Six Sigma
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Lecture - 55
Design for Acceptance Sampling Plans for Attribute (Part-1)

Hello friends, once again I welcome you to our ongoing journey on Six Sigma. And I would like to remind you that we are in the final phase of our DMAIC cycle. And we are discussing the various issues in the control phase; mainly we are focusing on statistical quality control, as a part of that we have talked about statistical process control, control charts for variables and control charts for attributes. Now we are talking about acceptance sampling. We have seen the key concepts like produces risk, consumers risk AQL AOQL and couple of other things in the last lecture. Now this particular lecture 55, we will focus on design of acceptance sampling plan for attributes.

As we need to discuss in detail and there are lot many things to cover, this particular lecture is divided into part 1 and part 2.

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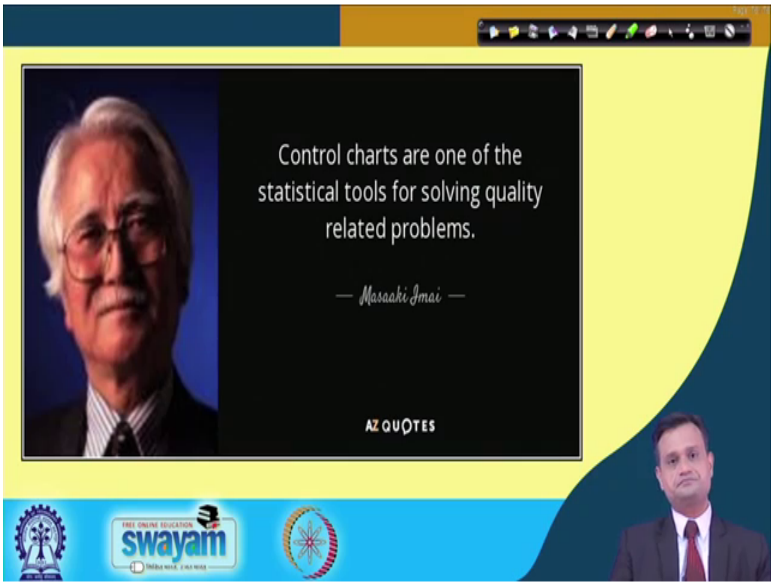

So let us begin with lecture 55. Once again I would emphasize that, there is no substitute of control chart and statistical process control and acceptance sampling are must in order to have fact based management in the organisation.

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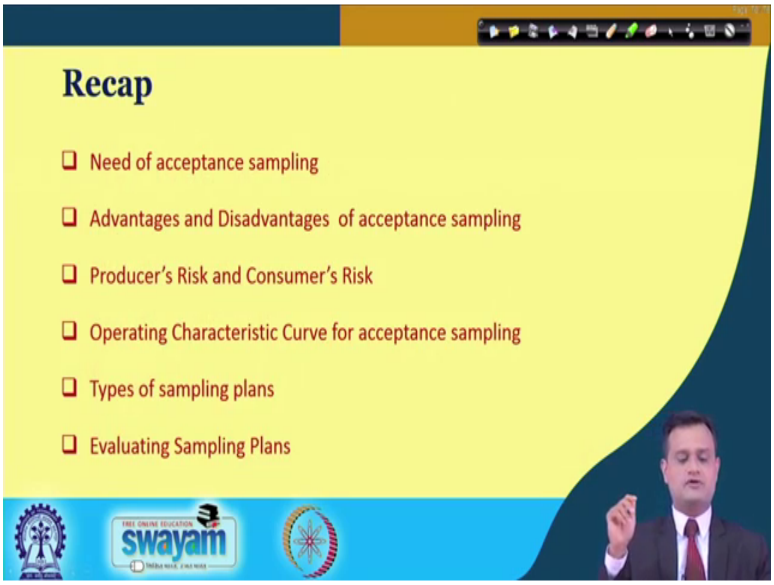

So, this is what we have covered, and we also had little bit glimpses on types of sampling plan and their key characteristics and we had seen the sum of the evaluating criteria for sampling plan.

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So, this lecture we will basically focus on lot by lot attribute sampling plan; and we will talk about single sampling plan, double sampling plan and multiple sampling plan. Our focus mainly is on the design of the sampling plan typically under the category of lot by lot attributes sampling plans.

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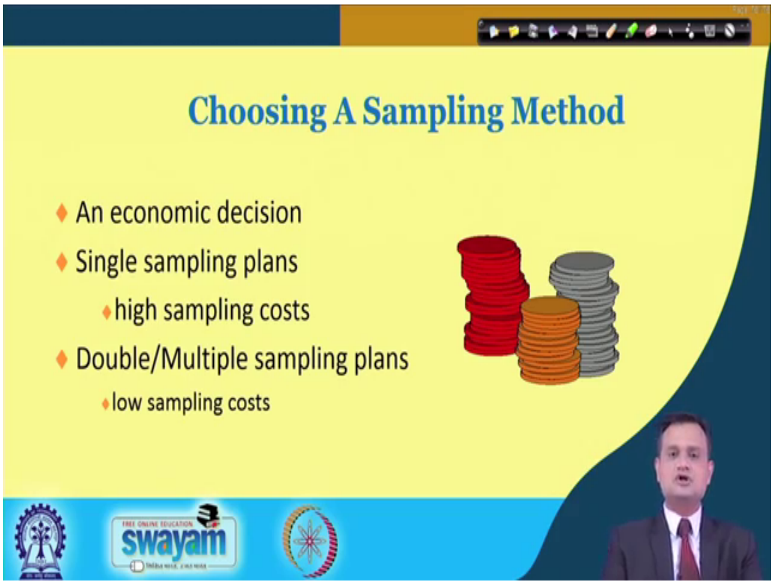

So, choosing a sampling method is an economic decision I cannot go with 100 percent inspection. But obviously, say I have to see the trade off between producers risk and consumers risk; and single sampling plan, there is a high sampling cost; double multiple sampling plan, I may take a decision on a very first sample and my sample size would be less.

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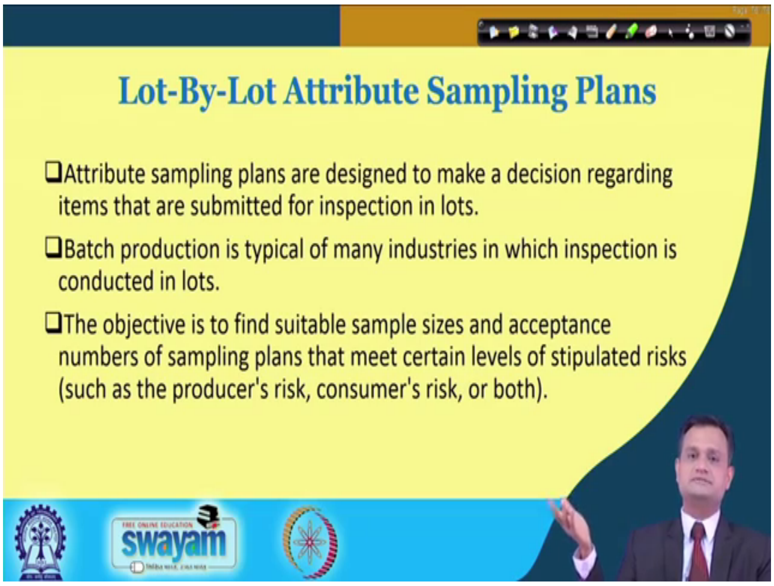

So, my sampling cost is little bit lower compared to the single sampling plan.

Now, if you see the lot by lot attributes sampling plan, then this plans are designed to make a decision regarding items that are submitted for the inspections in lot. So typically if you see, the batch products and then, items are produced in lot and then they are submitted for the sampling and inspection decision making in a particular lot. And the objective is to find here, sample size and acceptance number of a sampling plan which is acceptable to both producer and consumer. Once again I would like to remind you that producers risk is type one error denoted as alpha and producer would not like his good quality lot to be rejected. Consumers risk is beta type two error and consumer would not like the bad quality lot to be accepted.

So, we have to see the trade off between producers risk and consumer's risk; and unless you consider the judicious trade off between these two you cannot really design a sampling plan with sample size, an acceptance number and rejection number and other criteria. So, the whole exercise, we will do in this particular lecture is all about to see that how producers stipulated risk and consumers stipulated risk can be appropriately accommodated, so that I can devise a judicious, logical and mutually acceptable sampling plan to both the parties.

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So, this is exactly we will try to do. So, we have broadly say four different categories of lot by lot attributes sampling plan single sampling plan, double sampling plan, multiple sampling plan and standard sampling plans. So, there are totally four. In this particular lecture, we will focus on these three that is single sampling plan, double sampling plan, single, double and multiple and we will focus on standard sampling plan in the next lecture.

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So, this is what we want to do. Now let us see the first the design of the single sampling plan, which is easiest to employee but because we only rely on single sample and sample size may be larger so, the overall cost of inspection maybe more compared to the double sampling and the multiple sampling. But so for the administration is concerned single sampling plan is the best to use and implement.

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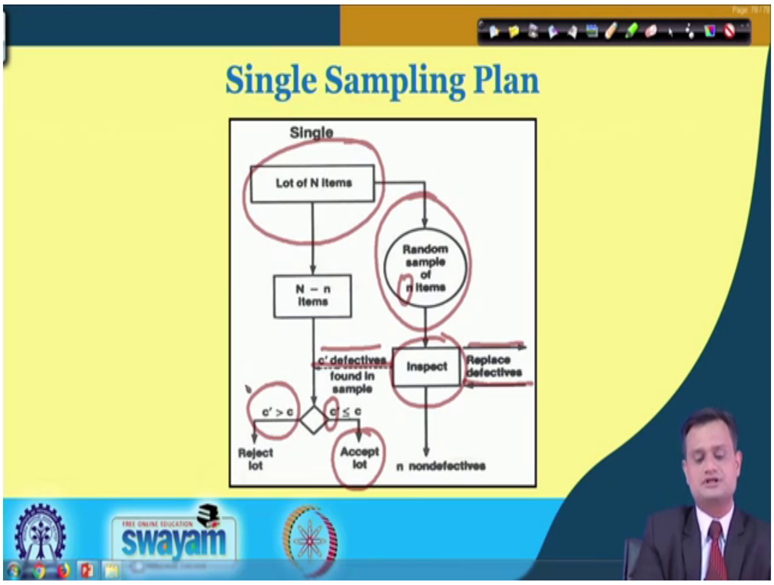

So, the overall picture goes like this in single sampling, I have the lot of N items and then I have random sample of n item that is the small n. Now I inspect it, after inspection I will find that there are c dash defectives found in the sample. I can also replace the defectives as a part of my rectification process, but here just try to focus on these that c dash is the defectives found in a sample. Now if this c dash is less than or equal to c which is the acceptance number, I will accept the lot, because it is less than the acceptance number and I will accept the lot. If it is greater than c means number of defectives, then I will reject the lot and you can have n non defectives from a lot of say capital N in a sample of n.

So, this is what exactly I do in a single sampling plan and it is called single sampling because I just draw collect one sample can then based on that, I try to compare the number of defectives in the sample with the acceptance number and make my decision.

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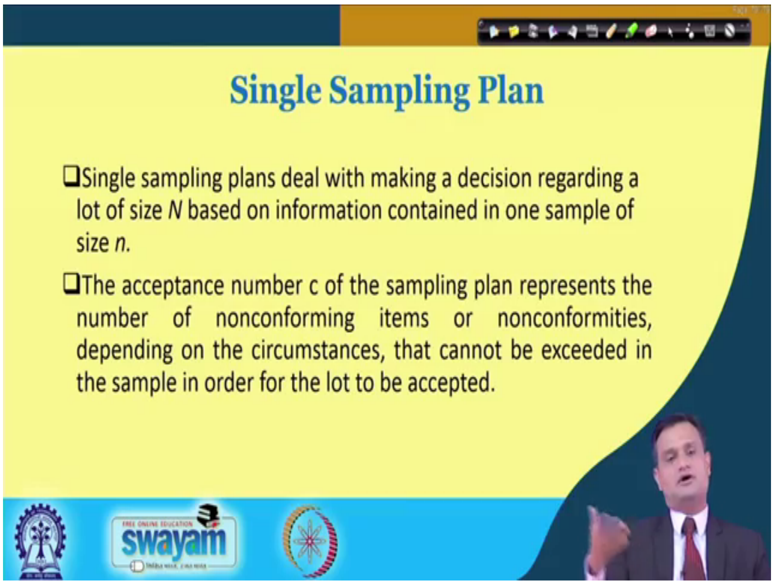

So, single sampling plan basically deals with the decision regarding a lots of size N and sample size n and you have the acceptance number c to decide that, whether this is accepted or this is rejected.

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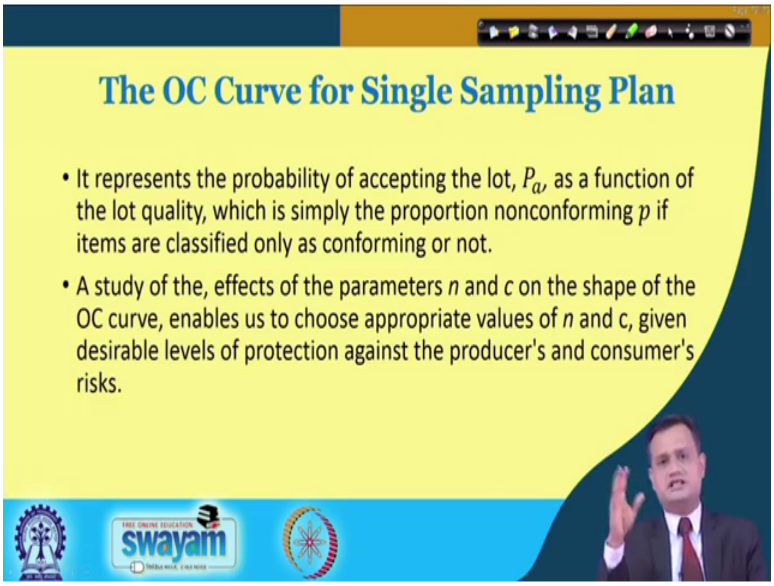

So, typically if you see the operating characteristic curve which is a measure of performance, either of control chart or the acceptance sampling plan. Basically it provides the discriminatory power I am repeating this statement again and again to my acceptance sampling plan. And how discriminatory it is in terms of say separating the good lot from the bad lot and this is what we try to appreciate as a part of operating characteristic curve.

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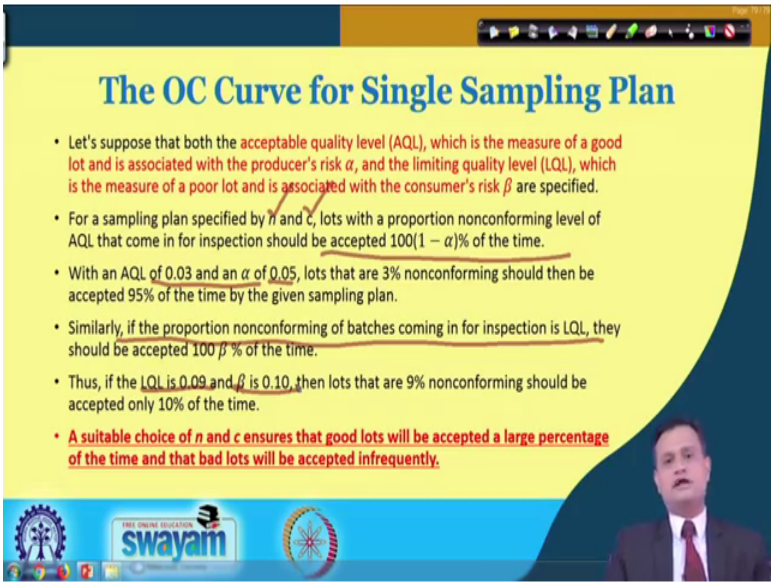

So, if you just see the couple of issues that we discussed in the last lecture, then one of them was AOQ or AQL that is acceptable quality level. So, acceptable quality level is the measure of a good lot and typically it is associated with the producers risk alpha. So, my producer would not like a good lot to be rejected, an AQL acceptable quality level is typically the one which is associated with producer's risk that is alpha. If you look at the another definition that is LQL this is limiting quality level, then this is the measure of a poor lot and typically it is associated with the consumers risk and consumer would not like a poor quality lot to be accepted.

So, in the previous lecture average outgoing quality, average outgoing quality level this concepts we have seen. Now I am introducing two more new terms, one is acceptable quality, other is say your limiting quality levels that is LQL. Now suppose I consider a sampling plan specified by n and c n and c, lots with a proportion nonconforming level of AQL, that come in for inspection should be accepted 100 1 minus alpha percent of the time. It means number of lots coming for the inspection and out of this many lots I will select 100 1 minus alpha percent of the time out of the submitted lots and this is something which refers to my AQL. So, same way I can say that AQL is 0.03 and alpha of 0.05.

So, I would say that lots that are 3 percent nonconforming should than be accepted 95 percent of the time by the given sampling plan. So, you may make a mistake; 3 percent is the level that you need to achieve that is the acceptable quality level. And let us say if it is 0.03, 3 percent and I operate with a particular level alpha, it means the lot is of good quality type one error, but I will reject it. So, I will say that if my alpha is 0.05 that is 5 percent, then the lot which contains 3 percent nonconforming or defective 95 percent of the time, it should be accepted and that is what my operating characteristic curve or my acceptance sampling plan must incorporate.

So, this is the clear cut understanding, similarly if the proportion of non-conforming of the batches in coming in for inspection is LQL. Now let us think from the consumers point of view, suppose I am referring the limiting quality level this is the quality of a poor lot. So, let us say LQL, they should be accepted by 100 beta percent, means the bad quality lot should be accepted by 100 beta percent. Let us try to appreciate LQL is 0.09 and beta is 0.10. So, what would I say that the lot that are 9 percent nonconforming 9 percent nonconforming should be accepted only 10 percent of the time.

So, the idea is very clear, let us say AQL 0.03 that is 3 percent. And if I have the alpha is equal to 0.05, I will say the number of lots submitted out of this number of lots the lots having 3 percent defective or nonconforming (Refer Time: 12:00) 95 percent of the time it should be accepted. It is only 5 percent time producers risk will come in picture and I will check the wrong decision. Similar way if I refer the bad quality lot let us say 9 percent is LQL and beta is my 10 percent 0.10, then I will say that the lots which has 9 percent defective this lot should only be accepted 10 percent of the time. So, that my consumers risk remains at 10 percent and he feels protected. So, it is all about protecting your producers and consumers and I would say that what is the level of protection, your acceptance sampling plan ensures to your consumer and producer and acceptable to them decides the goodness of your acceptance sampling plan.

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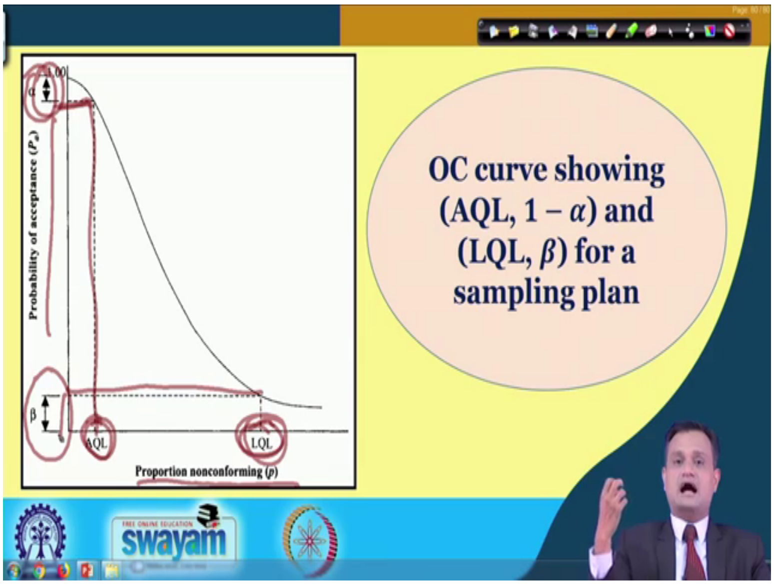

So, this is something which is very important and you must appreciate, just see the OC curve for AQL 1 minus alpha that is specific to your produces risk. And LQL beta that is specific to your consumers risk you can see it very well here, AQL is this and this is my alpha that is producers risk. It means that the lot which has this much percent you see proportion of non conforming the lot which has this much percent of non conforming AQL, it will be accepted 95 96 whatever is this, alpha as per that that many times. But the lots which has LQL quality level that is very poor lot maybe let us say 9 percent 8 percent whatever.

Then it would only be say accepted this much of the time and this is my basically probability of acceptance. So, I hope the idea is crystal clear that a producer would like the lot to be accepted at AQL; and he would not like any lot to be rejected. If it is below AQL or at AQL and similar way a consumer would not like to accept the lot which is of very poor quality LQL with beta percent risk.

So, again I would remind you that we are dealing with the probability, we are dealing with the sampling and we always take the decision or test the hypothesis by accepting some level of type one and type two error. We cannot get rid of say making the error, but we can just bass try to see that how this error can be kept at minimum level? And this is acceptable typical in acceptance sampling plan to my both producer and consumer. So, this is something that I wanted to emphasize.

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Now, if you see the design of a single sampling plan, then there are three issues. Number one stipulated producers risk. So, let us say AQL is something related to producers risk alpha we have seen in the figure and consumers risk that is beta specific to your consumer. And you may have stipulated producers and consumers risk. So now when I want to design the single sampling plan, meaning to say I want to decide about sample size and; and the acceptance numbers c, I have to consider three different situation or one of these three either I design it for stipulated producers risk. It means my plan will exactly try to meet the producers risk or I may try for consumers risk or I may try to incorporate both simultaneously.

So, let us try to see how we can design this single sampling plan with this three different criteria.

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So, let us see the first one design of single sampling plan based on stipulated producer's risk.

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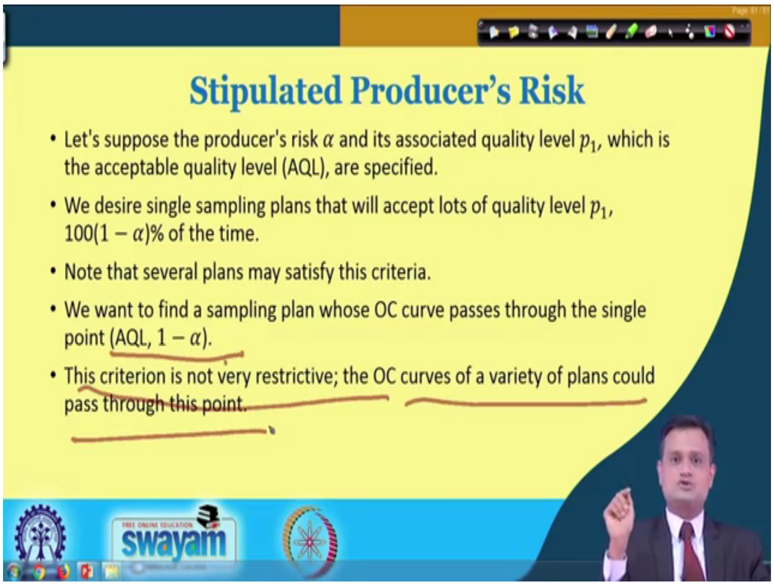

So, stipulated producers risk it is say given by alpha, associated quality level p 1 which is basically AQL. And I would like the lot containing p 1 non conforming to be accepted 100 1 minus alpha percentage of time in a very simple way. If your type one error alpha producers risk is 5 percent, then I would like the lot having AQL percent defective to be accepted 95 percent of the time.

Now, what would I will accept here that we want to find a sampling plan whose OC curve will basically pass through the point which is tagged as AQL 1 minus alpha. Now remember that this criteria is not very restrictive the OC curves of a variety of plans could pass through this point. So, you may have different sample size you may have different acceptance number. So, you cannot say there would be only one OC curve that will pass through this particular point AQL 1 minus alpha, but there could be multiple plans that you can devise, but my point here is to see that how best I can address the producers stipulated risk, in designing my single sampling plan.

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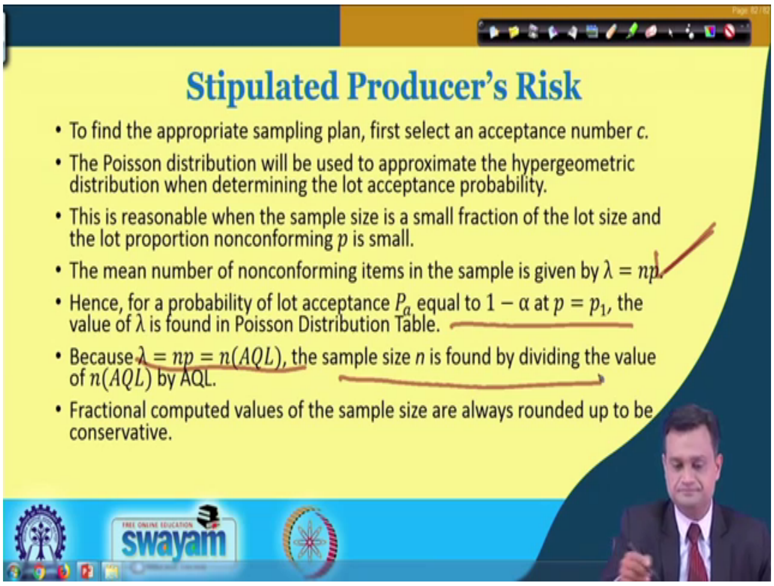

So, with this let us try to move it and what I would like to do to find the appropriate sampling plan for select an acceptance number that is the c the procedure I am just going through. The Poisson distribution will be used to approximate the hypergeometric distribution, when determining the lot acceptance probability. So, we have done this exercise previously also. So, your Poisson distribution table will have mean as well as the value individual value; by looking at these you can figure out what is the probability of acceptance. And this will give you the cumulative probability from the Poisson distribution table.

Now, mean number you can find for referring the Poisson distribution table, that is lambda is equal to n p and P a equals to 1 minus alpha. So, you have a probability of acceptance and p is equal to p 1, that is the value of lambda found in the Poisson distribution table. Now here because you have p that is AQL, so I will say lambda is equal to n p is equal to n into AQL and this is what you will try to refer when you will see the cumulative Poisson distribution table.

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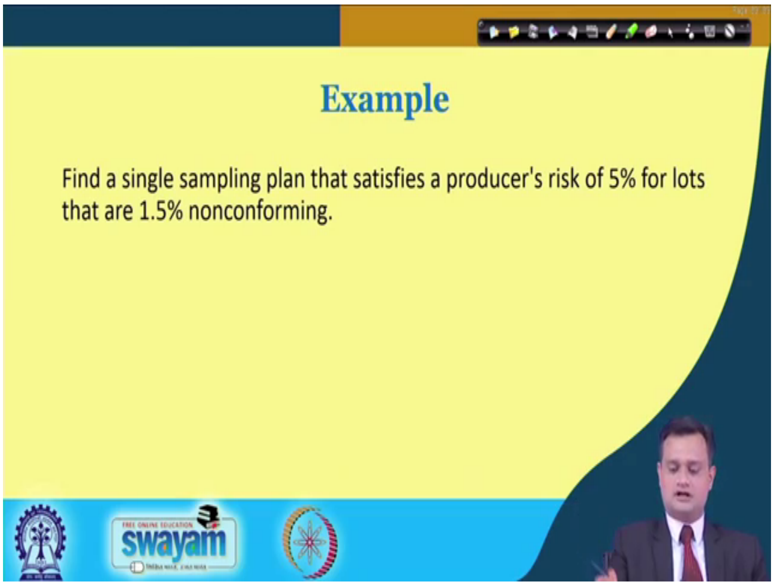

So, now let us try to see this through example. So, when you see the numbers the idea is better clear. Now find or design a single sampling plan that satisfies the producers risk 5 percent as usual and you have the lots with 1.5 percent non-conforming. So, you are given this two value alpha 5 percent and the lots with 1.5 percent non conforming, we are supposed to design single sampling plan.

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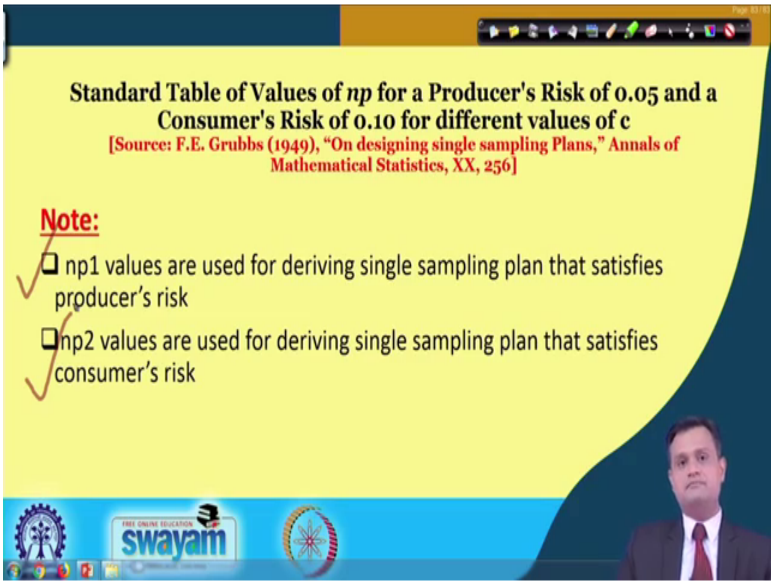

So, here I would refer a particular standard table, we are we are not interested in seeing how this table have come? But I will refer a particular standard table of values of n p for a producer's risk of 0.05 and consumers risk 0.10 this particular table is developed and published by F.E Grubbs in 1949 long back, on designing single sampling plans and it was published in Annals of Mathematical Statistics.

So, typically I will have two things here one is the np 1 values that are used for deriving single sampling plan, that satisfies the producers risk. And np 2 these are used for deriving single sampling plan that satisfies the consumers risk. So, this researcher F.E Grubb has designed a table, standard table which can help us to find the value of np 1 and np 2 for the producers risk and respective say consumers risk respectively. So, that we can easily figure out our single sampling plan for a stipulated producer risk or a stipulated consumer risk and also, we can use this table to find a plan which meets the stipulated producer and consumer risk both.

So, we have three conditions for which we will try to develop the single sampling plan

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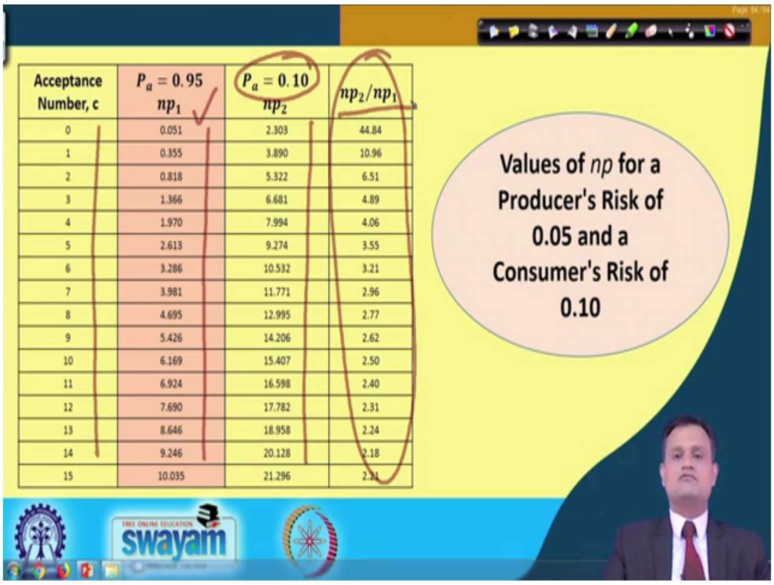

Acceptance Number, c	$P_a = 0.95$ np_1	$P_a = 0.10$ np_2	np_2/np_1
0	0.051	2.303	44.84
1	0.355	3.890	10.96
2	0.818	5.322	6.51
3	1.366	6.681	4.89
4	1.970	7.994	4.06
5	2.613	9.274	3.55
6	3.286	10.532	3.21
7	3.981	11.771	2.96
8	4.695	12.995	2.77
9	5.426	14.206	2.62
10	6.169	15.407	2.50
11	6.924	16.598	2.40
12	7.690	17.782	2.31
13	8.646	18.958	2.24
14	9.246	20.128	2.18
15	10.035	21.296	2.21

So, now just see the table and what it tells? You have the acceptance number in the first column and you have n p 1 for P a is equal to 0.95 so 1 minus alpha percentage. It means I am referring to the producers risk and these are the values specific to n p 1 it is a standard table. So, we just accept it in order to design our single sampling plan; n p 2 it is for 10 percent beta you can see here 10 percent beta.

And this is for used for my stipulated consumer risk based single sampling plan. You will see the third column where you have np 2 divided by n p 1, this we will use for devising or developing the single sampling plan which can accommodate stipulated producers as well as consumers risk.

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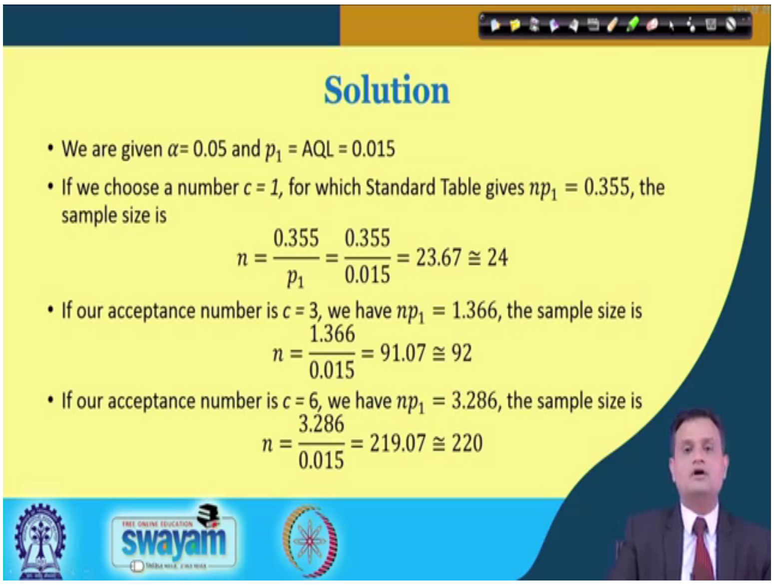

So, now, the thing is very simple that we have this table. And now the data given is like this alpha is equal to 0.05 and you have p 1 is equal to AQL 0.015. So, 1.5 percent defective is 0.015. So, this is my p 1 is equal to AQL. And I have alpha is equal to 0.05.

Now just refer the table and for c is equal to 1 what do you find? So, let me go back to help you. So, for c is equal to 1 what you find here is 0.355, we will only refer this particular column which I have highlighted in pink. When we will talk about the next case I will highlight this second column, third case I will highlight third column in pink, so that you can focus on that particular column. So, here you can see that the value of n p 1 basically 0.355. So, once you get this value from the table, you just see here that for c is equal to 1 n p 1 is 0.355.

So, I can just rewrite it like this and my p 1 is 0.015 so I will have 24 as the sample size. Now let us say, if our acceptance plan is c is equal to 3 acceptance number is c equal to 3, then I will get corresponding np 1 1.366. Let us verify, so that you become crystal clear. So, for c is equal to 3 you can see that, c is equal to three it is 1.366. So, I hope now it is clear that I am checking the value for different c the value of n p 1 and this is 92 same way this will be 220 for c is equal to 6.

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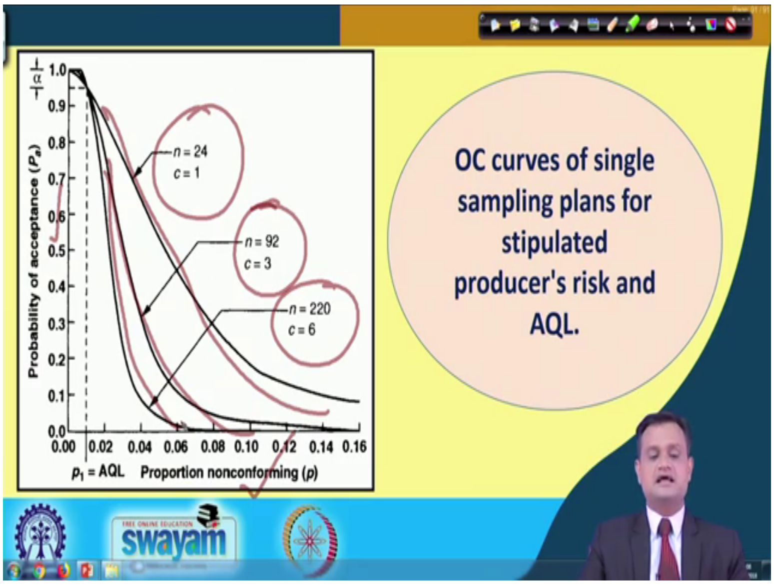

Now once I done this calculation, you can easily plot on your operating characteristic curve for single sampling plan for stipulated producers risk and AQL. So, I referred AQL I referred Grubbs table for typically stipulated produces risk and this is where I get the probability of acceptance, probability of non conforming I get n is equal to 24 for c is equal to 1 this is the OC curve, then n is equal to 92 c is equal to 3 this is the OC curve n is equal to 220 c is equal to 3, this is the OC curve. You can see that, as the sample size increases I am going for steeper and steeper operating characteristic curve. It means my discriminatory power increases and this is exactly I told that not necessary that you will have only one OC curve passing through a point AQL and 1 minus alpha percent. But you can have multiple OC curve depending upon sample size n and acceptance number c.

So, now, out of these which one to choose depends on the criticality of the quality characteristic and the consequences that it may have on the final quality of the product. So, depending upon that you can decide or you can choose judiciously the discriminatory power of the single sampling plan.

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So, when you have understood this, let us try to apply the same for the second case and second case is stipulated consumers risk.

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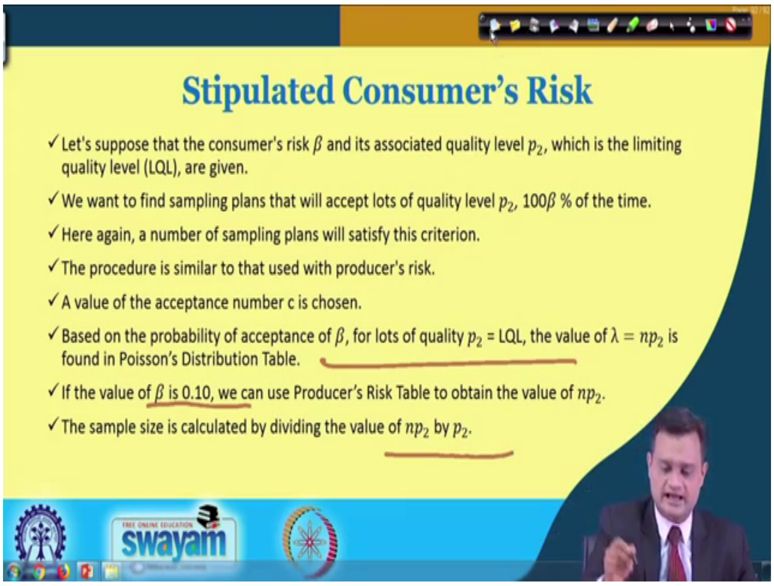

Now, here same way I will try to follow the procedure, but I will refer the beta and p 2 is equal to LQL. So, my limiting quality level typically indicates that the bad quality lot at some beta percent consumers risk and this is what I will try to use. So, my beta is 0.10.

And I will try to find n p 2 from the Grubbs table so that I can figure out my particular acceptance sampling plan.

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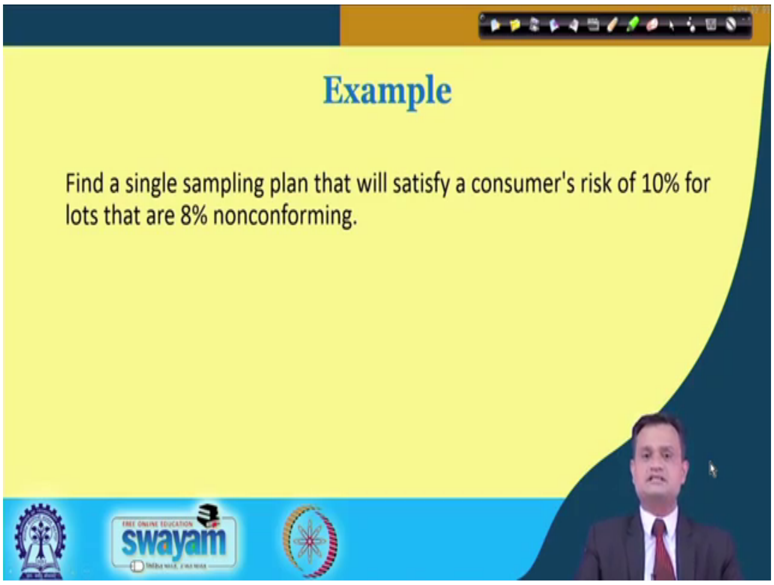

So, let us see the example and it would be clear, that single sampling plan that will satisfy a consumer's risk of 10 percent for lots that are 8 percent nonconforming. So, I will once again refer this particular table and if you see I have now highlighted the second column in pink. So, I have to refer these because np 2 is specific to consumers risk and I have been given the value of consumer's risk 0.10.

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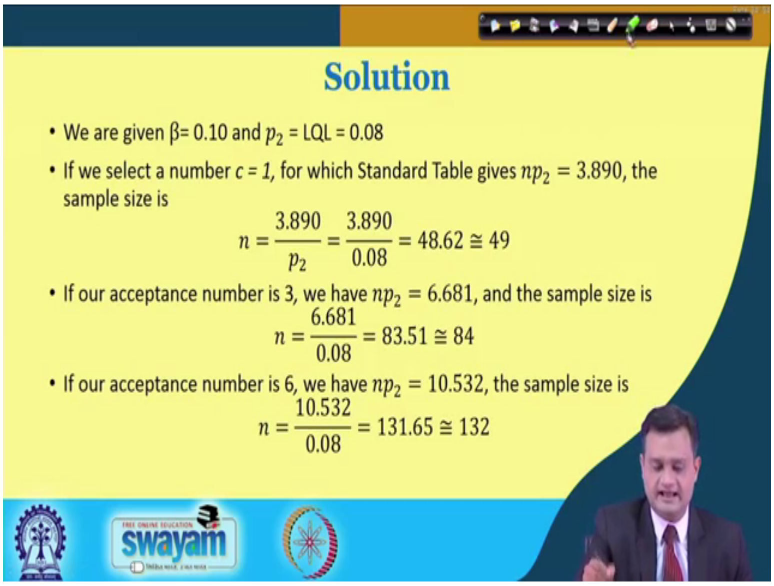

So, what I get here? When I refer the table for c is equal to 1 I get the value n p 2 3.890.

I get n by putting value in this similar way for c is equal to 3, I get n is equal to 84 6 I get this.

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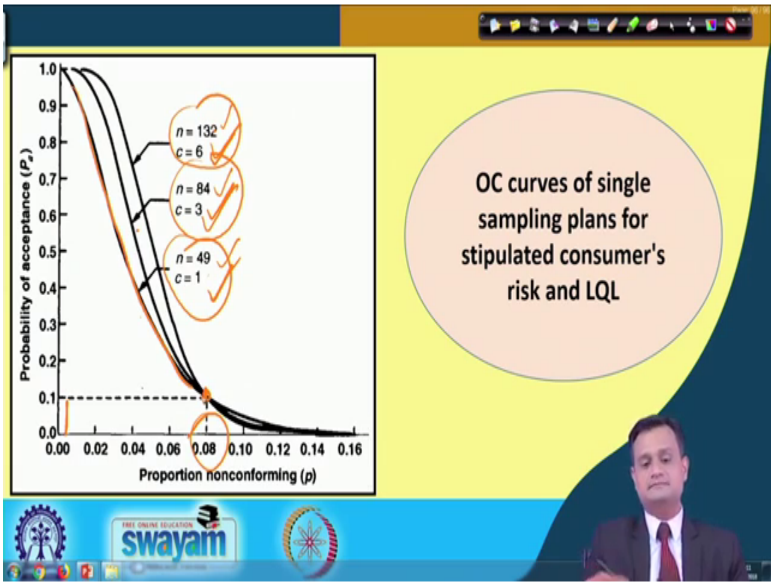

So, finally, you can figure out an acceptance sampling plan for single sampling with n is equal to 132 and n is equal to 84, n is equal to 49 and what do you see here? My sample size is reducing, but this is 132 this is 84 this is 49, but simultaneously my value of c is also reducing and you will find that the combination of these two is giving me a better discriminatory power. So, here it is not straight forward, what we have seen that you increase the sample size for producer’s risk and your curve becomes steeper.

Here even though the sample size is decreasing or if you say increasing, my curve is not behaving like that, but simultaneously you should see the effect of c and you will find that for this beta my consumers risk and this is the limiting quality level, I have three different OC curves which is passing through this LQL point. So, LQL and beta basically say designs my single sampling plan for stipulated producer’s consumers risk.

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Now, let us see the third case I want to design a plan for stipulated producers and consumers risk both.

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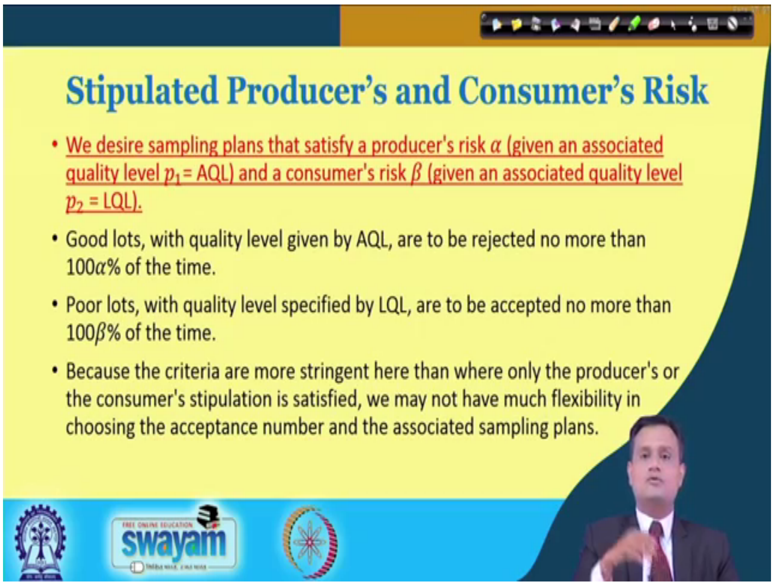

So, now what I want? I want a plan basically which satisfy a producer risk alpha given an appropriate quality that is p 1 is equal to AQL; and also consumers risk beta given an associated p 2 that is LQL. So, now, I want to consider both previously I considered only AQL that is p 1 and alpha in the second one I considered LQL and then, beta here I want to consider both my LQL as well as AQL beta and alpha.

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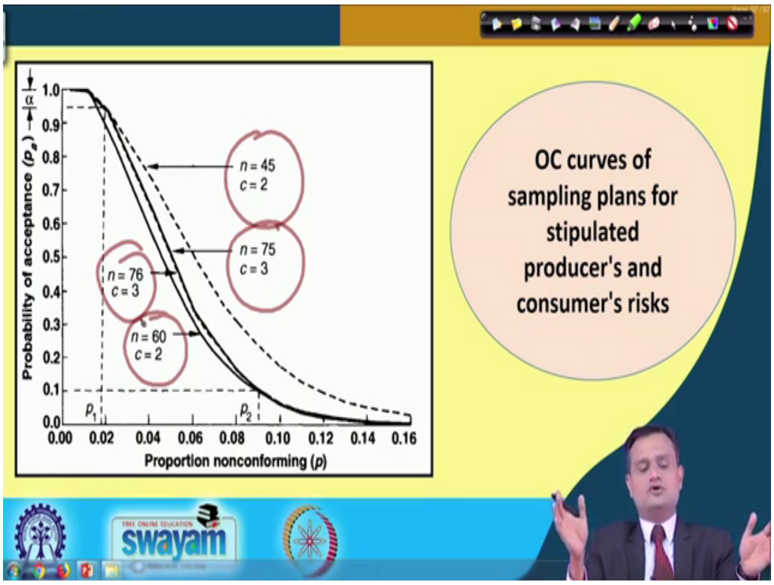

So, for this I would first try to present the operating characteristic curve and what it indicates that, my sampling plan whatever I have. Let us say n is equal to 45 and c is equal to 2 n is equal to 75 and c is equal to 3 and n is equal to 76 c is equal to 3 n is equal to 60 and c is equal to 2. So, now, here you will see that there are two plans which are indicated in dotted and there are two plans which are indicated in full black line. So, if you see the plans which are indicated in dotted they are passing through this particular that is the AQL, and the plans which are in dark line they are passing through this. Now I have to select both in such a way that a proper trade off can be set when I select a particular plan.

So, this is really interesting and you have two plans which satisfies the stipulated risk indicated in dotted, stipulated risk of producer the plan indicated in full black stipulated risk of my consumer. So, now, let us see how we can goead.

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So, I want to figure out basically the plans which can best satisfy the stipulated risk of producer and stipulated risk of consumer. So, two plans need the producers stipulated risk as I mentioned in dotted line. And two plans in full line they meet the consumer stipulated risk exactly and come close to meeting the producer's stipulation.

So, now, what happens that? If I look at the dotted line for example, here then you have two dotted curves, let us say this is the one and this is the second one which is little bit hidden. These are the plans which exactly meets the producer of stipulated risk, because they are passing through this AQL and I have the full line plan which are passing through this LQL so exactly they meet.

Now; obviously, I have to meet the expectation of both producers and this consumer so what I would like to do? I can design a plan which exactly meet the producers risk and would be very close to my consumer stipulation. Similar way I can do otherwise, I would like to choose a plan which exactly means the stipulation of the consumer and would be very close to my producer.

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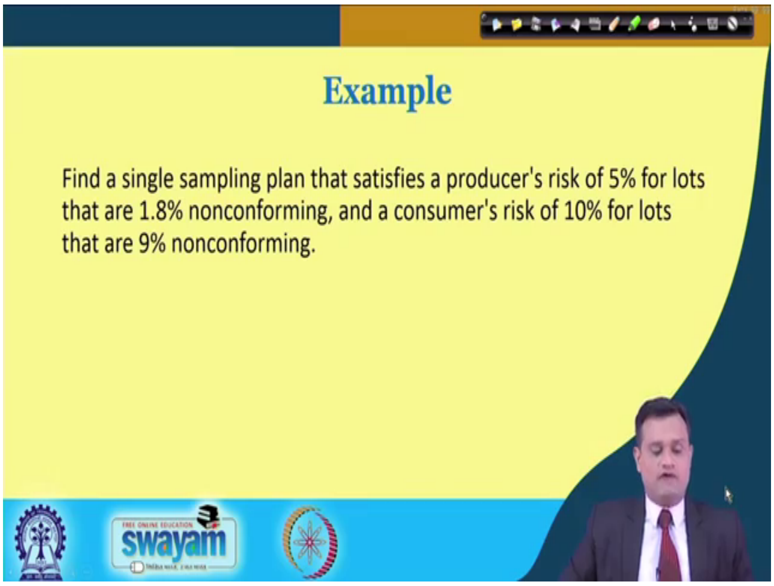

So, this is the choice what I have, now if I just try to see it through example then let us try to see, find a single sampling plan that satisfy the producers risk of 5 percent for lots that are 1.8 percent nonconforming, and consumer risk of 10 percent for lots that are 9 percent nonconforming. So, 5 percent is the producer's risk that is the alpha 1.8 percent non conforming that is your related to producer. Consumer risk 10 percent and lots that are 9 percent nonconforming. So, I will again use this particular table and you can see that now I have highlighted this particular last column in pink. So, I would be simultaneously using n p 2 and n p 1 in order to find a plan which satisfy my producer and consumer both.

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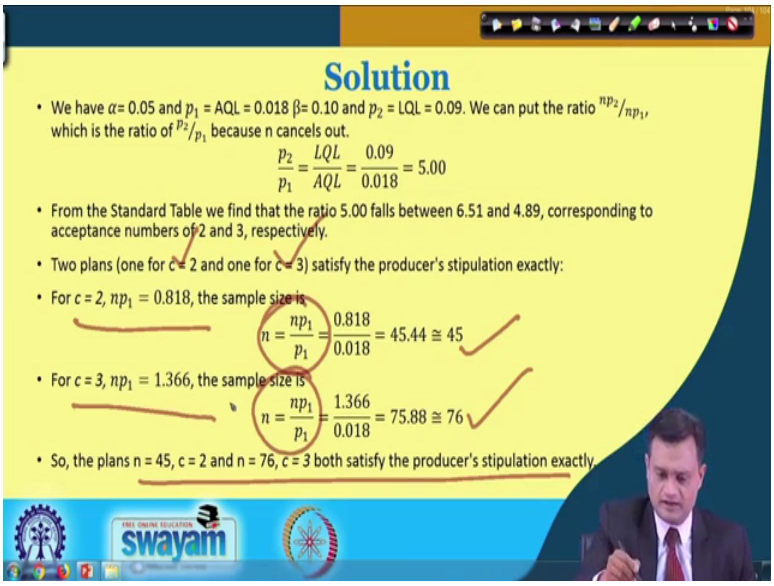

So, just see what is the data given you have alpha is equal to 0.05 p 1 is AQL 0.018 beta is 0.10 p 2 LQL is 0.09 and we can find the ratio n p 2 by n p 1. So, p 2 by p 1 is basically LQL by AQL and this is 5. Now let us see that where this ratio false? So, this particular ratio 5 falls between 6.51 and 4.89 corresponding to acceptance number 2 and 3. So, you want to see then let me go back, this is the say table and just see the ratio 5 so this ratio 5 will fall somewhere here, 5 will fall somewhere here.

So, this is specific to acceptance number 2 and acceptance number 3. So, now, what I have I have two acceptance number; c is equal to 2 and c is equal to 3. So, I can now find c is equal to 2 n p 1 value from the same table, and I can find the value of n similar way c is equal to 3 n p 1 and I can find the value of n. So what I get here is n is equal to 45 and c is equal to 2 n is equal to 76 and c is equal to 3 both satisfy the producers stipulation exactly. So, this is something because I have used this np 1 which is specific to producer so these plans exactly satisfy the producers stipulation.

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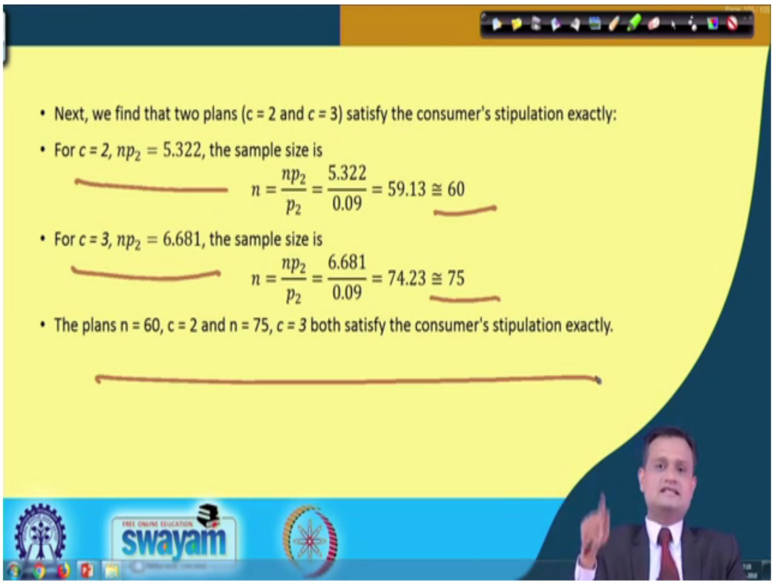

Now let me go ahead to this really interesting just do it comfortably and it would be crystal clear. Now if I refer the column two in order to consider the consumers stipulation, I will get c is equal to 2 for n p 2 5.322 and I will get n is equal to 60 for c is equal to 3. I will have n p 2 6.681 and n is equal to 75. So, I can chock out two plans from consumer stipulation point of view one is n is equal to 60 c is equal to 2 n is equal to 75 c is equal to 3.

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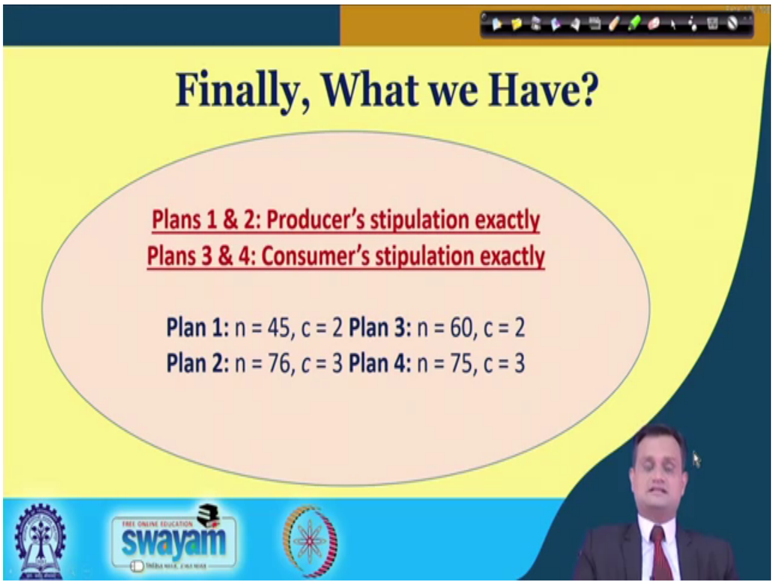

So, now its really interesting that I am left with now four plans plan 1 and 2 producers stipulation exactly, plan 3 and 4 consumers stipulation exactly. So, plan 1 n is equal to 45 c is equal 2, plan 2 76 and 3 plan 3 and plan 4.

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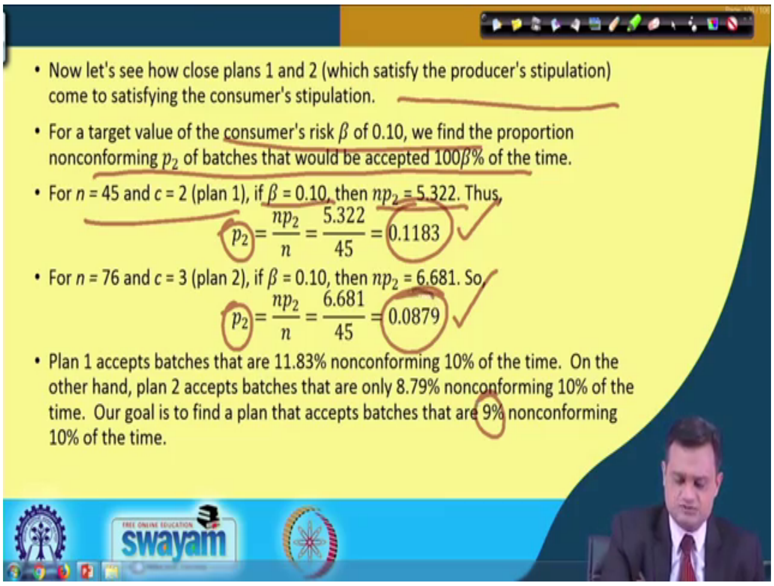

Now what I want to figure out? I want to go by either of the criteria and the criteria is like this, I want a plan which exactly meets the producers stipulation and would be very close to my consumer or a plan which would be exactly meeting the stipulation of consumer and would be very close to producer you cannot satisfy both exactly.

So, it is something like if you have a two child and they have different requirement you will see that ok, the requirement of child one is extremely critical let me try to meet it exactly; and if the child two then let me try to go as close as possible. So, it is exactly like that. So, here what I will do that for a target value of consumer is beta 0.10 we find the proportion of non conforming p 2 or batches that would be accepted 100 beta percent of the time. So, plan 1 n is equal to 45 and c is equal to 2.

So, I am taking two plans which exactly meets the producers stipulation, satisfies the producer simulation and for beta is equal to 0.10 n p 2 is equal to 5.322 I can find the value of p 2 value of p 2. Now what do you see here? Plan 1 gives you value of p 2 0.1183, plan 2 gives you value of p 2 0.0879. So, my condition here is that, plan 1 accepts batches that are 11.83 percent non conforming 10 percent of the time, because beta is 10 percent on the other hand plan 2 accept the batches that are only 8.79 non conforming 10 percent of the time.

Our goal is to find that plan can accept the batches that are 9 percent nonconforming 10 percent of the time. So, 10 percent is beta do not get confused. So, now, what is close to 9 percent I am exactly considering the stipulation of producer and I want to go very close to consumer. So, here this plan 2 is very close to 9 percent.

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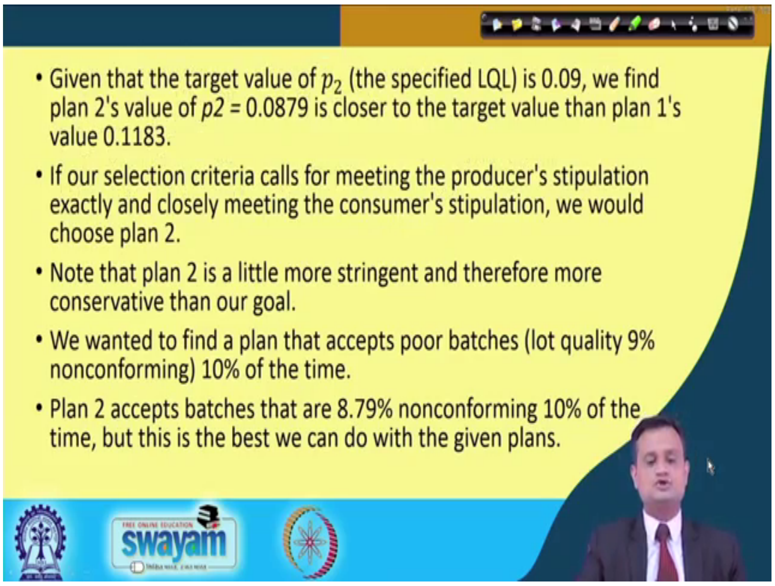

And I will say that if I select plan 2 it would be very close. So, it our selection criteria calls for meeting the producer’s stipulation exactly and closely meeting the consumer stipulation I will go with plan 2. So, plan 2 is little bit more stringent because my expectation is 9 percent it is 8.9 percent. So, it is little bit more stringent, but it will help me to satisfy the exactly producers stipulation and very close consumer stipulation.

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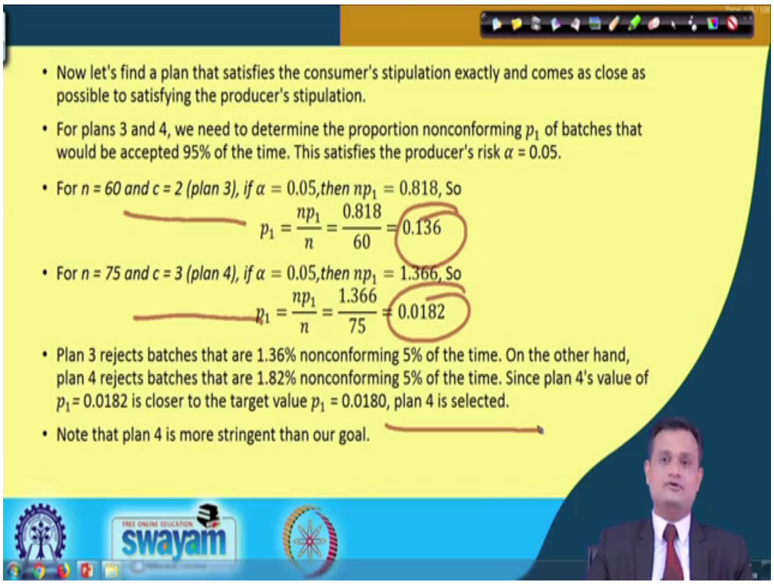

Similar way I can analyse my plan 3 and plan 4, here and what you can see that I am getting p 1 0.136 and p 1 0.0182. So, you will select plan 4 which is very close to the expected value of my producer's stipulation and I can here me to my consumer stipulation exactly in plan 3 and 4, and I could be very close to the stipulation of my producer.

So, maybe little bit sounding confusing, but the concept is very simple I explained

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Now let us look at the double sampling plan.

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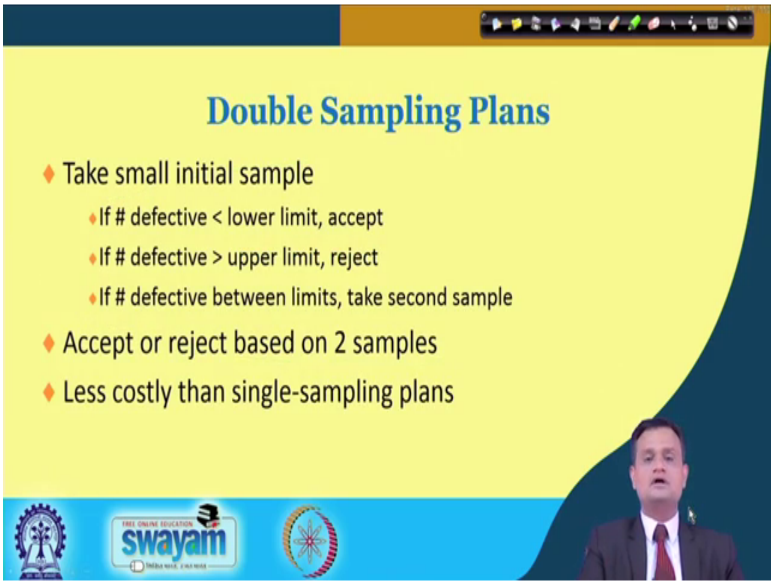

So the idea is very simple, if you have defective and lower less than the lower limit accept no issue first go, if you have a defective greater than upper limit reject. And if you have number of defective between limit, take the second sample accept or reject based on 2 samples not on one sample loss less costly than the single sampling plan, because the average sample number in double sampling would be little bit say lesser than or drastically less than the single sampling plan.

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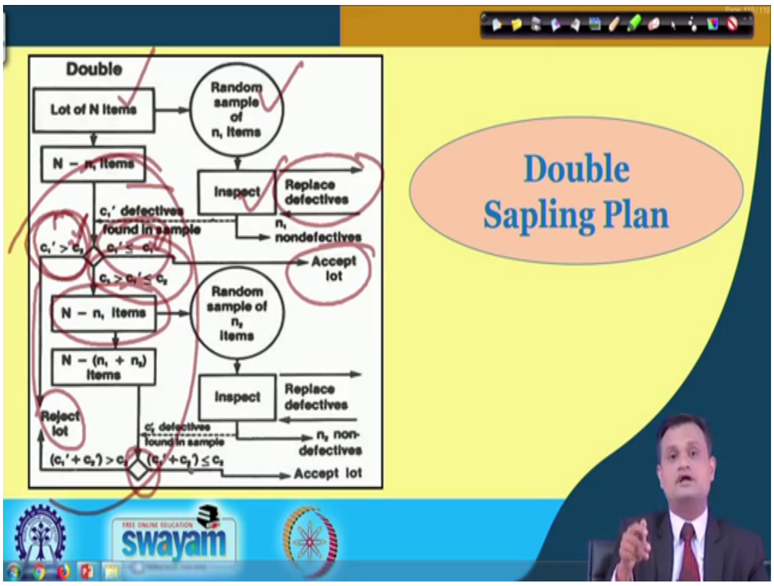

So, just see how it works double sampling plan? I have the lot of N items I randomly select a sample of an one item I inspect it, I may replace defectives. Now let us say this is the main part say I am coming here c 1 dash is the defective find in a sample, suppose c 1 is greater than c 2 and c 1 is less than or equal to c 1, find this c 1 is my one limit this c 2 is another limit. If exits this I will simply reject. If it is less than this I will not bother I will accept. Now if it is in between your c 1 and c 2 I will go for this is in between c 1 and c 2 second sample as already I have drawn one sample of n 1. So, I am left with N minus n 1, I will draw another sample. And now I will take a decision here that, c 1 dash plus c 2 dash.

So, number of defectives found in the first sample plus number of defectives found in the second sample, if it is less than this particular value c 3 or if it is greater than I will now stop by taking a decision either reject or accept.

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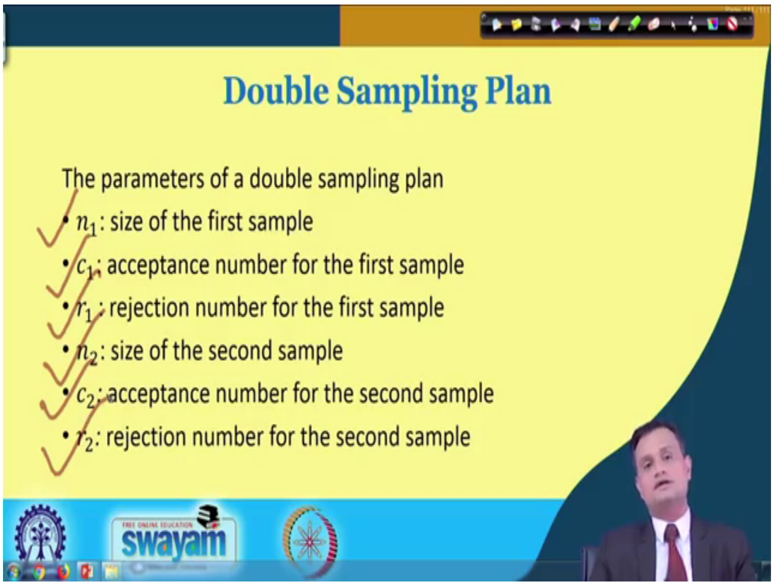

So, this is how my double sampling plan works, and only thing that I am going for one more equation. And I am considering the number of defectives found in the first sample as well as second, in order to make my final decision after taking the second sample. So, the notations which we used generally may differ from book to book, but it may be like n 1 size of the first sample, c 1 acceptance number of the first sample, r 1 rejection for the first sample, n 2 size of second sample c 2 acceptance number of the second sample and r 2 rejection number of the second sample.

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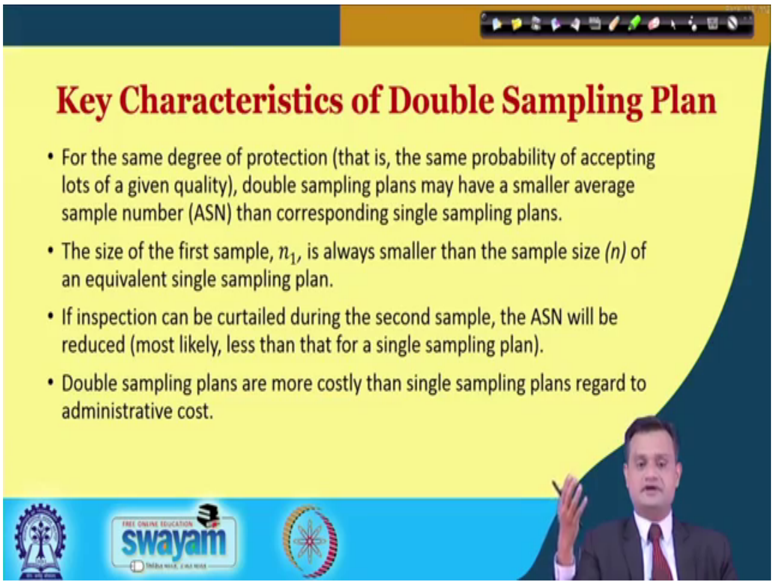

So, this is what exactly we do, and typically there are some key characteristic I mentioned previously also that every sample number, in case of double sampling plan may be smaller compared to the single and the size of the first sample is always smaller than the sample size of the equivalent single sampling plan. If inspection can be curtailed during second sample, the ASN will be reduced and double sampling plans are more costly than single sampling plan in terms of the record keeping and the administrative cost.

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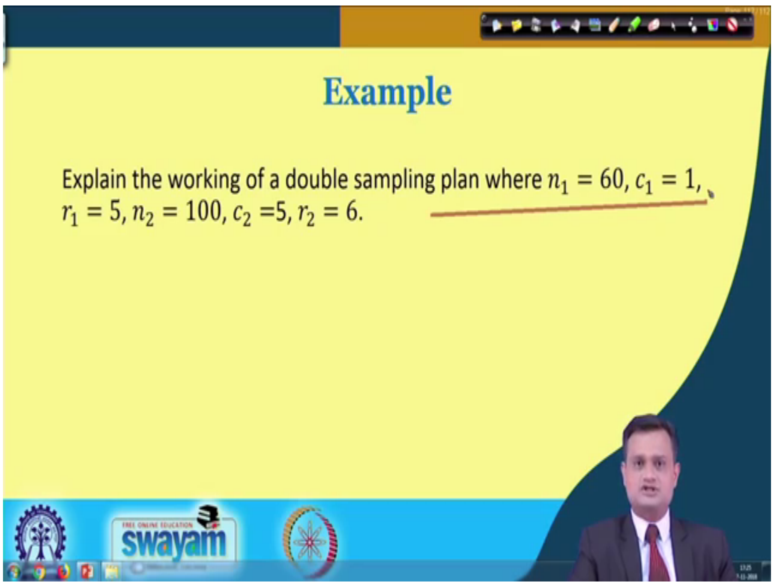

So, let us see quickly what it means so let us say I have n 1 is equal to 60 first sample first acceptance number 1 rejection number 5 n 2 100 c 2 5 r 2 6 and how it works?

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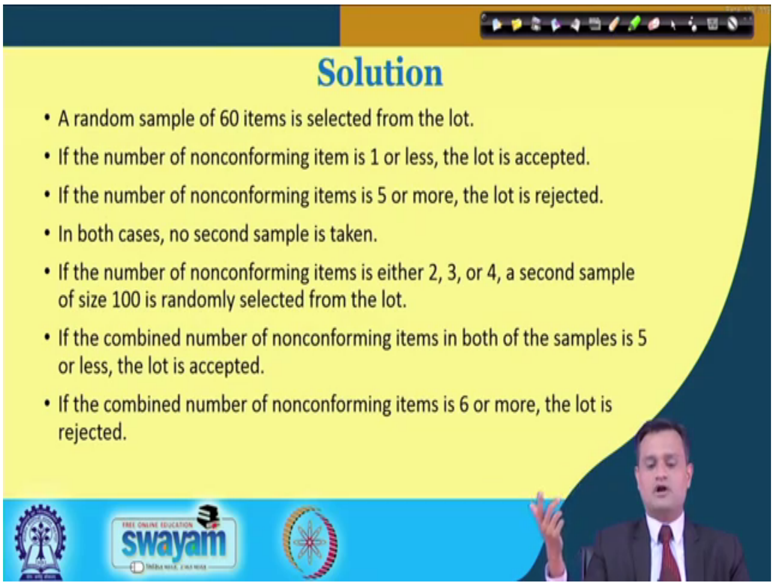

So, the solution is very simple, random sample of 60 is taken, number of non conforming found one or less lot is accepted if it is nonconforming is 5 or more lot is rejected. If it is anywhere in between 2 3 4, then I will take the second sample if the number of non conforming item 2 3 4, let us say I take the second sample of 100 n 2 is equal to 100.

If the combined number of nonconforming, nonconforming found in the first nonconforming found in the second is 5 or less lot is accepted, if it is say more than 6 the lot is rejected.

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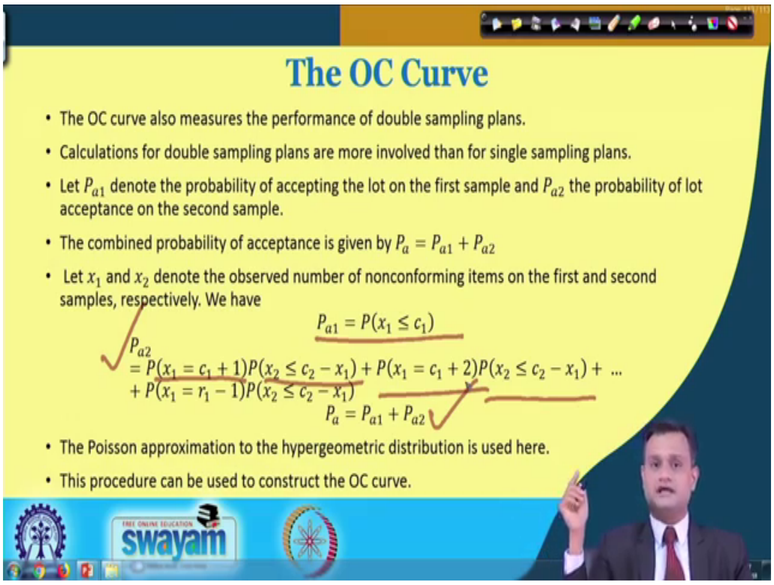

So, this is what I try to do, I can also plot the OC curve and you can have a very simple expression P a 1 that is referring to the probability of acceptance in the first say sample. So, probability that x 1 is less than or equal to c 1 c 1 is my acceptance number. And P a 2 I will have x 1 is equal to c 1 plus 1, because already you have taken the first sample and now you are in between. So in between your acceptance and rejection, so c 1 plus 1 and probability of x 2 less than or equal to c 2 minus x 1, plus probability of x 1 is equal to c 1 plus 2.

And probability into probability of x 2 less than or equal to c 2 minus x 1, we discuss the probability issues in detail and you can appreciate this. So, total probability is P a 1 plus P a 1 probability of acceptance in first sample probability of acceptance in second sample.

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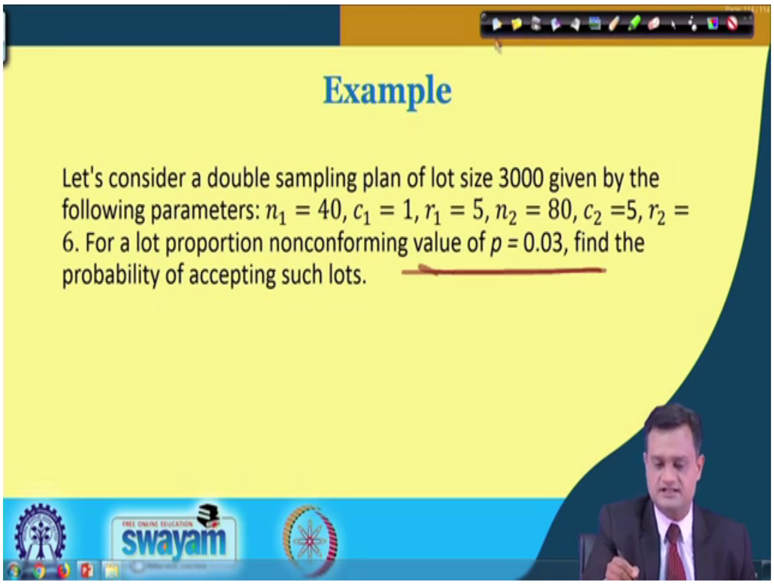

So, when we try to do it for a particular example let us say the lot size is 3000 n 1 is 40 c 1 is 1, r 1 is 5, n 2 is 80, c 2 is 5, r 2 is 6 and nonconforming value is p is equal to 0.03, find the probability of accepting such lots.

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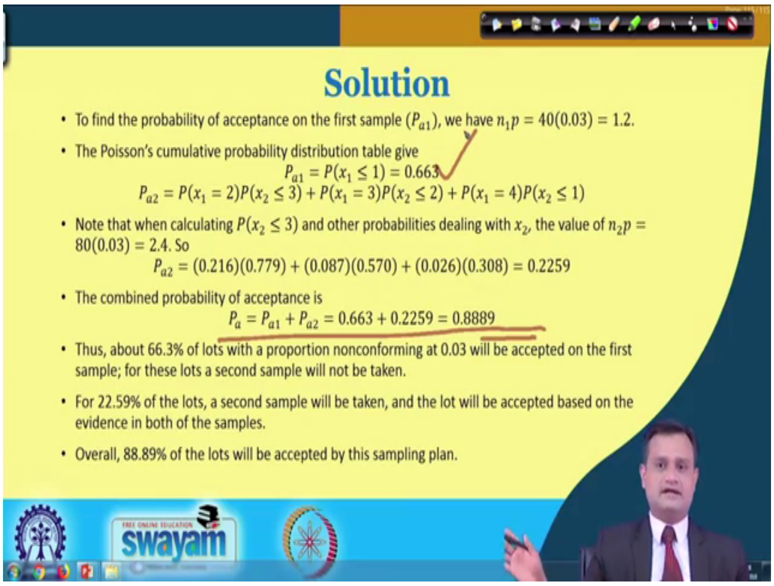

So, you can just plug in the values and find the probabilities. So, this you can find that is the P a 0.8889 and what does it say? That about 66.33 percent that is P a 1 of lot with a proportion nonconforming 3 percent will be expected on the first sample. So, out of so many lot 66.3 percent of the lot that they will be cleared on the first sample with 3 percent defective; 22.59 percent lot a second sample will be taken and overall 88.59 percent of the lot will be accepted by the sampling plan. So, I think this is a quite interesting.

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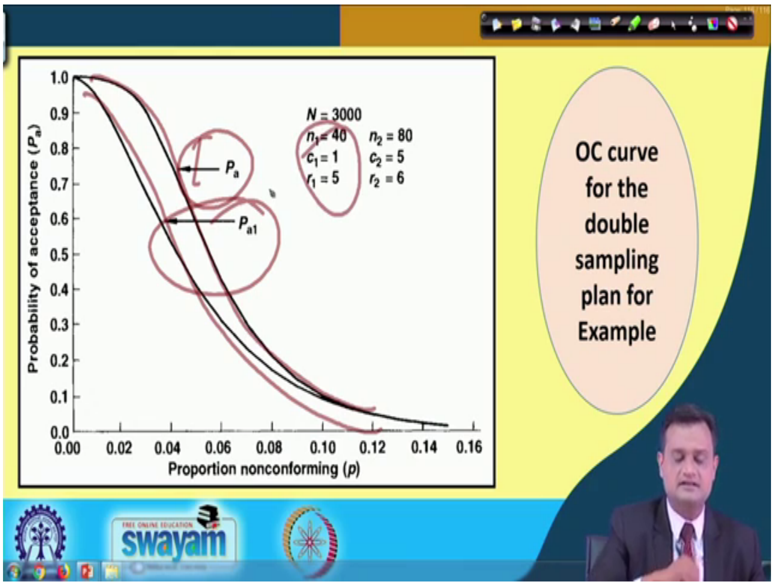

And what you can see here, that for double sampling plan this is my P a this is my P a and this is my P a 1. So, you can see that the operating characteristic curve for total probability is on the upper side with and the lower one is your P a 1 this is my single for drawn based on the first sampling. So, discriminatory power of this one is higher and it is cheaper compared to the total probability of acceptance.

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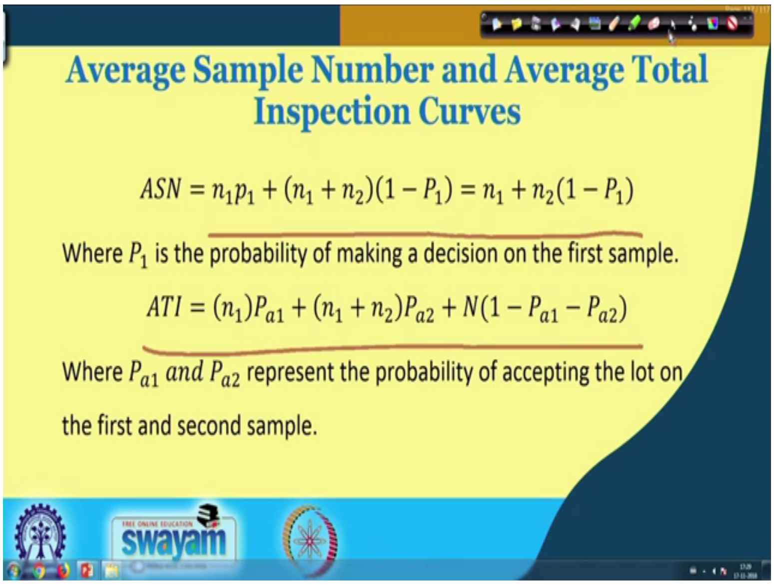

So, you can also find ASN by using the expression n 1 p 1 n plus n 2 minus p 1, and average total inspection for the double sampling plan.

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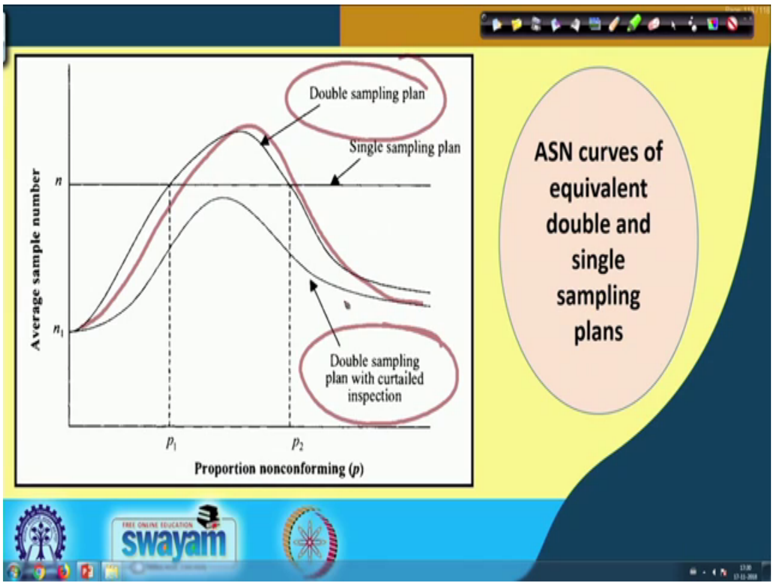

So, I think this thing we have discussed. So, you can just apply and just try to appreciate that double sampling plan your average sample number will be on higher side, double sampling plan with curtailed inspection. Once you know that in the first sample it is done; then you will curtail you will not go for the inspection. And this is less average sample number in case of single sample it is only n because you are only going for one stage decision.

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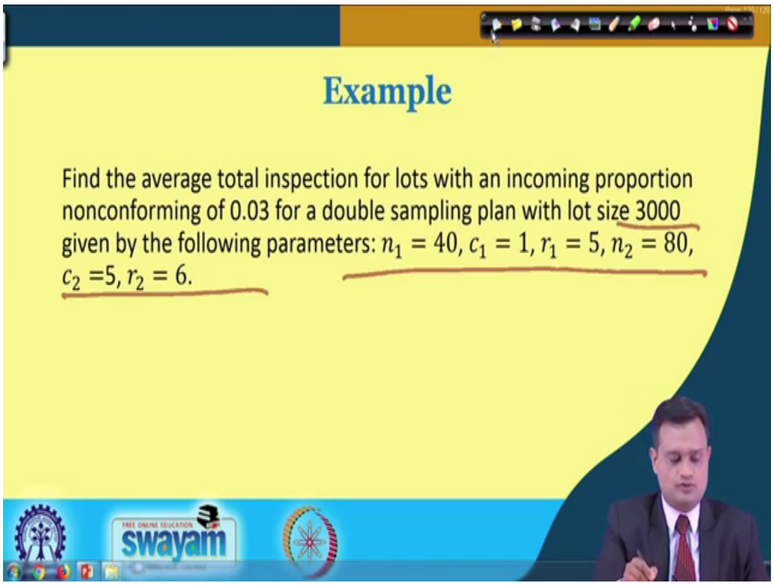

So, you can consider again a data of lot size 3000 and h parameters these n 1 c one r 1 n 2 and c 2 and r 2.

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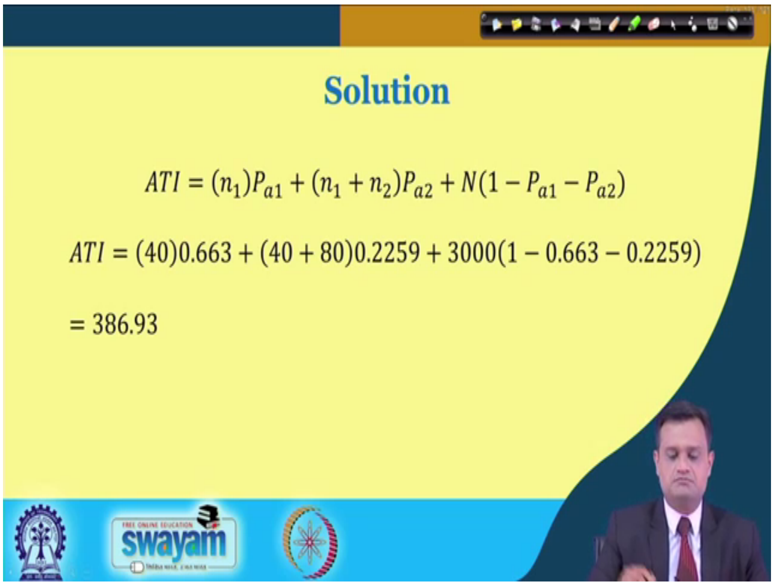

And you can plug in this value, so you will get average total inspection 386.93.

So, for a given double sampling plan your average inspection total inspection will be of 386.93 parts or units.

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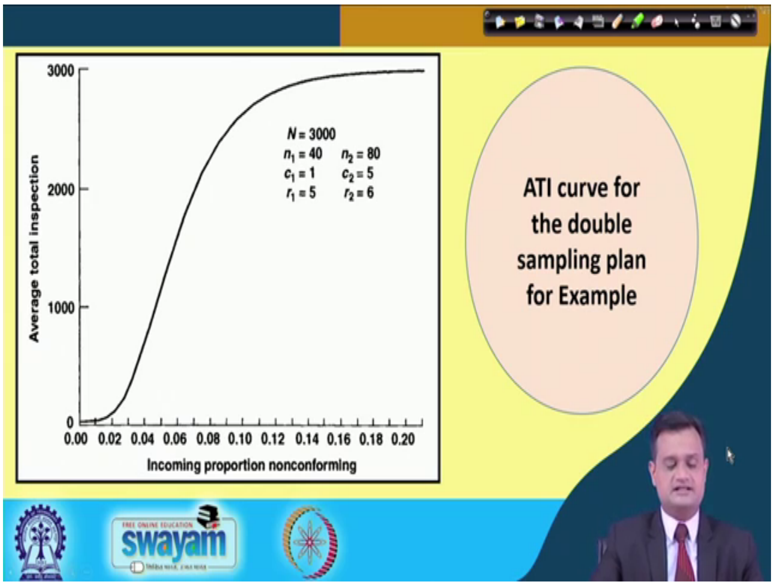

And you can see the ATI. So, as your incoming proportion nonconforming increases your average total inspection is also increasing.

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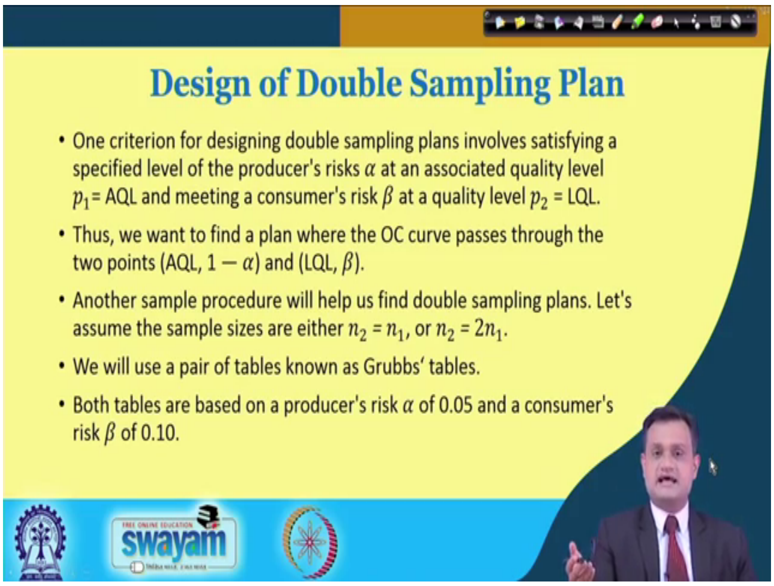

You can design this double sampling plan in the same way for AQL 1 minus alpha LQL beta.

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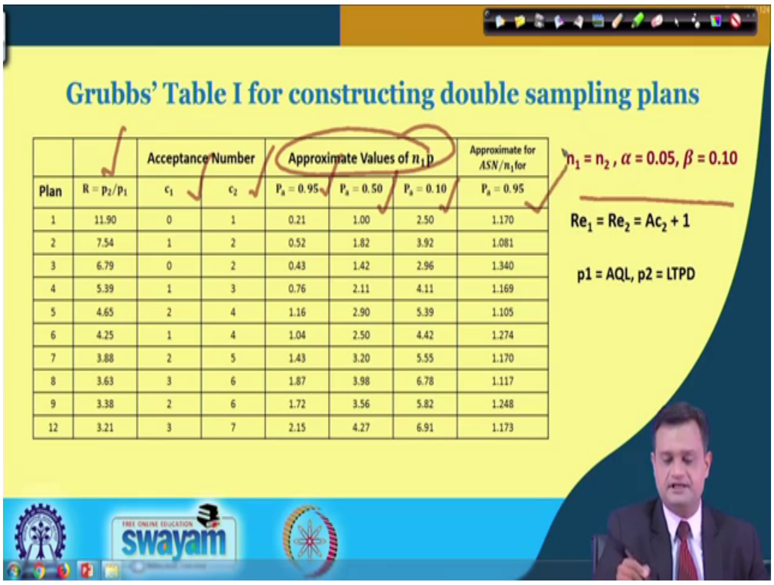

		Acceptance Number		Approximate Values of n_1p			Approximate for ASN/n_1 for
Plan	$R = p_2/p_1$	c_1	c_2	$P_a = 0.95$	$P_a = 0.50$	$P_a = 0.10$	$P_a = 0.95$
1	11.90	0	1	0.21	1.00	2.50	1.170
2	7.54	1	2	0.52	1.82	3.92	1.081
3	6.79	0	2	0.43	1.42	2.96	1.340
4	5.39	1	3	0.76	2.11	4.11	1.169
5	4.65	2	4	1.16	2.90	5.39	1.105
6	4.25	1	4	1.04	2.50	4.42	1.274
7	3.88	2	5	1.43	3.20	5.55	1.170
8	3.63	3	6	1.87	3.98	6.78	1.117
9	3.38	2	6	1.72	3.56	5.82	1.248
12	3.21	3	7	2.15	4.27	6.91	1.173

And you have the Grubbs table so there are two tables; the first table accepts n 1 is equal to n 2 alpha is equal to 0.05 and beta is equal to 0.10 and you have the r is equal to p 2 by p 1 c 1 c 2 P a 0.95 P a 0.50 p a 0.10. So, approximate values of n 1 p and P a is 0.95.

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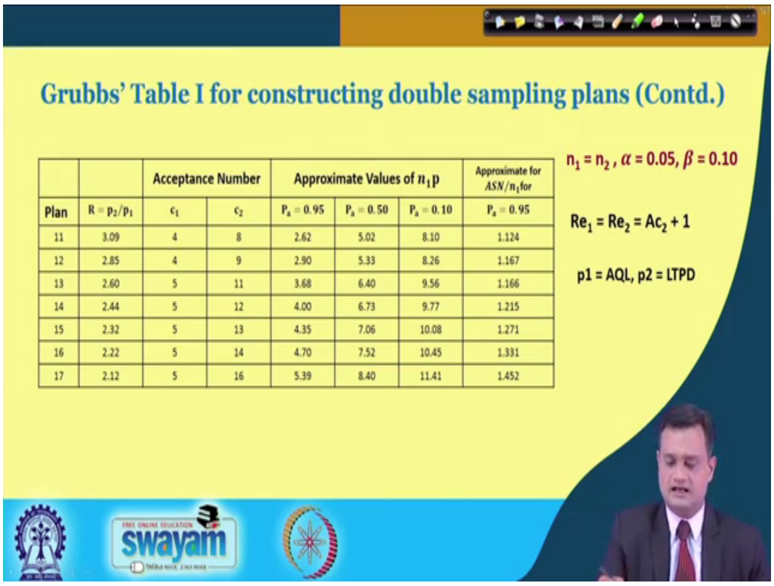

		Acceptance Number		Approximate Values of n_1p			Approximate for ASN/n_1 for
Plan	$R = p_2/p_1$	c_1	c_2	$P_a = 0.95$	$P_a = 0.50$	$P_a = 0.10$	$P_a = 0.95$
11	3.09	4	8	2.62	5.02	8.10	1.124
12	2.85	4	9	2.90	5.33	8.26	1.167
13	2.60	5	11	3.68	6.40	9.56	1.166
14	2.44	5	12	4.00	6.73	9.77	1.215
15	2.32	5	13	4.35	7.06	10.08	1.271
16	2.22	5	14	4.70	7.52	10.45	1.331
17	2.12	5	16	5.39	8.40	11.41	1.452

So, you have this table this is in continuation for n 1 is equal to n 2, and similar way that table two Grubbs has develop that is for n 2 is equal to 2 n 1.

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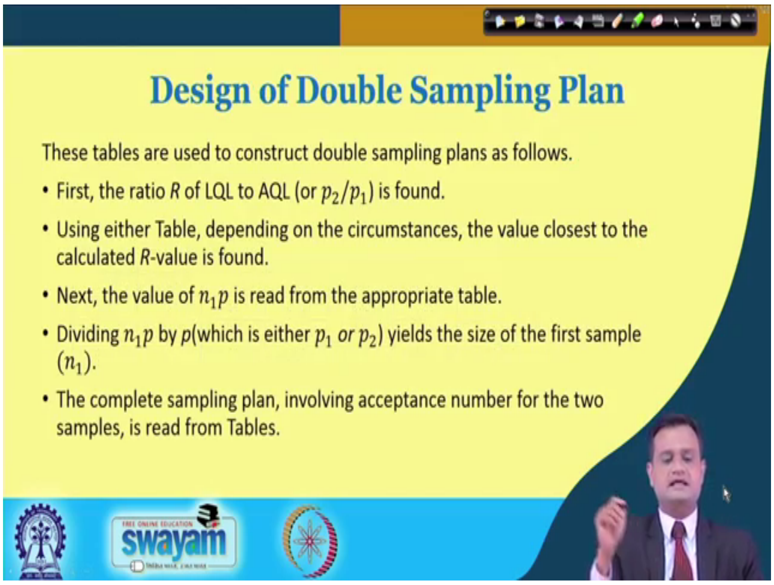

So, we can easily make use of this table for designing a sampling plan and the procedure is very simple. First the ratio R which is LQL to RQL should be calculated p 2 by p 1 using either of the table the value closest to the R should be found. Then value of n 1 p is read from the appropriate table divide n 1 p by p which either p 1 or p 2 yields the size of the first sample the complete sampling plan involving acceptance number for the two sample can be read from the table.

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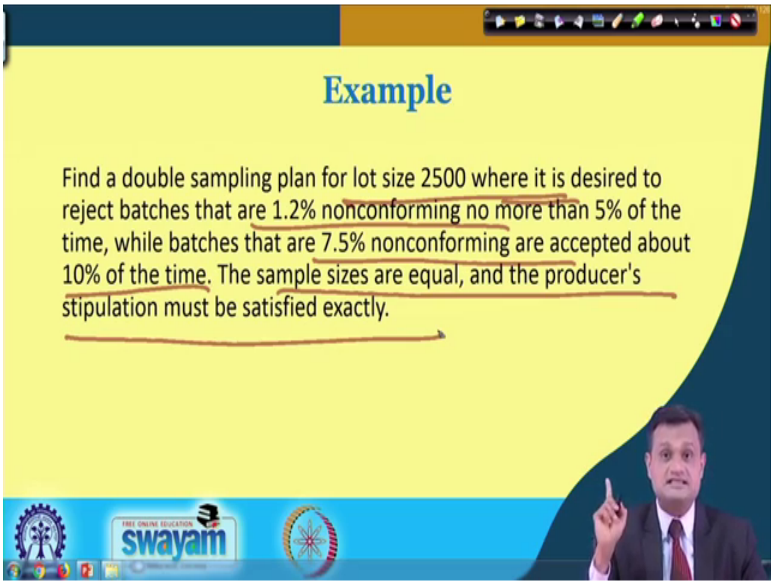

So if we just see the example, then lot size is 2500 1.2 percent nonconforming 7.5 percent nonconforming are accepted at some beta. And the sample size are equal and the producers stipulation must be satisfied exactly.

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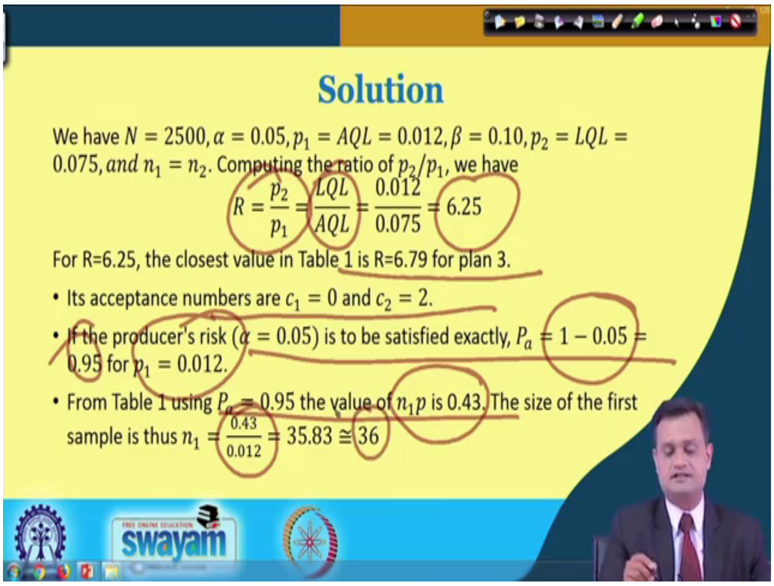

So, I have to design a plan which can satisfy so as I mention find the value of R p 2 by p 1 p 2 by p 1 which is LQL by AQL 6.25 you refer table one R is equal to 6.79 for plan 3. So, the acceptance number c 1 is equal to 0 c 2 is equal to 2. And for alpha is equal to 0.05 you have P a 1 minus 0.05 that is 95 percent and for p 1 0.012. So, with this particular values ; n 1 p is 0.43 and you can figure out what is the sample size of the first sample in my double sampling plan?

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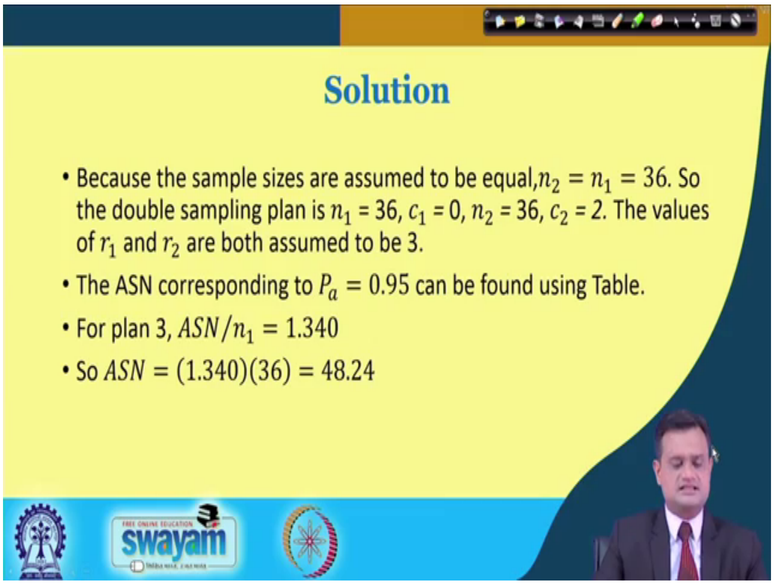

So, this is the solution that you can find ASN also because ASN by n 1 ratio is given in the Grubbs table and this is how you can design the double sampling plan.

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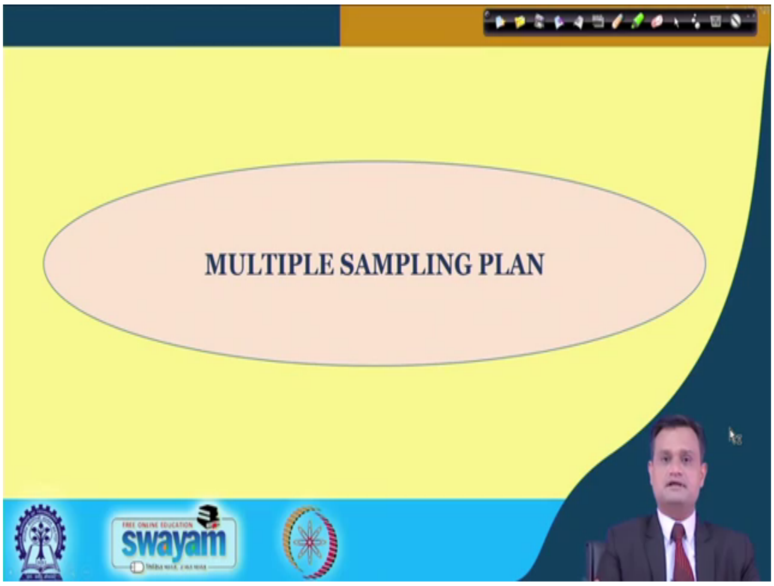

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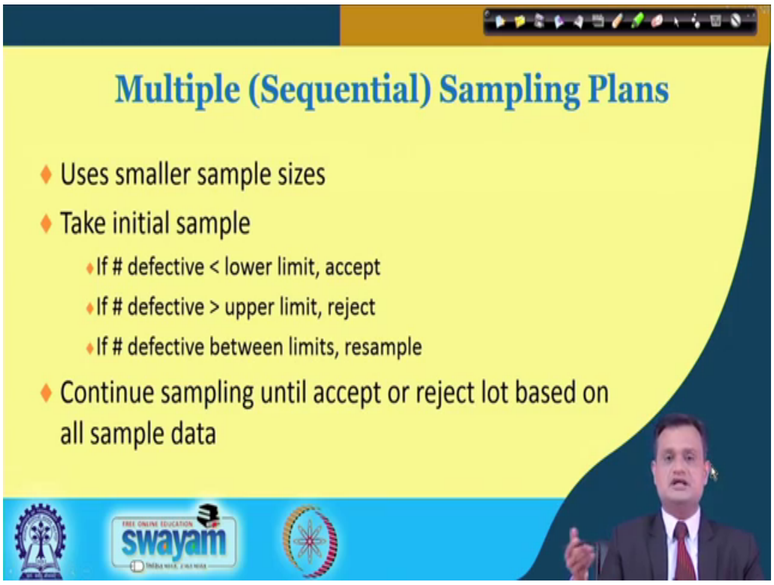

We are not going into the detail of multiple sampling plan, it is just an extension of double sampling, but you take the initial sample defective is lower than less than lower limit accept, greater than upper limit reject, in between go for the another sample and continue for n number of stages and this is your multiple or sequential sampling plan.

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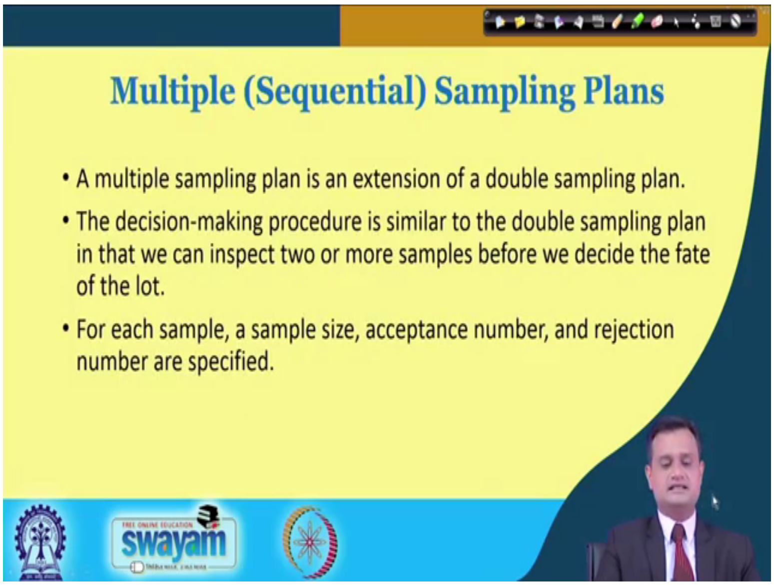

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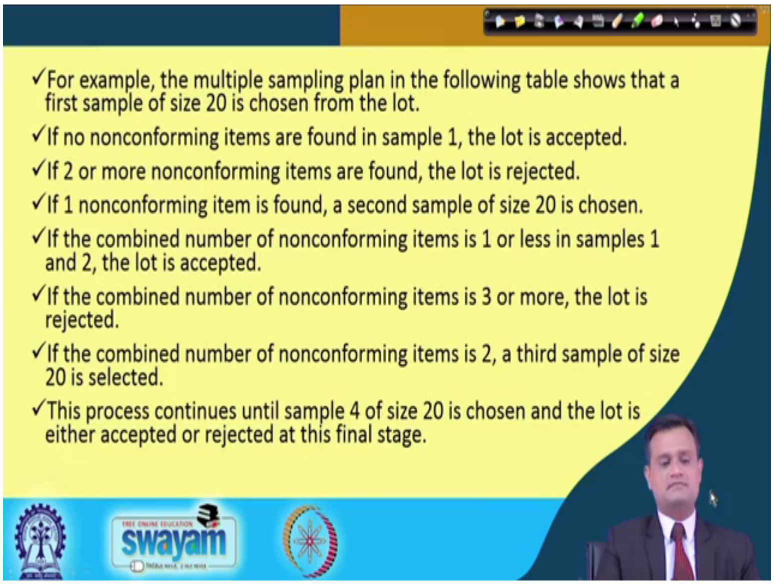

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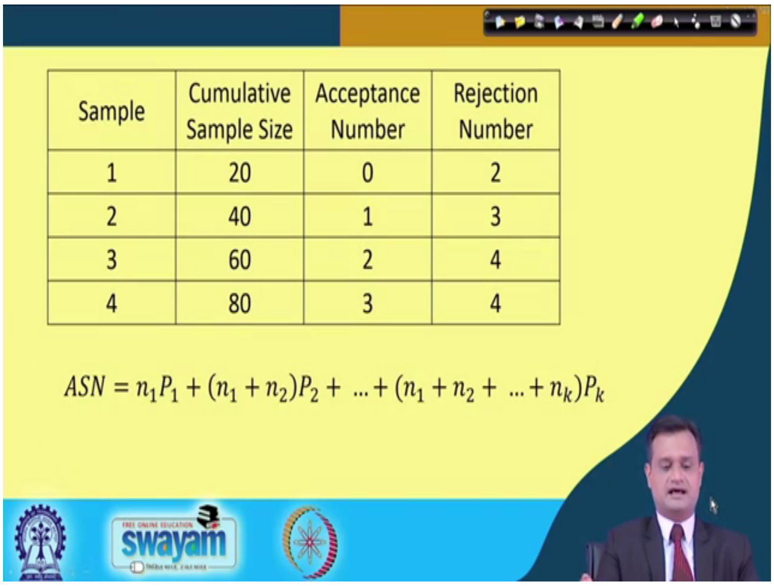

So, just see this example sample one you have cumulative sample size 20 and acceptance number 0 rejection 2 40 1 3 60 2 4.

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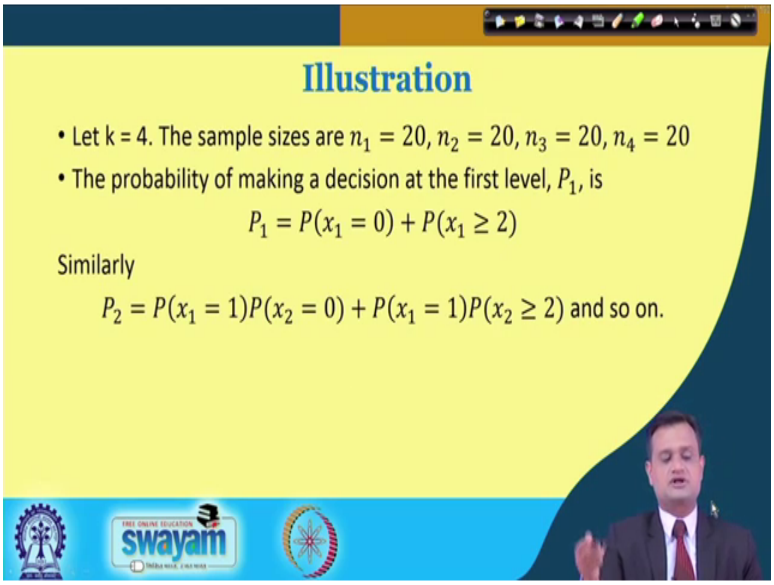

And you can easily take the decision about the acceptance or rejection of the sample at an appropriate level. So, you can find the probability at P 1 your stage 1 p 2 at stage 2 and so on.

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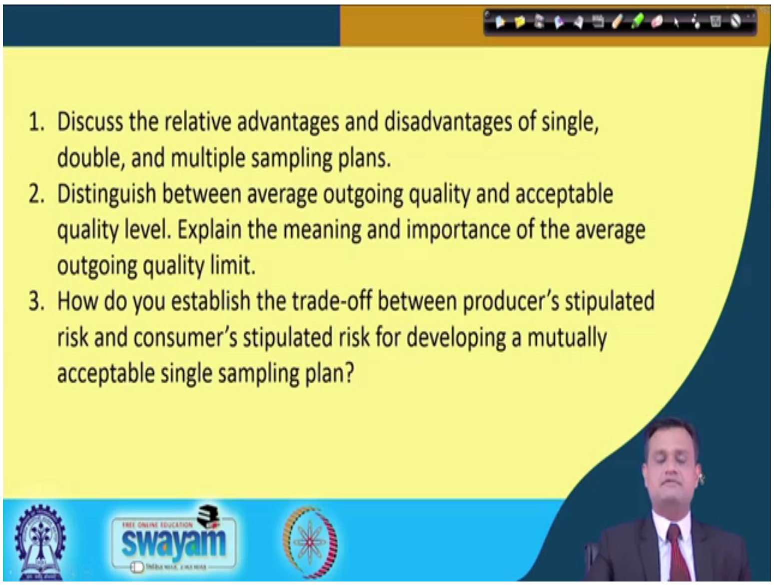

So, just before we end let me float couple of think it, discuss the relative advantage and disadvantage of single double and multiple sampling plan, distinguish between average outgoing quality and acceptable quality level.

Explain meaning and importance of the average outgoing quality limit. And how do you see the trade-off between producers stipulated risk and consumers stipulated risk and device a plan which is mutually acceptable to both.

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So, these are the references mainly I have used Mitra for this particular lecture.

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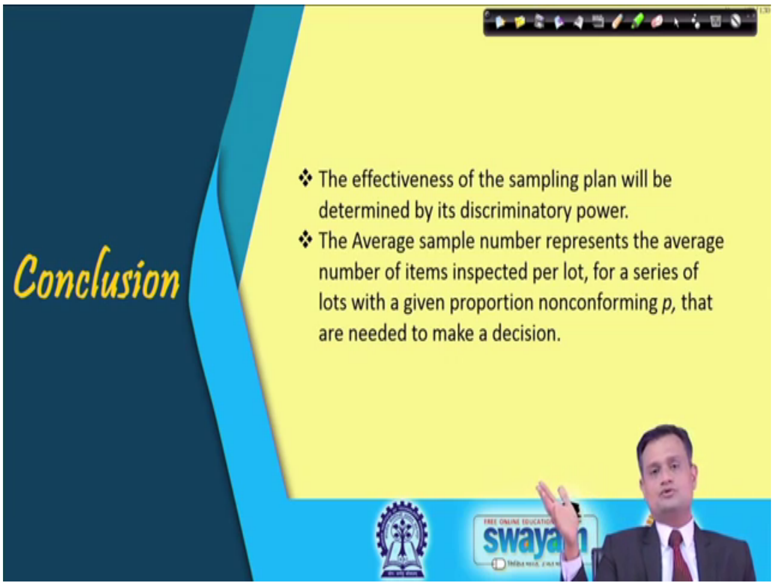

And we have to device a effective discriminatory sampling plan which helps producers and consumers both to have the consensus on a particular value of n c and so on. So, thank you very much for your interest in learning this particular lecture on design of acceptance sampling plan for attributes. And we will continue our discussion on this with couple of more plans till that time keep revising introspecting with me enjoy.