are in dark line they are passing through this. Now I have to select both in such a way that a proper trade off can be set when I select a particular plan.

So, this is really interesting and you have two plans which satisfies the stipulated risk indicated in dotted, stipulated risk of producer the plan indicated in full black stipulated risk of my consumer. So, now, let us see how we can go ahead.

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Stipulated Producer's and Consumer's Risk

- ❑ Let's consider the plans shown in Figure.
- ❑ Two plans meet the producer's stipulation exactly and come close to meeting the consumer's stipulation.
- ❑ Two other plans meet the consumer's stipulation exactly and come close to meeting the producer's stipulation.

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So, I want to figure out basically the plans which can best satisfy the stipulated risk of producer and stipulated risk of consumer. So, two plans need the producers stipulated risk as I mentioned in dotted line. And two plans in full line they meet the consumer stipulated risk exactly and come close to meeting the producer's stipulation.

So, now, what happens that? If I look at the dotted line for example, here then you have two dotted curves, let us say this is the one and this is the second one which is little bit hidden. These are the plans which exactly meets the producer of stipulated risk, because they are passing through this AQL and I have the full line plan which are passing through this LQL so exactly they meet.

Now; obviously, I have to meet the expectation of both producers and this consumer so what I would like to do? I can design a plan which exactly meet the producers risk and would be very close to my consumer stipulation. Similar way I can do otherwise, I would like to choose a plan which exactly means the stipulation of the consumer and would be very close to my producer.

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Stipulated Producer's and Consumer's Risk

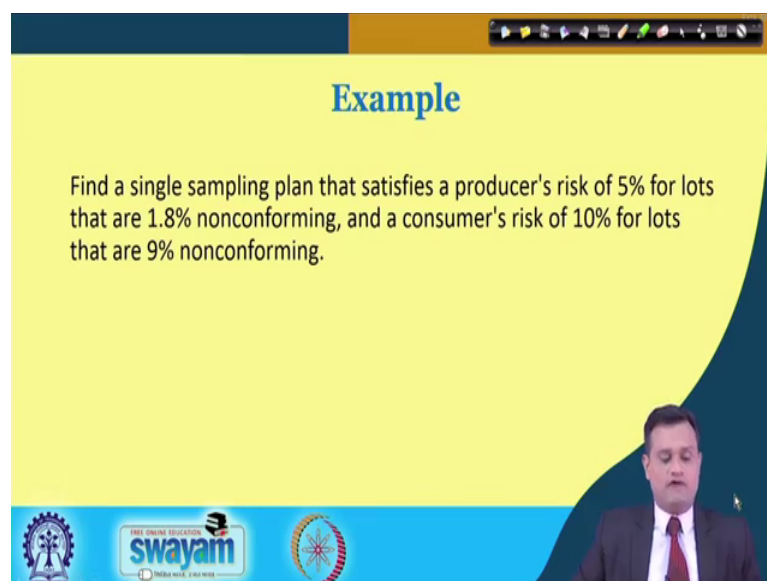
Which Plan to Select?:

Of these four plans, one must be selected based on additional criteria of concern to the user.

- ✓ It may be of interest, for example, to choose the plan with the smallest sample size to minimize inspection costs, or the one with the largest sample size to provide the most protection.
- ✓ Alternatively, a preference to satisfy either the producer's or consumer's risk could be incorporated in the decision-making framework. That is, a user may desire the producer's stipulation to be satisfied exactly, with the consumer's stipulation satisfied as closely as possible, or vice versa.

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Example

Find a single sampling plan that satisfies a producer's risk of 5% for lots that are 1.8% nonconforming, and a consumer's risk of 10% for lots that are 9% nonconforming.

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So, this is the choice what I have, now if I just try to see it through example then let us try to see, find a single sampling plan that satisfy the producers risk of 5 percent for lots that are 1.8 percent nonconforming, and consumer risk of 10 percent for lots that are 9 percent nonconforming. So, 5 percent is the producer's risk that is the alpha 1.8 percent non conforming that is your related to producer. Consumer risk 10 percent and lots that are 9 percent nonconforming. So, I will again use this particular table and you can see that now I have highlighted this particular last column in pink. So, I would be

simultaneously using np_2 and np_1 in order to find a plan which satisfy my producer and consumer both.

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Solution

- We have $\alpha = 0.05$ and $p_1 = AQL = 0.018$, $\beta = 0.10$ and $p_2 = LQL = 0.09$. We can put the ratio np_2/np_1 , which is the ratio of p_2/p_1 because n cancels out.

$$\frac{p_2}{p_1} = \frac{LQL}{AQL} = \frac{0.09}{0.018} = 5.00$$
- From the Standard Table we find that the ratio 5.00 falls between 6.51 and 4.89, corresponding to acceptance numbers of 2 and 3, respectively.
- Two plans (one for $c = 2$ and one for $c = 3$) satisfy the producer's stipulation exactly:
- For $c = 2$, $np_1 = 0.818$, the sample size is

$$n = \frac{np_1}{p_1} = \frac{0.818}{0.018} = 45.44 \approx 45$$
- For $c = 3$, $np_1 = 1.366$, the sample size is

$$n = \frac{np_1}{p_1} = \frac{1.366}{0.018} = 75.88 \approx 76$$
- So, the plans $n = 45, c = 2$ and $n = 76, c = 3$ both satisfy the producer's stipulation exactly.

So, just see what is the data given you have α is equal to 0.05 p_1 is AQL 0.018 β is 0.10 p_2 LQL is 0.09 and we can find the ratio np_2 by np_1 . So, p_2 by p_1 is basically LQL by AQL and this is 5. Now let us see that where this ratio falls? So, this particular ratio 5 falls between 6.51 and 4.89 corresponding to acceptance number 2 and 3. So, you want to see then let me go back, this is the say table and just see the ratio 5 so this ratio 5 will fall somewhere here, 5 will fall somewhere here.

So, this is specific to acceptance number 2 and acceptance number 3. So, now, what I have I have two acceptance number; c is equal to 2 and c is equal to 3. So, I can now find c is equal to 2 np_1 value from the same table, and I can find the value of n similar way c is equal to 3 np_1 and I can find the value of n . So what I get here is n is equal to 45 and c is equal to 2 n is equal to 76 and c is equal to 3 both satisfy the producers stipulation exactly. So, this is something because I have used this np_1 which is specific to producer so these plans exactly satisfy the producers stipulation.

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• Next, we find that two plans ($c = 2$ and $c = 3$) satisfy the consumer's stipulation exactly:

• For $c = 2$, $np_2 = 5.322$, the sample size is

$$n = \frac{np_2}{p_2} = \frac{5.322}{0.09} = 59.13 \cong 60$$

• For $c = 3$, $np_2 = 6.681$, the sample size is

$$n = \frac{np_2}{p_2} = \frac{6.681}{0.09} = 74.23 \cong 75$$

• The plans $n = 60, c = 2$ and $n = 75, c = 3$ both satisfy the consumer's stipulation exactly.

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Now let me go ahead to this really interesting just do it comfortably and it would be crystal clear. Now if I refer the column two in order to consider the consumers stipulation, I will get c is equal to 2 for $n p_2 \leq 5.322$ and I will get n is equal to 60 for c is equal to 3. I will have $n p_2 \leq 6.681$ and n is equal to 75. So, I can chock out two plans from consumer stipulation point of view one is n is equal to 60 c is equal to 2 n is equal to 75 c is equal to 3.

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Finally, What we Have?

Plans 1 & 2: Producer's stipulation exactly
Plans 3 & 4: Consumer's stipulation exactly

Plan 1: $n = 45, c = 2$ Plan 3: $n = 60, c = 2$
Plan 2: $n = 76, c = 3$ Plan 4: $n = 75, c = 3$

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So, now its really interesting that I am left with now four plans plan 1 and 2 producers stipulation exactly, plan 3 and 4 consumers stipulation exactly. So, plan 1 n is equal to 45 c is equal 2, plan 2 76 and 3 plan 3 and plan 4.

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- Now let's see how close plans 1 and 2 (which satisfy the producer's stipulation) come to satisfying the consumer's stipulation.
- For a target value of the consumer's risk β of 0.10, we find the proportion nonconforming p_2 of batches that would be accepted $100\beta\%$ of the time.
- For $n = 45$ and $c = 2$ (plan 1), if $\beta = 0.10$, then $np_2 = 5.322$. Thus,

$$p_2 = \frac{np_2}{n} = \frac{5.322}{45} = 0.1183$$
- For $n = 76$ and $c = 3$ (plan 2), if $\beta = 0.10$, then $np_2 = 6.681$. So

$$p_2 = \frac{np_2}{n} = \frac{6.681}{76} = 0.0879$$
- Plan 1 accepts batches that are 11.83% nonconforming 10% of the time. On the other hand, plan 2 accepts batches that are only 8.79% nonconforming 10% of the time. Our goal is to find a plan that accepts batches that are 9% nonconforming 10% of the time.

Now what I want to figure out? I want to go by either of the criteria and the criteria is like this, I want a plan which exactly meets the producers stipulation and would be very close to my consumer or a plan which would be exactly meeting the stipulation of consumer and would be very close to producer you cannot satisfy both exactly.

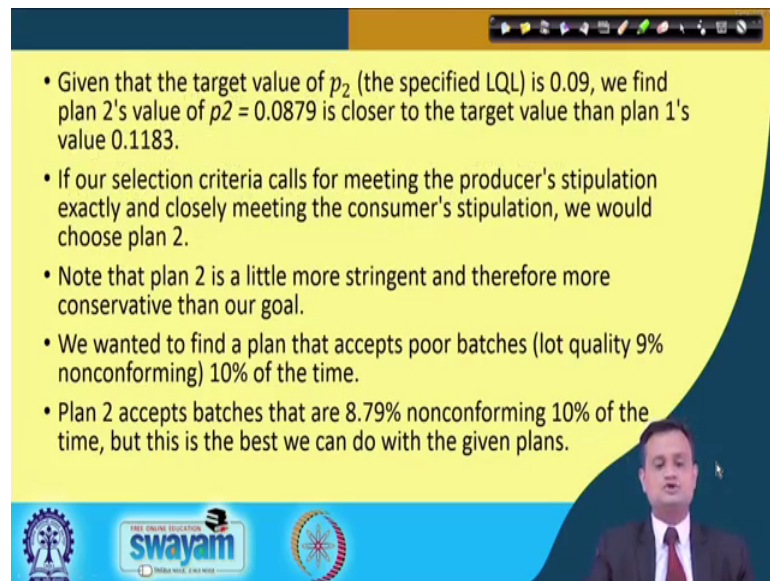
So, it is something like if you have a two child and they have different requirement you will see that ok, the requirement of child one is extremely critical let me try to meet it exactly; and if the child two then let me try to go as close as possible. So, it is exactly like that. So, here what I will do that for a target value of consumer is beta 0.10 we find the proportion of non conforming p_2 or batches that would be accepted 100 beta percent of the time. So, plan 1 n is equal to 45 and c is equal to 2.

So, I am taking two plans which exactly meets the producers stipulation, satisfies the producer simulation and for beta is equal to 0.10 np_2 is equal to 5.322 I can find the value of p_2 value of p_2 . Now what do you see here? Plan 1 gives you value of p_2 0.1183, plan 2 gives you value of p_2 0.0879. So, my condition here is that, plan 1 accepts batches that are 11.83 percent non conforming 10 percent of the time, because

beta is 10 percent on the other hand plan 2 accept the batches that are only 8.79 non conforming 10 percent of the time.

Our goal is to find that plan can accept the batches that are 9 percent nonconforming 10 percent of the time. So, 10 percent is beta do not get confused. So, now, what is close to 9 percent I am exactly considering the stipulation of producer and I want to go very close to consumer. So, here this plan 2 is very close to 9 percent.

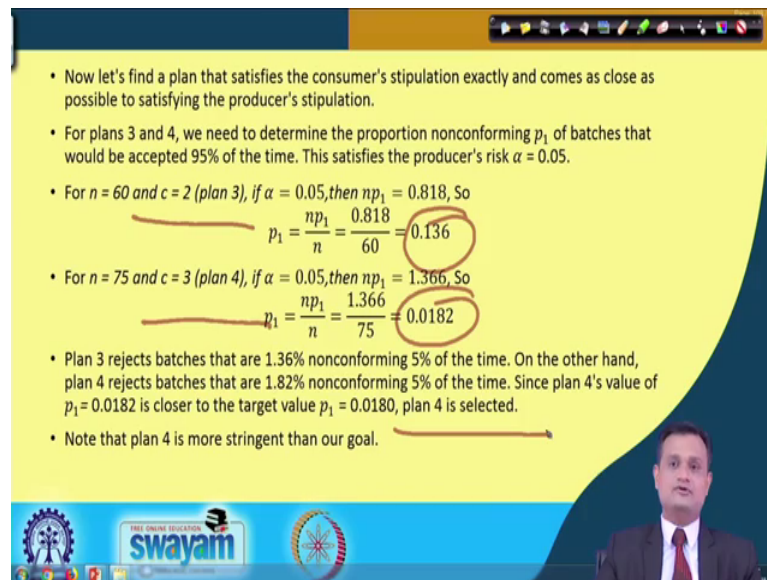
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- Given that the target value of p_2 (the specified LQL) is 0.09, we find plan 2's value of $p_2 = 0.0879$ is closer to the target value than plan 1's value 0.1183.
- If our selection criteria calls for meeting the producer's stipulation exactly and closely meeting the consumer's stipulation, we would choose plan 2.
- Note that plan 2 is a little more stringent and therefore more conservative than our goal.
- We wanted to find a plan that accepts poor batches (lot quality 9% nonconforming) 10% of the time.
- Plan 2 accepts batches that are 8.79% nonconforming 10% of the time, but this is the best we can do with the given plans.

And I will say that if I select plan 2 it would be very close. So, if our selection criteria calls for meeting the producer's stipulation exactly and closely meeting the consumer stipulation I will go with plan 2. So, plan 2 is a little bit more stringent because my expectation is 9 percent it is 8.9 percent. So, it is a little bit more stringent, but it will help me to satisfy the exact producer's stipulation and very close consumer stipulation.

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• Now let's find a plan that satisfies the consumer's stipulation exactly and comes as close as possible to satisfying the producer's stipulation.

• For plans 3 and 4, we need to determine the proportion nonconforming p_1 of batches that would be accepted 95% of the time. This satisfies the producer's risk $\alpha = 0.05$.

• For $n = 60$ and $c = 2$ (plan 3), if $\alpha = 0.05$, then $np_1 = 0.818$, So

$$p_1 = \frac{np_1}{n} = \frac{0.818}{60} = 0.136$$

• For $n = 75$ and $c = 3$ (plan 4), if $\alpha = 0.05$, then $np_1 = 1.366$, So

$$p_1 = \frac{np_1}{n} = \frac{1.366}{75} = 0.0182$$

• Plan 3 rejects batches that are 1.36% nonconforming 5% of the time. On the other hand, plan 4 rejects batches that are 1.82% nonconforming 5% of the time. Since plan 4's value of $p_1 = 0.0182$ is closer to the target value $p_1 = 0.0180$, plan 4 is selected.

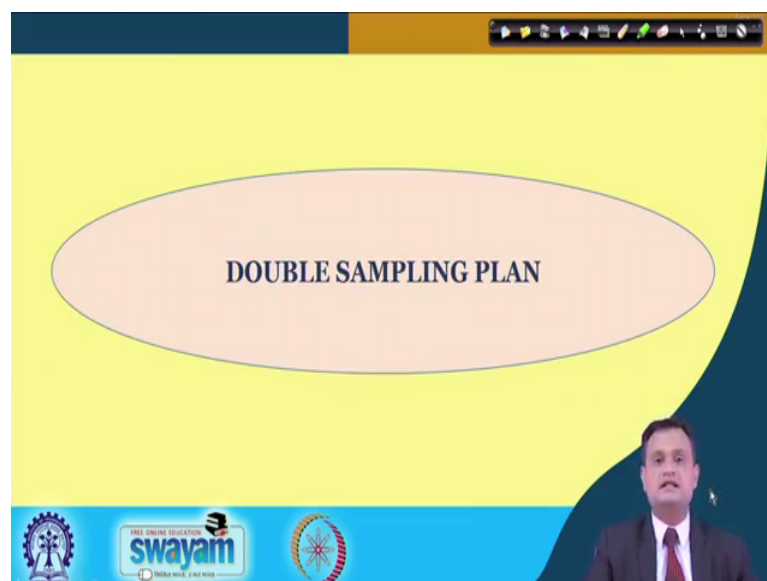
• Note that plan 4 is more stringent than our goal.

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Similar way I can analyse my plan 3 and plan 4, here and what you can see that I am getting $p_1 = 0.136$ and $p_1 = 0.0182$. So, you will select plan 4 which is very close to the expected value of my producer's stipulation and I can here me to my consumer stipulation exactly in plan 3 and 4, and I could be very close to the stipulation of my producer.

So, maybe little bit sounding confusing, but the concept is very simple I explained

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DOUBLE SAMPLING PLAN

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Now let us look at the double sampling plan.

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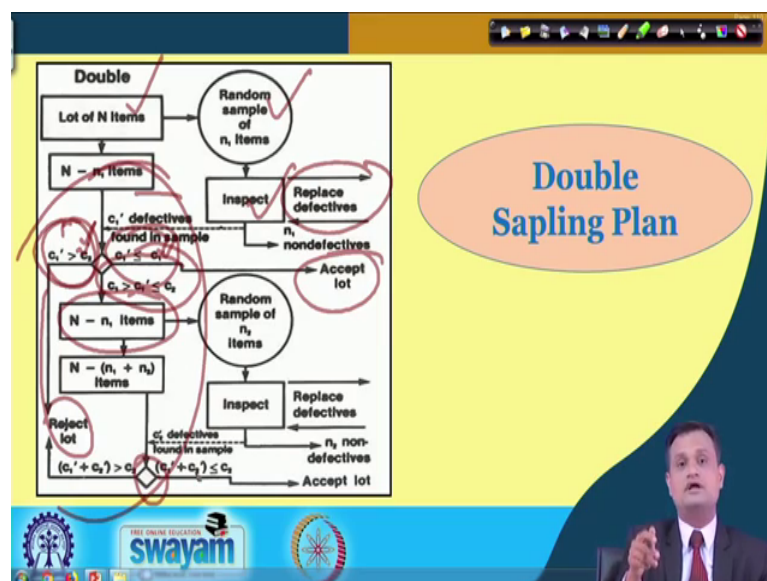
Double Sampling Plans

- ♦ Take small initial sample
 - ♦ If # defective < lower limit, accept
 - ♦ If # defective > upper limit, reject
 - ♦ If # defective between limits, take second sample
- ♦ Accept or reject based on 2 samples
- ♦ Less costly than single-sampling plans



So the idea is very simple, if you have defective and lower less than the lower limit accept no issue first go, if you have a defective greater than upper limit reject. And if you have number of defective between limit, take the second sample accept or reject based on 2 samples not on one sample loss less costly than the single sampling plan, because the average sample number in double sampling would be little bit say lesser than or drastically less than the single sampling plan.

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So, just see how it works double sampling plan? I have the lot of N items I randomly select a sample of an one item I inspect it, I may replace defectives. Now let us say this is the main part say I am coming here c_1 dash is the defective find in a sample, suppose c_1 is greater than c_2 and c_1 is less than or equal to c_1 , find this c_1 is my one limit this c_2 is another limit. If exits this I will simply reject. If it is less than this I will not bother I will accept. Now if it is in between your c_1 and c_2 I will go for this is in between c_1 and c_2 second sample as already I have drawn one sample of n_1 . So, I am left with N minus n_1 , I will draw another sample. And now I will take a decision here that, c_1 dash plus c_2 dash.

So, number of defectives found in the first sample plus number of defectives found in the second sample, if it is less than this particular value c_3 or if it is greater than I will now stop by taking a decision either reject or accept.

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Double Sampling Plan

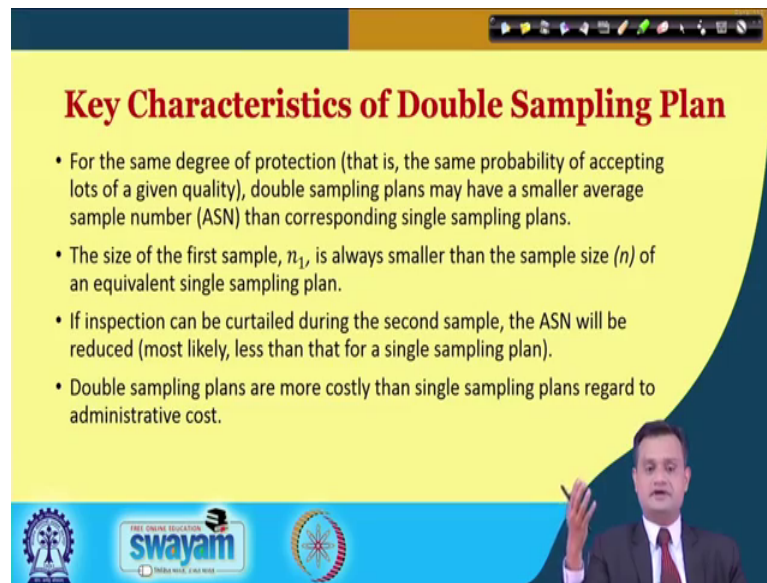
The parameters of a double sampling plan

- n_1 : size of the first sample
- c_1 : acceptance number for the first sample
- r_1 : rejection number for the first sample
- n_2 : size of the second sample
- c_2 : acceptance number for the second sample
- r_2 : rejection number for the second sample

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So, this is how my double sampling plan works, and only thing that I am going for one more equation. And I am considering the number of defectives found in the first sample as well as second, in order to make my final decision after taking the second sample. So, the notations which we used generally may differ from book to book, but it may be like n_1 size of the first sample, c_1 acceptance number of the first sample, r_1 rejection for the first sample, n_2 size of second sample c_2 acceptance number of the second sample and r_2 rejection number of the second sample.

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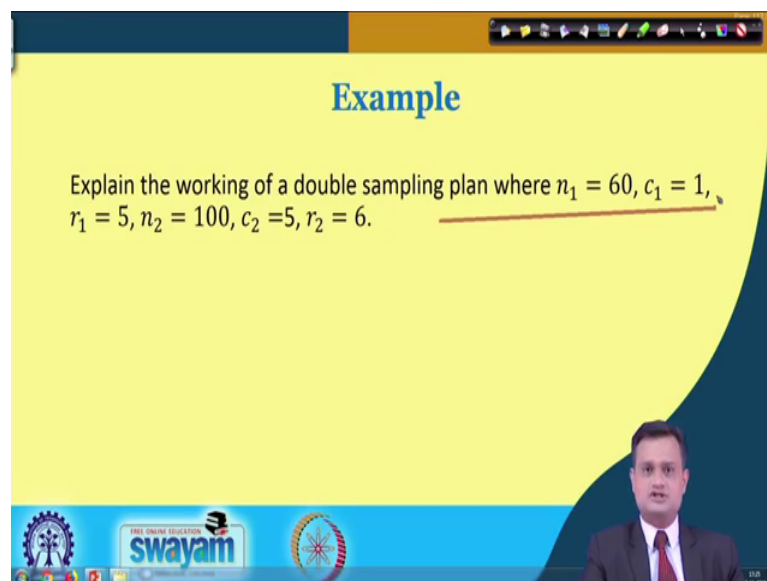
Key Characteristics of Double Sampling Plan

- For the same degree of protection (that is, the same probability of accepting lots of a given quality), double sampling plans may have a smaller average sample number (ASN) than corresponding single sampling plans.
- The size of the first sample, n_1 , is always smaller than the sample size (n) of an equivalent single sampling plan.
- If inspection can be curtailed during the second sample, the ASN will be reduced (most likely, less than that for a single sampling plan).
- Double sampling plans are more costly than single sampling plans regard to administrative cost.

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So, this is what exactly we do, and typically there are some key characteristic I mentioned previously also that every sample number, in case of double sampling plan may be smaller compared to the single and the size of the first sample is always smaller than the sample size of the equivalent single sampling plan. If inspection can be curtailed during second sample, the ASN will be reduced and double sampling plans are more costly than single sampling plan in terms of the record keeping and the administrative cost.

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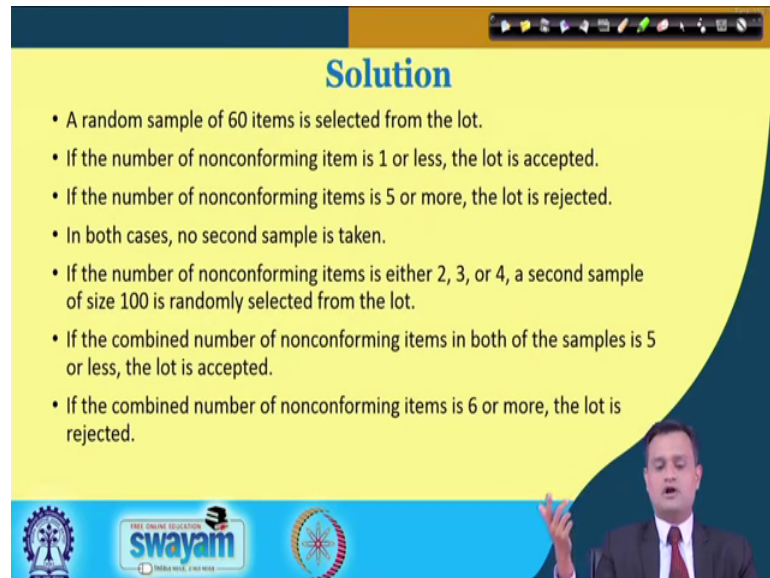
Example

Explain the working of a double sampling plan where $n_1 = 60$, $c_1 = 1$, $r_1 = 5$, $n_2 = 100$, $c_2 = 5$, $r_2 = 6$.

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So, let us see quickly what it means so let us say I have n_1 is equal to 60 first sample first acceptance number 1 rejection number 5 n_2 100 c_2 5 r_2 6 and how it works?

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Solution

- A random sample of 60 items is selected from the lot.
- If the number of nonconforming item is 1 or less, the lot is accepted.
- If the number of nonconforming items is 5 or more, the lot is rejected.
- In both cases, no second sample is taken.
- If the number of nonconforming items is either 2, 3, or 4, a second sample of size 100 is randomly selected from the lot.
- If the combined number of nonconforming items in both of the samples is 5 or less, the lot is accepted.
- If the combined number of nonconforming items is 6 or more, the lot is rejected.

So, the solution is very simple, random sample of 60 is taken, number of non conforming found one or less lot is accepted if it is nonconforming is 5 or more lot is rejected. If it is anywhere in between 2 3 4, then I will take the second sample if the number of non conforming item 2 3 4, let us say I take the second sample of 100 n_2 is equal to 100.

If the combined number of nonconforming, nonconforming found in the first nonconforming found in the second is 5 or less lot is accepted, if it is say more than 6 the lot is rejected.

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The OC Curve

- The OC curve also measures the performance of double sampling plans.
- Calculations for double sampling plans are more involved than for single sampling plans.
- Let P_{a1} denote the probability of accepting the lot on the first sample and P_{a2} the probability of lot acceptance on the second sample.
- The combined probability of acceptance is given by $P_a = P_{a1} + P_{a2}$
- Let x_1 and x_2 denote the observed number of nonconforming items on the first and second samples, respectively. We have

$$P_{a1} = P(x_1 \leq c_1)$$

$$P_{a2} = P(x_1 = c_1 + 1)P(x_2 \leq c_2 - x_1) + P(x_1 = c_1 + 2)P(x_2 \leq c_2 - x_1) + \dots$$

$$+ P(x_1 = r_1 - 1)P(x_2 \leq c_2 - x_1)$$

$$P_a = P_{a1} + P_{a2}$$
- The Poisson approximation to the hypergeometric distribution is used here.
- This procedure can be used to construct the OC curve.

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So, this is what I try to do, I can also plot the OC curve and you can have a very simple expression P_{a1} that is referring to the probability of acceptance in the first say sample. So, probability that x_1 is less than or equal to c_1 c_1 is my acceptance number. And P_{a2} I will have x_1 is equal to c_1 plus 1, because already you have taken the first sample and now you are in between. So in between your acceptance and rejection, so c_1 plus 1 and probability of x_2 less than or equal to c_2 minus x_1 , plus probability of x_1 is equal to c_1 plus 2.

And probability into probability of x_2 less than or equal to c_2 minus x_1 , we discuss the probability issues in detail and you can appreciate this. So, total probability is P_{a1} plus P_{a2} probability of acceptance in first sample probability of acceptance in second sample.

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Example

Let's consider a double sampling plan of lot size 3000 given by the following parameters: $n_1 = 40$, $c_1 = 1$, $r_1 = 5$, $n_2 = 80$, $c_2 = 5$, $r_2 = 6$. For a lot proportion nonconforming value of $p = 0.03$, find the probability of accepting such lots.

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So, when we try to do it for a particular example let us say the lot size is 3000 n_1 is 40 c_1 is 1, r_1 is 5, n_2 is 80, c_2 is 5, r_2 is 6 and nonconforming value is p is equal to 0.03, find the probability of accepting such lots.

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Solution

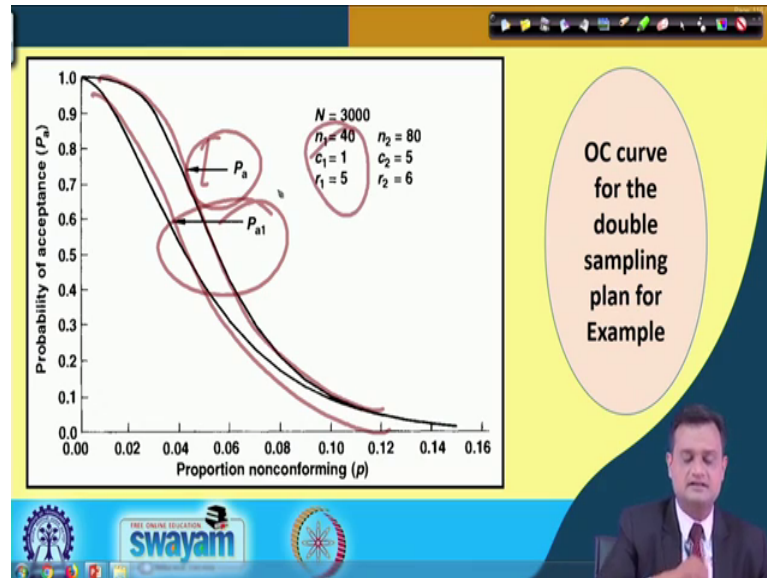
- To find the probability of acceptance on the first sample (P_{a1}), we have $n_1 p = 40(0.03) = 1.2$.
- The Poisson's cumulative probability distribution table give
 $P_{a1} = P(x_1 \leq 1) = 0.663$
- $P_{a2} = P(x_1 = 2)P(x_2 \leq 3) + P(x_1 = 3)P(x_2 \leq 2) + P(x_1 = 4)P(x_2 \leq 1)$
- Note that when calculating $P(x_2 \leq 3)$ and other probabilities dealing with x_2 , the value of $n_2 p = 80(0.03) = 2.4$. So
 $P_{a2} = (0.216)(0.779) + (0.087)(0.570) + (0.026)(0.308) = 0.2259$
- The combined probability of acceptance is
 $P_a = P_{a1} + P_{a2} = 0.663 + 0.2259 = 0.8889$
- Thus, about 66.3% of lots with a proportion nonconforming at 0.03 will be accepted on the first sample; for these lots a second sample will not be taken.
- For 22.59% of the lots, a second sample will be taken, and the lot will be accepted based on the evidence in both of the samples.
- Overall, 88.89% of the lots will be accepted by this sampling plan.

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So, you can just plug in the values and find the probabilities. So, this you can find that is the P_a 0.8889 and what does it say? That about 66.33 percent that is P_{a1} of lot with a proportion nonconforming 3 percent will be expected on the first sample. So, out of so many lot 66.3 percent of the lot that they will be cleared on the first sample with 3

percent defective; 22.59 percent lot a second sample will be taken and overall 88.59 percent of the lot will be accepted by the sampling plan. So, I think this is a quite interesting.

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And what you can see here, that for double sampling plan this is my P_a this is my P_a and this is my P_{a1} . So, you can see that the operating characteristic curve for total probability is on the upper side with and the lower one is your P_{a1} this is my single for drawn based on the first sampling. So, discriminatory power of this one is higher and it is cheaper compared to the total probability of acceptance.

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Average Sample Number and Average Total Inspection Curves

$$ASN = n_1 p_1 + (n_1 + n_2)(1 - P_1) = n_1 + n_2(1 - P_1)$$

Where P_1 is the probability of making a decision on the first sample.

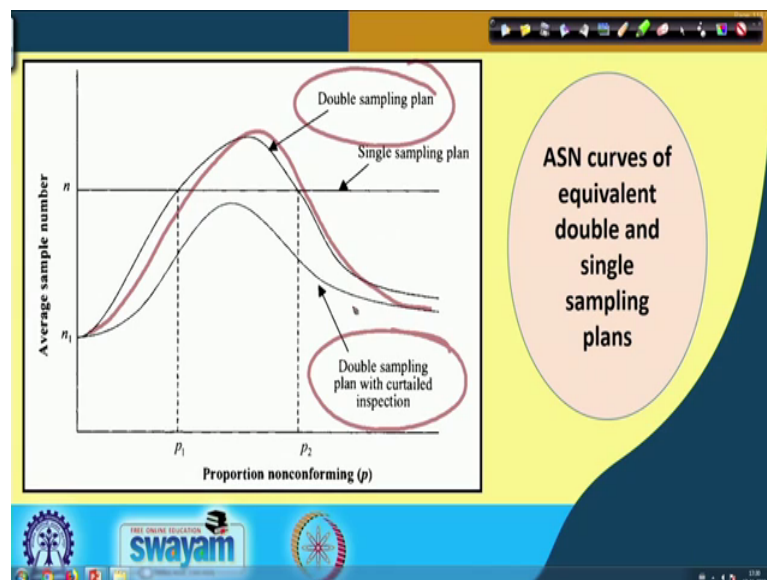
$$ATI = (n_1)P_{a1} + (n_1 + n_2)P_{a2} + N(1 - P_{a1} - P_{a2})$$

Where P_{a1} and P_{a2} represent the probability of accepting the lot on the first and second sample.



So, you can also find ASN by using the expression $n_1 p_1 + n_1 + n_2 - p_1$, and average total inspection for the double sampling plan.

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So, I think this thing we have discussed. So, you can just apply and just try to appreciate that double sampling plan your average sample number will be on higher side, double sampling plan with curtailed inspection. Once you know that in the first sample it is done; then you will curtail you will not go for the inspection. And this is less average

sample number in case of single sample it is only n because you are only going for one stage decision.

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Example

Find the average total inspection for lots with an incoming proportion nonconforming of 0.03 for a double sampling plan with lot size 3000 given by the following parameters: $n_1 = 40$, $c_1 = 1$, $r_1 = 5$, $n_2 = 80$, $c_2 = 5$, $r_2 = 6$.

So, you can consider again a data of lot size 3000 and the parameters these n_1 , c_1 , r_1 , n_2 , c_2 and r_2 .

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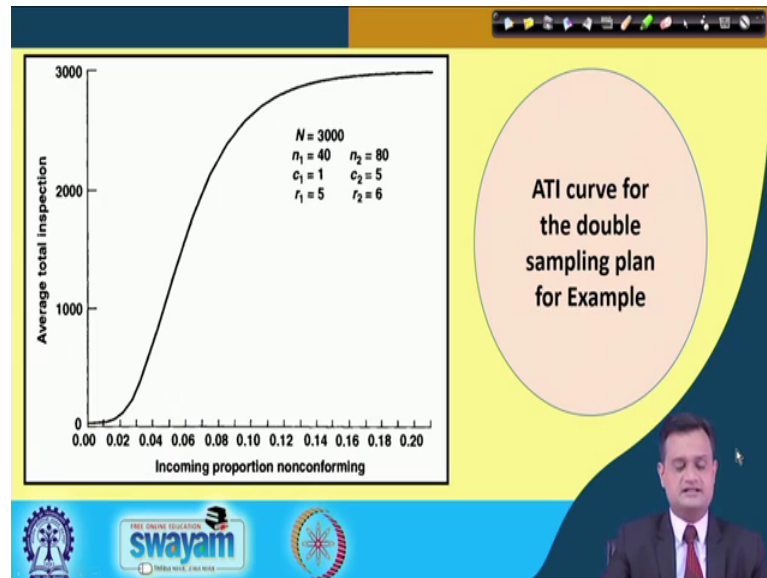
Solution

$$ATI = (n_1)P_{a1} + (n_1 + n_2)P_{a2} + N(1 - P_{a1} - P_{a2})$$
$$ATI = (40)0.663 + (40 + 80)0.2259 + 3000(1 - 0.663 - 0.2259)$$
$$= 386.93$$

And you can plug in this value, so you will get average total inspection 386.93.

So, for a given double sampling plan your average inspection total inspection will be of 386.93 parts or units.

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And you can see the ATI. So, as your incoming proportion nonconforming increases your average total inspection is also increasing.

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Design of Double Sampling Plan

- One criterion for designing double sampling plans involves satisfying a specified level of the producer's risks α at an associated quality level $p_1 = \text{AQL}$ and meeting a consumer's risk β at a quality level $p_2 = \text{LQL}$.
- Thus, we want to find a plan where the OC curve passes through the two points $(\text{AQL}, 1 - \alpha)$ and (LQL, β) .
- Another sample procedure will help us find double sampling plans. Let's assume the sample sizes are either $n_2 = n_1$, or $n_2 = 2n_1$.
- We will use a pair of tables known as Grubbs' tables.
- Both tables are based on a producer's risk α of 0.05 and a consumer's risk β of 0.10.

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You can design this double sampling plan in the same way for AQL $1 - \alpha$ LQL β .

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Grubbs' Table I for constructing double sampling plans

Plan	$R = p_2/p_1$	Acceptance Number		Approximate Values of $n_1 p$			Approximate for ASN/n_1 for
		c_1	c_2	$P_a = 0.95$	$P_a = 0.50$	$P_a = 0.10$	
1	11.90	0	1	0.21	1.00	2.50	1.170
2	7.54	1	2	0.52	1.82	3.92	1.081
3	6.79	0	2	0.43	1.42	2.96	1.340
4	5.39	1	3	0.76	2.11	4.11	1.169
5	4.65	2	4	1.16	2.90	5.39	1.105
6	4.25	1	4	1.04	2.50	4.42	1.274
7	3.88	2	5	1.43	3.20	5.55	1.170
8	3.63	3	6	1.87	3.98	6.78	1.117
9	3.38	2	6	1.72	3.56	5.82	1.248
12	3.21	3	7	2.15	4.27	6.91	1.173

$n_1 = n_2, \alpha = 0.05, \beta = 0.10$
 $Re_1 = Re_2 = Ac_2 + 1$
 $p_1 = AQL, p_2 = LTPD$

And you have the Grubbs table so there are two tables; the first table accepts n_1 is equal to n_2 alpha is equal to 0.05 and beta is equal to 0.10 and you have the r is equal to p_2 by p_1 c_1 c_2 P_a 0.95 P_a 0.50 p_a 0.10. So, approximate values of $n_1 p$ and P_a is 0.95.

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Grubbs' Table I for constructing double sampling plans (Contd.)

Plan	$R = p_2/p_1$	Acceptance Number		Approximate Values of $n_1 p$			Approximate for ASN/n_1 for
		c_1	c_2	$P_a = 0.95$	$P_a = 0.50$	$P_a = 0.10$	
11	3.09	4	8	2.62	5.02	8.10	1.124
12	2.85	4	9	2.90	5.33	8.26	1.167
13	2.60	5	11	3.68	6.40	9.56	1.166
14	2.44	5	12	4.00	6.73	9.77	1.215
15	2.32	5	13	4.35	7.06	10.08	1.271
16	2.22	5	14	4.70	7.52	10.45	1.331
17	2.12	5	16	5.39	8.40	11.41	1.452

$n_1 = n_2, \alpha = 0.05, \beta = 0.10$
 $Re_1 = Re_2 = Ac_2 + 1$
 $p_1 = AQL, p_2 = LTPD$

So, you have this table this is in continuation for n_1 is equal to n_2 , and similar way that table two Grubbs has develop that is for n_2 is equal to $2 n_1$.

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Design of Double Sampling Plan

These tables are used to construct double sampling plans as follows.

- First, the ratio R of LQL to AQL (or p_2/p_1) is found.
- Using either Table, depending on the circumstances, the value closest to the calculated R -value is found.
- Next, the value of $n_1 p$ is read from the appropriate table.
- Dividing $n_1 p$ by p (which is either p_1 or p_2) yields the size of the first sample (n_1).
- The complete sampling plan, involving acceptance number for the two samples, is read from Tables.



So, we can easily make use of this table for designing a sampling plan and the procedure is very simple. First the ratio R which is LQL to RQL should be calculated p_2 by p_1 using either of the table the value closest to the R should be found. Then value of $n_1 p$ is read from the appropriate table divide $n_1 p$ by p which either p_1 or p_2 yields the size of the first sample the complete sampling plan involving acceptance number for the two sample can be read from the table.

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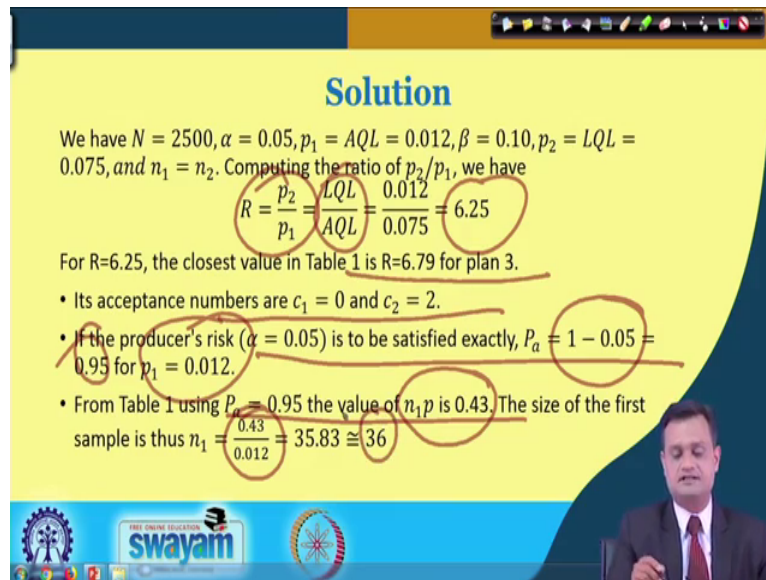
Example

Find a double sampling plan for lot size 2500 where it is desired to reject batches that are 1.2% nonconforming no more than 5% of the time, while batches that are 7.5% nonconforming are accepted about 10% of the time. The sample sizes are equal, and the producer's stipulation must be satisfied exactly.



So if we just see the example, then lot size is 2500 1.2 percent nonconforming 7.5 percent nonconforming are accepted at some beta. And the sample size are equal and the producers stipulation must be satisfied exactly.

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Solution

We have $N = 2500, \alpha = 0.05, p_1 = AQL = 0.012, \beta = 0.10, p_2 = LQL = 0.075$, and $n_1 = n_2$. Computing the ratio of p_2/p_1 , we have

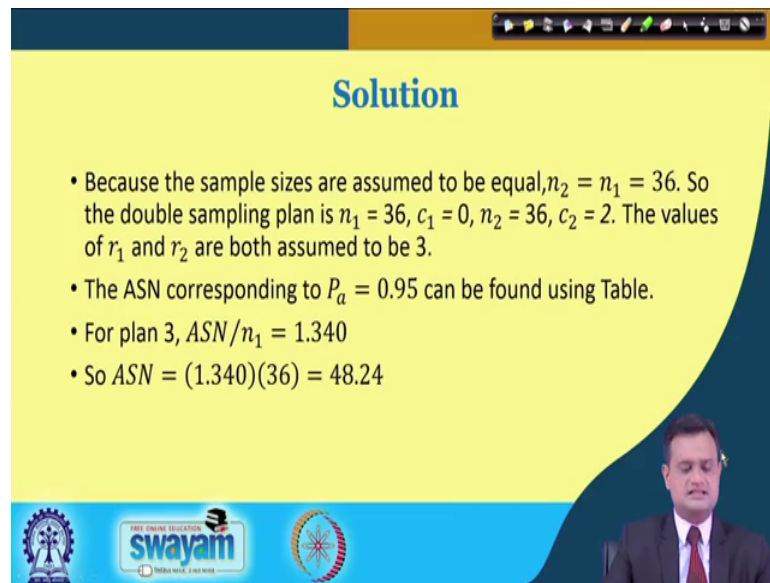
$$R = \frac{p_2}{p_1} = \frac{LQL}{AQL} = \frac{0.075}{0.012} = 6.25$$

For $R=6.25$, the closest value in Table 1 is $R=6.79$ for plan 3.

- Its acceptance numbers are $c_1 = 0$ and $c_2 = 2$.
- If the producer's risk ($\alpha = 0.05$) is to be satisfied exactly, $P_a = 1 - 0.05 = 0.95$ for $p_1 = 0.012$.
- From Table 1 using $P_a = 0.95$ the value of $n_1 p$ is 0.43. The size of the first sample is thus $n_1 = \frac{0.43}{0.012} = 35.83 \approx 36$

So, I have to design a plan which can satisfy so as I mention find the value of R p_2 by p_1 p_2 by p_1 which is LQL by AQL 6.25 you refer table one R is equal to 6.79 for plan 3. So, the acceptance number c_1 is equal to 0 c_2 is equal to 2. And for α is equal to 0.05 you have $P_a = 1 - 0.05$ that is 95 percent and for $p_1 = 0.012$. So, with this particular values ; $n_1 p$ is 0.43 and you can figure out what is the sample size of the first sample in my double sampling plan?

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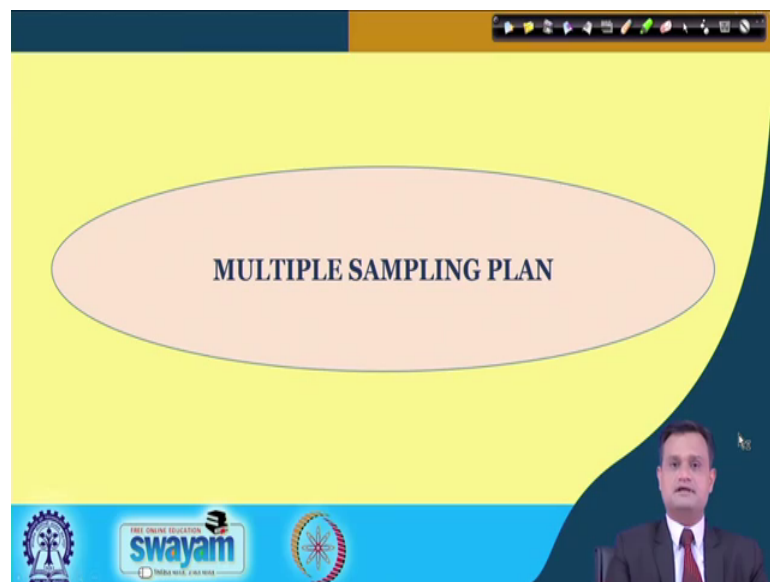
Solution

- Because the sample sizes are assumed to be equal, $n_2 = n_1 = 36$. So the double sampling plan is $n_1 = 36$, $c_1 = 0$, $n_2 = 36$, $c_2 = 2$. The values of r_1 and r_2 are both assumed to be 3.
- The ASN corresponding to $P_a = 0.95$ can be found using Table.
- For plan 3, $ASN/n_1 = 1.340$
- So $ASN = (1.340)(36) = 48.24$

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So, this is the solution that you can find ASN also because ASN by n_1 ratio is given in the Grubbs table and this is how you can design the double sampling plan.

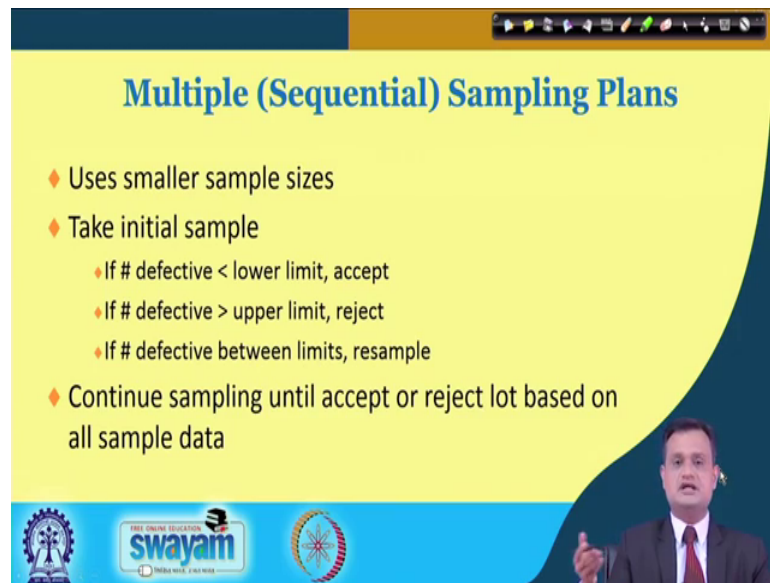
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MULTIPLE SAMPLING PLAN

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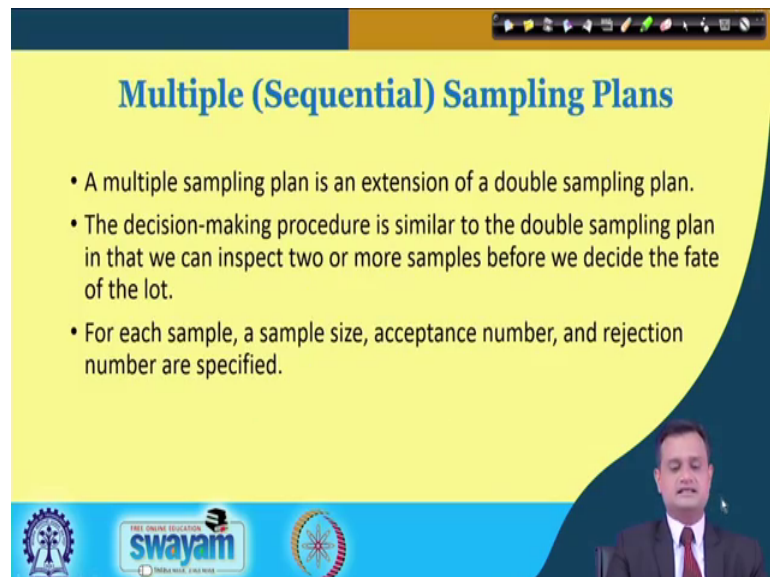
Multiple (Sequential) Sampling Plans

- ♦ Uses smaller sample sizes
- ♦ Take initial sample
 - ♦ If # defective < lower limit, accept
 - ♦ If # defective > upper limit, reject
 - ♦ If # defective between limits, resample
- ♦ Continue sampling until accept or reject lot based on all sample data

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We are not going into the detail of multiple sampling plan, it is just an extension of double sampling, but you take the initial sample defective is lower than less than lower limit accept, greater than upper limit reject, in between go for the another sample and continue for n number of stages and this is your multiple or sequential sampling plan.

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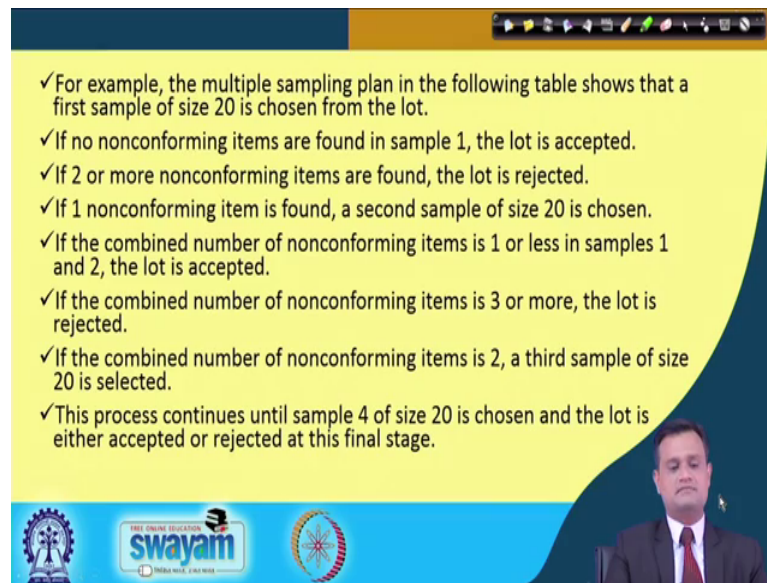


Multiple (Sequential) Sampling Plans

- A multiple sampling plan is an extension of a double sampling plan.
- The decision-making procedure is similar to the double sampling plan in that we can inspect two or more samples before we decide the fate of the lot.
- For each sample, a sample size, acceptance number, and rejection number are specified.

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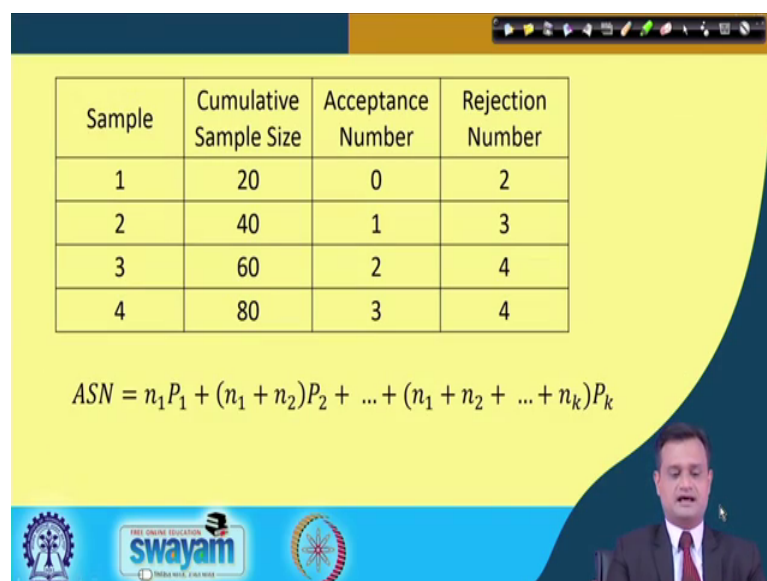


✓ For example, the multiple sampling plan in the following table shows that a first sample of size 20 is chosen from the lot.

- ✓ If no nonconforming items are found in sample 1, the lot is accepted.
- ✓ If 2 or more nonconforming items are found, the lot is rejected.
- ✓ If 1 nonconforming item is found, a second sample of size 20 is chosen.
- ✓ If the combined number of nonconforming items is 1 or less in samples 1 and 2, the lot is accepted.
- ✓ If the combined number of nonconforming items is 3 or more, the lot is rejected.
- ✓ If the combined number of nonconforming items is 2, a third sample of size 20 is selected.
- ✓ This process continues until sample 4 of size 20 is chosen and the lot is either accepted or rejected at this final stage.

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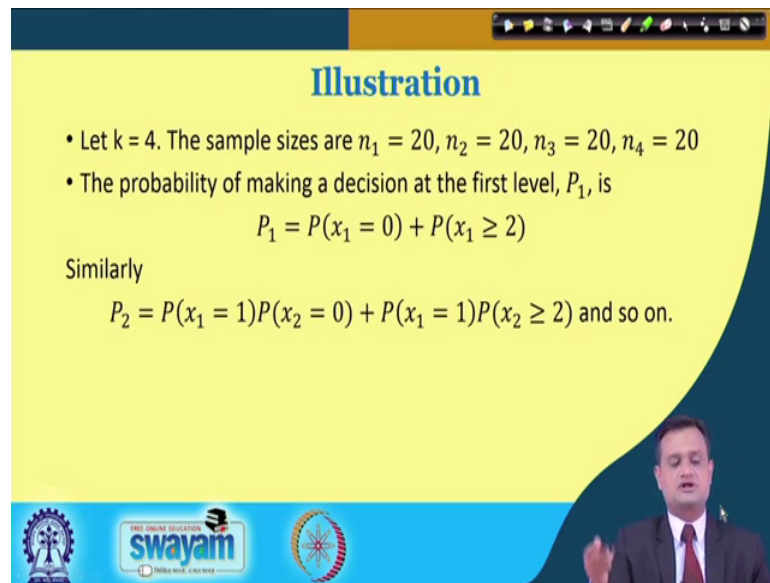
Sample	Cumulative Sample Size	Acceptance Number	Rejection Number
1	20	0	2
2	40	1	3
3	60	2	4
4	80	3	4

$$ASN = n_1 P_1 + (n_1 + n_2) P_2 + \dots + (n_1 + n_2 + \dots + n_k) P_k$$

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So, just see this example sample one you have cumulative sample size 20 and acceptance number 0 rejection 2 40 1 3 60 2 4.

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Illustration

- Let $k = 4$. The sample sizes are $n_1 = 20, n_2 = 20, n_3 = 20, n_4 = 20$
- The probability of making a decision at the first level, P_1 , is
$$P_1 = P(x_1 = 0) + P(x_1 \geq 2)$$

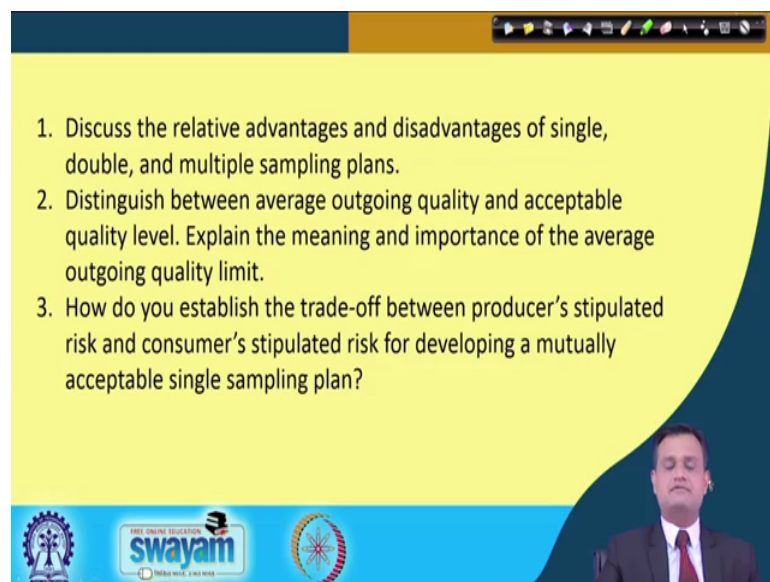
Similarly

$$P_2 = P(x_1 = 1)P(x_2 = 0) + P(x_1 = 1)P(x_2 \geq 2) \text{ and so on.}$$

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And you can easily take the decision about the acceptance or rejection of the sample at an appropriate level. So, you can find the probability at P_1 your stage 1 p_2 at stage 2 and so on.

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1. Discuss the relative advantages and disadvantages of single, double, and multiple sampling plans.
2. Distinguish between average outgoing quality and acceptable quality level. Explain the meaning and importance of the average outgoing quality limit.
3. How do you establish the trade-off between producer's stipulated risk and consumer's stipulated risk for developing a mutually acceptable single sampling plan?

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So, just before we end let me float couple of think it, discuss the relative advantage and disadvantage of single double and multiple sampling plan, distinguish between average outgoing quality and acceptable quality level.

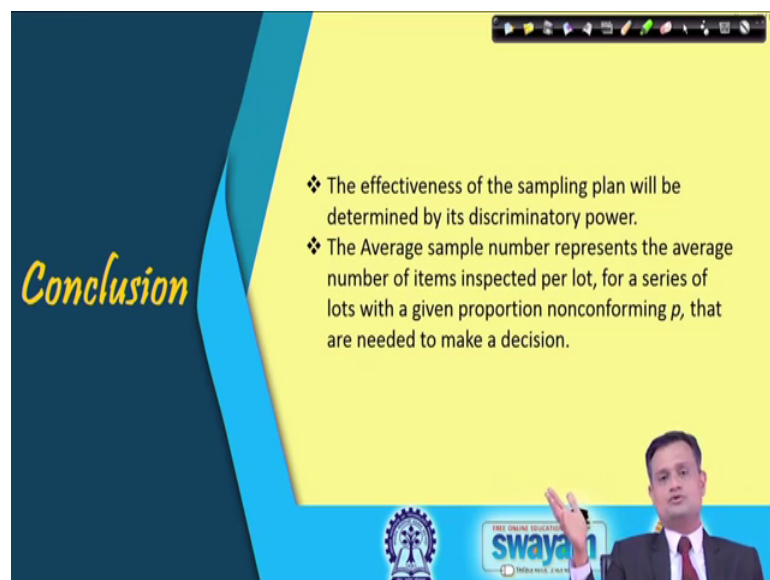
Explain meaning and importance of the average outgoing quality limit. And how do you see the trade-off between producers stipulated risk and consumers stipulated risk and device a plan which is mutually acceptable to both.

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So, these are the references mainly I have used Mitra for this particular lecture.

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And we have to device a effective discriminatory sampling plan which helps producers and consumers both to have the consensus on a particular value of n_c and so on. So, thank you very much for your interest in learning this particular lecture on design of

acceptance sampling plan for attributes. And we will continue our discussion on this with couple of more plans till that time keep revising introspecting with me enjoy.