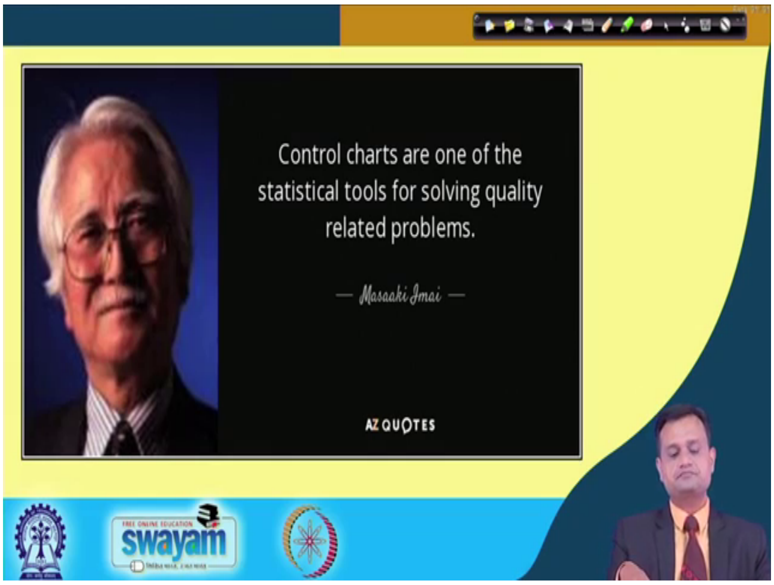
Six Sigma

Prof. Jitesh J Thakkar
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 50
Operating Characteristics (OC) Curve for Variable Control Charts

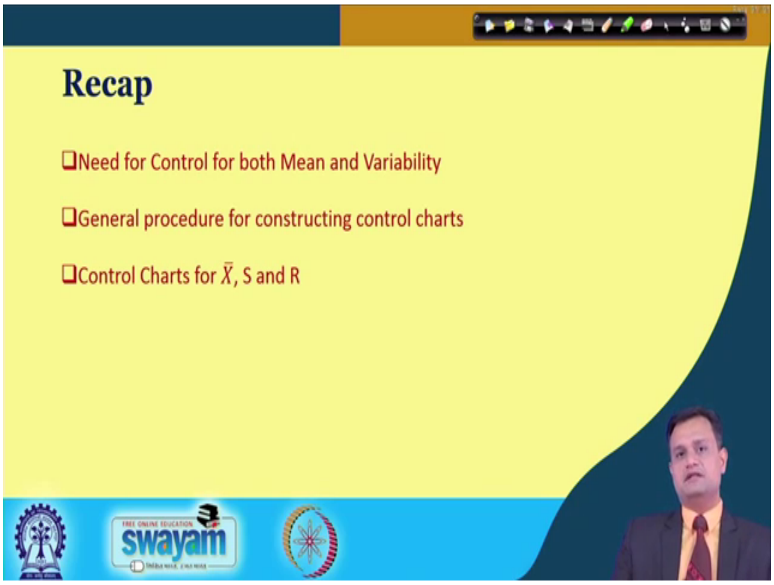
Hello friends, how are you? Hope you are doing well in your studies, in universities and your profession typically industry practitioners and hope and the concepts we are discussing as a part of Six Sigma course must be helping you. So, now as a part of lecture 50, I want to talk about Operating Csharacteristic Curve for Variable Control Charts. So, we have seen in detail the design of variable control chart in the previous lecture. Now I want to discuss the operating characteristic curve for variable control chart as a part of lecture 50.

(Refer Slide Time: 01:02)



Once again I would refer this statement and faith of a great, quality guru Imai that control charts are one of the statistical tool, for solving the quality related problem. He shows faith that yes, control chart can definitely help you to solve the problem and do the fact based analysis investigation.

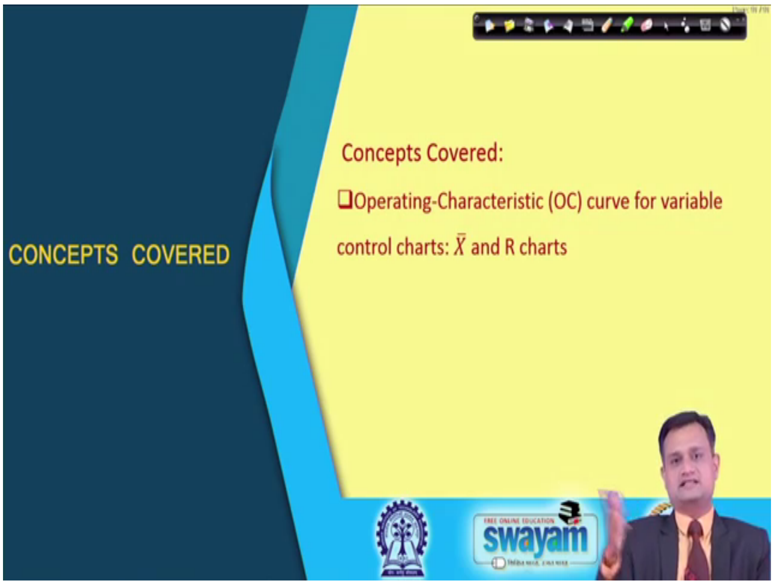
(Refer Slide Time: 01:28)



So, we had a detailed discussion on need for control for both mean and variability. I may have both at nominal say mean as well as variability, but I may have larger mean larger variability for both. So, in that case my products or services will fall in the rejection region and basically I would be producing defective which is not acceptable to the market and customer.

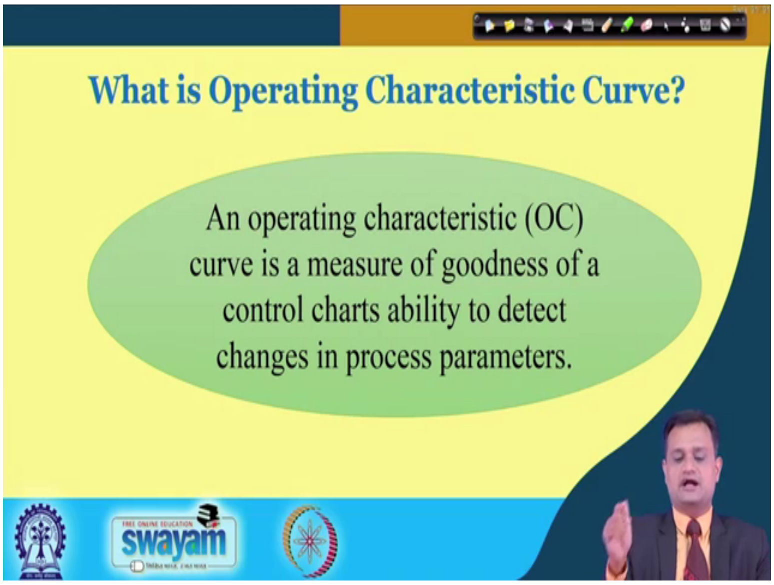
So, if I have scrape the entire value is lost, if I have re work then again I have to do some processing so that will further add the value and then else I can sell it to the market. So, I will have extra additional cost to be incurred. So, both the situations, I do not want to enter. So, it is better that let me have my process within the control and we had seen the procedure for constructing X bar and r chart.

(Refer Slide Time: 02:34)



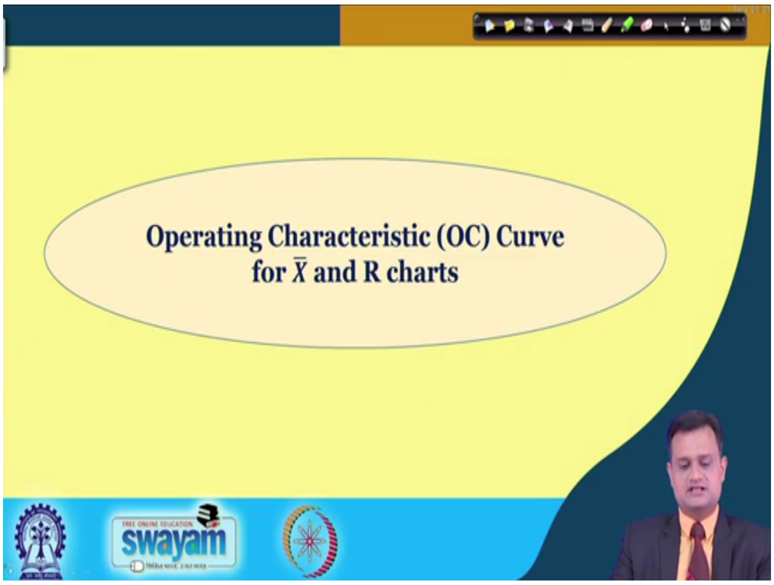
And the detailed example on X bar, S and r chart.

(Refer Slide Time: 02:42)



So, now this lecture basically will talk about OC curve for the X bar and r chart.

(Refer Slide Time: 02:46)



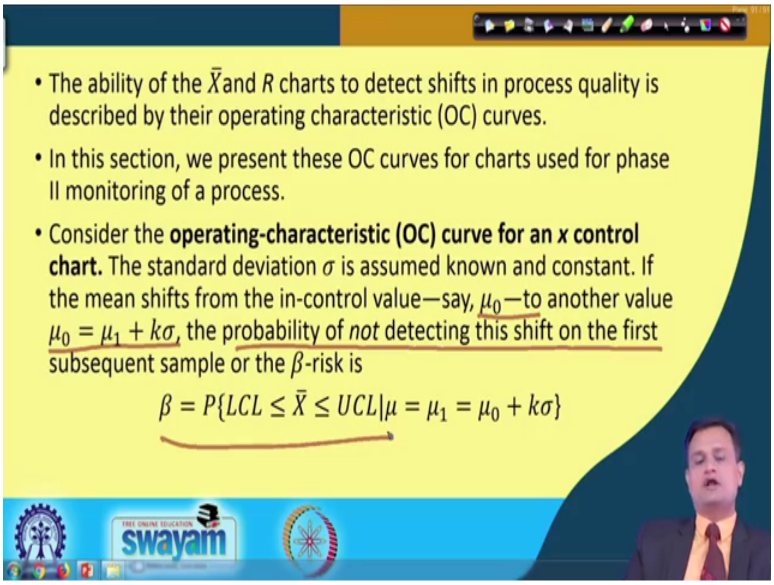
So, now just if I just hide this particular slide then tell me what is the OC curve? We have already discussed when we started with the preliminary concepts and basics on control charts, I have already talked about OC curve. Can you recall? What is operating characteristic curve, known as OC curve? So, fine I hope you have some ideas in your mind. So, let us try to brush it up. So, OC curve and operating characteristic curve is a measure of goodness of a control chart ability to detect changes in the process parameter. You have a gun but it cannot fire, you have a sword it cannot be operated; it is blunt, you have a knife it cannot cut.

So, now we need to have a tool which has the power typically called discriminatory power really to separate out good and bad. So, operating characteristic curve is a measure of goodness, how good my control chart is about the detection of change in the process parameter. There is a change in the process parameter maybe mean or maybe variance, variability and my chart is not able to detect it. Then what is the use of this chart? Unnecessarily I will be spending money on the maintenance of such chart and it is just like vital effect. So, something can really be beneficial, it can have a utility if it serves the intended purpose for which it is developed.

So, here my control chart is developed to tell me that something is going wrong, or something is going right and if it cannot tell me then there is a question suspicion on the goodness of the tool goodness of the control chart and I have no use of maintaining such kind of system. So, typically OC curve is the discriminatory power captures the discriminatory power of my control chart.

So, operating characteristic curve for X bar and r chart we will see, we are talking about the operating characteristic curve for variables.

(Refer Slide Time: 05:27)



Now, just see that there is a type 2 error and typically we call it as a beta. So, the ability of the X bar and the r chart to detect shifts in the process quality is described by their OC curve and we will discuss on this part. So, just think that there could be 2 things that can change. One is the shift in mean other is shift in the variance.

So, your variability has got shifted from let us say sigma o to sigma 1 and your mean has shifted let us say from mu o to mu 1, both the things are possible do not get confused by the notation its very simple.

So, my process may have shifted to a new mean maybe my bearings are worn out, maybe there is a wear and tear, or maybe there are vibrations, or maybe there is an issue with the scale of the operator and same is true with the variability. So, now, I have beta that is type 2 error and let us say mu 0 is shifted to say another value mu 0 is mu 1 plus k sigma, the probability of not detecting this shift is basically the beta risk. So, is type 2 error. Again I will like to remind you that, a jury is listening the case and there is a person who is guilty.

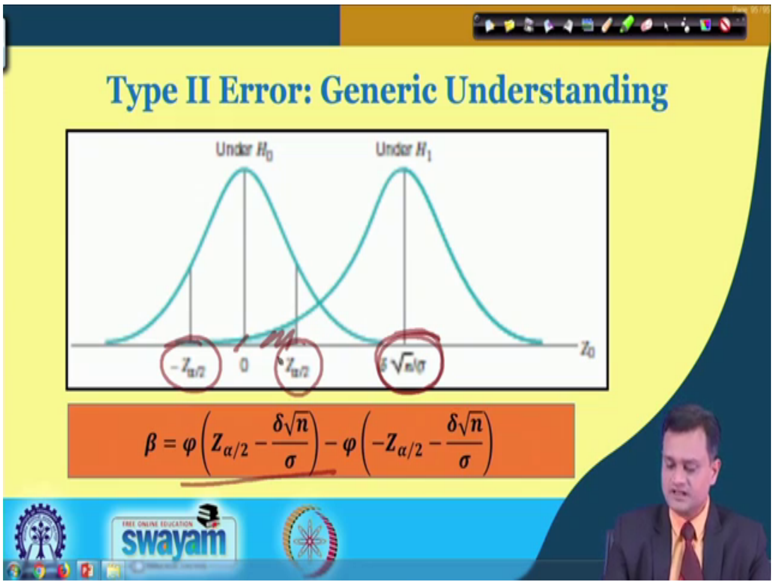
So; obviously, he must be punished, but based on whatever evidences they have they conclude that the person who is really guilty is innocent. So, in this case I am trying to make type 2 error that is beta if I convert now this into the process, I will say the process which is really out of control or the process which is showing the assignable cause, it means it is guilty. So, the process where the mu is got shifted or sigma has got shifted I say that the process is in control, it is innocent.

So, this is where you will find that I am committing, the type 2 error. We cannot hundred percent get rid of type 1 and type 2 error, please remember we always try to see the best trade off. When we are dealing with the sample we are not dealing with the hundred percent data so; obviously, my say analysis is based on the probability and I cannot get rid of type 1 and type 2 error, but yes I want to see the impact of these on my ability to detect and what can I do to improve the discriminatory power of my control chart.

So, I hope the idea is clear that why I am doing this. So, let us say my beta is basically my beta is probability that LCL is less than equal to X bar is less than equal to UCL, for given mu is equal to mu 1 is equal to mu 0 plus k sigma.

So, now, the mean has shifted to mu 1 and mu 1 is basically mu 0 for a given mu, mu 0 plus k sigma mu 0 is your original position, centre of your process. So, now, there is a shift. Now what is the probability that my X bar will fall within LCL and UCL? It means I am accepting that my reading is well within the specification, but actually there is something wrong. I say that it is the person is innocent, but it is guilty. I say that the process is in control, but actually process has gone out of control because there is a shift in the mean.

(Refer Slide Time: 09:47)



So, now with this understanding I think the remaining thing would be extremely clear. Just see I am repeating the generic understanding because the moment you have to plot the OC curve you need to know properly the type 2 error, your beta rays and just see this is the process under H 0, null hypothesis this is under H 1.

So, when I say H 0 means there is no shift in mean, my process is as it is status q is maintained. When I say H 1 yes there is a shift in mean and now this has got shifted from these to these and the difference or shift is delta square root of n divided by sigma; we have already seen this it is just a revision.

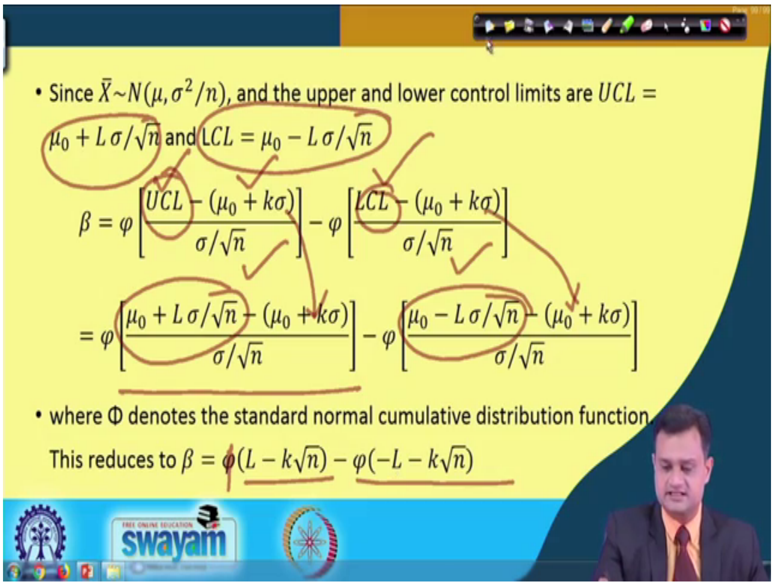
So, now I want to find the probability beta that is type 2 error and for this particular diagram H 0 H 1 my processes got shifted from here to here, I want to find the probability that tells me that when there is a shift in the process what is the region in this or what is the probability which tells me that the process has not shifted or my process is well with under control.

So, typically you will see that this is the region which will give me the type 2 error because it falls within, the acceptance, but actually my process has got shifted. So, after shifting please remember, if I am not considering the case of shifting it would be a different diagram. After shifting what is that region or probability which tells me the beta risk give me the beta risk that I cannot detect shift or there is no shift and status q is maintained?

So, this is my type 2 error and this is the region which I am focusing on. So, now, easily you can compute. We have already done this and just see that beta is equal to this is the cumulative probability, between Z alpha by 2 minus delta square root n divided by sigma.

So, it is the region between these two minus cumulative probability minus Z alpha by 2 and minus delta square root n by sigma. So, easily you can see that this is the region I am referring. So, fine this is my type 2 probability and it is called beta risk if you have understood it, I think we are done.

(Refer Slide Time: 12:26)



So, now, let us see I am just trying to convert the basic equation generic understanding into more specific one, otherwise the concept remain same.

So, let us say X bar which is the mean of my individual sample it is normally distributed with mu and sigma, variance sigma square divided by n. So, my upper and lower control limits are upper control limit is mu 0 that is as usual plus L sigma divided by square root n. So, this is the upper control limit. So, I am taking L as the constant to find the upper control limit sigma divided by square root n and mu 0 minus L divided by say my sigma square root n.

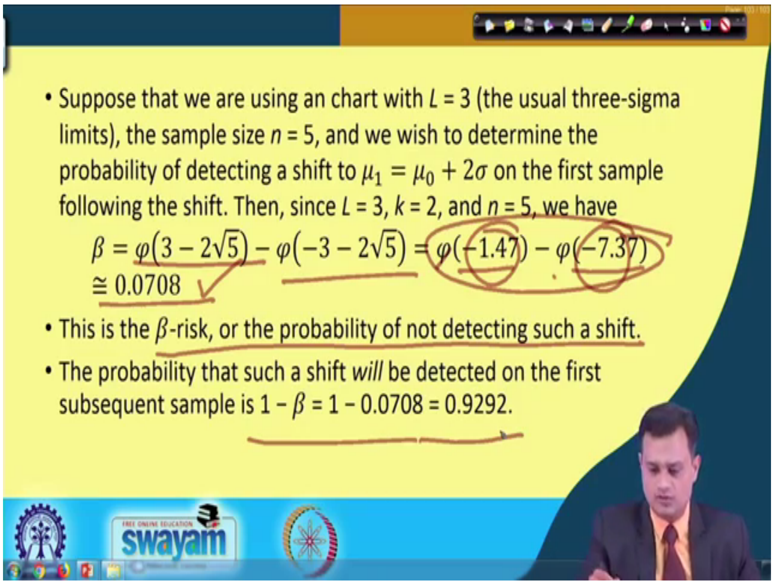
So, when you do this you get the upper and lower control limit and I am just trying to put these values into previous equations. So, these are the values. I can just go back to help you. I am just trying to put these values in the previous equation beta cumulative value of this.

So, this will basically give me if you carefully observe put them one to one; UCL minus mu o plus k sigma divided by sigma square root n, LCL minus mu o plus k sigma divided by square root sigma divided by square root n. So, I can now further replace the values of UCL by this particular one.

So, mu o plus L sigma square root n LCL is this, I am just putting this value here and this is already there this is the new mean. So, you have basically the expression X minus mu by sigma. So, now with this when I simplify this particular term is reduced to L minus k square root n, this is the cumulative probability my phi and this is minus L minus k square root n.

So, exactly this expression is nothing but it is just an outcome of what we have referred here, it is this equation. So, please do not get confused this is very simple and now let me try to see the development of OC curve and how I can interpret it for X bar and r chart.

(Refer Slide Time: 15:02)



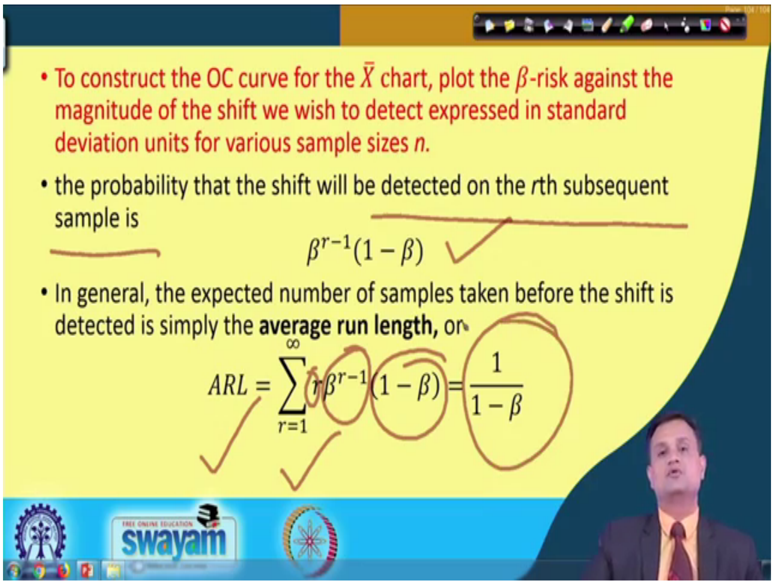
So, I will just put the values. Let us say I am taking the value. Suppose that we are using a chart with L is equal to 3, the usual 3 sigma limits, sample size is n is equal to 5; we wish to determine the probability of detecting mu 1 is equal to mu 0 plus 2 sigma. So, what does it mean? It means that my mu from the original position it has shifted to 2 sigma, and mu 1 is mu 0 which is the original one plus 2 sigma.

So, I want to detect the probability, I want to check that what is the potential discriminatory power of my control chart in detecting such kind of shift. So, I have L equal to 3, k is equal 2, n and is equal to 5 and just see this that in this expression what you need? You need L, you need k, you need n and I have all the 3 values available. So, I will just substitute these values here.

So, I have 5 3 minus 2 square root 5 minus phi minus 3 this and this is minus 1.47 minus 7.37 and you can get the crisp values of this from the table Z table and you will get the probability value these are the crisp values. You will get the cumulative probability for this from this Z table and your beta is basically type 2 error that is the probability 0.0708.

Now, what is this probability? It is the beta risk and typically it is the probability of not detecting such a shift. Person is guilty; I am saying innocent, process is out of control; I am saying it is in control. So, it is my beta risk inability to detect change in process mean shift or change in my variability. So, I will say 1 minus beta is just the complementary event, that the probability that such a shift will be detected on the first subsequent sample. So, 1 minus beta is my 0.9292.

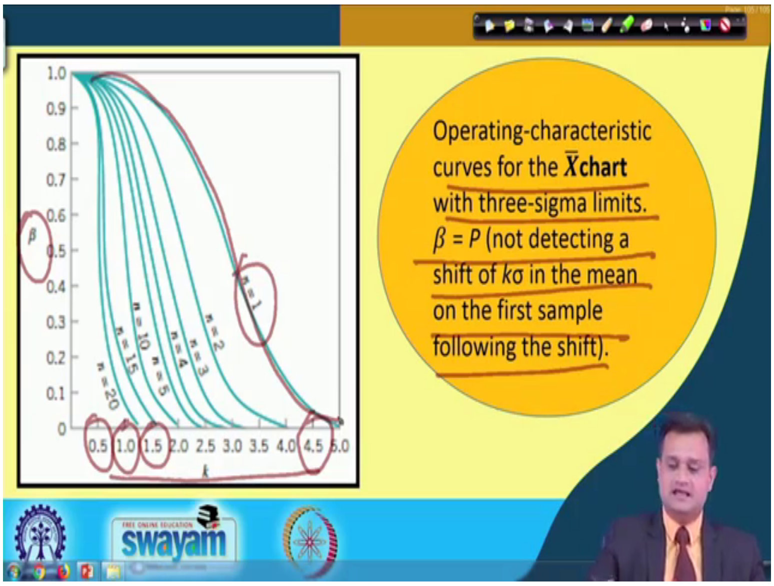
(Refer Slide Time: 17:28)



Now, once you are done this the remaining task is very simple. So, you have to construct the control chart sorry OC chart OC curve for the X bar. So, you need to plot the beta risk against the magnitude of the shift. We wish to detect in standard deviation units, for various sample size and in standard deviation unit means my mean got shifted by 2 sigma, 3 sigma, 4 sigma what?

So, the probability that shift will be detected on the rth subsequent sample can be easily expressed as beta r minus 1 1 minus beta, I am considering both the event as well as complementary events and my average run length will be you have already seen this in detail sigma r is equal to 1 to infinity r beta r s 2 r minus 1, 1 minus beta and the simplified version of this is 1 upon 1 minus beta.

(Refer Slide Time: 18:27)



So, let us try to see the graphs that will help us. Now this is the operating characteristic curve for X bar chart, with 3 sigma limit and beta is equal to probability of not detecting is shift on the first sample, following the shift means, shift has occurred. After shift has occurred I took the sample and I am not able to detect this on the first sample after shift has occurred this is typically my beta probability of non detection type 2 error.

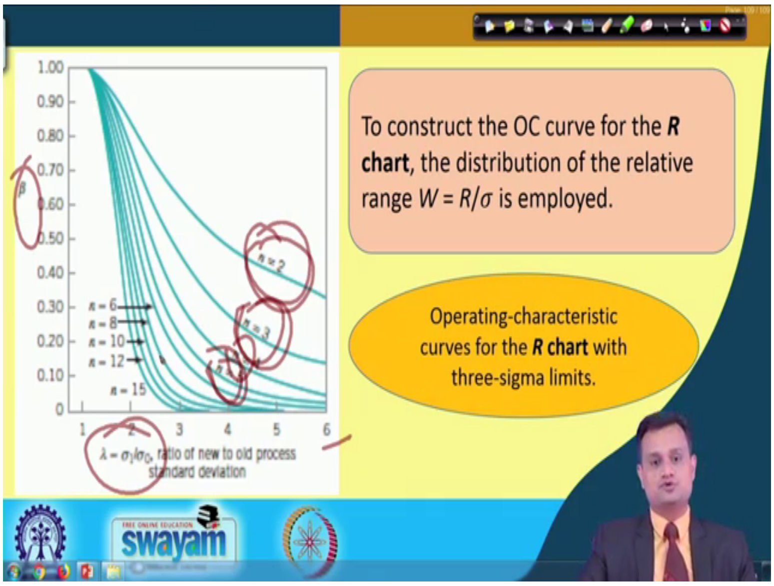
So, you have beta and this values you can compute as we have seen these sample calculation this is k. So, what does it mean? There is k sigma shift. When I say 0.5, it means you have 0.5 sigma shift in mean, 1.0 means 1 sigma shift in mean, 1.5 means 1.5 sigma shift in mean, 4.5 means 4.5 shift in mean, now you have the sample.

So, you can easily plot this curve which is called operating characteristic curve by finding the value of various values of beta type 2 error for your shift in say mean by k sigma and beta expressions basically, if you recall it needs your value of n, it needs your sigma, it needs the basically k shift sigma shift, I can just show you and it needs the n, it needs the k, it needs the n. So, you have these values and for different values you can plot a particular OC curve; particular OC curve. Now each OC is for particular sample size. Now what do you see here?

I have the OC curve for n is equal to 1, n is equal to 2, n is equal to 3, n is equal to 4, 5, 10, 15, and 20. Just let me draw a line and let us say there is the shift of 2.5 then I am just drawing a line. Suppose I am referring n is equal to 1 OC curve this is the type 2 error this is the type 2 error for n is equal to 2, this is the type 2 error for n is equal to 3, this is the type.

So, now, what you can see that as I am increasing the sample size my curve is become steeper and steeper, and this says that the discriminatory power of my OC curve has increased with increase in the sample size and hence I would be committing less type 2 error. So, this is a very important observation please remember.

(Refer Slide Time: 21:25)



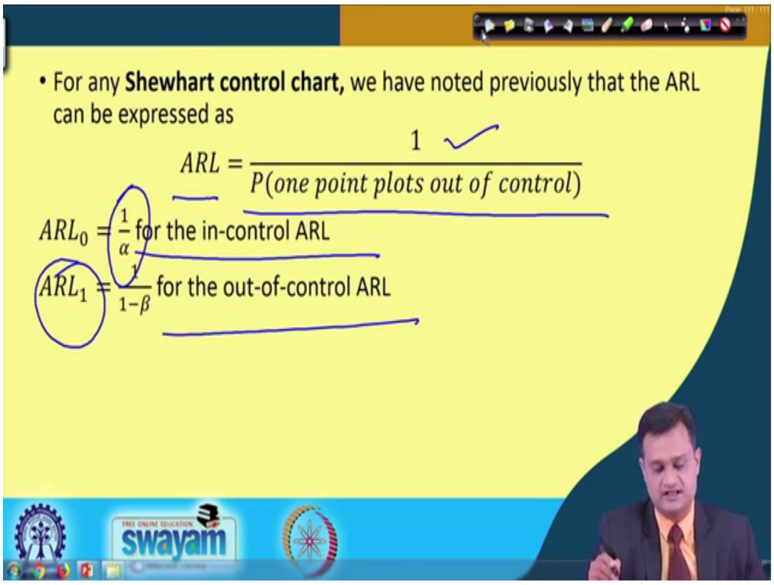
Similar way I can plot the OC co for R and here I am considering lambda is equal to sigma 1 divided by sigma 0, because for R chart I am considering that there is a shift in the variability my original variability let us say sigma 0 it has got shifted to sigma 1.

So, now only this value will change remaining calculation will follow the same expression, and you can compute the value of beta for the given n 2, 3, 4, 5, 6, 7, 8, 9, 10. Again you will see as the n increases the discriminatory power means stiffness of my OC curve increases, and hence the probability of type 2 error that is beta it decreases.

(Refer Slide Time: 22:22)

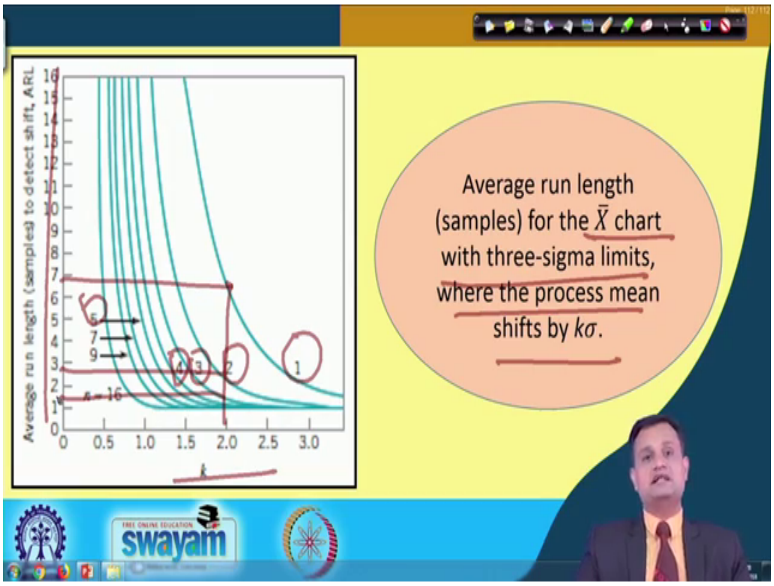
So, I think it is a very interesting discussion. Now the another performance measure, there are 2 performance measures, we discussed; number 1 is the operating characteristic curve and number 2 is the average run length. On an average how many sample I need to take before I really detect the change in the process and this is typically called my average run length. So, I have to performance measures; one is OC other is ARL.

(Refer Slide Time: 22:50)



So, here your ARL is basically 1 divided by we have seen this in detail. So, ARL is 1 by probability, one point plots out of control. So, this can be given by 1 by alpha, and this is in control ARL. So, this is for in control ARL and when I say ARL 1 for the out of control 1 upon 1 minus beta.

(Refer Slide Time: 23:21)

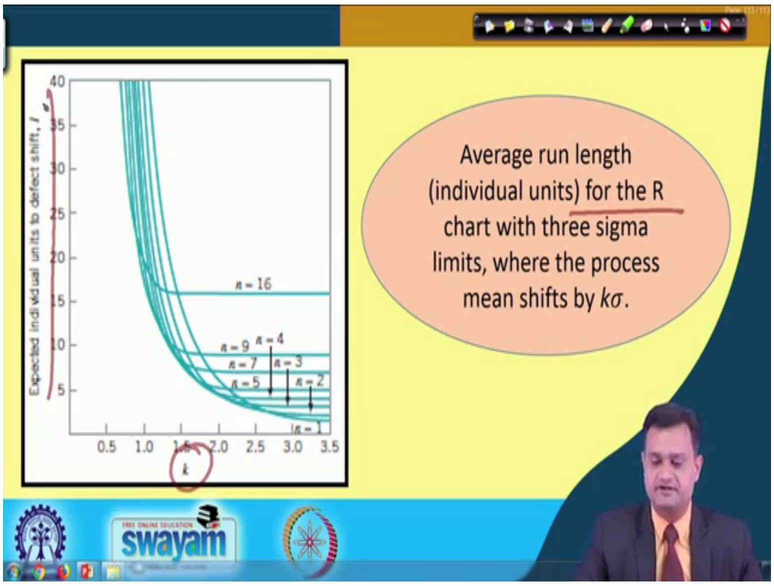


So, we have already seen this expressions in detail. Now just let us try to appreciate this in terms of the OC curve. So, or typically ARL curve here. So, average run length curve for X bar chart with 3 sigma limit by k sigma shift this remains same the logic remain same, here instead of beta I am putting the average run length. So, now, what you see here these are the values of n.

So, as my n increases again I am getting the better discriminatory power my say average run length for 2 let us say here for this OC curve n is equal to 1, average run length is let us say about 6.5, but if refer 2, it is 2.5 if I refer these maybe it is 1.5. So, what does it mean? It tell that if I am increasing the sample size then on an average number of sample required to take to detect a change in process reduces, it means I would be able to detect the change early.

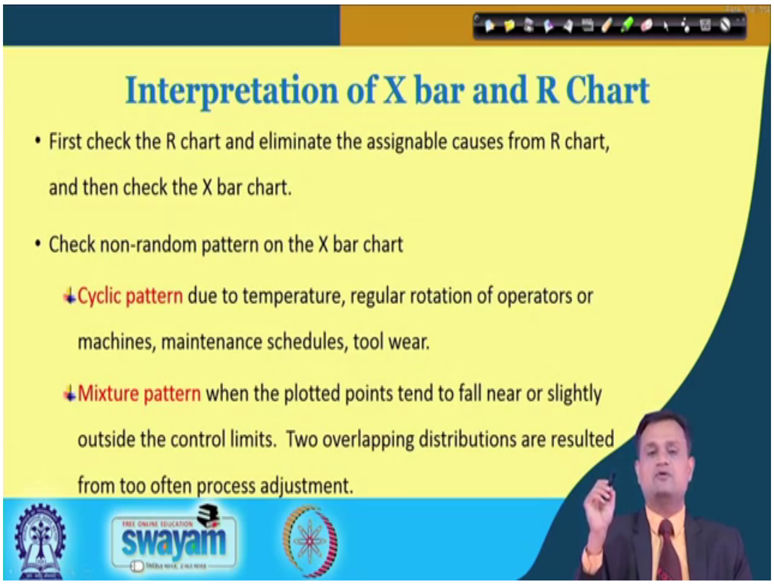
So, here again say with increasing n my discriminatory power in terms of ARL is increasing. So, obviously, when you increase the n, the cost also is the important consideration. So, I will not say that, if you go for more and more sample size higher and higher sample size it is better, you need to be judicious in the sample size and the discriminatory power you need for a given quality characteristics.

(Refer Slide Time: 25:08)



So, fine, this is something that is really interesting and same way you can put it for R, but here for a k I am just trying to see the expected individual limits to be detected shape and this you can do very easily.

(Refer Slide Time: 25:27)

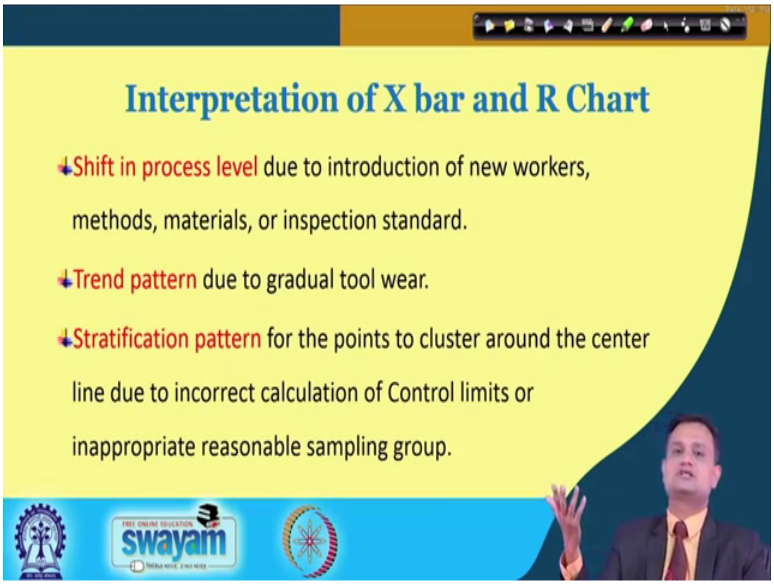


So, interpretation of X bar R chat, I have couple of interpretation in terms of some of the non random pattern prevailing. So, there could be a cyclic pattern. Your chart is showing all the points within the control limit, but there could be some cyclic pattern present. So, this may be due to temperature, regular rotation of operator or machine maintenance schedule tool we are, there could be something called mixture pattern.

So, when the plotted points tend to fall near or slightly outside the control limit little bit, so, near or outside the control limit 2 overlapping distributions are resulted from too often process adjustment.

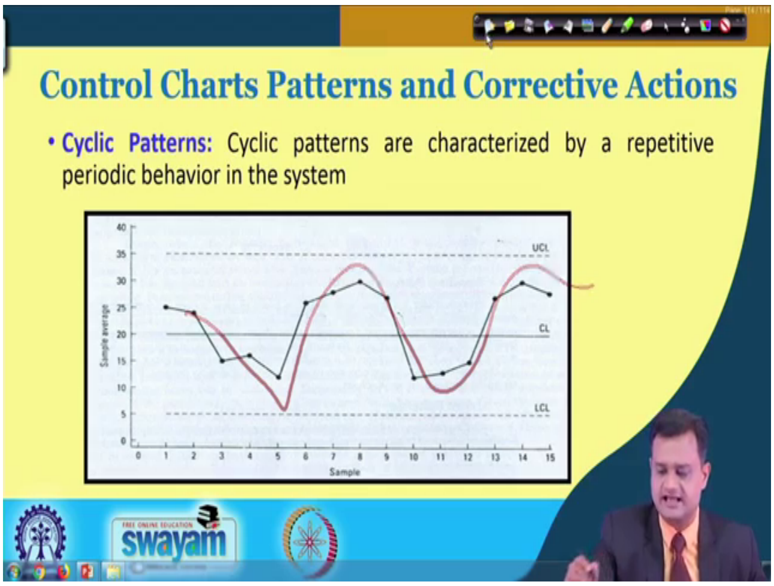
So, if you are adjusting your process too often then it means, that the resulting output may follow different distribution 2 different distribution and sometimes you may get a point very close to control limit you may get a point slightly outside the control limit.

(Refer Slide Time: 26:29)



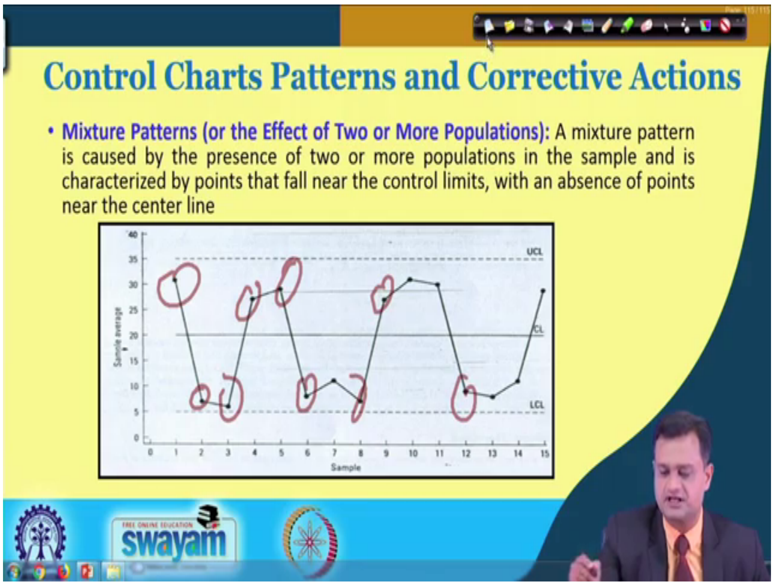
So, this is the mixture pattern. Shift in process level this may be because of say introduction of new worker, method, material, inspection standard. So, there could be shift in the process level. You may find trend pattern and this is mainly because of some wear and tear or gradual tool wear, which is a very obvious phenomena in machining. And there could be a stratification pattern. So, points to cluster around the centre line. So, you have a central line and you will find that points they are forming a cluster around the central line due to incorrect calculation of the control limits or inappropriate say or reasonable sampling group. So, these are couple of reasons.

(Refer Slide Time: 27:18)



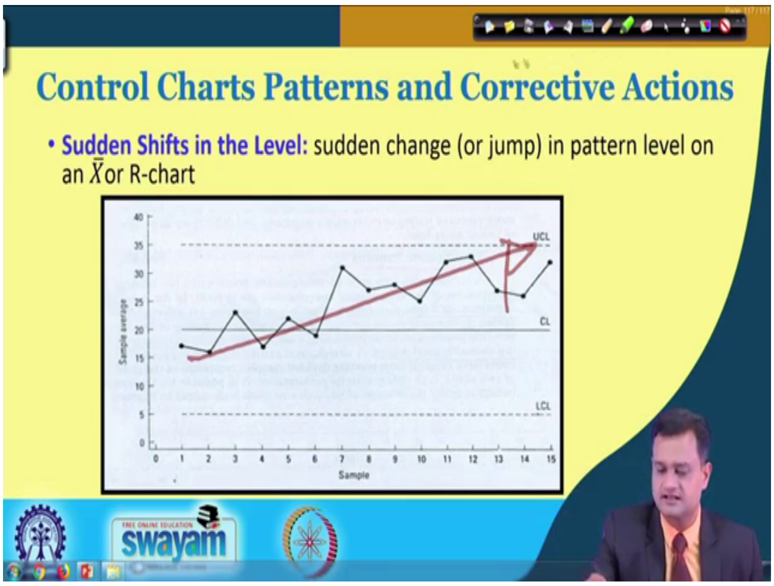
So, let us try to see each 1 through a picture so that idea becomes extremely clear. It is a cyclic pattern you can see, the cycle is formed the reasons I told.

(Refer Slide Time: 27:33)



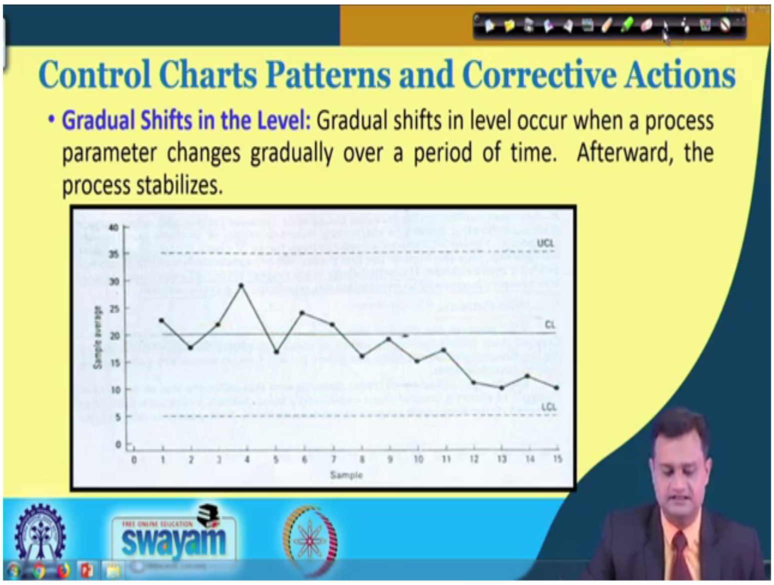
You can see another that is the mixture pattern and you see the points are falling very close to the control lines maybe upper or lower and this is typically called mixture pattern.

(Refer Slide Time: 27:46)



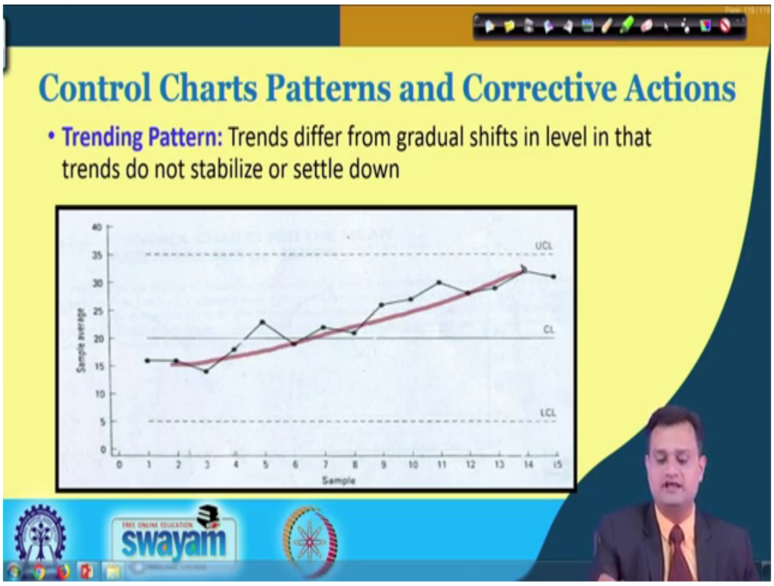
So, 2 populations are mixed, 2 distributions are mixed and sudden shift in the level you can see there is a gradual increase and sudden shift in the level.

(Refer Slide Time: 27:58)



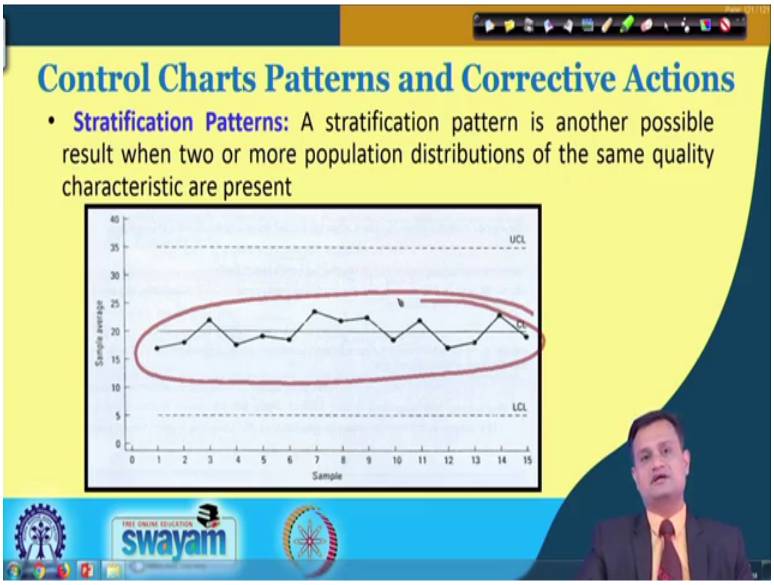
You can see gradual shift in the level. So, shift is taking place, but it is gradually and this is your gradual shift in level.

(Refer Slide Time: 28:08)



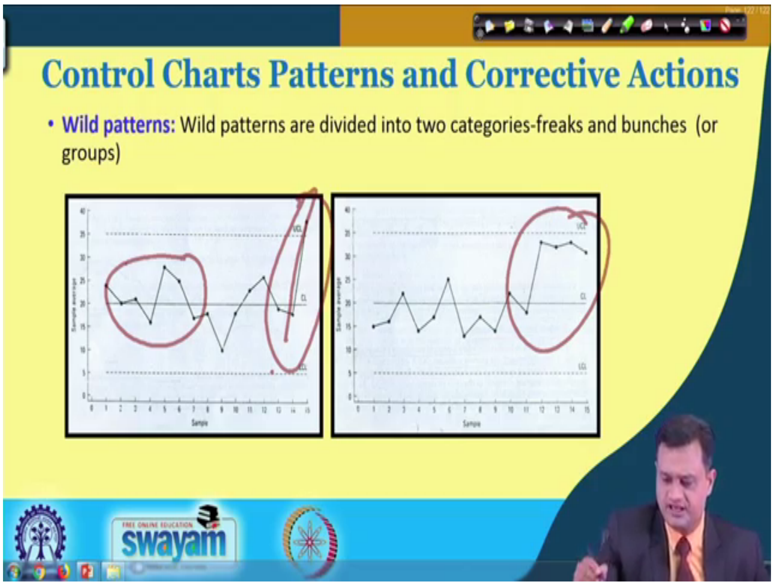
There is it trending pattern. So, some trend you can observe maybe because of wear and tear of the tool bearing or some other say component or there maybe some error in the measuring instrument.

(Refer Slide Time: 28:22)



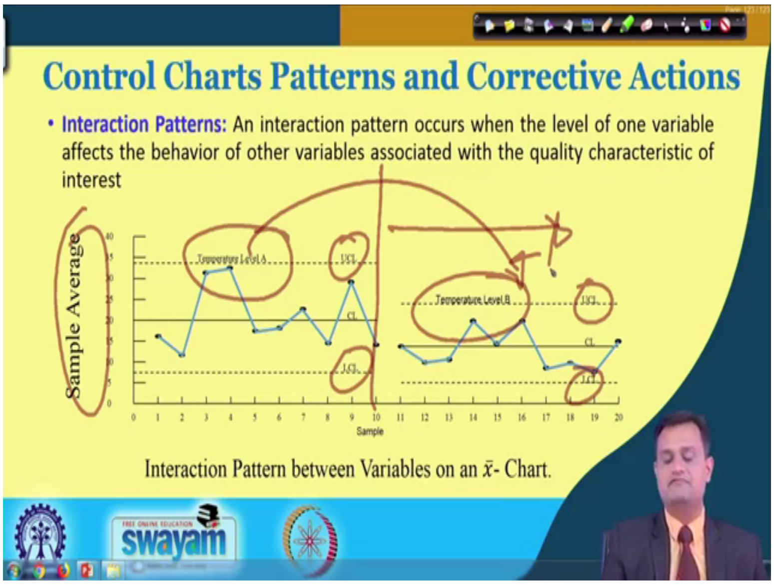
There is a stratification pattern and what do you say that points are more or less concentrated around the centre line and this kind of stratification pattern maybe because of wrongly calculated upper control or lower control limits or maybe some problem in the sample size or reasonability you have criteria you considered for number of sample and sample size.

(Refer Slide Time: 28:52)



So, this is what you can appreciate. There could be a wild pattern and what you see that up to some extent it is well with you and then suddenly it jumps and there is a wild pattern, again suddenly it jumps and there is a wild pattern. So, there are some of the typical causes may be let us say because of some reason that is an excessive vibration in the machine and you may find that there is a sudden jump or there is a wild wear patterns.

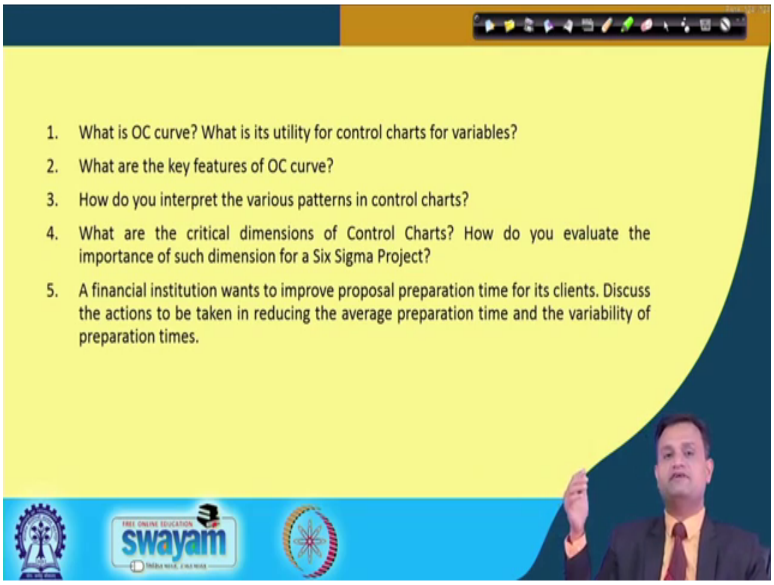
(Refer Slide Time: 29:20)



So, typically we go through different patterns and we need to appropriately interpret it. One more pattern that you can see is the interaction pattern. So, here what do you see that you have 2 variable, and if let us say one variable affects the behaviour of the another variable, then there is some say interconnected or interaction pattern present. We have seen in ANOVA that there could be an interaction effect can be observed it is statistically.

So, let us say these are the sample averages as usual, this is my upper and lower control limit this is my upper and lower control limit. Now this is temperature level A and this is temperature level B and suppose if change in level can have some introduction or impact you would find that after sometime say when you have change from level A to level B, after let us say 10 sample you have different pattern to observe. So, this could be because of interaction.

(Refer Slide Time: 30:33)

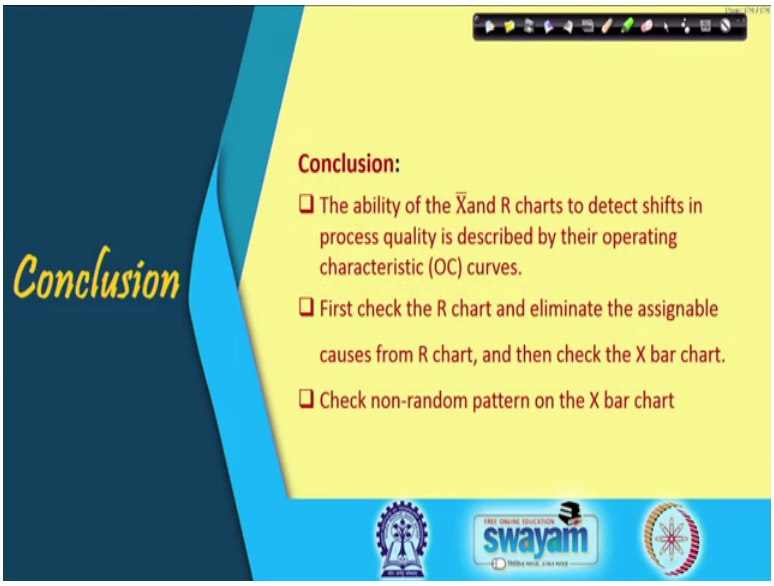


Now, before we end, let me put couple of thing in question that can help you to introspect and revised, what is OC curve, what is its utility for control charts for variables, what are the key features of OC curve, how do you interpret the various patterns in the control chart, and what are the critical dimensions of the control chart? There is a small situation. A financial institution wants to improve proposal preparation time for its client. Discuss the actions to be taken in reducing the average preparation time and the variability of the preparation times.

(Refer Slide Time: 31:11)

So, I think with this you have acquired the adequate knowledge on operating characteristic curve for control charts for variables and these are the references you can use to have better understanding on this particular topic.

(Refer Slide Time: 31:27)



So, OC curve is a discriminatory power and is a measure of goodness of my control chart and typically it relate the specific shifting in mean or variability with the type 2 error. As I increase the sample size the discriminatory power increases, but also I have to see the economics and the one which can give me the best trade off I must go for that particular operating characteristic curve.

So, thank you very much for your interest in learning operating characteristic curve for variables control charts and keep revising take some data hypothetical or real I emphasise again and again and try to internalize the concept.

Thank you very much, be with me, enjoy.