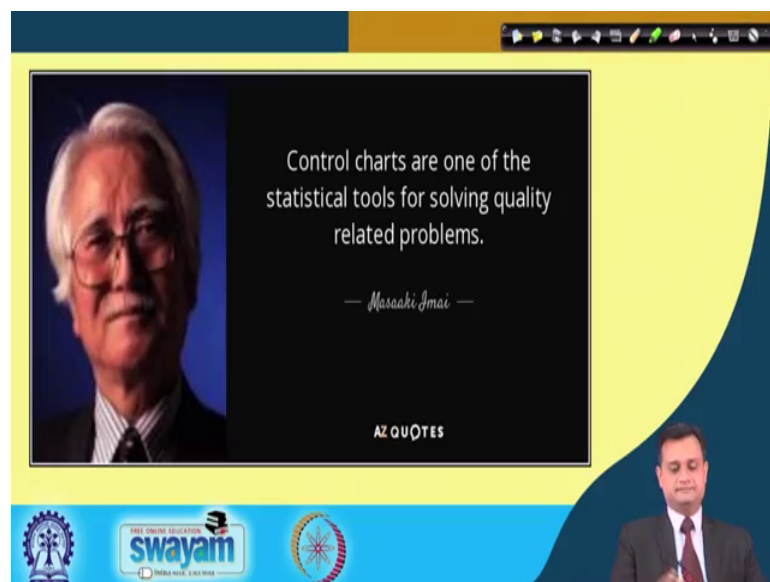


Six Sigma
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Lecture - 54
Acceptance Sampling: Key Concepts

Hello friends, hope you are doing well in your day-to-day life and profession, and we are moving ahead in our Six Sigma journey. So, I welcome you to lecture 54 that is on Acceptance Sampling Key Concepts. This week we will discuss the various issues specific to acceptance sampling, and we are beginning with some key concepts on acceptance sampling.

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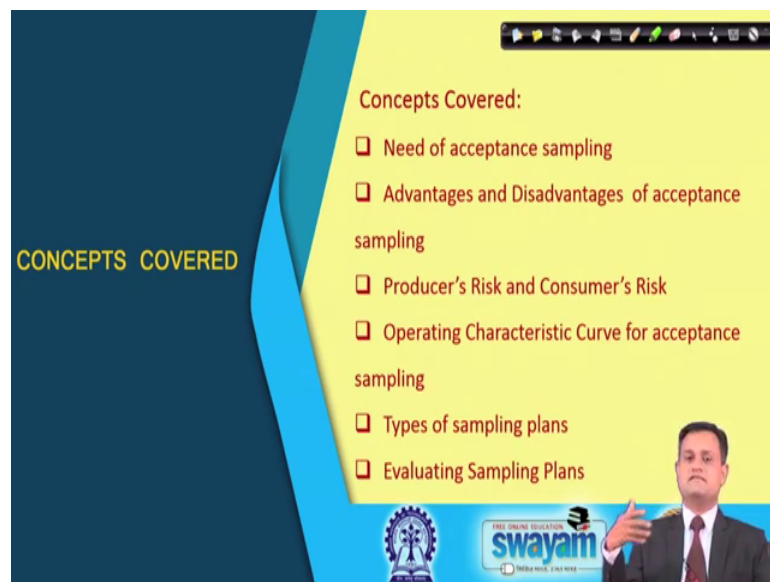
So, I will keep reminding you that control chart are the most important tool, statistical basis fact based management to have control over my processes and take the appropriate action at the right time.

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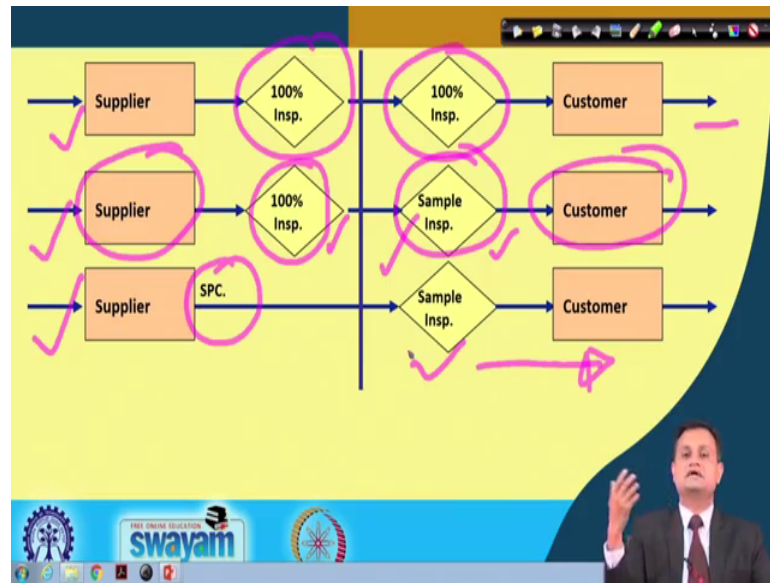
So, last lecture if you recall, we had seen the detailed application of Minitab for variable control chart as well as attribute control chart.

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And now in this lecture, we will try to appreciate the various concepts specific to acceptance sampling. So, just see need of acceptance sampling, advantages, disadvantages, producers risk and consumers risk, some new terminology you will be familiarizing operating characteristic curve for acceptance sampling, types of sampling plan and evaluating sampling plans.

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So, now, just try to appreciate this figure and what does it tell. So, let us say I have a supplier and 100 percent inspection is taking place at suppliers end, and then again I am taking the 100 percent inspection of the incoming material, and then it goes to customer. So, this is the situation 1. Now, let us say supplier as produce situation 2, 100 percent inspection is taking place. Now, supply is giving the certificate that 100 percent inspection is taking place; I am just doing the sampling inspection, and then it goes to my customers.

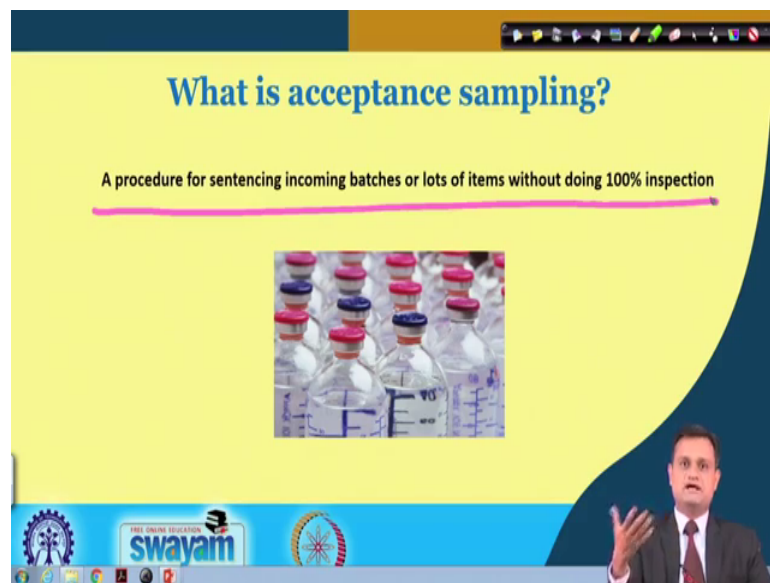
Sampling inspection means I am not doing 100 percent inspection, I will just take sample you all know what does it mean, sample a word we have used extensively in control chart. So, even if you go to purchase wheat or rice, do you check the entire lot, not possible, you will just take a small sample and try to check it. So, same applies over here that my supplier has done the 100 percent inspection, I may go with sampling inspection accept or reject it, and then it will go to customer.

Now, just see that third situation in which my supplier as already established an external process control through control charts, statistical process control, and then I am just taking the sample and then passing the units to my customer. So, you will see that the amount of effort is going down, because I am trying to exercise better control over the process, but you will also see that when I am going for sampling inspection at this stage, second stage, then I need to devise my sampling plan in such a way that it really has a

good discriminatory power, it means it should be able to say discriminate, separate out, good lot from the bad lot, and give me the right information.

So, if I can design such kind of system, a sampling plan then I can avoid doing the 100 percent inspection. Similar way, if my supplier establishes SPC, then also with better control I can reduce my inspection effort. And I can feel equally confident in accepting or rejecting the incoming or outgoing quality.

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So, this is where we try to appreciate the importance of acceptance sampling. Just try to see little bit a simple definition what is the acceptance sampling. So, I would just like to emphasize that a procedure for sentencing, sentencing incoming batches or lots of items without doing 100 percent inspection. So, typically somebody is sentenced or sentencing the word we use in the judicious system somebody is punished or somebody is exempted. But here what I say that without doing 100 percent inspection I will either reject the lot based on my sampling plan or I will accept it, and this decision I take based on my acceptance sampling. So, this is what exactly we want to discuss as a part of this lecture.

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Why not 100% inspection?

- ✓ Very expensive
- ✓ Can't use when product must be destroyed to test
- ✓ Handling by inspectors can itself induce defects
- ✓ Inspection becomes tedious in order to prevent defective items from slipping through inspection

The slide features a cartoon illustration of a man in a suit and glasses, looking thoughtful with his hand on his chin. At the bottom, there is a logo for 'swayam' (Free Online Education) and a small inset image of a man in a suit pointing upwards.

Now we can appreciate that why not 100 percent inspection? The reasons are very obvious it is extremely expensive, suppose non-destructive testing is required, you cannot do it 100 percent; otherwise you will spoil all the products. Then handling by inspector can itself induce some defects, many a times when you are handling very precise parts and every part you are trying to inspect then inspector can induce some defect that is not acceptable. And inspection typically becomes the tedious task. And in that case your inspector may make an error and defective items maybe say accepted. So, these are couple of reasons that I would not like to go for 100 percent inspection.

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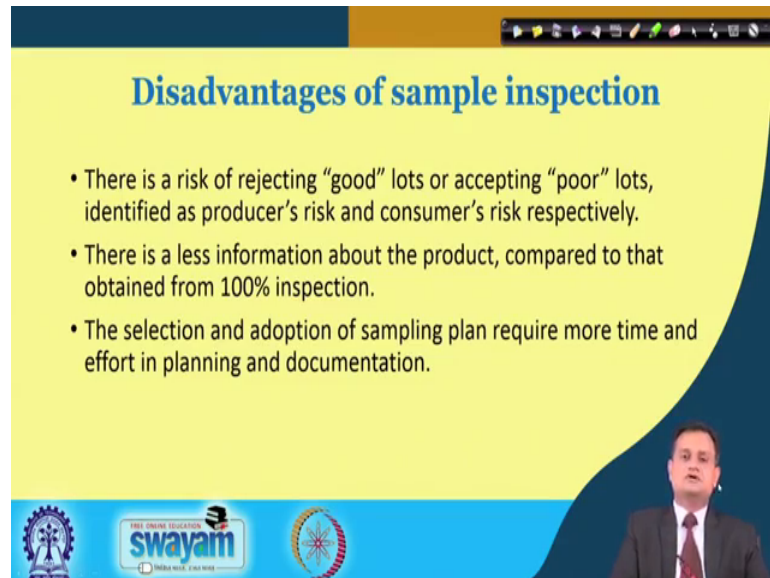
Advantages of sample inspection

- Lower costs
- Smaller staff
- Less risk of handling damages
- Less time
- Helps in preparing for quality planning

The slide features a small inset image of a man in a suit pointing upwards. At the bottom, there is a logo for 'swayam' (Free Online Education) and a small inset image of a man in a suit pointing upwards.

So; obviously, I have the advantages lower cost, smaller staff, less risk of handling damages, less time helps in preparing the quality planning.

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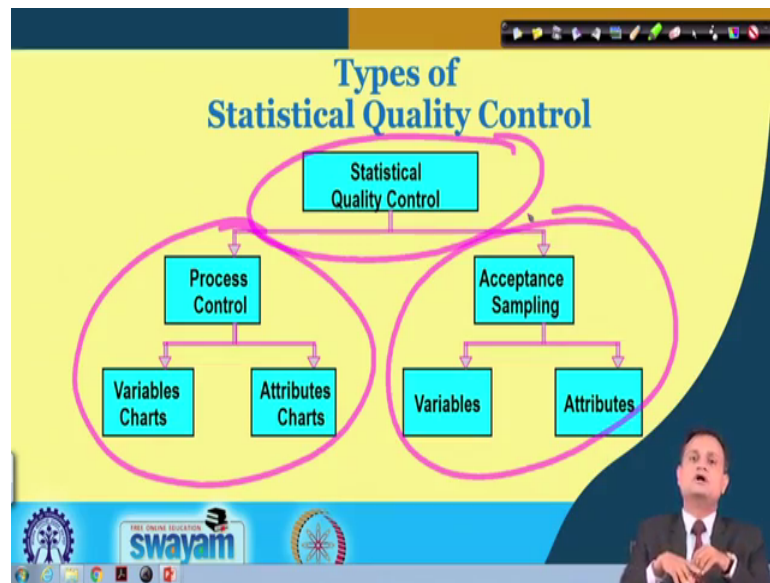
Disadvantages of sample inspection

- There is a risk of rejecting “good” lots or accepting “poor” lots, identified as producer’s risk and consumer’s risk respectively.
- There is a less information about the product, compared to that obtained from 100% inspection.
- The selection and adoption of sampling plan require more time and effort in planning and documentation.

swayam
INDIA RISE, CHINA RISE

So, disadvantages are obvious with the acceptance sampling that when you are only taking the decision based on sample, you may make a mistake in accepting a lot which is not good, but a bad quality or you may reject a lot which is actually good, but you will declare it as a bad. So, this disadvantage and risk we have to accept. And you cannot realize the benefit of 100 percent inspection when you are going for the sampling. But if you design your sampling plan appropriately, then you can really have better control over such decisions and you can reduce the probability of making say wrong decision in either case whether accepting the bad lot or rejecting the good lot, and that can really give you more or less the same confidence that otherwise you can have through 100 percent inspection.

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So, let us try to see the broader domain. So, when I say statistical quality control, we have seen the statistical process control, variable chart and control charts for attributes. And now we are discussing the second pillar of this that is acceptance sampling, acceptance sampling for variables and acceptance sampling for attributes. So, the simple is very system, very simple that you have statistical quality control, process control, acceptance sampling within process control, you have the control charts for variables, control charts for attributes, similar way attribute sampling you have the attributes sampling plan for variables as well as attributes.

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So, this is how we try to appreciate the concept of statistical quality control. And there could be couple of typical points where you would like to employ the acceptance sampling plan or the inspection level as desired. So, it maybe raw materials and purchase parts, it is coming to your processes. So, if there is some bad quality that will have a significant impact on the quality of your produce, final product, and your process is will also not be able to accept such variation.

Finished products, before costly operation because your entire value will be lost if your input material is not good, then before an irreversible process, all these are the examples where you have very high risk of losing the value in it is always better to employee inspection or an adequate sampling plan, so that I actually process the good quality component or work in process inventory or the raw material, so that my operations cost, value addition is not wasted.

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What Is an Acceptance Plan?

- Set of procedures for inspecting incoming materials or finished goods
- Identifies
 - Type of sample
 - Sample size (n)
 - Criteria (c) used to reject or accept a lot
- Producer (supplier) & consumer (buyer) must negotiate

So, this is where we can realize the importance of acceptance sampling. So, what is an acceptance sampling in a very simple way, set a procedure for inspecting incoming material or finished goods based on some rules, not the 100 percent inspection. So, typically couple of points that I consider when I talk about the design of the acceptance sampling plan, these includes my type of sample, sample size, criteria used to reject or accept the lot, and most importantly producer and consumer must negotiate. Because when you go for sampling, they both have to accept some amount of risk that is the

stipulated risk and this risk is called consumers risk and producers risk and they must agree negotiate on this, so that even if there is a probability of making some mistake, wrong decision, it should be acceptable to both the party. So, there are some of the things that we try to incorporate in the design of acceptance sampling plan.

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Sampling Plan

- ♦ Guidelines for accepting lot
- ♦ Single sampling plan
 - ♦ N = lot size ✓
 - ♦ n = sample size (random) ✓
 - ♦ c = acceptance number ✓
 - ♦ d = number of defective items in sample ✓
- ♦ If $d \leq c$, accept lot; else reject

So, I will just prefer the notations like this capital N when I say it is the lot size; small n is the sample size; c is the acceptance number and d is the number of defective items in sample. So, the rule is very simple d is less than or equal to c , so number of defectives is less than or equal to c , you accept the lot; if it is greater than, reject the lot. So, I have predefined acceptance number, rejection number. And if let us say number of defective d is less than or equal to acceptance number, then I will accept; otherwise I will reject. So, this is how I try to develop and design the sampling plan.

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Sampling Errors

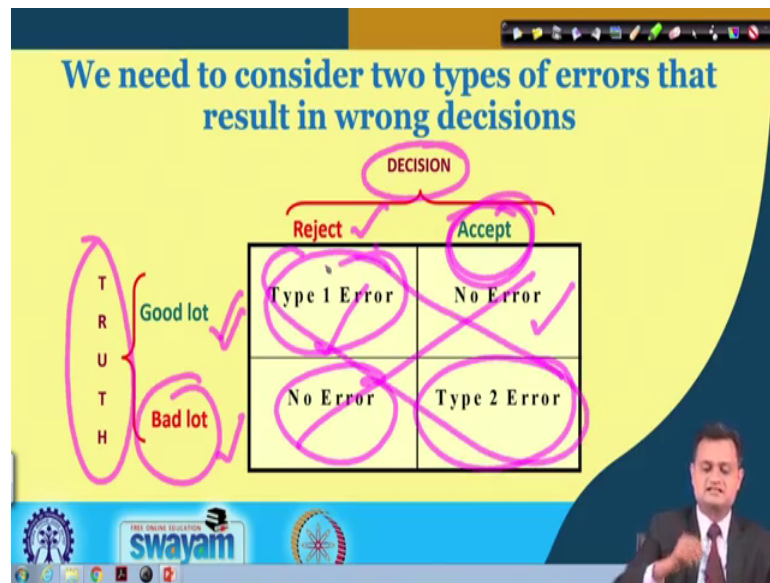
- ♦ Type I (α Error or Producer's Risk)
 - ♦ Occurs when a sample says part are bad or the process is out of control when the opposite is true.
 - ♦ The probability of rejecting good parts as scrap.
- ♦ Type II (β error or Consumer's Risk)
 - ♦ Occurs when a sample says parts are good or the process is in control when the reverse is true.
 - ♦ The probability of a customer getting a bad lot represented as good.

But we need to go deeper in order to appreciate the inclusion of producers risk and consumers risk and design, a sampling plan which has a good discriminatory power meaning to say separating the bad lot from the good lot, and satisfy the requirement, risk acceptance of the producers as well as consumer. So, just try to see it would just be a revision type 1 error we have already discussed. Punishing an innocent percent when he is actually an innocent not guilty is type 1 error and not punishing a guilty is type 2 error.

So, same concept I will use here. Instead of a person, here it is a lot. So, I will say type 1 error or alpha error or produces risk. So, this occurs when my sample says that parts are bad or the process is out of control when the opposite is true. I am rejecting the good quality lot based on sampling, and this is called type 1 error. My producer is at risk. My producer has shift good quality component, but based on a sampling plan, I am rejecting it. So, this type 1 error.

If you see the type 2 error, beta error or consumer risk, whatever you say it is the same thing. In this case typically, I am accepting the bad quality product and say that the bad quality product is of acceptable kind and this is where I as a consumer is at risk. So, we must appreciate producers and consumer risk very well, so that this can be appropriately incorporated in design of sampling plan.

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So, just to see this figure and that will give you the fantastic clear cut idea on producers risk and consumers risk. Just see this is the decision, and this is the truth. So, your decision and truth must align; when they do not align, you make a mistake.

So, let us say I have a good lot and I say my decision is reject, this means it is a producers risk. Producer has put all the effort value addition, gave me the good lot, I am saying it is reject. So, this is my type 1 error. Now, I have a bad lot, and I accept. So, I as a consumer is at risk, and this is called consumers risk for type 2 error. And if you see this diagonal, then my decision and the truth, they are matching. When I say bad lot, I reject; so no error. When I say good lot and I accept; it is no error zone. So, these are the two points which are of my concern and that really creates the problem.

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The slide is titled "Errors and Risks" in blue text. It contains two main bullet points, each with handwritten checkmarks and additional notes:

- TYPE I ERROR = P (reject good lot)
 - α or Producer's risk
 - 5% is common
- TYPE II ERROR = P (accept bad lot)
 - β or Consumer's risk
 - 10% is typical value

A cartoon character with glasses and a red headband is positioned to the right of the text. The bottom of the slide features a blue banner with the "swayam" logo and a small image of a person in a suit.

So, I have to have the say typical tradeoff between my producers risk and consumers risk in order to devise a good discriminatory sampling plan. So, just see type 1 error is probability of reject good lot, and typically this is called producers risk, alpha same notations. And if nothing is given, I will accept 5 percent as a common producer's risk. Similar way type 2 error probability of accepting bad lot, so this is called beta or consumer risk, and 10 percent is a typical value where nothing is given. So, 5 percent as producer risk is the standard value; 10 percent beta consumer risk is the standard value, where nothing is given. And I would like to design develop my acceptance sampling plan with this alpha and beta.

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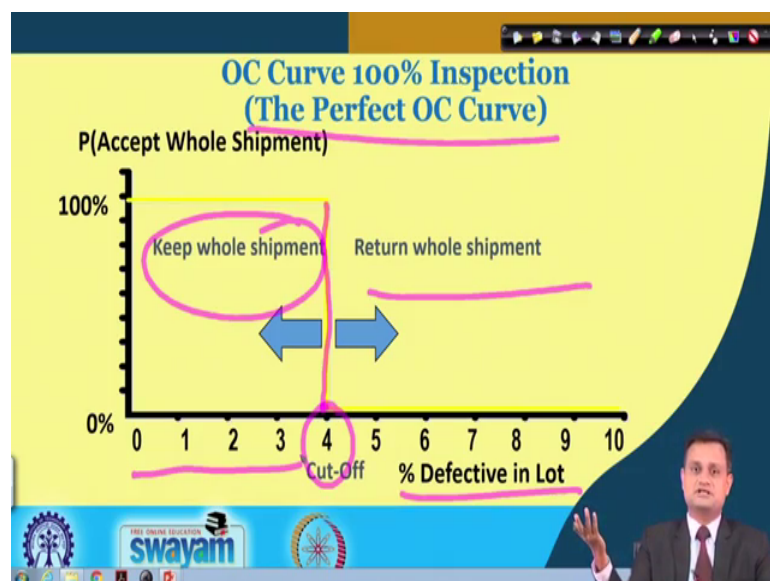
What is Operating Characteristic (OC) Curve?

- ✓ Curves that illustrate graphically the probability of accepting lots that contain different percent defectives
- ✓ Shows how well a sampling plan discriminates between good & bad lots (shipments)

The slide features a yellow background with a large light pink oval containing the text. At the bottom, there are logos for 'swayam' and 'INDIA WIDE, FREE WIDE' along with a small video of a presenter in a suit.

So, now let us move ahead that again when I have to talk about the discriminatory power, I have to talk about the goodness of my control system, I must talk about the operating characteristic curve. And exactly the way we have develop the operating characteristic curve for the control charts, I also need to use this concept of operating characteristic curve in order to check the performance of my say acceptance sampling plan. So, curves that illustrate graphically the probability of accepting the lot that contain different percent defective discriminatory power is basically the operating characteristic curve for acceptance sampling.

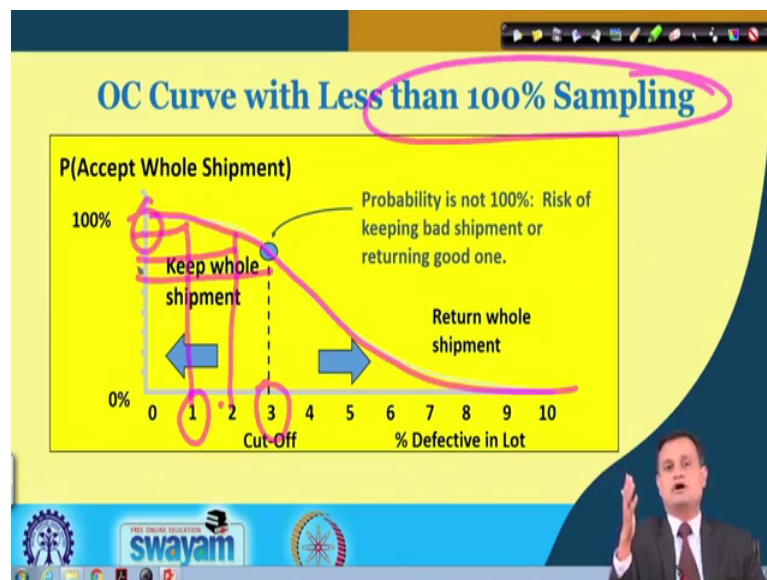
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Now just see this and it would really be interesting to appreciate. Now, let us say I have say cut off point 4, it means anything which is less than 4, I will keep the whole shipment. And 4 is what percentage defective in a lot. Suppose let us say you have a lot of 100 item, out of these 4 items are defective or 4 items are defective. If it falls below 4, I will accept the entire lot. Suppose if it exists this cut off the say threshold point, I will return the whole shipment, and my decision is based on sample I am not doing 100 percent inspection.

Now, do you see the policy with this concept, do you see any policy with this concept? So, this is called perfect OC curve, because there is one line when exactly you reach to this line, it tells you that either you accept or you reject. But do you think that when you move from 0 percent defective to 1 percent to 2 percent to 3 percent your probability of acceptance will remain same. I do not think so. As the percentage defective increases, you must have changing your probability of acceptance which is a quiet say wise and realistically design, operating characteristic curve rather than I go for the perfect OC curve based on 100 percent inspection.

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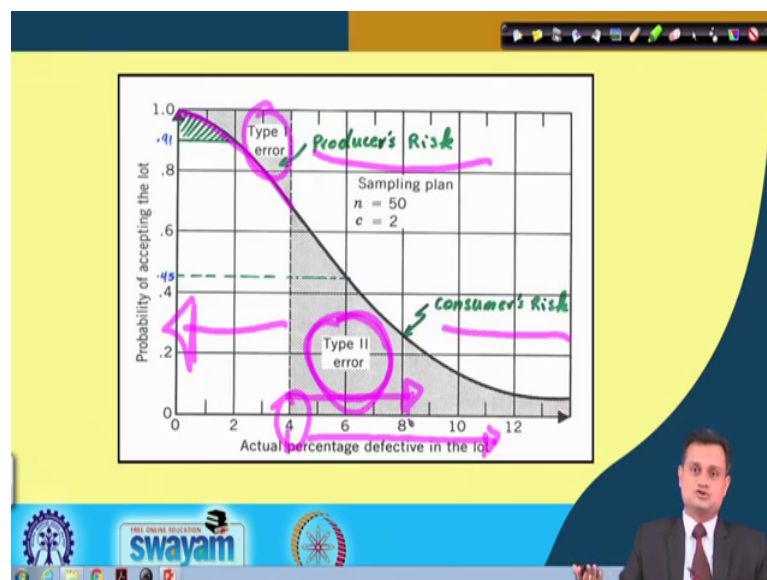


So, now, when, when I cannot operate with these, and I have to go for the sampling, just see this and this would really help you to appreciate, I have a cut off, and I am going for less than 100 percent inspection. And I have a cut off let us say 3 now what do you see here I have the OC curve. And 100 percent means when there is 0 defective in my lot

when I am doing based on sampling, I will accept, 100 percent acceptance. When there is 1 percent the probability changes, and I will be maybe close to 95 percent, where it is to my probability will further reduce it would be maybe 93 percent or 90 percent same way I will have reduced probability.

So, here what you can appreciate that I have reducing probability as the number of defective increases in the lot, and this is where I can really feel happy and realistic about my operating characteristic curve for acceptance sampling plan. So, this is for less than 100 percent sampling.

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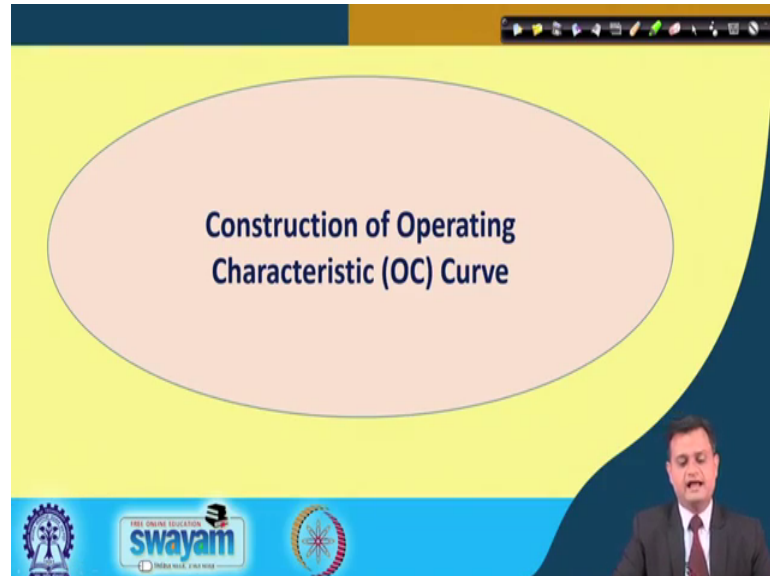


The previous one was for 100 percent inspection. Now, just see this in order to make it more elaborative. Here my cut off is let us say somewhere 4, and now I am writing something here type 1 error, I am writing something here type 2 error. What does it mean? It means that actual percentage defective in lot is let us say 4 percent and then I have a particular region. So, here if I reject the lot, my producer is at risk. So, this is called my producer's risk, because percentage defective is less, then also I reject the lot, then my producer is at risk.

When I go for higher percent defective, and suppose if I accept the lot, then I as a consumer is at risk, and this is typically call type 2 error, and it is my consumers risk. So, when I refer with respect to cut off, then both the party they have to accept some risk. When any lot which is rejected which is less than cutoff, this side, then my producer is at

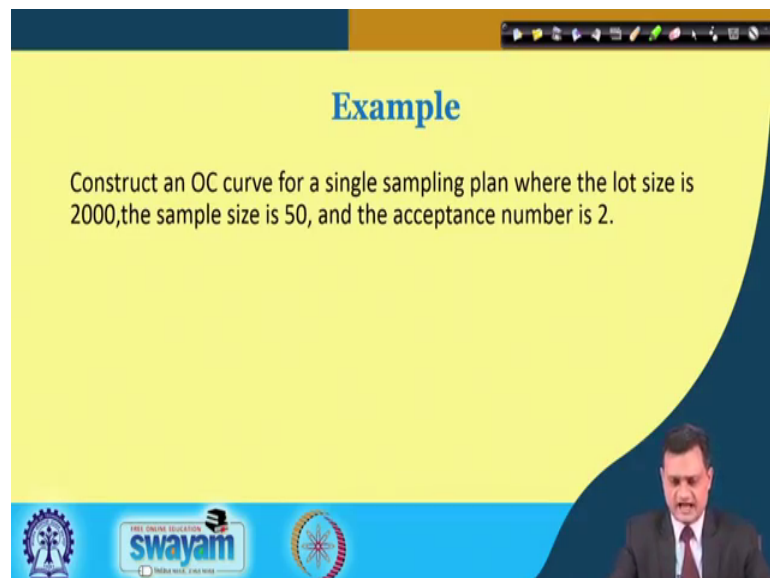
risk. So, producer's risk any lot which is accepted by the consumer which has more percent defective than the cutoff, it is my consumer's risk.

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So, this is exactly where I have to see the trade off. And now let us see how would you like to construct the OC curve for particular acceptance sampling plan.

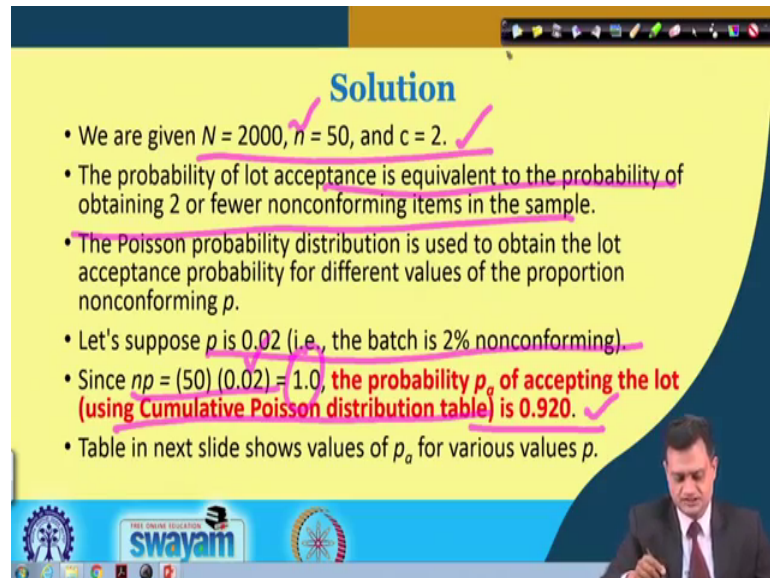
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So, let us take some data. Construct an OC curve for a single sampling plan where the lot size is 2000, this is the complete lot. And from that lot you are taking the sample of 20.

So, there could be many samples. You may take, but the lot is of 2000 and acceptance number is 2.

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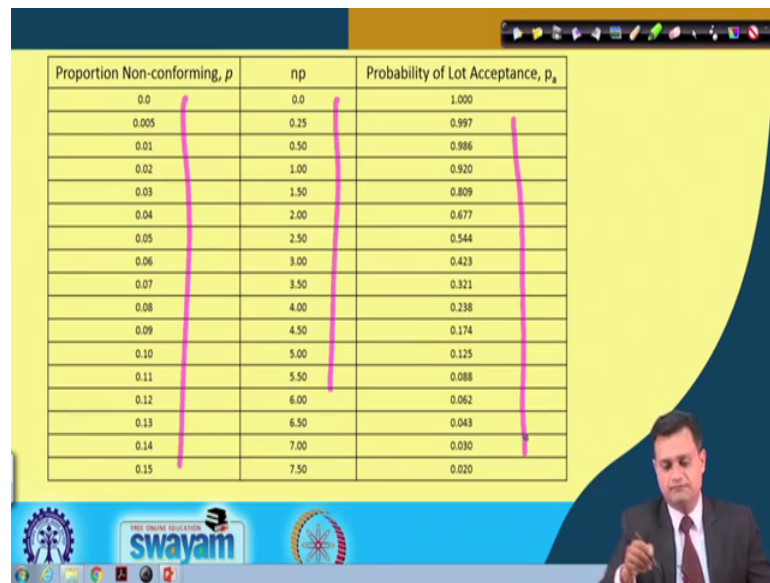
Solution

- We are given $N = 2000$, $n = 50$, and $c = 2$.
- The probability of lot acceptance is equivalent to the probability of obtaining 2 or fewer nonconforming items in the sample.
- The Poisson probability distribution is used to obtain the lot acceptance probability for different values of the proportion nonconforming p .
- Let's suppose p is 0.02 (i.e., the batch is 2% nonconforming).
- Since $np = (50)(0.02) = 1.0$, the probability p_a of accepting the lot (using Cumulative Poisson distribution table) is 0.920.
- Table in next slide shows values of p_a for various values p .

So, now let us see how can I construct the OC curves for this. So, the given data is like this N is equal to 2000, n is equal to 50 and c is equal to 2. So, what I say that let us say my p is 0.02 that batch is 2 percent defective. So, the probability of lot acceptance is equal to the probability of obtaining 2 or fewer nonconforming items and I am assuming my p as 2 percent that is 0.02.

Now, if I just compute np , so your n is let us say 50 that is the sample size, and p is 0.02 when I multiply this I get 1. Now np is 1. So, once again we discussed previously I will refer the cumulative Poisson distribution table which basically relates the value of mean with respect to the value of x or particular number, here my number would be c . So, let us see that for the given acceptance number c my probability will be 0.920, and this probability is called p_a probability of acceptance.

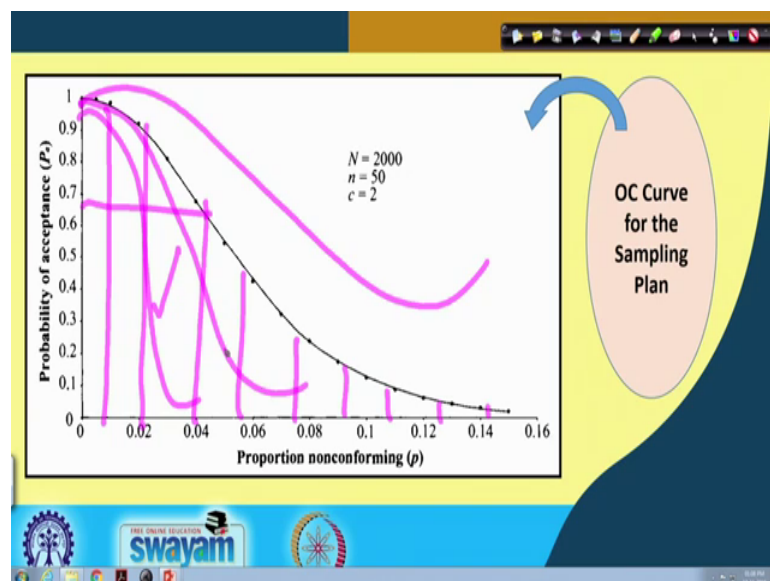
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Proportion Non-conforming, p	np	Probability of Lot Acceptance, p_a
0.0	0.0	1.000
0.005	0.25	0.997
0.01	0.50	0.986
0.02	1.00	0.920
0.03	1.50	0.809
0.04	2.00	0.677
0.05	2.50	0.544
0.06	3.00	0.423
0.07	3.50	0.321
0.08	4.00	0.238
0.09	4.50	0.174
0.10	5.00	0.125
0.11	5.50	0.088
0.12	6.00	0.062
0.13	6.50	0.043
0.14	7.00	0.030
0.15	7.50	0.020

So, now I can just try to calculate the probability in the same way and what do you see here that let us say I am checking the different values of p initially 0, then 0.005, then 0.01, 0.02, it means 2 percent, 3 percent, 4 percent, 5 percent same way my np will change. And for the given np , I can find out my probability of acceptance from the probability poisson probability table. So, once I found this p_a , which is the probability of acceptance, and I have the value of p , then my task would be simple and I will plot the OC curve.

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So, you can see here that on x-axis you have the proportion non conformance. So, 2 percent, 4 percent, 6 percent, 8 percent and so on, and you have the probability of acceptance which you have determined from the cumulative Poisson probability table, in this particular acceptance sampling plan is for n is equal 2000 that is the lot size, small n is equal to 50 that is your sample size, and c is equal to 2 that is the acceptance number.

So, now, what do you see here? You see a curve and this curve has a discriminatory power. So, what exactly you say that when you have 0.04, when you have 0.04 means four percent defective in your lot, proportion nonconforming or proportion defective then what is the probability of acceptance. So, you can very well see that when I have less percentage of defective or the nonconforming proportion my probability of acceptance is high my probability of acceptance is high gradually. My probability of acceptance is reducing which is obviously a desired feature of my operating characteristic curve.

But obviously, you cannot have same shape, you may think about a shape like this, you may think about a shape like this, you may think about a shape like this. And if you go mores cheaper, then your discriminatory power increases, so it all depends that what is your quality characteristic and what could be the consequences of accepting a bad quality part or rejecting a good quality lot, depending upon that you need the discriminatory power of your acceptance sampling plan.

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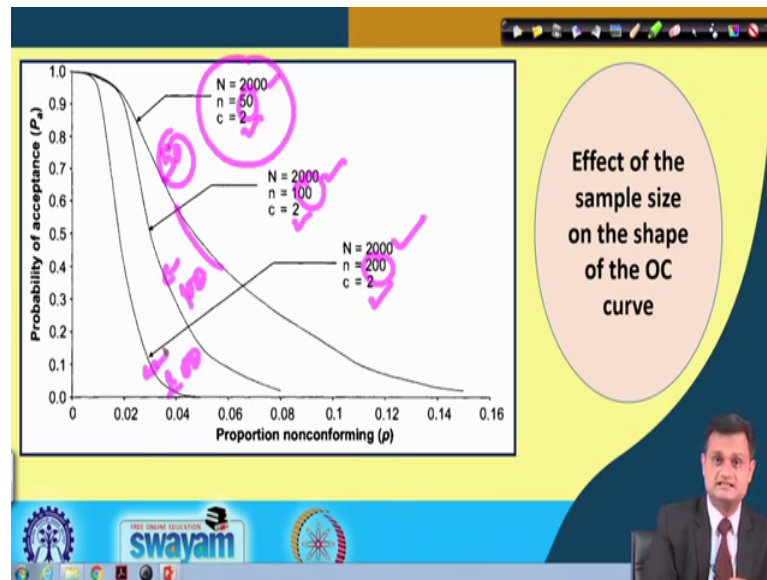
The slide is titled "Effect of the Sample Size and Acceptance Number" in blue text on a yellow background. It contains three bullet points: "The parameters n and c of the sampling plan affect the shape of the OC curve.", "As long as the lot size N is significantly large compared to the sample size n , the lot size does not have an appreciable impact on the shape of OC curve.", and "For fixed values of N and c , as the sample size becomes larger, the slope of the OC curve becomes steeper, implying a greater discriminatory power." The slide is part of a presentation, as evidenced by the navigation icons at the top and the presenter's video feed in the bottom right corner. The bottom of the slide features logos for "swayam" and other educational institutions.

Effect of the Sample Size and Acceptance Number

- The parameters n and c of the sampling plan affect the shape of the OC curve.
- As long as the lot size N is significantly large compared to the sample size n , the lot size does not have an appreciable impact on the shape of OC curve.
- For fixed values of N and c , as the sample size becomes larger, the slope of the OC curve becomes steeper, implying a greater discriminatory power.

So, once you have appreciated this, then you can also see the effect of sample size and acceptance number. I can change my sample size, I can change my acceptance number, and I can just try to say that what is its impact on the discriminatory power it means slope of my OC curve.

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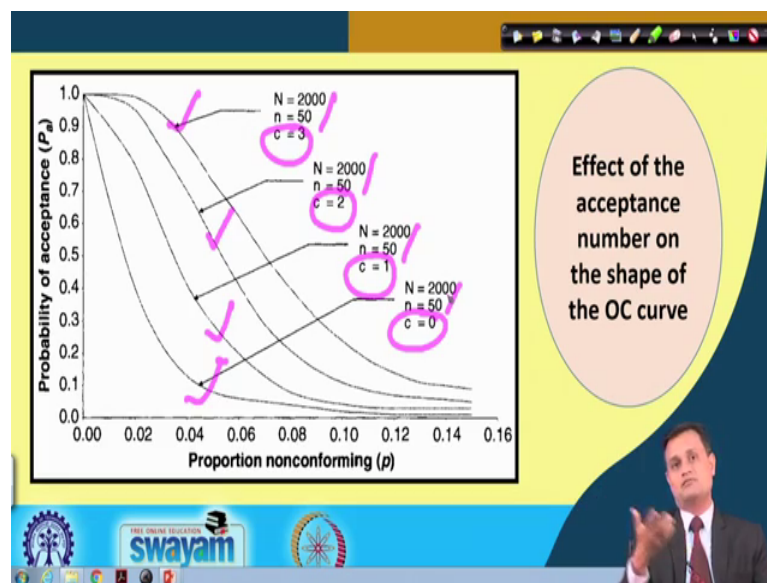


So, just see this and the idea will be extremely clear. So, you have a sampling plan where N is equal to 2000, small n is 50, c is equal to 2. So, I have a lot of 2000, I have a sample size 50, I have acceptance number 2, you just see what I am doing I am just keeping this values same, I am just keeping this value same. So, capital N and c , I am keeping constant, I am just changing the value of sample size, and you will see that for 50 sample size it is this one. For 100, it is this one; and for 200, it is this one. It means if I just draw a line here, then for same percentage proportion nonconforming if my OC curve is highly discriminatory which is this one having let us say higher sample size, then probability of acceptance is very low.

Probability of acceptance is little higher and probability of acceptance is very high. So, it all depends what is your quality characteristic, the kind of product you are dealing with. Suppose you are making a very precise component, purchasing it from a supplier. Typically I visited HAL - Hindustan Aeronautics Limited, and they have many indigenous suppliers as well as they have many foreign suppliers for the purchase of highly precise components for aircrafts. Now, you can see that the aircraft safety is

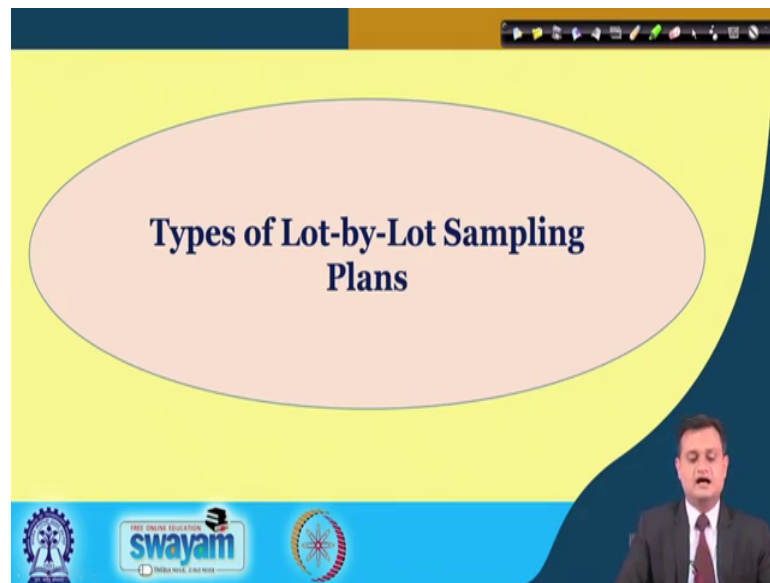
something which is of highest order and you cannot compromise with the quality. But if you think about some nuts and bolts used in some ordinary manufacturing company, then you may not need that much of discriminatory power of the OC curve. But if you talk about aircraft then you may like to go for little higher discriminatory power of the OC curve. So, I just want to show you that there is the impact of the sample size on the discriminatory power of the OC curve; obviously, my cost of inspection will increase. So, this is something that you can appreciate.

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Now, just see what I am doing effect of the acceptance number on the step of the OC curve. I am just changing the acceptance number 3, this is the one I am tightening it to greater discriminatory power, 1 greater discriminatory power, 0 greater discriminatory power. These values are kept constant, so that I can see the impact of c on change in c . So, as I am reducing the acceptance number, I am becoming developing more and more tighter acceptance sampling plan, the discriminatory power increases, obviously, this will have an impact on my producers risk as well as consumers risk. So, this is something that we can appreciate.

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Now, if you see the types of lot by lot sampling plan, and what is the variety.

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Then you have single sampling plan, double sampling plan and multiple sampling plan.

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Single Sampling plan

- ✓ The information obtained from one sample is used to make a decision to accept or reject a lot.
- ✓ There are two parameters in this sampling plan: the sample size n and the acceptance number c .

Single sampling plan, I will take the decision based on a single sample. And information obtained from one sample is used to make the decision about accept and reject. And typically your sampling plan will have sample size n and the acceptance number c .

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Double Sampling plan

- ✓ It involves making a decision to accept the lot, reject the lot, or take a second sample.
- ✓ If the inference from the first sample is that the lot quality is quite good, the lot is accepted.
- ✓ If the inference is poor lot quality, the lot is rejected.
- ✓ If the first sample gives an inference of neither good nor poor quality, a second sample is taken.
- ✓ Thereafter, based on the combined number of nonconforming items or nonconformities in both samples, a decision is made to accept or reject the lot.

If I go to double sampling plan, I will check the decision in the first go if my number of defective parts are less than the acceptance number, fine, it is a good quality lot, I will accept it. But if not I will not stop that I will check the another sample, and then I will check the cumulative number of defective, both sample 1 and sample 2 to compare it

with the say acceptance number for the second one, second sample, and then I will take the decision.

Multiple sampling plan, you can extend your double sampling plan keep taking. So, basically you will have c_1 and small n_1 you can c_2 and small n_2 or if you keep n same, then you will have c_1, c_2, c_3, c_4 , and you can just extend your double sampling plan to see that what decision you can take appropriately about the acceptance or the reactions of the lot. So, I am just giving you the introductory idea in detail we will see the design of single and double sampling plan in the subsequent lectures.

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Advantages and Disadvantages of Various Sampling Plans

- ✓ As far as simplicity is concerned, the single sampling plan is the best, followed by double and then multiple sampling plans.
- ✓ Administrative costs for record keeping, training, and inspection are the least for single and the highest for multiple sampling plans.
- ✓ On average, for equivalent plans, the number of items inspected to make a decision regarding the lot is usually more for a single sampling plan. This is because double and multiple sampling plans use fewer items in their samples, so if the lots are of very good or poor quality, a decision to accept or reject them is made quickly.
- ✓ Inspection costs will therefore be the most for single, and the least for multiple sampling plans.

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So, now there are obviously, some advantages and disadvantages with the plans. Single sampling plan are very simple, and maybe the record keeping, training and other thing is very less which is quite high in double sampling and multiple sampling. But on an average the for equivalent plan means the discriminatory power, the number of items inspected to make a decision regarding the lot is usually more for a single sampling plan. This is because double and multi sampling plan, use fewer items in their sample, if the lots are very good or poor, so obviously, single sampling plan my inspection cost will be little bit higher when I take only single sample which is of a larger size.

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Average Outgoing Quality (AOQ)

- The average outgoing quality (AOQ) is the average quality level of a series of batches that leave the inspection station, assuming rectifying inspection, after coming in for inspection at a certain quality level p .
- The AOQ is not the quality level of a single batch that leaves the inspection station.

$$AOQ = \frac{P_a p (N - n)}{N}$$

- Where N – Lot Size, n – sample size, p – incoming lot quality, P_a – the probability of accepting the lot using the given sampling plan.

Evaluating sampling plans this is an important thing. So, we will try to see couple of performance measures number 1, which is very important is average outgoing quality. So, please try to appreciate this properly, average outgoing quality is defined as the average outgoing quality is the average quality level of a series of batches, I am taking series of batches, please emphasize on this, series of batches that leaves the inspection station assuming rectifying inspection. It means suppose a particular batch is rejected then rectification means I will replace the bad quality product with the new one, and then only I will allow it to go to the next stage.

So, after coming in for inspection at certain quality level that is percentage defective proportion defective. So, AOQ is defined as please refer it carefully P a probability of acceptance, p is your incoming lot quality proportion nonconforming, capital N is my lot size, small n is simple divided by N .

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The AOQ measures the average quality level of a large number of such batches of incoming quality p , the proportion nonconforming in the lots, assuming rectification.

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So, this is a very important definition, average outgoing quality, it incorporates to remind you the concept of rectification. So, AOQ measures the average quality level of a large number of such batches not one batch, large number of series of series batches in your such batches is of incoming quality with p , proportion nonconforming in the lots, assuming rectification.

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Example

Construct the AOQ curve for the sampling plan $N = 2000$, $n = 50$, $c = 2$

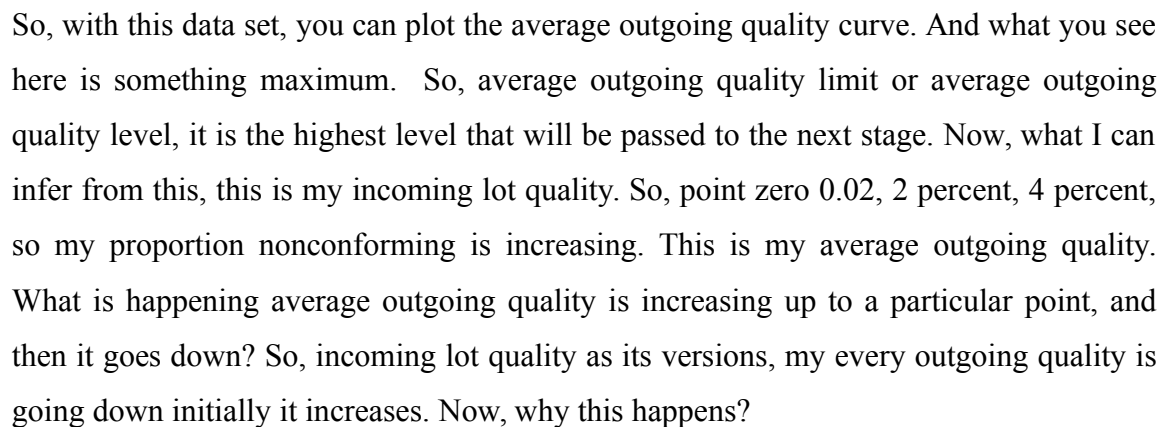
Solution

Incoming Lot Quality, p	Probability of Lot Acceptance, P_a	Average Outgoing Quality, AOQ
0.01	0.986	0.0096
0.02	0.920	0.0179
0.03	0.809	0.0237
0.04	0.677	0.0264
0.05	0.544	0.0265
0.06	0.423	0.0247
0.07	0.321	0.0219
0.08	0.238	0.0186
0.09	0.174	0.0153
0.10	0.125	0.0122
0.11	0.088	0.0094
0.12	0.062	0.0073
0.13	0.046	0.0055
0.14	0.030	0.0041
0.15	0.020	0.0029

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So, now, if we just do a little bit computation, then I have let us say incoming lot quality 0.01, 0.02, 1 percent, 2 percent and so on, and I can compute P a probability of

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Note

- ✓ When the incoming quality is very good, the average outgoing quality is also very good.
- ✓ When the incoming quality is very poor, the average outgoing quality is good because most of the lots are rejected by the sampling plan and go through screening. In between these extremes, the AOQ curve reaches a maximum, AOQL.

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So, it is interesting to see that when the incoming quality is very good, fantastic, means your proportion nonconforming is less the average outgoing quality is also very good that is obvious. Now, consider the concept of rectification, and what happens that when the incoming quality is poor the average outgoing quality is good because most of the lots are rejected and through a sampling plan go through screening. In between and you will have something like AOQL between these two extremes which is the maximum limit of AOQ average outgoing quality.

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Average Outgoing Quality Limit (AOQL)

- The *average outgoing quality limit* (AOQL) is the maximum value, or peak, of the AOQ curve.
- It represents the worst average quality that would leave the inspection station, assuming rectification, regardless of the incoming lot quality.
- The AOQL value is also a measure of goodness of a sampling plan.
- Note that the protection offered by the sampling plan, in terms of the AOQL value, does not apply to individual lots. It holds for the average quality of a series of batches.

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So, typically I can define this maximum limit AOQL is average outgoing quality limit is the maximum value or peak of the AOQ curve and typically it represents the worst average quality, please remember it is the worst average quality that would leave the inspection station, then after when the lots have started rejecting, I am applying the same rectification and replacing the bad quality products with the good one. So, my curve is going down, but AOQL is the peak which is the worst average quality that would leave the inspection assuming rectification, regardless of the incoming quality of the lot. So, this is something that we can appreciate that protection offered by the sampling plan, in terms of AOQL value does not applied to individual lots. It holds the average quality of a series of batches. So, this part we much appreciate.

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Average Total Inspection (ATI)

- The ATI represents the average number of items inspected per lot.
- If a lot has no nonconforming items, it will obviously be accepted by the chosen sampling plan, and only n items (the sample size) will be inspected for a lot.
- At the other extreme, if the lot has 100% nonconforming items, the number inspected per lot will be N (the lot size) assuming that rejected lots are screened.
- For a lot quality between these extremes, the average amount inspected per lot will vary between these two values.
- For single sampling plans, the average total inspection per lot for lots with an incoming quality level p is given by

$$ATI = n + (1 + P_a)(N - n)$$
- For a double sampling plan, the ATI is given by

$$ATI = n_1 P_{a1} + (n_1 + n_2) P_{a2} + N(1 - P_{a1} - P_{a2})$$

Now, there is another performance measures for my operating characteristic curve or acceptance sampling plan typically and this is called average total inspection. So, average number of items to be inspected for lot, and you can put it like this that suppose I have sample size n , then n is inspected plus 1 plus P_a n minus n . So, n is your total lot minus n which is already respected inspected n plus P_a is the probability of acceptance. So, I can expand it for the double sampling also, and I can have $n_1 P_{a1} + n_1 + n_2 P_{a2} + N(1 - P_{a1} - P_{a2})$ and then capital N 1 minus P_{a1} minus P_{a2} , it is a double sampling plan.

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Example

Construct the ATI curve for the sampling plan where $N = 2000$, $n = 50$, $c = 2$.

Solution

Consider the calculations for a given value of the lot quality p of 0.02.
The probability of accepting such a lot using the sampling plan is $P_a = 0.920$.
The ATI for this value of p is
$$ATI = 50 + (1 - 0.920)(2000 - 50) = 206$$

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So, when you analyze this data, let us say you have N is equal to 2000, n is equal to 50 and c is equal to 2, I can just plug in the values in my equation. So, 50 plus 1 minus 0.920, P_a is 0.920, 200 minus 50.

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Example

Construct the ATI curve for the sampling plan $N = 2000$, $n = 50$, $c = 2$

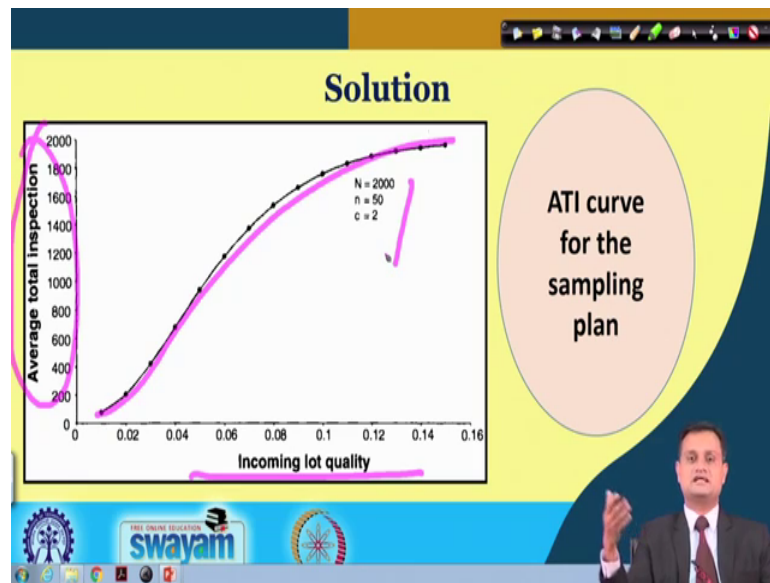
Solution

Incoming Lot Quality, p	Probability of Lot Acceptance, P_a	Average Total Inspection, ATI
0.01	0.986	77.30
0.02	0.920	206.00
0.03	0.809	422.45
0.04	0.677	679.85
0.05	0.544	939.20
0.06	0.423	1175.15
0.07	0.321	1374.05
0.08	0.238	1535.90
0.09	0.174	1660.70
0.10	0.125	1756.25
0.11	0.088	1828.40
0.12	0.062	1879.10
0.13	0.046	1916.15
0.14	0.030	1941.50
0.15	0.020	1961.00

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So, I can plug in these values here. And what I can found that I can compute the series of values of ATI for the incoming quality and probability of acceptance.

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So, when you do this, when you do this, you will have a typical curve which is called average total inspection. And what you can see here that my average total inspection increases for a given N is equal to 2000, n is equal to 50 and c is equal to 2 as my incoming lot quality is becoming bad. So, fine, it is obvious, but again you can choose the appropriate level of inspection depending upon the criticality of the quality characteristic and the constituencies.

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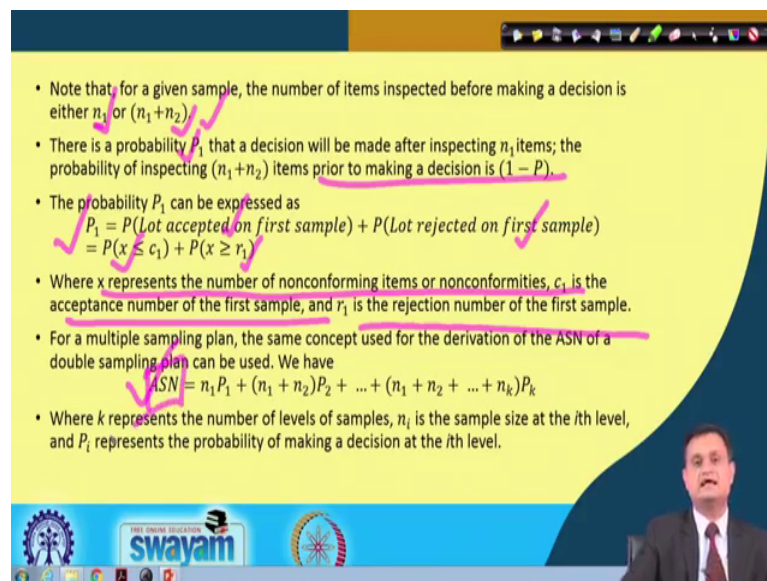
Average Sample Number (ASN)

- ✓ The average number of items inspected for a series of lots with a given incoming lot quality in order to make a decision is known as the *average sample number* (ASN).
- Assume that inspection is not curtailed for a single sampling plan when making a decision.
- For example, if 3 nonconforming items are found by the twentieth unit when using a single sampling plan $N = 800$, $n = 60$, $c = 2$, even though a decision can be made after the twentieth unit to reject the lot, inspection continues for all 60 items in the sample.
- Under this assumption, the average sample number for a single sampling plan is equal to the sample size n .
- For a double sampling plan, the ASN is given by
$$ASN = n_1 P_1 + (n_1 + n_2)(1 - P_1) = n_1 + n_2(1 - P_1)$$
Where P_1 is the probability of making a decision on the first sample.

So, this is what we can talk about ATI, there is another measure which is called ASN. So, ASN is basically average sample number. This refers to average number of items inspected for a series of lots with a given incoming lot quality to order to make a decision is known as the average sample number. You need to take the decision whether it is of good quality or bad quality. So, it is the average number of item inspected for a series of lots with a given incoming lot quality in order to make the decision.

So, suppose I have 3 nonconforming items are found by twentieth unit when using a single sampling plan, when you have N is equal to 800, n is equal to 50 and c is equal to 2, what will you do? n is equal to 3, so you will say that you can take the decision on twentieth because it is exceeding your c is equal to 2, but you will not stop here, you will continue after the twentieth unit to reject, you will continue your inspection up to 60. So, I want to understand that, under these assumptions the average sample number for a single sampling plan is equal to the sample size n . So, ASN in case of double sampling, you can just expand it and you will get the value of ASN.

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- Note that, for a given sample, the number of items inspected before making a decision is either n_1 or $(n_1 + n_2)$.
- There is a probability P_1 that a decision will be made after inspecting n_1 items; the probability of inspecting $(n_1 + n_2)$ items prior to making a decision is $(1 - P_1)$.
- The probability P_1 can be expressed as

$$P_1 = P(\text{Lot accepted on first sample}) + P(\text{Lot rejected on first sample})$$

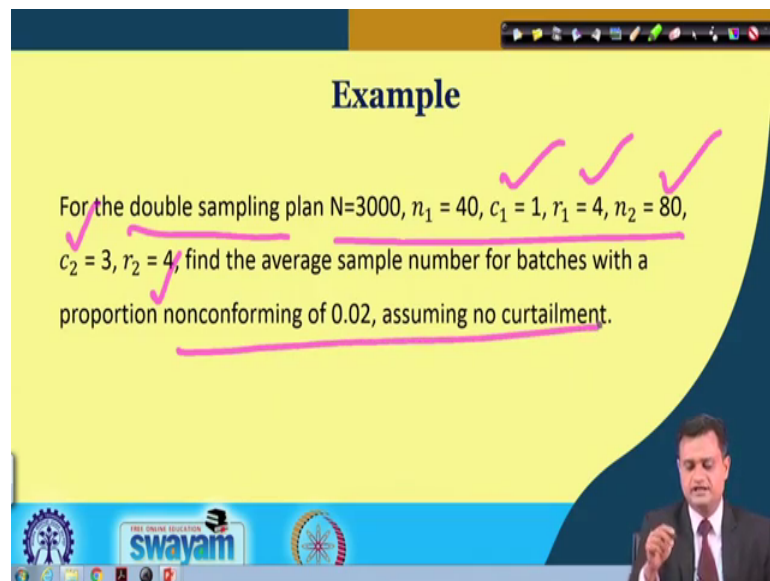
$$= P(x \leq c_1) + P(x \geq r_1)$$
- Where x represents the number of nonconforming items or nonconformities, c_1 is the acceptance number of the first sample, and r_1 is the rejection number of the first sample.
- For a multiple sampling plan, the same concept used for the derivation of the ASN of a double sampling plan can be used. We have

$$ASN = n_1 P_1 + (n_1 + n_2) P_2 + \dots + (n_1 + n_2 + \dots + n_k) P_k$$
- Where k represents the number of levels of samples, n_i is the sample size at the i th level, and P_i represents the probability of making a decision at the i th level.

So, now if we just try to do little bit analysis, then it is not that difficult, suppose let us say the number of item inspected before making a decision ASN 1 or n_1 plus n_2 . Now P_1 is the probability that a decision will be made after n_1 items are inspected, and the probability of inspecting n_1 plus n_2 is say items prior making decision is 1 minus P_1 .

Now, if I find the probability P_1 that is equal to probability of lot accepted on first sample plus probability lot rejected on the first sample, so both I can consider c_1 is the acceptance number, r_1 is the rejection, x is less than or equal to c_1 that is the probability specific to lot accepted on first sample; x is greater than or equal to r_1 lot rejected on the first sample. When I do this x represents the number of nonconforming items, c_1 is acceptance number and r_1 is the rejection number, you can also do it for the multiple sampling.

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Example

For the double sampling plan $N=3000$, $n_1 = 40$, $c_1 = 1$, $r_1 = 4$, $n_2 = 80$, $c_2 = 3$, $r_2 = 4$, find the average sample number for batches with a proportion nonconforming of 0.02, assuming no curtailment.

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So, basically my purpose here is to find the ASN - average sample number, and if I just do the analysis for this particular sampling plan, where N is equal to 300 I am talking about the double sampling, c_1 is equal to 1, r_1 is equal to 4, n_2 is equal to 80, c_2 is equal to 3, and r_2 is equal to 4. So, you can see that you have c_1 , r_1 , n_1 , c_2 , r_2 and n_2 , this is my double sampling plan. And proportion nonconforming is 0.02 assuming low curtailment I will continue. Even if I can detect that my lot or my particular sample is of reject level.

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Solution

First, calculate P_1 , the probability of making a decision after the first sample:

$$P_1 = P(x \leq c_1) + P(x \geq r_1)$$

- where x represents the observed number of nonconforming items. From the cumulative Poisson tables, we get

$$\begin{aligned}
 P_1 &= [x \leq 1 | n_1 p = 40(0.02)] + [x \geq 4 | n_1 p = 40(0.02)] \\
 &= [x \leq 1 | n_1 p = 0.8] + [x \geq 4 | n_1 p = 0.8] \\
 &= 0.809 + (1 - 0.991) = 0.818
 \end{aligned}$$

The average sample number for batches with a proportion nonconforming of 0.02 is

$$ASN = n_1 + n_2(1 - P_1) = 40 + 80(1 - 0.818) = 54.56$$

So, I will apply this P_1 is equal to probability $x \leq c_1$ plus probability $x \geq r_1$, I can plug in the values for $n_1 p$ this $n_1 p$ this, and you can again refer the q_1 cumulative Poisson table where you can refer the mean as well as the number of x and you will get the values of probability. So, you can put these values and you will find that ASN is 54.56.

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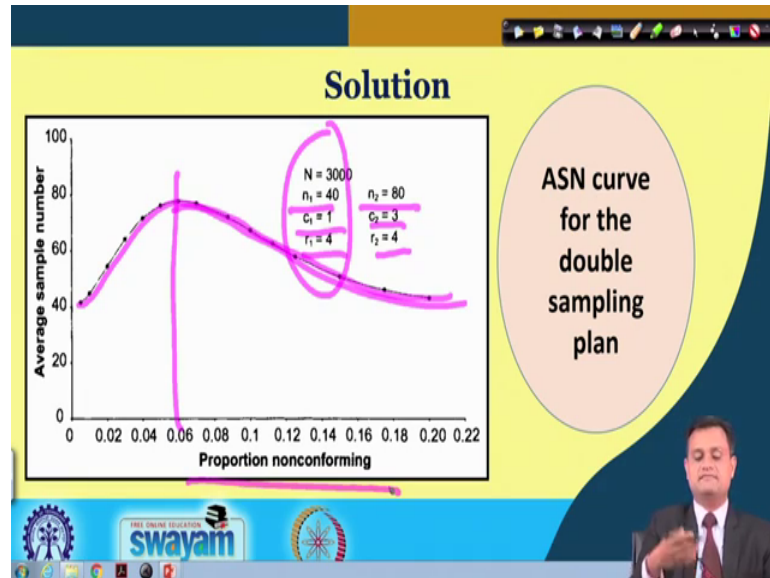
Incoming Lot Quality, p	Probability of Lot Acceptance, P_a	Average Sample Number, ASN
0.01	0.986	41.44
0.02	0.920	44.88
0.03	0.809	54.56
0.04	0.677	64.24
0.05	0.544	71.68
0.06	0.423	76.08
0.07	0.321	77.68
0.08	0.238	76.88
0.09	0.174	72.08
0.10	0.125	67.28
0.11	0.088	62.48
0.12	0.062	58.00
0.13	0.046	50.72
0.14	0.030	46.00
0.15	0.020	43.12

Solution

So, with this understanding I can just extend my calculation for multiple points. And what you can do here that your incoming lot quality p , 2 percent, 3 percent, 4 percent, 5

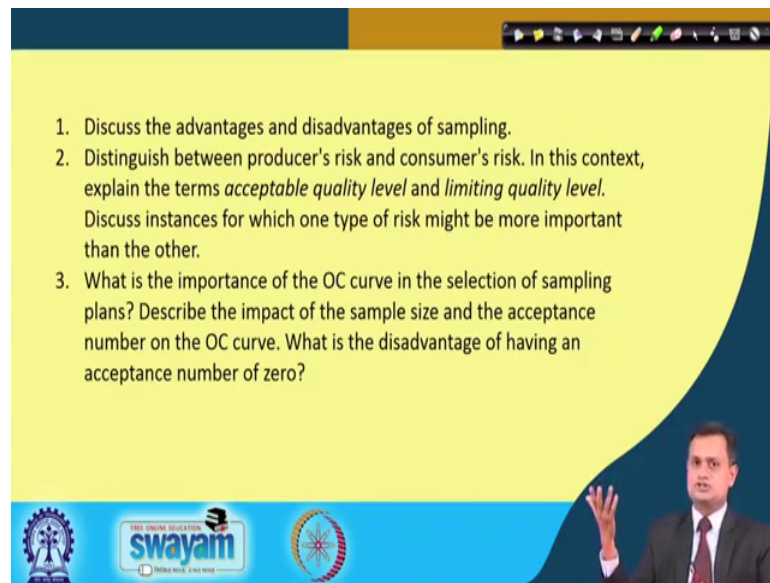
percent, you have probability of acceptance that you can find and you have average sample number. So, once you have the average sample number, you can plot this on a graph, and you would get something like this.

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So, what does it mean that N is equal to 3000, n_1 is equal to 40, second to 80, 1, 3, 4, 4. So, I have the average sample number to be inspected in this fashion. So, initially when my proportion of nonconforming increases up to this point, average sample number is increasing, then my average sample number to be inspected is gradually going down with proportion of nonconforming maybe more number of lots are rejected, and that could be the reason that my average sample number is going down.

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1. Discuss the advantages and disadvantages of sampling.

2. Distinguish between producer's risk and consumer's risk. In this context, explain the terms *acceptable quality level* and *limiting quality level*. Discuss instances for which one type of risk might be more important than the other.

3. What is the importance of the OC curve in the selection of sampling plans? Describe the impact of the sample size and the acceptance number on the OC curve. What is the disadvantage of having an acceptance number of zero?

So, before I end let me float couple of think it. Discuss the advantages and disadvantages of the sampling, distinguish between producer's risk and consumer's risk, and try to explain it with some suitable example. What is the importance of OC curve and describe the impact of sample size and acceptance number on the OC curve. So, try to answer these questions that will help you to have a good internalization of the concept and better clarity on the concepts covered.

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References

- Montgomery, D C. Design and Analysis of Experiments, Wiley.
- Mitra, Amitava. Fundamentals of Quality Control and Improvement, Wiley India Pvt Ltd.

So, these are the references I have used for this particular lecture material. You can also refer it.

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Conclusion

- ❖ Acceptance sampling can be performed during inspection of incoming raw materials, components, and assemblies, in various phases of in-process operations, or during final product inspection.
- ❖ Note that acceptance sampling does not control or improve the quality level of the process.
- ❖ Curves that illustrate graphically the probability of accepting lots that contain different percent defectives.

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So, acceptance sampling is a branch of your statistical quality control and extremely useful in deciding the discriminatory power of my acceptance sampling. When I do not want to go with 100 percent, I want to go with the sampling procedure, acceptance sampling gives me a confidence by designing an appropriate sampling plan in my decision of accepting or rejecting the lot.

So, thank you very much for your interest in learning the concept on acceptance sampling, the basics we have discussed. Keep revising, try to introspect, solve with some hypothetical data as I recommend, and practice the concepts covered in various sessions for your better introspection. So, be with me, enjoy.