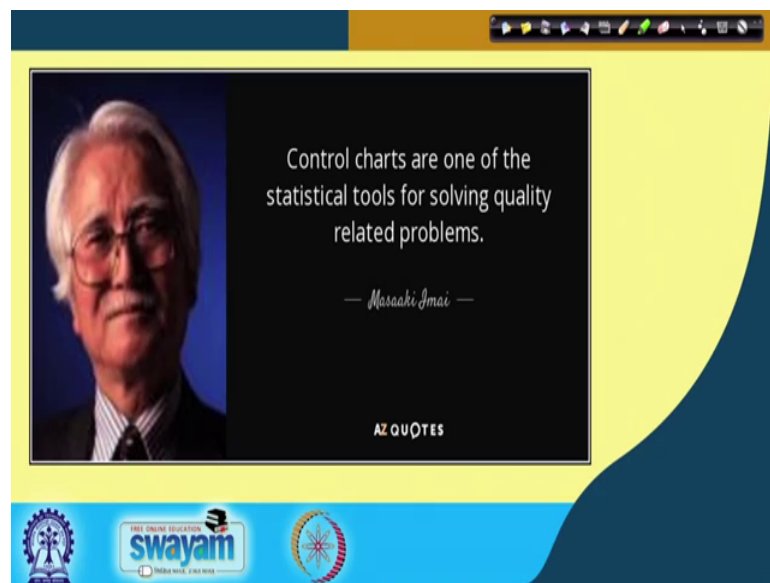


Six Sigma
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Lecture - 51
Statistical Process Control: Control Charts for Attributes

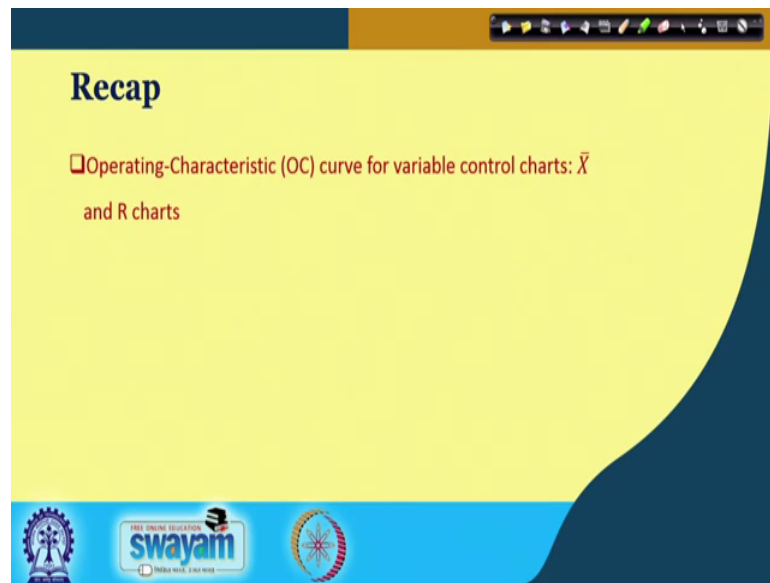
Hello, friends. I once again welcome you to lecture 51 that is Statistical Process Control: Control Chart for Attributes. I would like to remind you that we are in the final phase of our DMAIC cycle of Six Sigma and we are discussing the various topics in the control phase of our six sigma cycle. So, today we will see lecture 51 that is statistical process control chart for attributes we already had a detailed discussion on control charts for variables and then operating characteristic of for variables.

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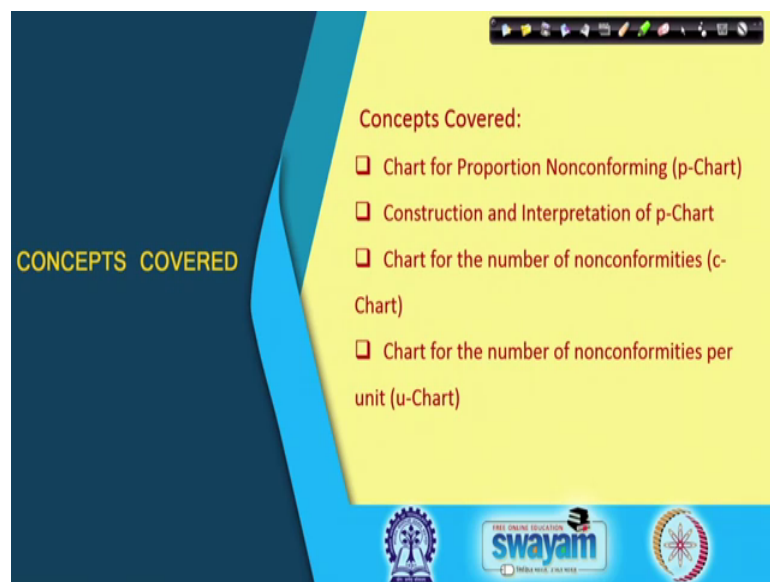
Now, we will see the control chart for attributes. So, let us appreciate this statement again that control charts are seen as the most important tool for solving the quality problem through a diagnostic approach and we can really figure out that what is going wrong and where and we can check the corrective action timely.

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So, if you see we have talked about the operating characteristic curve and basically it indicates the goodness of my control chart ability to detect when the change in the process is taking place and we had seen the OC curve for my $\bar{X}$ and R charts.

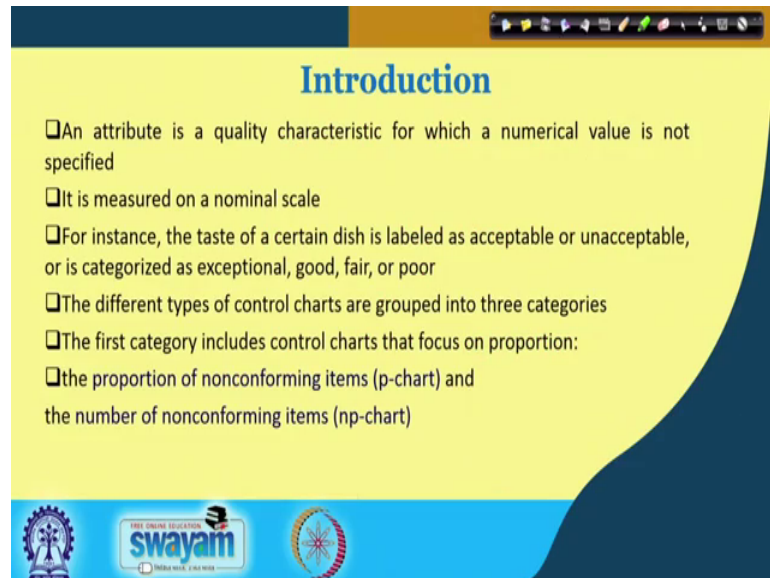
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Now, this particular lecture 51, we would like to see the design and construction of charts for attributes typically p-Chart that stands for Proportion Nonconforming, interpretation of this chart, then number of nonconforming that is c-Chart and chart for number of

nonconforming per unit u-Chart. So, we will try to appreciate this concept as a part of this lecture.

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Introduction

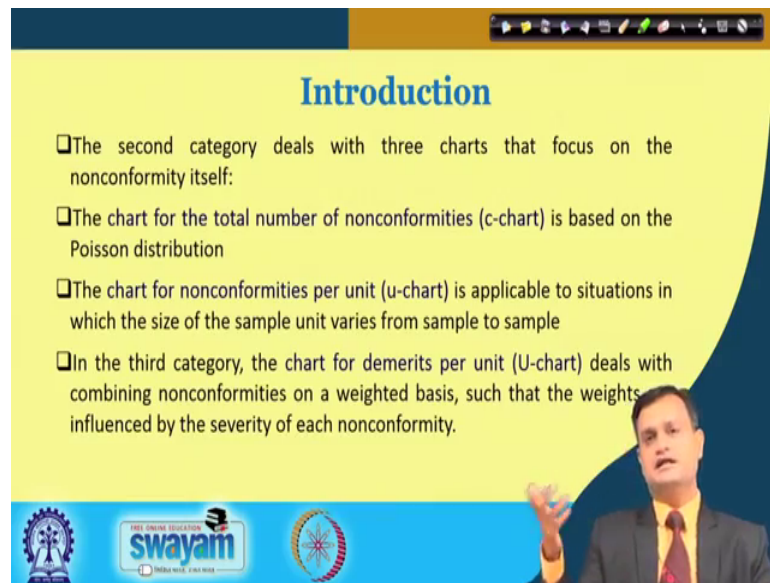
- ❑ An attribute is a quality characteristic for which a numerical value is not specified
- ❑ It is measured on a nominal scale
- ❑ For instance, the taste of a certain dish is labeled as acceptable or unacceptable, or is categorized as exceptional, good, fair, or poor
- ❑ The different types of control charts are grouped into three categories
- ❑ The first category includes control charts that focus on proportion:
 - ❑ the proportion of nonconforming items (p-chart) and
 - ❑ the number of nonconforming items (np-chart)

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Now, just to remind you we have two different types of quality characteristic; one is variable, other is the attribute and the quality characteristics which is called attribute we do not say define the numerical value for this, but usually say it is measured on a nominal scale and you say good, fair, bad or poor and something like this you make a conclusion.

So, basically we have the different types of control chart. We can group it into broadly three categories. Number 1 is proportion of non conforming items p-chart also we can say number of non conforming items if I multiply p by n, so, then it becomes the np-chart.

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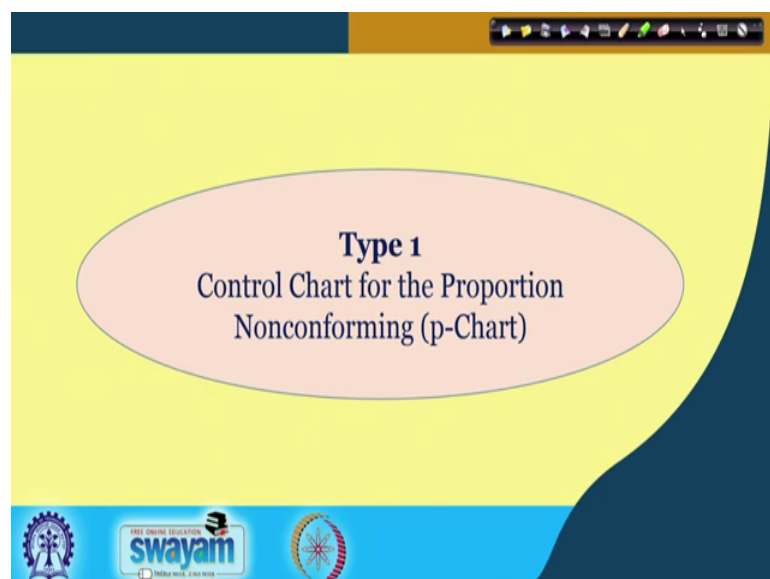
Introduction

- ❑ The second category deals with three charts that focus on the nonconformity itself:
- ❑ The chart for the total number of nonconformities (c-chart) is based on the Poisson distribution
- ❑ The chart for nonconformities per unit (u-chart) is applicable to situations in which the size of the sample unit varies from sample to sample
- ❑ In the third category, the chart for demerits per unit (U-chart) deals with combining nonconformities on a weighted basis, such that the weights are influenced by the severity of each nonconformity.

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Then, another variety I have second category that is the c-chart and this is based on Poisson distribution. So, number of nonconformities. So, typically the chart for nonconformities per unit; so, that is your u-chart is typically applicable to the situation where size of the sample unit varies from sample to sample. So, suppose you collected 25 samples, then the first sample maybe having size 4, second sample 5, third sample say 3 fourth sample may be 7 and likewise. So, we have broadly 3 categories of chart p and np, c-chart and u-chart. So, we will try to see the construction of this various charts applied to attribute kind of quality characteristic and how this can be constructed.

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Type 1
Control Chart for the Proportion
Nonconforming (p-Chart)

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So, let us see type one that is control chart for the proportion nonconforming.

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Chart for Proportion Nonconforming (p-Chart)

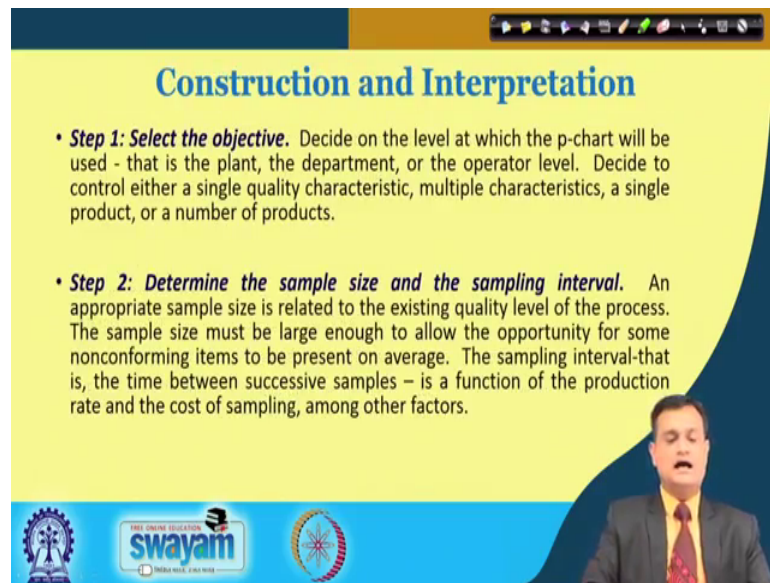
- ❑ A chart for the proportion of nonconforming items (p-chart) is based on a binomial distribution
- ❑ For a given sample, the proportion nonconforming is defined as
$$\hat{p} = \frac{X}{n}$$
where, x = number of nonconforming items in the sample
 n = sample size
- ❑ A p-chart is one of the most versatile control charts
- ❑ A p-chart can be used as a measure of the performance of top management
- ❑ For instance, the management of the chief executive officer of a company can be evaluated by considering the proportion of nonconforming items produced at the company level
- ❑ A p-chart can provide a source of information for improving product quality

UGC swayam

So, the key word is p proportion nonconforming and what I can do here is that for a given sample the proportion of nonconforming can be defined as p hat X by n. So, you have X number of non conforming items in the sample. Suppose, you took the sample and you have inspected and your sample size is 5 and out of this let us say 2 items are defective. So, you will say 2 by 5 that is the proportion your nonconforming, n is the sample size.

So, this is the most versatile control chart and this can be used as a measure of performance for the top management because directly it will help the management to understand that how much amount of nonconforming items are produced and that can help the management to glance the overall quality of the plant or operation and that is why this chart is widely used. So, a p-chart can provide source of information for improving the product quality and this is something which always design department and top management they look for.

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Construction and Interpretation

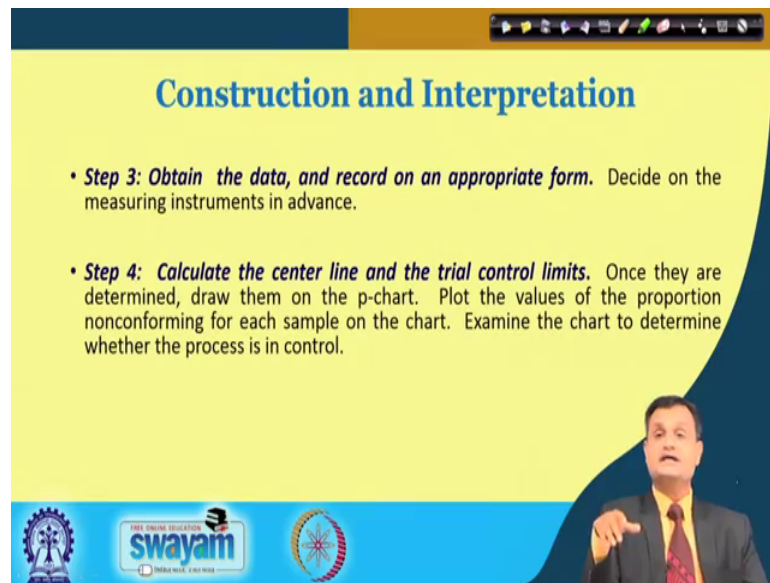
- **Step 1: Select the objective.** Decide on the level at which the p-chart will be used - that is the plant, the department, or the operator level. Decide to control either a single quality characteristic, multiple characteristics, a single product, or a number of products.
- **Step 2: Determine the sample size and the sampling interval.** An appropriate sample size is related to the existing quality level of the process. The sample size must be large enough to allow the opportunity for some nonconforming items to be present on average. The sampling interval-that is, the time between successive samples – is a function of the production rate and the cost of sampling, among other factors.

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So, now, the construction and interpretation goes like this. We have already seen the steps more or less we will follow the same steps, let us try to see step one select the objective. So, decide on the level at which the p-chart will be maintained whether you want to maintain it the plant level, department level, operator level and decide to control either single quality characteristics or me you may choose if you think that multiple quality characteristics are critical then for a particular product you can also choose that.

Step 2: Determine the sample size and sampling interval. So, we have discussed already that a sample size when you typically study the OC curve it has lot of impact on the discriminatory power of your control chart, but, obviously, economics associated with sample size cannot be ignored and this is something that we tried to decide right at the beginning as a part of preliminary decision. Also we think about the sampling interval. So, time between you will take the successive samples and again it depends upon the criticality of the quality characteristic.

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Construction and Interpretation

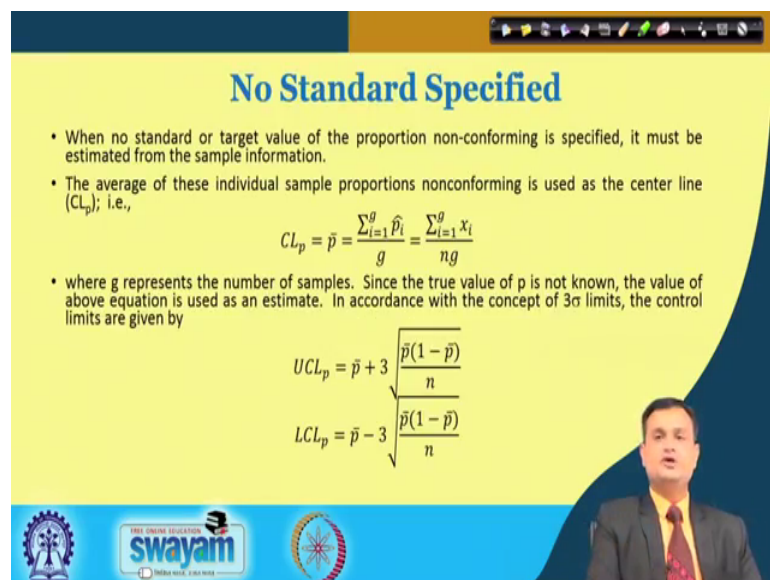
- **Step 3: Obtain the data, and record on an appropriate form.** Decide on the measuring instruments in advance.
- **Step 4: Calculate the center line and the trial control limits.** Once they are determined, draw them on the p-chart. Plot the values of the proportion nonconforming for each sample on the chart. Examine the chart to determine whether the process is in control.

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Step 3: Obtain the data, record on the appropriate form.

Step 4: same way it will go calculate central line and the trial control limit.

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No Standard Specified

- When no standard or target value of the proportion non-conforming is specified, it must be estimated from the sample information.
- The average of these individual sample proportions nonconforming is used as the center line (CL_p); i.e.,
$$CL_p = \bar{p} = \frac{\sum_{i=1}^g \hat{p}_i}{g} = \frac{\sum_{i=1}^g x_i}{ng}$$
- where g represents the number of samples. Since the true value of p is not known, the value of above equation is used as an estimate. In accordance with the concept of 3σ limits, the control limits are given by
$$UCL_p = \bar{p} + 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$
$$LCL_p = \bar{p} - 3 \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

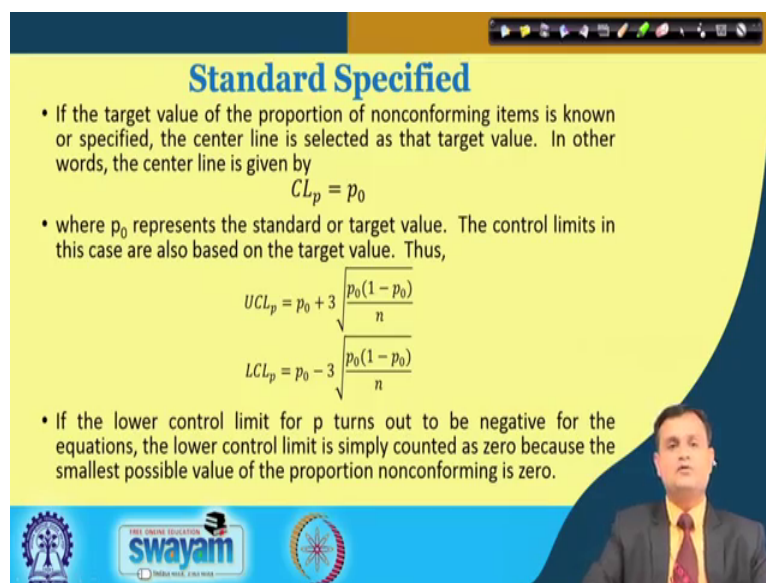
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Then you have the formulas basically for central line. So, this is very easy to interpret central line is basically nothing, but your p bar p bar and this is sigma i is equal to sigma i is equal to 1 to g p i divided by g. So, here we g represents the number of samples and you can have maybe g sample. So, I will have sigma i is equal to 1 to g x i divided by ng because pi hat is basically x i by n.

So, this is what I can do and now we know that your control limits are usually placed at plus or minus 3 sigma because this gives me the best (Refer Time: 07:58) between type 1 and type 2 error. So, you will have UCL p this is your mean value $\bar{p}$ and this is your 3σ . So, $\bar{p} + 3\sigma$ and this is the standard deviation UCL p, your $\bar{p} + 3\sigma$ and you have $\bar{p} - 3\sigma$.

So, this is how you can compute your upper control and lower control limit for the p-chart.

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Standard Specified

- If the target value of the proportion of nonconforming items is known or specified, the center line is selected as that target value. In other words, the center line is given by

$$CL_p = p_0$$
- where p_0 represents the standard or target value. The control limits in this case are also based on the target value. Thus,

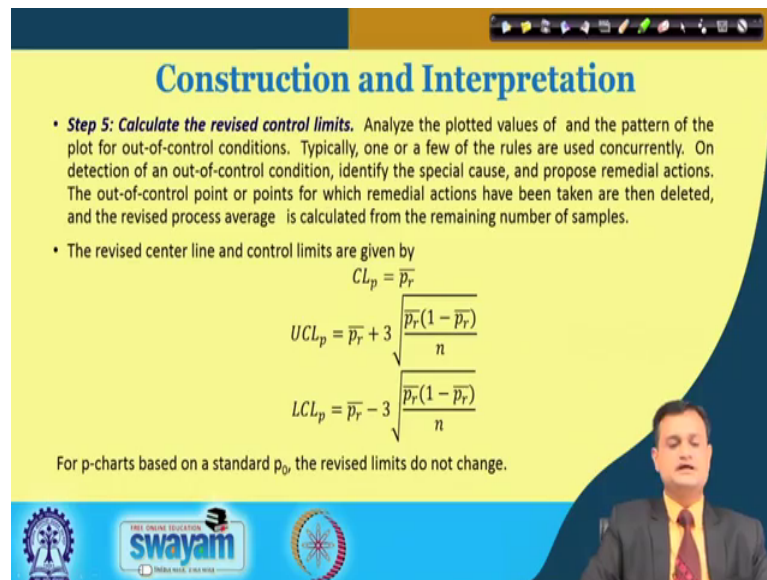
$$UCL_p = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}}$$

$$LCL_p = p_0 - 3\sqrt{\frac{p_0(1-p_0)}{n}}$$
- If the lower control limit for p turns out to be negative for the equations, the lower control limit is simply counted as zero because the smallest possible value of the proportion nonconforming is zero.

Now, you have CL p that is p_0 and if the target value of the proportion nonconforming is known or specified, then you take that value as the central line value otherwise you determine the centerline value as I suggested. And, if you are given the target value that is basically p_0 that is basically your p_0 , then this is the p_0 . You put this p_0 here and p_0 here and similar way you put p_0 here, p_0 here. So, that will basically help you to find the upper control limit and lower control limit for the p-chart.

Now, there is no point in having the negative lower control limit because proportion of nonconforming cannot be negative maximum it can go to 0, if you have the best quality lot. So, this is what exactly we do in the control chart.

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Construction and Interpretation

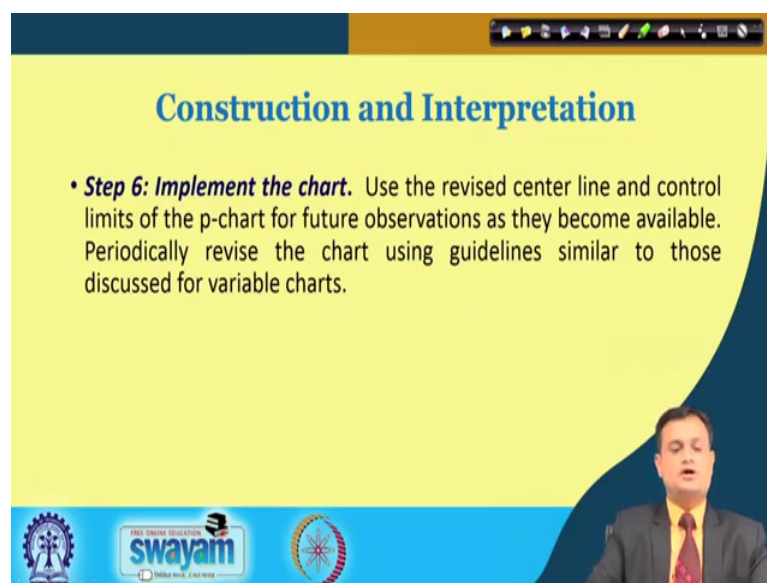
- **Step 5: Calculate the revised control limits.** Analyze the plotted values of $\bar{p}$ and the pattern of the plot for out-of-control conditions. Typically, one or a few of the rules are used concurrently. On detection of an out-of-control condition, identify the special cause, and propose remedial actions. The out-of-control point or points for which remedial actions have been taken are then deleted, and the revised process average $\bar{p}_r$ is calculated from the remaining number of samples.
- The revised center line and control limits are given by
$$CL_p = \bar{p}_r$$
$$UCL_p = \bar{p}_r + 3 \sqrt{\frac{\bar{p}_r(1 - \bar{p}_r)}{n}}$$
$$LCL_p = \bar{p}_r - 3 \sqrt{\frac{\bar{p}_r(1 - \bar{p}_r)}{n}}$$

For p-charts based on a standard p_0 , the revised limits do not change.

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And, calculate the revised control limit. So, as we have seen that if some of the points are going out then try to eliminate these points or take the corrective action and then you try to compute the revised control limits, once again check if your points are falling well within, then your control chart is ready for implementation. So, this is exactly we have seen in the $\bar{X}$ and r -chart also and we follow the same procedure in attribute chart. Only thing that my calculation for upper control limit, lower control limit and central line will be the different.

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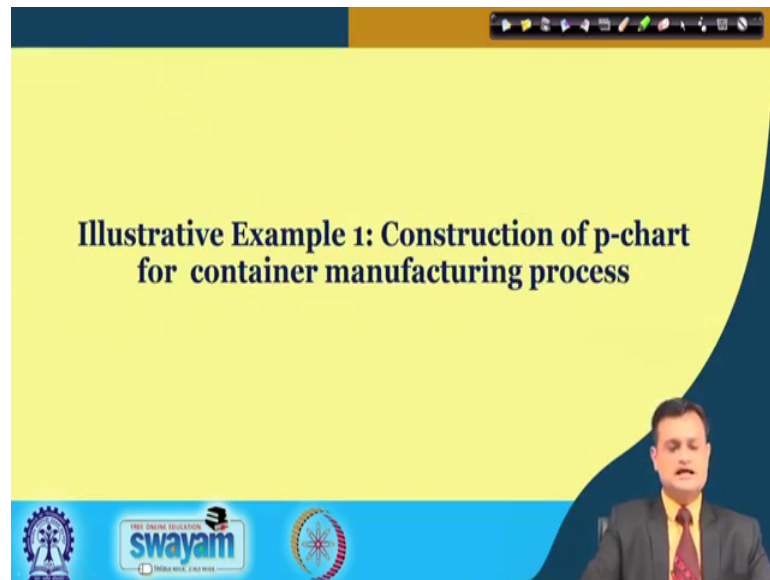
Construction and Interpretation

- **Step 6: Implement the chart.** Use the revised center line and control limits of the p-chart for future observations as they become available. Periodically revise the chart using guidelines similar to those discussed for variable charts.

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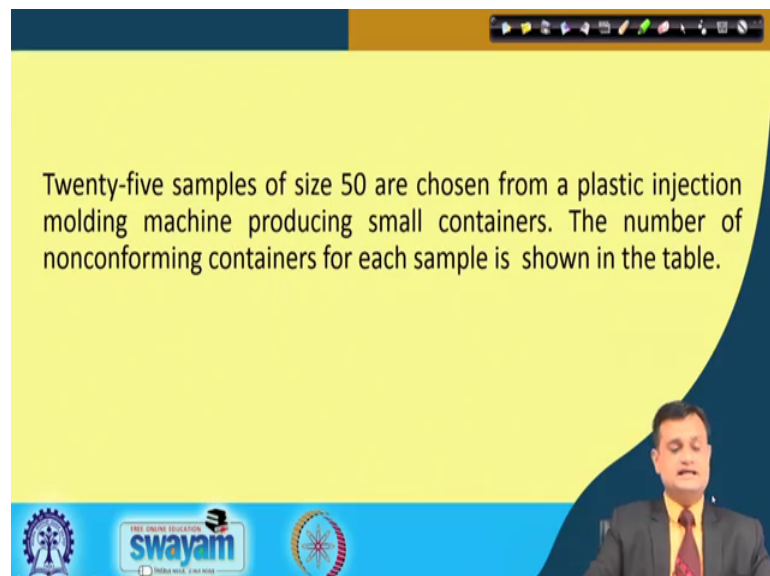
So, step 6 as I said once you feel that your process well within the control of your control chart and you have the stable upper control and lower control limit you implement it in order to set the statistical control over your process and take the corrective action at an appropriate time if the assignable cause is present.

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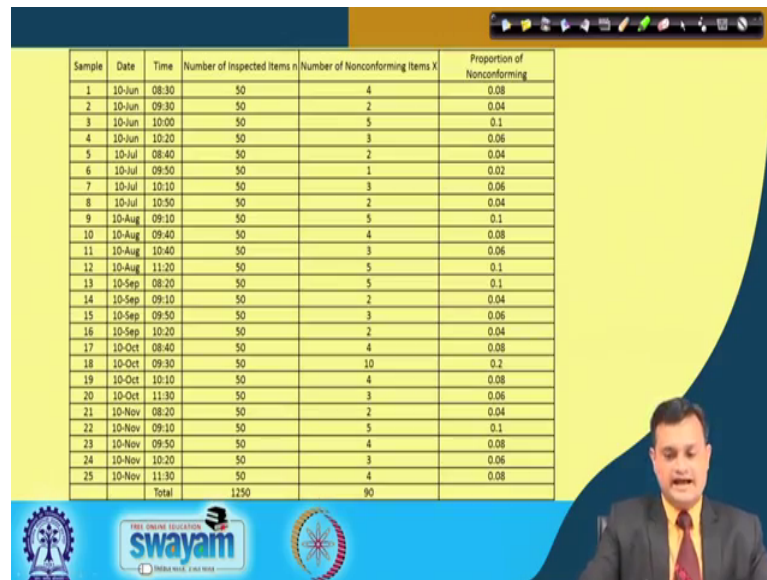
So, this is the example 1, illustrative example 1 for construction chart or p-chart and this will help you to appreciate the use of the formula and the concept which we have discussed.

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So, let us say there is a twenty-five samples and of size 50. So, each sample is of 50 size, twenty-five samples of size 50 at chosen from a plastic injection molding machine producing small containers and the number of non conforming containers for each sample is shown in the subsequent table.

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Sample	Date	Time	Number of Inspected Items n	Number of Nonconforming Items X	Proportion of Nonconforming
1	10-Jun	08:30	50	4	0.08
2	10-Jun	09:30	50	2	0.04
3	10-Jun	10:00	50	5	0.1
4	10-Jun	10:20	50	3	0.06
5	10-Jul	08:40	50	2	0.04
6	10-Jul	09:50	50	1	0.02
7	10-Jul	10:10	50	3	0.06
8	10-Jul	10:50	50	2	0.04
9	10-Aug	09:10	50	5	0.1
10	10-Aug	09:40	50	4	0.08
11	10-Aug	10:40	50	3	0.06
12	10-Aug	11:20	50	5	0.1
13	10-Sep	08:20	50	5	0.1
14	10-Sep	09:10	50	2	0.04
15	10-Sep	09:50	50	3	0.06
16	10-Sep	10:20	50	2	0.04
17	10-Oct	08:40	50	4	0.08
18	10-Oct	09:30	50	10	0.2
19	10-Oct	10:10	50	4	0.08
20	10-Oct	11:30	50	3	0.06
21	10-Nov	08:20	50	2	0.04
22	10-Nov	09:10	50	5	0.1
23	10-Nov	09:50	50	4	0.08
24	10-Nov	10:20	50	3	0.06
25	10-Nov	11:30	50	4	0.08
Total			1250	90	

So, your data looks like this that you have sample. You have sample and you have date time number of inspected items number of nonconforming items and proportion nonconforming. So, you have say collected the data you have also mention the date. So, total you have collected twenty-five samples and you have also recorded the time and then you can see that your sample size is 50 everywhere and you have found the number of non conforming items. For example, sample one it is 4. So, 4 divided by 50 is nothing, but 0.08. So, you have 8 percent nonconforming items in sample one. So, this is what you have done.

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Solution

The average proportion nonconforming is $\bar{p} = \frac{90}{1250} = 0.072$

This is the center line of the p-chart.

The trial control limits are found to be

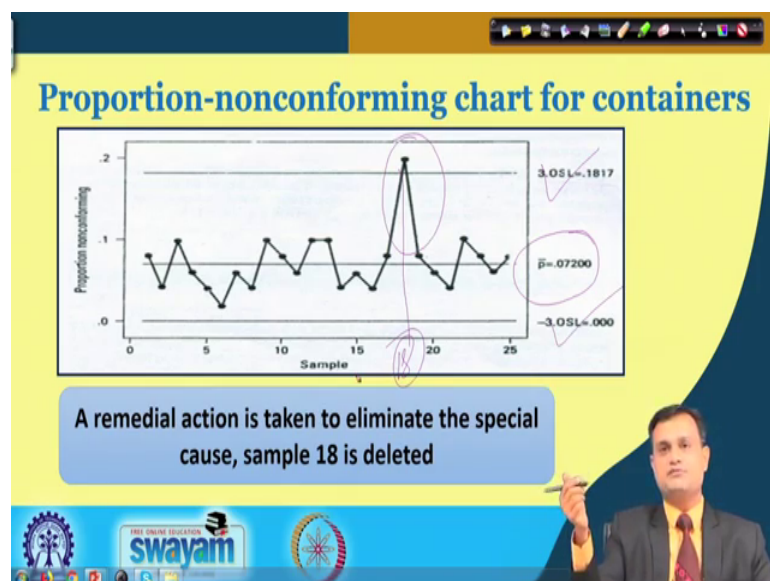
$$CL_p = 0.072$$
$$UCL_p = 0.072 + 3 \sqrt{\frac{(0.072)(1 - 0.072)}{50}} = 0.182$$
$$LCL_p = 0.072 - 3 \sqrt{\frac{(0.072)(1 - 0.072)}{50}} = -0.038 \rightarrow 0$$

Since the calculated value of the lower control limit is negative, it is converted to zero.



Now, once you have the values of percent nonconforming for individual sample you just check the sum and divide it by 1250, that is the total number of items inspected twenty-five sample, each sample has 50. So, you will have 25 into 1250. So, 90 divided by 1250 you will have $\bar{p}$ and this is 0.072. So, now, you compute the UCL_p and LCL_p and you will find that this comes out to be 0.182 and this is 0. So, because it is negative I cannot deal with the negative so, I will take 0.

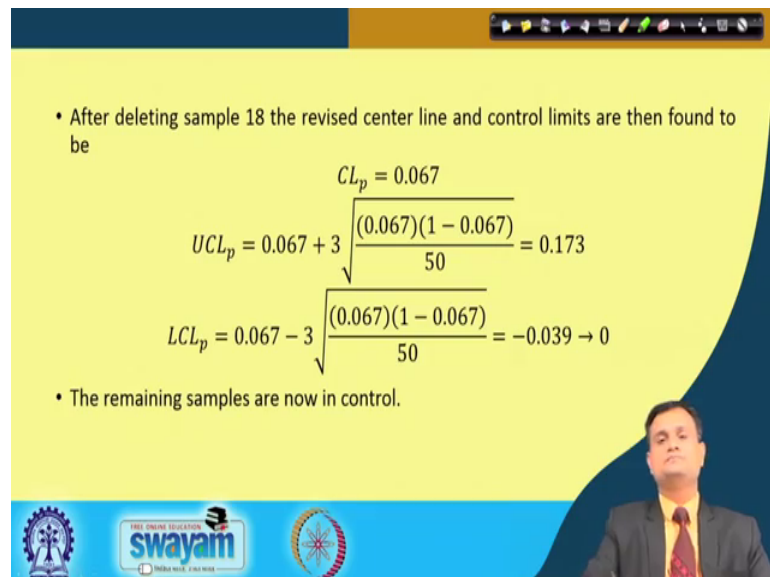
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So, now, once you have done this, you plot the control chart and what do you see here that you have the upper control limit, you have the upper control limit, you have the lower control limit, you have the p-bar value and you will see that one point that is 18 point sample 18 that is basically going out of the upper control limit. So, it is an indication that that could be some assignable cause.

So, you need to investigate through a cross functional team or using your process knowledge and see that a corrective action can be taken to put your process back into the control.

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- After deleting sample 18 the revised center line and control limits are then found to be

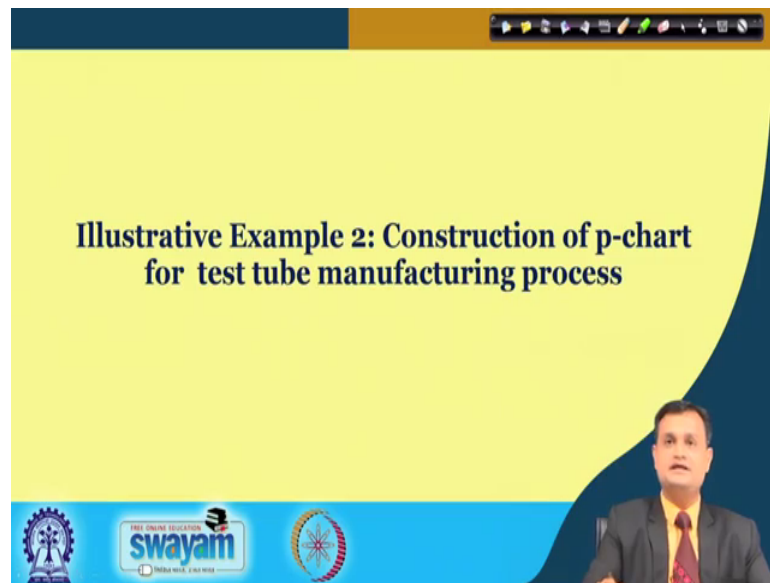
$$CL_p = 0.067$$
$$UCL_p = 0.067 + 3 \sqrt{\frac{(0.067)(1 - 0.067)}{50}} = 0.173$$
$$LCL_p = 0.067 - 3 \sqrt{\frac{(0.067)(1 - 0.067)}{50}} = -0.039 \rightarrow 0$$

- The remaining samples are now in control.

The slide also features a small video inset of a man in a suit in the bottom right corner, and logos for IIT Bombay and Swamyam in the bottom left corner.

So, now you have deleted this 18 point because I want to develop the stabilize my control limit, so that it can be used and implemented for establishing a statistical control on the process. So, once this is done, I will again compute my CL p and this CL p is basically 0.067. I will compute the UCL p. So, CL p is 0.067, UCL p and I will also compute the LCL p as usual.

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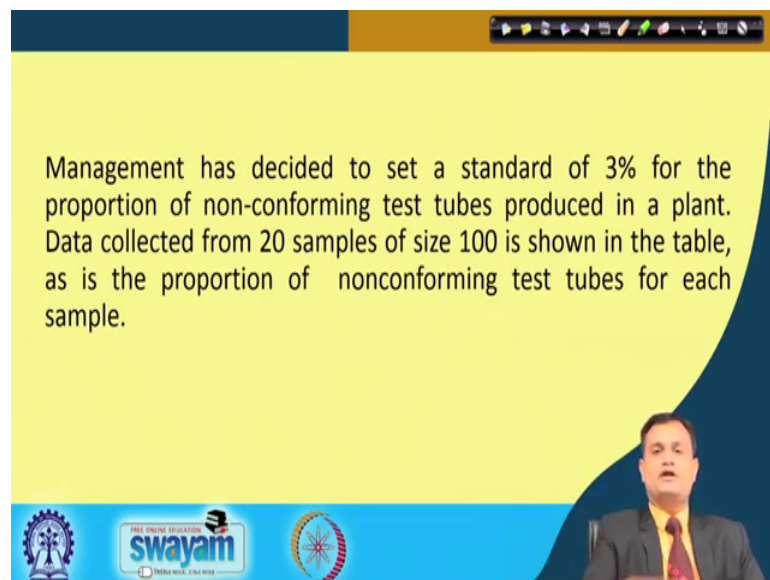


**Illustrative Example 2: Construction of p-chart
for test tube manufacturing process**

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So, now the remaining samples are in control and I can implement the control chart. Let us see the another example construction of p-chart for test tube manufacturing process.

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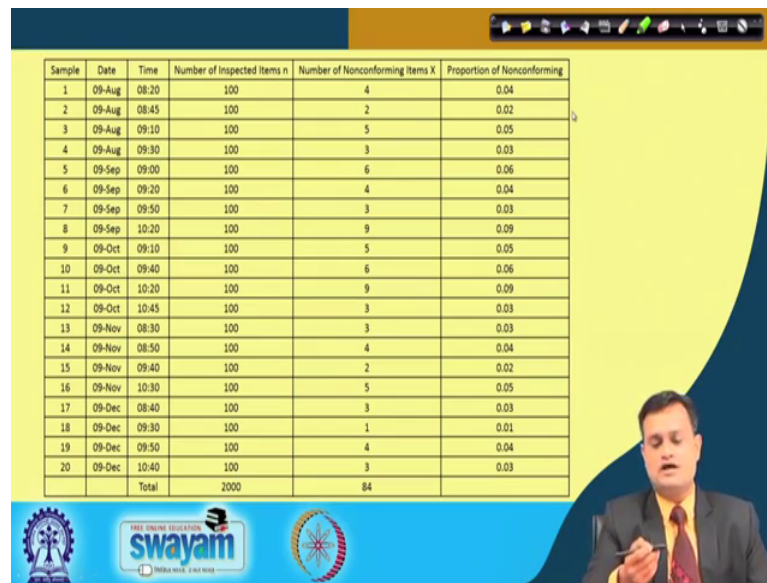


Management has decided to set a standard of 3% for the proportion of non-conforming test tubes produced in a plant. Data collected from 20 samples of size 100 is shown in the table, as is the proportion of nonconforming test tubes for each sample.

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So, here now what I am trying to do is that management has a company a management has decided to set a standard of 3 percent for the proportion of non-conforming test tubes produced in a plant and they have collected the data 20 samples of each sample of size 100 and this is the data is shown in the next table and proportion of non conforming tubes for each particular sample were identified.

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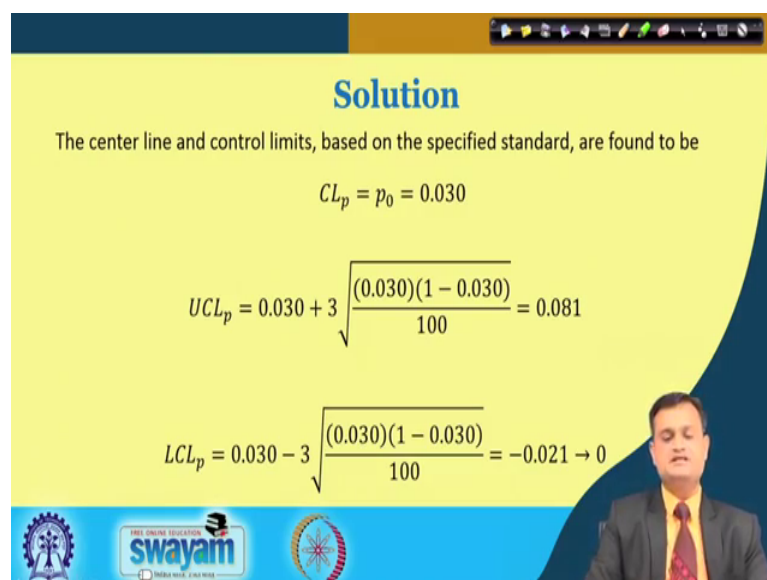


The slide displays a table with 6 columns: Sample, Date, Time, Number of Inspected Items n, Number of Nonconforming Items X, and Proportion of Nonconforming. The table contains 20 rows of data, followed by a total row. The presenter is visible in the bottom right corner.

Sample	Date	Time	Number of Inspected Items n	Number of Nonconforming Items X	Proportion of Nonconforming
1	09-Aug	08:20	100	4	0.04
2	09-Aug	08:45	100	2	0.02
3	09-Aug	09:10	100	5	0.05
4	09-Aug	09:30	100	3	0.03
5	09-Sep	09:00	100	6	0.06
6	09-Sep	09:20	100	4	0.04
7	09-Sep	09:50	100	3	0.03
8	09-Sep	10:20	100	9	0.09
9	09-Oct	09:10	100	5	0.05
10	09-Oct	09:40	100	6	0.06
11	09-Oct	10:20	100	9	0.09
12	09-Oct	10:45	100	3	0.03
13	09-Nov	08:30	100	3	0.03
14	09-Nov	08:50	100	4	0.04
15	09-Nov	09:40	100	2	0.02
16	09-Nov	10:30	100	5	0.05
17	09-Dec	08:40	100	3	0.03
18	09-Dec	09:30	100	1	0.01
19	09-Dec	09:50	100	4	0.04
20	09-Dec	10:40	100	3	0.03
		Total	2000	84	

So, you can just see now that how the data looks like. So, you have the sample, you have the date, you have the time, you have the number of inspected item and you have the number of nonconforming item and proportion of nonconforming. So, once you have say computed proportion of nonconforming and you have this then you are ready to follow the computations further.

(Refer Slide Time: 16:00)



The slide is titled 'Solution' and shows the calculation of control limits for a p-chart. The center line and control limits are based on the specified standard, $p_0 = 0.030$. The formulas for UCL_p and LCL_p are shown, with the final result for LCL_p being $-0.021 \rightarrow 0$.

Solution

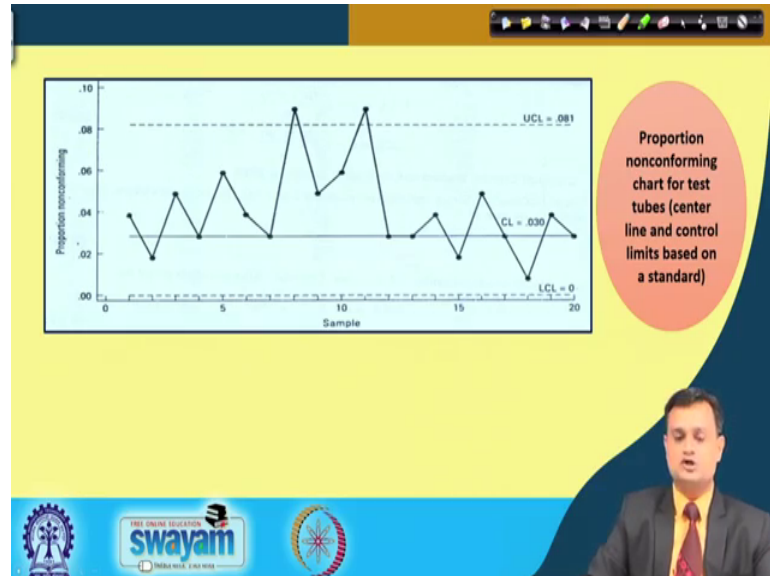
The center line and control limits, based on the specified standard, are found to be

$$CL_p = p_0 = 0.030$$
$$UCL_p = 0.030 + 3 \sqrt{\frac{(0.030)(1 - 0.030)}{100}} = 0.081$$
$$LCL_p = 0.030 - 3 \sqrt{\frac{(0.030)(1 - 0.030)}{100}} = -0.021 \rightarrow 0$$

So, you have CL p, p 0, 0.03. So, here I would like to remind you that I am not finding p bar. I have the set value to achieve and this set value is p 0.03. This is the expectations of

the management and that is why I will consider the target value specified value as my central line value and I will also use this value in order to find my UCL p and LCL p.

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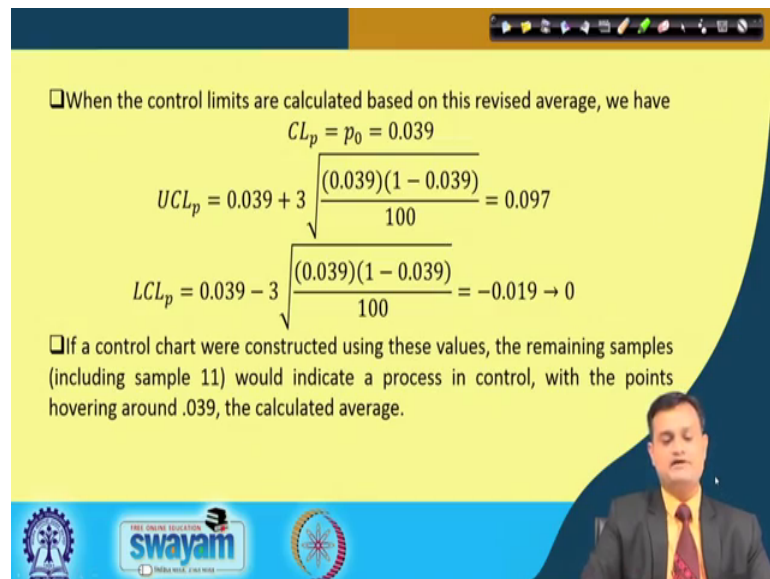
So, with this I have plotted the control chart UCL is 0.081 and you have LCL 0. So, this will set my control chart limits you can see here that you have point outside the upper control limit and going outside the lower control limit sorry upper control limit. So, these two points basically they are going outside the upper control limit and I have to eliminate these points in order to revise my control limits.

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- ❑ It is observed that the general level of the proportion nonconforming does not match the standard value of 3%; the actual proportion appears higher.
- ❑ The process average proportion nonconforming is $\bar{p} = \frac{84}{2000} = 0.042$
- ❑ This value exceeds the desired standard of 3%
- ❑ If sample 8 is eliminated following removal of its special cause, the revised process average is $\bar{p} = \frac{84-9}{1900} = 0.039$

So, same way you do and then you will have the $\bar{p}$ and desired standard is 3 percent.

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When the control limits are calculated based on this revised average, we have

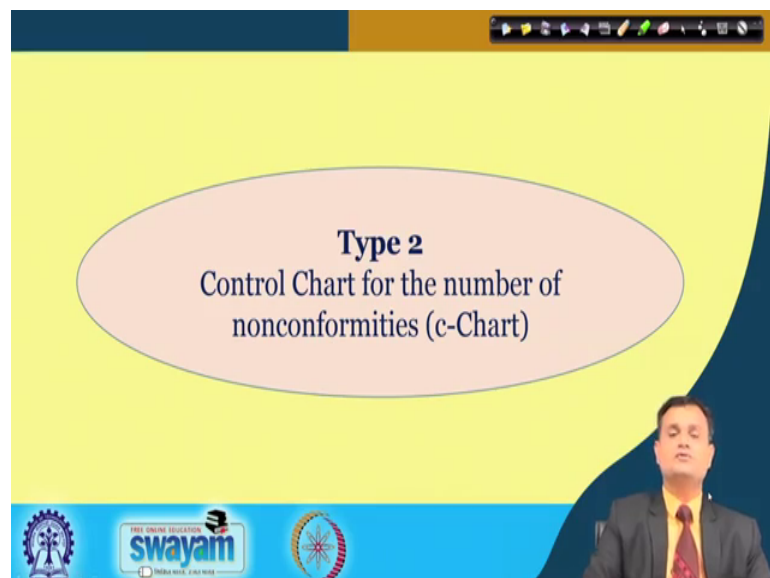
$$CL_p = p_0 = 0.039$$
$$UCL_p = 0.039 + 3 \sqrt{\frac{(0.039)(1 - 0.039)}{100}} = 0.097$$
$$LCL_p = 0.039 - 3 \sqrt{\frac{(0.039)(1 - 0.039)}{100}} = -0.019 \rightarrow 0$$

If a control chart were constructed using these values, the remaining samples (including sample 11) would indicate a process in control, with the points hovering around .039, the calculated average.

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So, I will once again compute my; this limits and then I will have the control chart ready for implementation. So, this is the procedure that you follow.

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Type 2
Control Chart for the number of nonconformities (c-Chart)

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Now, let us see the type 2. So, control chart for number of nonconforming that is c-Chart. So, here there is a difference I am not saying proportion nonconforming; I am saying number of nonconformities and this is my c-Chart.

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Chart for the number of nonconformities (c-Chart)

- ✓ A nonconformity is defined as a quality characteristic that does not meet some specification.
- ✓ A nonconforming item has one or more nonconformities that make it nonfunctional.
- ✓ It is also possible for a product to have one or more nonconformities and still conform to standards.
- ✓ A c-chart is used to track the total number of nonconformities in samples of constant size.
- ✓ The occurrence on nonconformities is assumed to follow a Poisson distribution.
- ✓ This distribution is well-suited to modeling the number of events that happen over a specified amount of time, space, or volume.

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So, typically say a non conforming item has one or more non nonconformities that make it nonfunctional. So, suppose you think about let us say mixture. Now, your switch is not working or your gear is broken or maybe your jar is not fitting then there could be many nonconformities and ultimately this nonconformities make sure mixture nonfunctional. So, a product may have number of non conformities and typically that we are trying to capture in this c-chart.

So, the occurrence on non conformities is assumed to follow a Poisson distribution and this distribution is typically well suited to modeling the number of events that happen over a specified amount of time or a space or volume. You can also think about the textile industry. Suppose, they are analyzing let us say a cloth maybe as a simple they have taken maybe 1 meter by 1 meter and they want to check that how many non conformities are there means where the colour is faded, the design is not appropriate or some threads are coming out. So, there could be different kinds of nonconformity and they will get 1 meter by 1 meter what are the number of nonconformities.

So, space wise, time wise, volume wise you can find the number of nonconformities.

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☐ If x represents the number of nonconformities in the sample unit and c is the mean, then Poisson distribution yields

$$p(x) = \frac{e^{-c} c^x}{x!}$$

No Standard Given

☐ The average number of nonconformities per sample unit is found from the sample observations and is denoted by $\bar{c}$. The center line and control limits are

$$CL_c = \bar{c}$$
$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$
$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

☐ If the lower control limit is found to be less than zero, it is converted to zero.



So, as I mentioned that typically this particular say chart follows the Poisson distribution and so, this is type 2 and this follows typically the Poisson distribution. So, this is my Poisson distribution. So, if average number of non conformities let us say $\bar{c}$ or sample unit you may determine is form from the sample you can compute the central line for this particular control chart that is CL_c which is $\bar{c}$ and you can have upper control limit and lower control limit. So, obviously, a lower control limit is found to be less than 0, I will consider that zero because my number of nonconformities cannot be less than 0.

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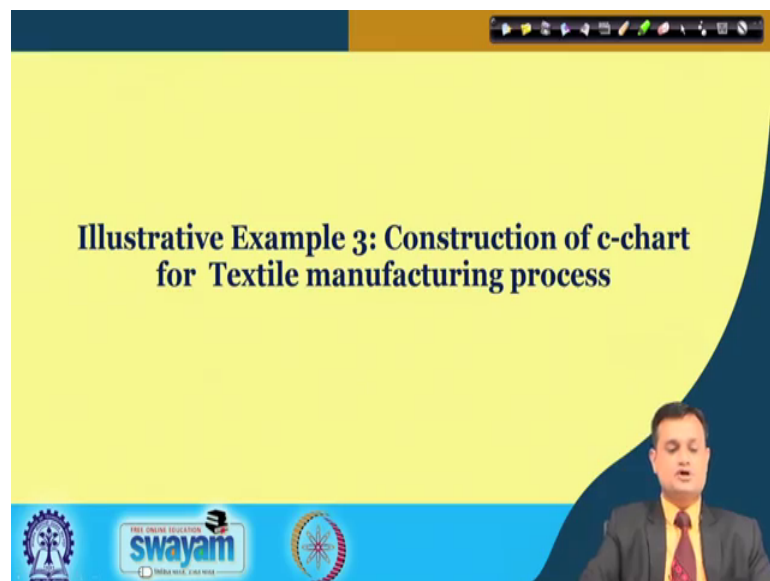
Standard Given

☐ Let the specified goal for the number of nonconformities per sample unit be c_0 . The center line and control limits are then calculated from

$$CL_c = c_0$$
$$UCL_c = c_0 + 3\sqrt{c_0}$$
$$LCL_c = c_0 - 3\sqrt{c_0}$$


So, with this we have now, if the standard is given same procedure follows like we did p 0 here I will put c 0, it means in a particular space or time some number of nonconformities are accepted as the standard and it cannot go beyond that, my management is emphasizing on a particular standard. So, if it is already given then I will consider that as my central line, then I will try to find upper control limit and lower control limit with respect to that particular standard.

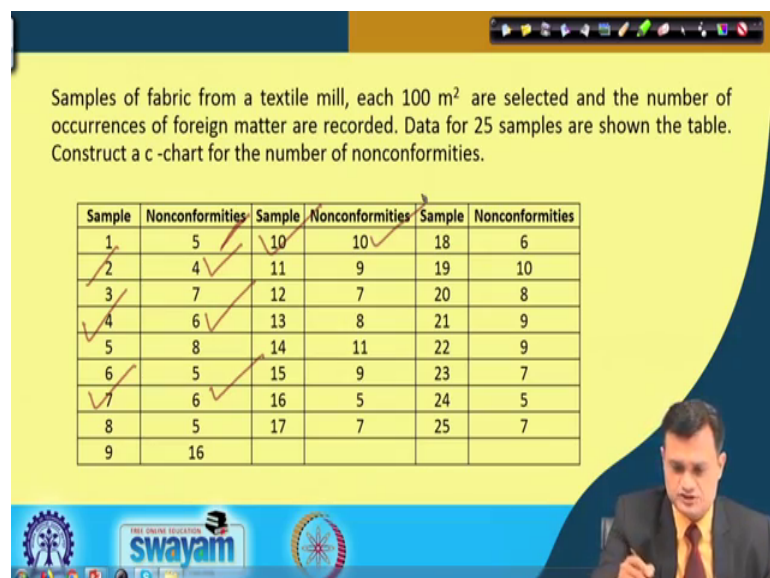
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Illustrative Example 3: Construction of c-chart for Textile manufacturing process

So, let us see the illustrative example for construction of c-chart.

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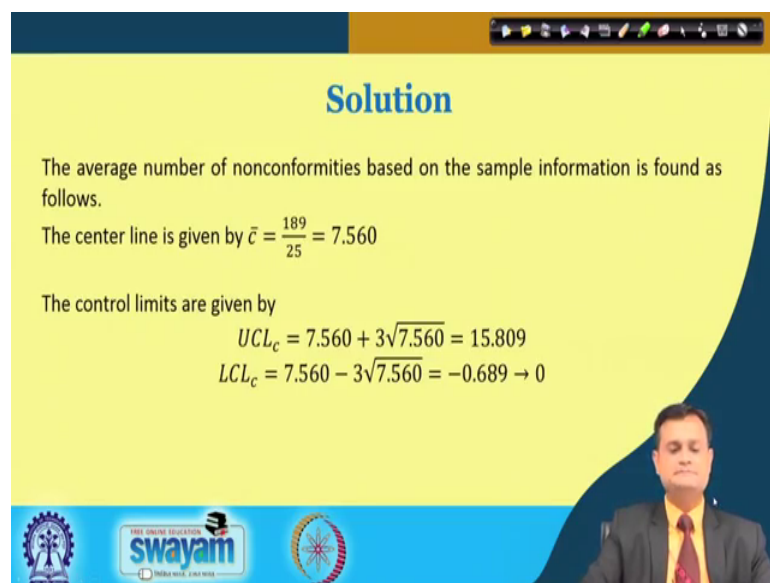
Samples of fabric from a textile mill, each 100 m² are selected and the number of occurrences of foreign matter are recorded. Data for 25 samples are shown the table. Construct a c -chart for the number of nonconformities.

Sample	Nonconformities	Sample	Nonconformities	Sample	Nonconformities
1	5	10	10	18	6
2	4	11	9	19	10
3	7	12	7	20	8
4	6	13	8	21	9
5	8	14	11	22	9
6	5	15	9	23	7
7	6	16	5	24	5
8	5	17	7	25	7
9	16				

And, what you have here is sample and as I mentioned that you have the sample and you have nonconformities present. So, each particular sample you have the nonconformities and you can see that sample 1 has 5 nonconformities, sample 4 has 6 nonconformities, sample 7 has 6 nonconformities, then sample 10 has 10 nonconformities; it means that your unit is basically a particular object and in this object they observe maybe 5, 4, 7, 10 whatever number of nonconforming units.

So, here as I mention it is an example of textile fabric and they were taken the 100 meter square say area 100 meter square cloth and they are trying to find the number of nonconformities in this particular space. So, it could be faded color or it could be your fade coming out or maybe there is some tearing of the cloth or it could be let us say some kind of haziness in the design.

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Solution

The average number of nonconformities based on the sample information is found as follows.

The center line is given by $\bar{c} = \frac{189}{25} = 7.560$

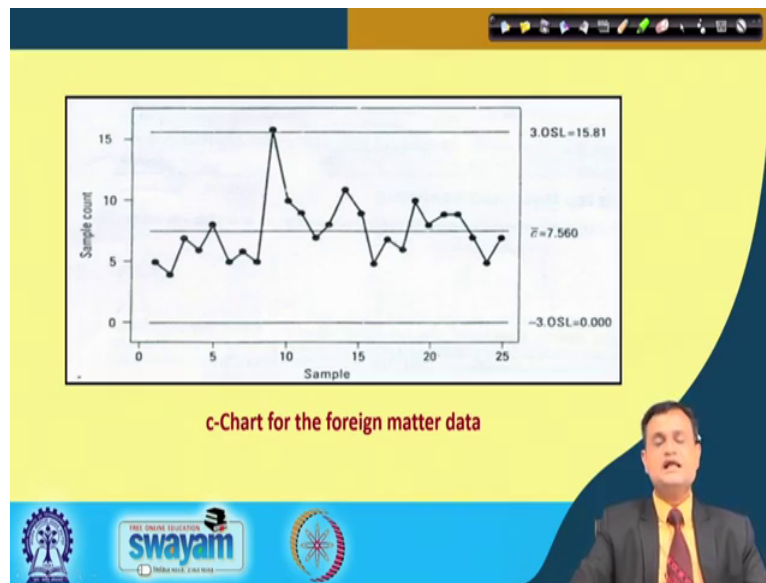
The control limits are given by

$$UCL_c = 7.560 + 3\sqrt{7.560} = 15.809$$
$$LCL_c = 7.560 - 3\sqrt{7.560} = -0.689 \rightarrow 0$$

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So, this kind of nonconformities you can observe and you can count it. So, it is very simple to tabulate the data. Now, you find the c bar, sum it up all the nonconformities divided by total number of samples. So, you will have say c bar that is 189 divided by 25 that is 7.56. You can find the upper control and lower control limit if it is negative convert it to 0, rule applies.

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Now, I am plotting the control chart by setting the upper control limit, lower control limit and what you will see here that a particular point is just falling on the upper control limit. This may be excluded and I can we calculate the limits in order to finalize my limits for maintaining the statistical control on the process.

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☐ Assuming remedial action has been taken for the special causes, the center line and control limits are revised as follows (sample 9 is deleted):

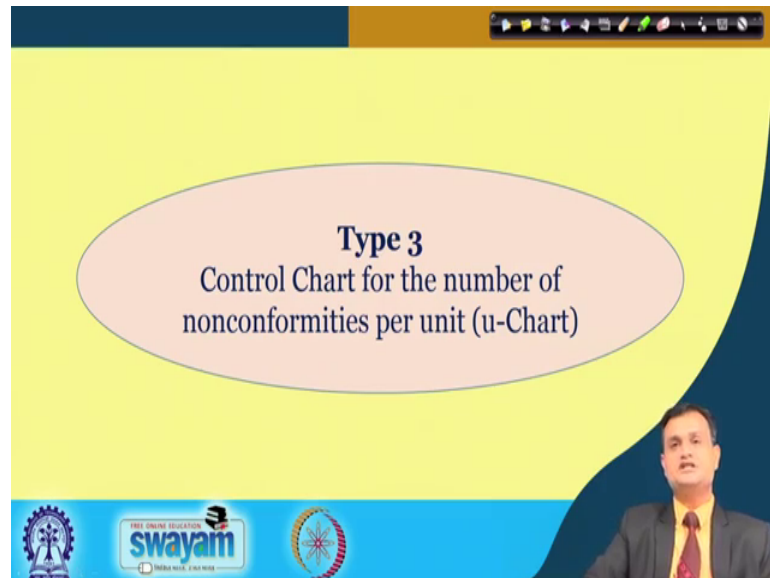
$$\bar{c} = \frac{189 - 16}{24} = 7.208$$
$$UCL_c = 7.208 + 3\sqrt{7.208} = 15.262$$
$$LCL_c = 7.208 - 3\sqrt{7.208} = -0.846 \rightarrow 0$$

☐ After this revision, the remaining sample points are found to be within the limits

So, this is what a sample 9 is detected, I am just trying to take it out. Once again I am computing $\bar{c}$, UCL_c , LCL_c and this I will use when my all the points are within this

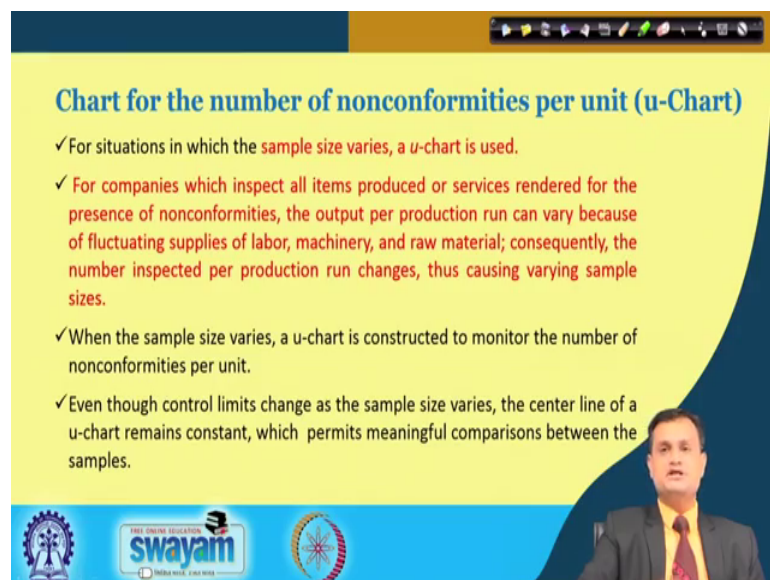
control limit I will use this as the finalized control limit in order to establish the control over my process.

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So, this is the type 3 of the control chart for attribute and this is about number of nonconforming per unit.

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But, the issue here is that that many companies which inspect all items produced or services typically rendered for the presence of nonconformities, the output for production run can vary. So, not necessary that you can have the same sample size or you can have

the same production output per production run. So, in this situation you may have fluctuating labor supply, fluctuating material supply, machinery, raw material and this will end up in the varying sample size.

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• When the sample size varies, the number of nonconformities per unit for the i^{th} sample is given by

$$u_i = \frac{c_i}{n_i}$$

• where c_i is the number of nonconformities in the i^{th} sample, and n_i is the size of the sample. Note that the sample size n_i need not always be an integer.

• The average number of nonconformities per unit ($\bar{u}$), which is also the center of a u-chart, is given by

$$\bar{u} = \frac{\sum_{i=1}^g c_i}{\sum_{i=1}^g n_i}$$

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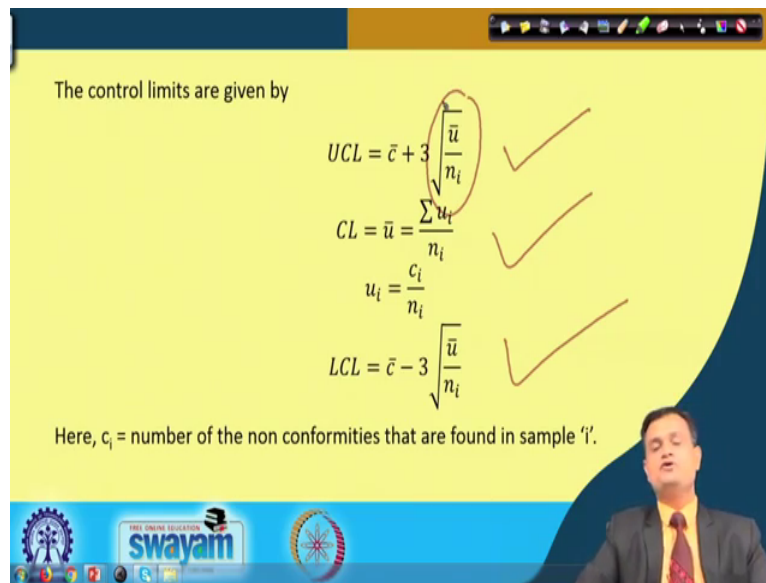
So, typically when I am trying to also address this particular issue I will use the u-chart and you can see here that u_i is equal to c_i divided by n_i because my sample size is varying I am dividing it by n_i . So, each particular sample my sample size maybe different and you can compute $\bar{u}$ as sigma is equal to 1 to g ; g is your number of samples, c_i divided by sigma is equal to 1 to g n_i . So, this is the simple of calculation for my u-chart.

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The control limits are given by

$$UCL = \bar{c} + 3 \sqrt{\frac{\bar{u}}{n_i}}$$
$$CL = \bar{u} = \frac{\sum u_i}{n_i}$$
$$u_i = \frac{c_i}{n_i}$$
$$LCL = \bar{c} - 3 \sqrt{\frac{\bar{u}}{n_i}}$$

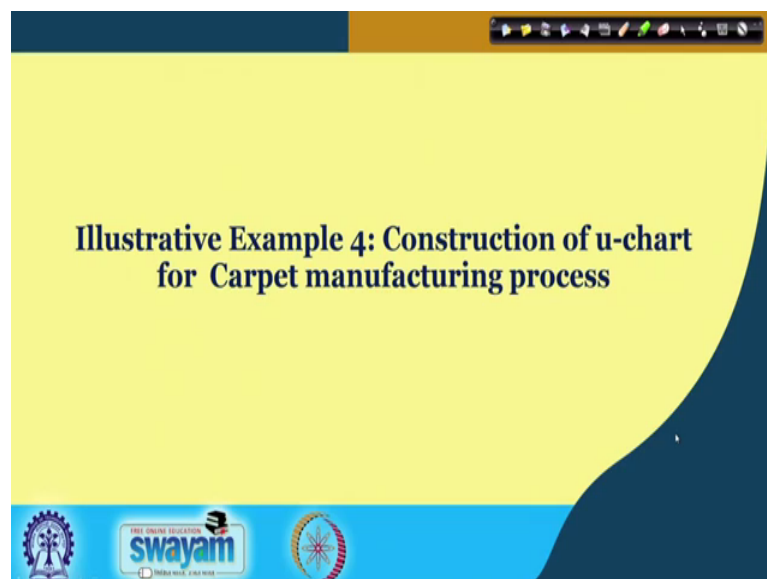
Here, c_i = number of the non conformities that are found in sample 'i'.



And, you can compute your UCL, CL and LCL. So, now, the idea is clear that it is basically mean plus or minus 3 sigma. Here your sigma is u square root of u bar divided by n i. So, you compute this by c bar plus this c bar minus.

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**Illustrative Example 4: Construction of u-chart
for Carpet manufacturing process**



And, once you have computed the control limits you can construct the control chart.

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The number of nonconformities in carpet is determined for 20 samples, but the amount of carpet inspected for each sample varies. Results of the inspection are shown in the table below. Construct a control chart for the number of nonconformities per 100 m².

Sample	Amount Inspected (in m ²)	Number of Nonconformities (c_i)	Sample	Amount Inspected (in m ²)	Number of Nonconformities (c_i)
1	200	5	11	300	9
2	300	14	12	250	16
3	250	8	13	200	12
4	150	8	14	250	10
5	250	12	15	100	6
6	100	6	16	200	8
7	200	20	17	200	5
8	150	10	18	100	5
9	150	6	19	300	14
10	250	10	20	200	8

So, you have let us say the illustrative example for the construction of u-chart and what you can see here that you have sample 1 and very well you can see that in this particular carpet example 20 samples were taken and say 100 meter square area was inspected. So, you will see that amount inspected is 200 in first sample. Now, I want to construct the control chart for the number of non conformities per 100 meter square.

So, I have checked the sample 2; for this it is 300 meter square; I have checked the sample 3, it is 3 250 meter square and you have total 20 samples. So, what I found that when I am checking meter square which is varying different I have some number of nonconformities per meter square area.

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Solution

With 100 m² as a unit, the sample sizes are computed for each sample.

The center line is found as $\bar{u} = \frac{\sum c_i}{\sum n_i} = \frac{192}{41} = 4.683$ ✓

The control limits are found to be

$$UCL_u = 4.683 + 3 \sqrt{\frac{4.683}{2}} = 9.273$$
$$LCL_u = 4.683 - 3 \sqrt{\frac{4.683}{2}} = 0.092$$

The control limits for other samples are calculated in the same manner



So, here my sample size is varying and this is where I am using the u-chart. So, $\bar{u}$ is equal to $\sum c_i / \sum n_i$. So, this comes out to be 4.683 and you find the UCL_u , LCL_u . So, now, the control limits for other samples are calculated in the same manner. Please remember in this chart you cannot have one control line for established considering all the sample you have to go sample by sample because the sample size is different.

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Control Limits for nonconformities per unit in carpets

Sample	Sample Size, n_i	Nonconformities per 100 m ² , u_i	Upper Control Limit	Lower Control Limit
1	2	2.500	9.274	0.092
2	3	4.667	8.431	0.935
3	2.5	3.200	8.789	0.577
4	1.5	5.333	9.984	0
5	2.5	4.800	8.789	0.577
6	1	6.000	11.175	0
7	2	10.000	9.274	0.092
8	1.5	5.667	9.984	0
9	1.5	4.000	9.984	0
10	2.5	4.000	8.789	0.577
11	3	3.000	8.431	0.935
12	2.5	6.400	8.789	0.577
13	2	6.000	9.274	0.092
14	2.5	4.000	8.789	0.577
15	1	6.000	11.175	0
16	2	4.000	9.274	0.092
17	2	2.500	9.274	0.092
18	1	5.000	11.175	0
19	3	4.667	8.431	0.935
20	2	4.000	9.274	0.092
Total	41			



So, once this is done then you can have something like this. This is the sample size nonconforming per 100 meter square sizes reduced it by considering 100 meter square which is the target upper control limit and lower control limit. So, now, you see sample 2, there is a different upper control limit, lower control limits; sample 3 there is a different upper control limit, lower control limit and same way.

So, you have different upper control and lower control limit for each sample because here your sample size is different and this is where I am using the u-chart.

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❑ After the special cause has been identified and appropriate corrective actions taken, the revised center line is found by deleting sample 7:

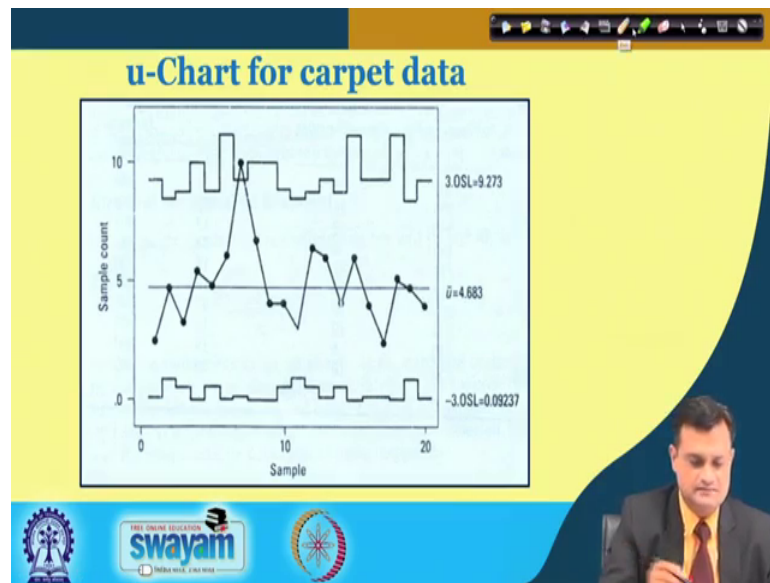
$$\bar{u} = \frac{192 - 20}{39} = 4.410$$

❑ The revised control limits are found using revised value. The remaining samples are within these revised control limits.

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So, I will compute the u bar that is 192 minus 20 divided by 39 and typically I will try to identify the special cause and appropriate corrective action and I will delete this particular sample.

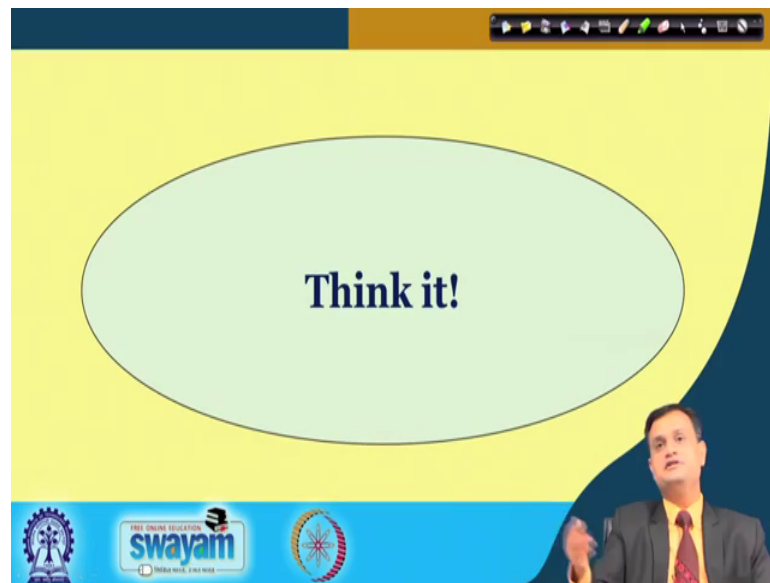
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So, you can just check that in my control chart you would find that somewhere here my sample 7 is going outside the control limit, ok. So, here it is going outside the control limit something like 7; sample. You can also see that my control line here are varying because sample size is different so, you will have zigzag lower control limit as well as upper control limit and this point is going out. So, I am say finding the cause for this and ah, I am removing this point from the my data set and computing the revised control limit as I am doing.

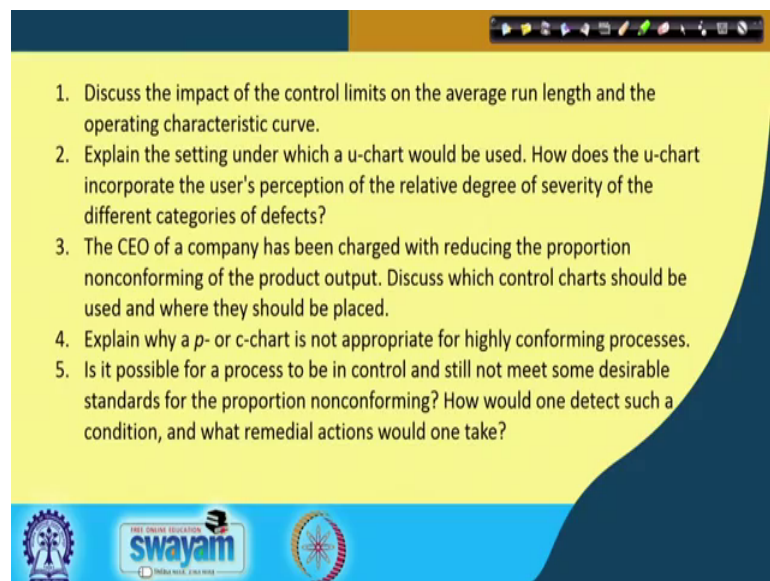
And, once this is done once again I would like to check that whether all the points are falling well within or not. If it is there then my tool, my gauge, my control chart for attribute is ready for establishing the statistical control on the process and then I can keep a track of my process and if there is any assignable cause I can always take the corrective action.

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So, before we end as a usual practice, let me float couple of think it.

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Discuss the impact of control limits on the average run length and the operating characteristic curve. So, just give a thought, now you are exposed to all the ideas. Explain the setting under which u-chart would be used. Just now we discussed. How does the u-chart incorporate the user's perception of the relative degree of severity of the different categories of defect. Just see the question 3, the CEO of a company has been charged with reducing the proportion nonconforming of the product output. Discuss

which control charts should be used and where they should be placed. This is also very important.

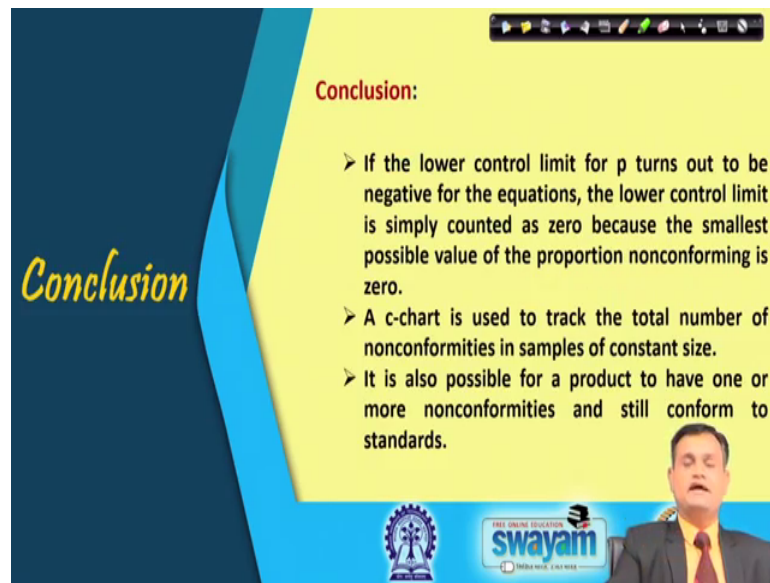
Explain why p or c-chart is not appropriate for highly conforming processes. Just give a thought what will happen. Is it possible for a process to be in control and still not need some desirable standards for the proportion nonconforming, is it possible? We have seen couple of cases that even if you are all the points are falling within the control limit, but still your process is out of control or it will say go out of control within a very small period of time and if this thing happens previous, then how will you detect such kind of condition.

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So, these are the references you can use and mainly I have used Mitra and Montgomery for say developing this lectures.

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Conclusion:

- If the lower control limit for p turns out to be negative for the equations, the lower control limit is simply counted as zero because the smallest possible value of the proportion nonconforming is zero.
- A c -chart is used to track the total number of nonconformities in samples of constant size.
- It is also possible for a product to have one or more nonconformities and still conform to standards.

And, finally, we can conclude that if the lower control limit for p -chart turn out to be negative we convert it to be zero because I cannot have negative nonconforming. A c -chart is used to track the total number of nonconformities in samples of constant size and it is also possible for a product to have one or more non conformities and still conform to the standards.

So, with this we can conclude our lecture on control charts for attributes. I hope you have understood the concepts as well as development procedure. I would advise you to take some simple example or real life data or hypothetical data and compute the control limits upper control limit, lower control limit, draw the control chart, eliminate the point going outside the control limit, recalculate and then try to develop a control chart which is ready for implementation and you can really implement it for establishing a statistical control on the process.

So, thank you very much. Be with me, enjoy.