Six Sigma
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Lecture – 49
Statistical Process Control: Control Charts for Variables

Hello friends, I welcome you once again to our ongoing journey on Six Sigma and we are in the final phase of over six sigma DMAIC cycle and we are discussing the control phase. We have already talked about control chart, preliminary concepts, 7 QC tools and now as a part of lecture 49 we would like to discuss the Statistical Process Control; Control Charts for Variables.

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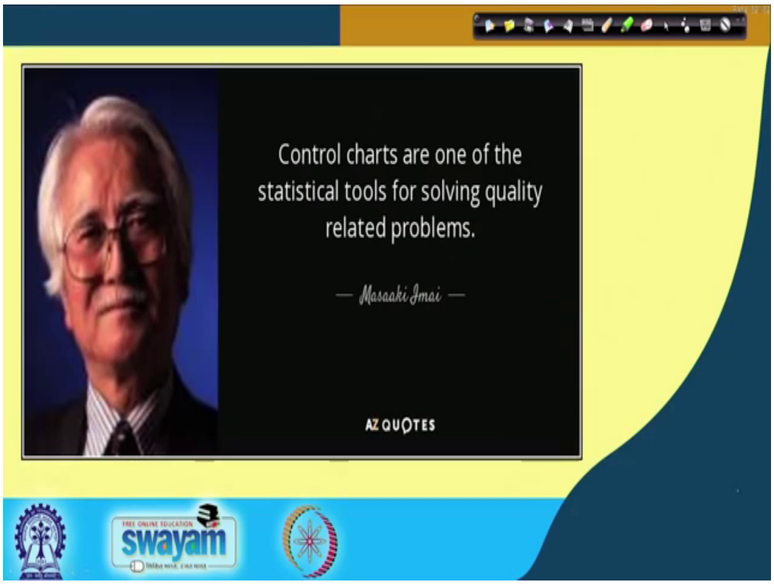

So, this is a very important topic and I hope you will appreciate it. So, control charts are one of the statistical tools for solving quality related problems and this is said by say Masaaki Imai a very well known person in the domain of quality and control charts they basically act as a tool to diagnose, that what is the problem? And then subsequently we can investigate the reason behind the problem.

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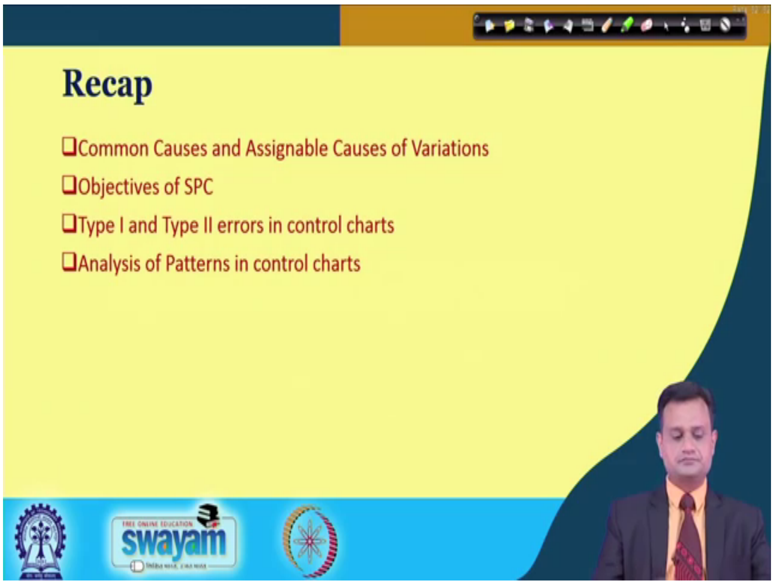

So, just to have recap we have talked about common causes, assignable causes of variation. Objectives of statistical process control, type I and type II error in control chart and analysis of patterns in control chart. So, please remember that my variability maybe because of some chance or random causes that is not very much problematic.

But there could be assignable cause because of which there is some change in the process, some shift in the process and unless I correct it I cannot have an appropriate control over the process. So, control chart basically draws my attention to such kind of problem and then I can take the corrective action to get hold on my processes.

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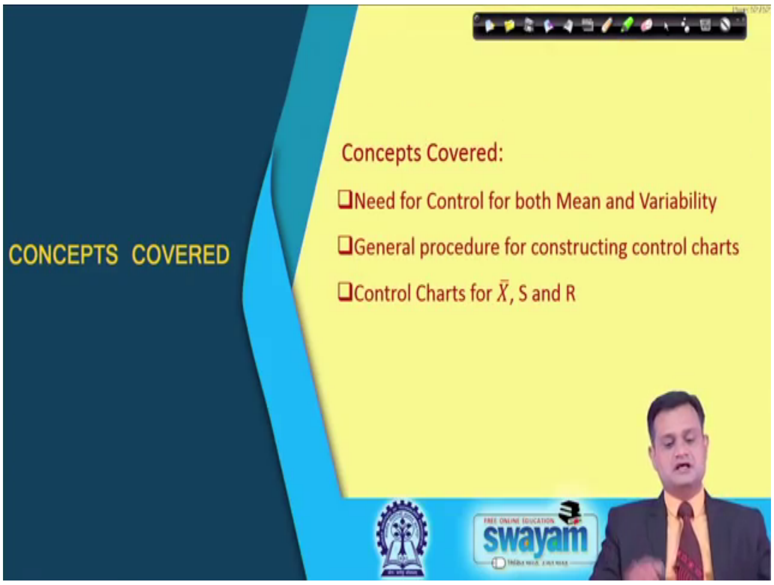

So, this particular lecture we will try to focus on need for control for both mean and variability. So, it is not only mean or variability we need to control mean as well as variability both in order to see that my process is operating at the target value, sat value, nominal value and the variability means the component to component, part to part variation is also well within the limit.

General procedure for constructing control chart we will see and that will remain same, for all different kinds of chart and typically this lecture basically we are discussing the control charts for variables, so we will talk about control charts for X bar, S and R.

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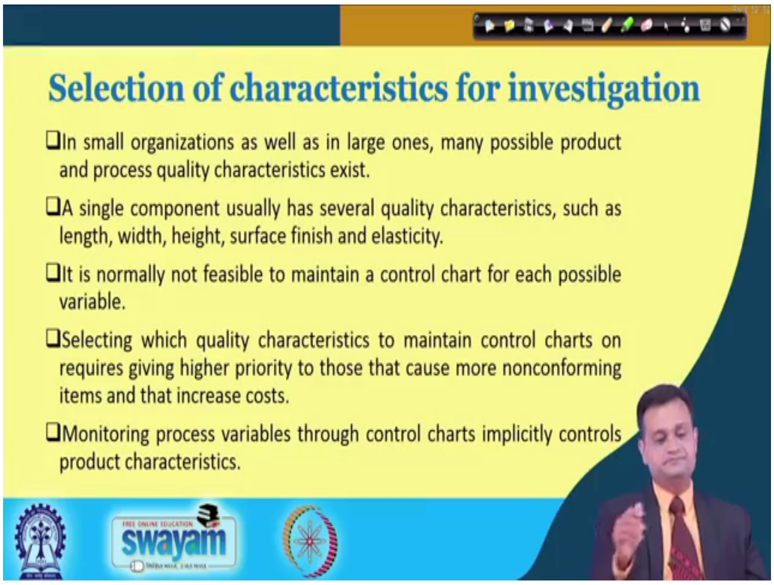

So, the first important thing is about the selection of the characteristic for investigation. Now you think about an organisation and suppose there even manufacturing se ball bearing or piston or engine or a maybe v belt or maybe the service organisation and their customer waiting time.

Now just think that they are lot many quality characteristics and its not possible or economical to focus on all the quality characteristics and maintain the control chart for every quality characteristics. So, we have to critically think and figure out that what are those quality characteristic which are really problematic and can a can cause a critical problem in the system.

So, once I identify such kind of quality characteristics, then I will only maintain the control chart for those characteristics and not for each and every characteristics specific to product or process.

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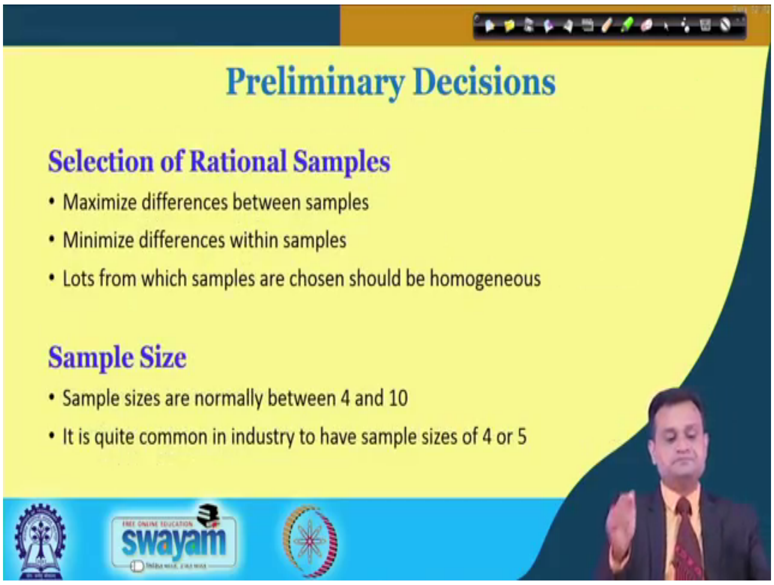

Then there no that is the preliminary decision before I go for the control chart development and design, so selection of the rational sample. So, I have already mentioned and once again I would emphasize that a simple must try to maximize differences between samples and within sample this variability must be minimized. So, let us say subgroup or simple 1, sample 2, sample 3 let there be say variability and this variability be maximized or must be captured because let us say sample 1 is drawn from the shift 1 production, sample 2 is from shift to and so on.

But within sample I expect that my sample should be such that within sample the variability is minimised. Lots from which the samples are chosen should be homogenous. So, any kind of heterogeneity let us say you are purchasing the material from 4 5 different vendors and then you are mixing it and then if you are drawing the sample I think it would not be a good idea. Sample size normally as a rule of thumb, we are happy with 4 and 10. So, 4 to 10 between I can choose the sample size of my sample and this is a quite common practice to go with 4 to 5.

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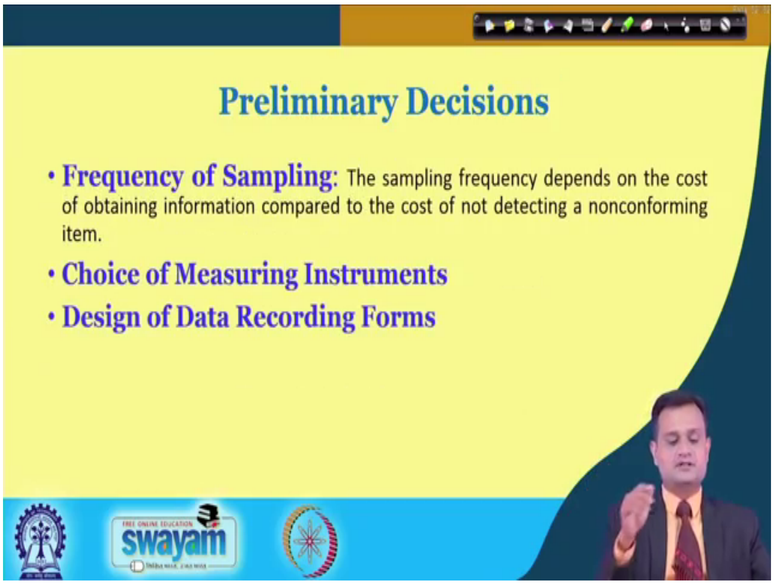

So, another important decision that I need to take before I go for the control chart is frequency of sampling. So, now, I will take a sample of sum size sample size maybe n is equal to 4 or 5, but what should be the frequency? Should I take the sample every 10 minute, every 30 minute, every hour once in a shift, once in a day.

So, here the logic is very simple the cost associated with frequency of the sample and hence the inspection and the cost associated with non detecting, the non conformity and its consequences must be critically seen and then depending upon the trade off between these two I must decide the frequency of sampling.

Then choice of measuring instrument, say you may have maybe different type of instrument maybe vernier calliper micrometre, maybe digital manual. Now you really need to see that what is that range within which a particular say measurement needs to be taken and if you want a title control than more precise instrument is recommended and also this instrument must be calibrated.

So, you must check, list count precision of the instrument whether it is a regular or digital or high end. So, depending upon the criticality of the quality characteristic and the band within which it should be measured or the tolerance is which you have to say fixing you must decide about the measuring instrument.

Design of data recording forms, see many times please understand that this data is basically recorded by the machine operator and then subsequently it is analysed by the quality inspector or the manager, but or data is recorded by the quality inspector depending upon your system. But whatever it is you need to have when simple form a systematic form so that an operator or maybe the quality inspector should not make any mistake in data recording.

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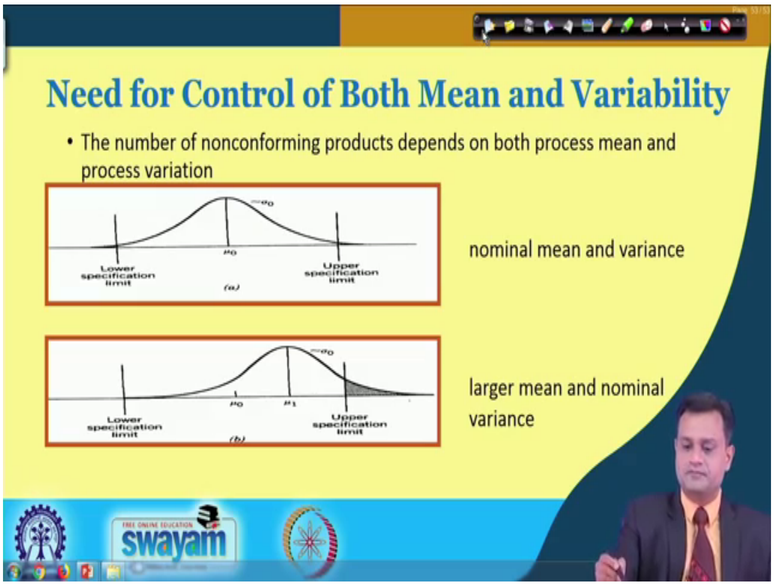

Now, let us see couple of cases to appreciate the need for control for both mean and variability. So, I am talking about the control chart for variables and I would emphasize that I must have X bar chart that used for mean and I must have R or sigma or S chart that is for variability and these two are complementary to each other. So, I will not say that only go by X bar or only go by R.

So, the reason is we need to control both mean and variability, if you just say the first one then you will say that lower specification limit, upper specification limit mu 0 there is no shift and the variability is well within these two upper specification and lower specification. So, you are not producing any defective mania achieving the nominal mean and variance, but if you look at the second case larger mean and nominal variance, then in this case there is a shift in mean mu 0 has got shifted to mu 1. So, there is the shifting and hence I am saying that larger mean there is a shift in mean, but variance is more or less same what we had here, but you are producing the defective component because it is going above upper specification limit. So, this is the second case, the first case both mean and variability are in the second case the problem is with mean got of shifted variability is.

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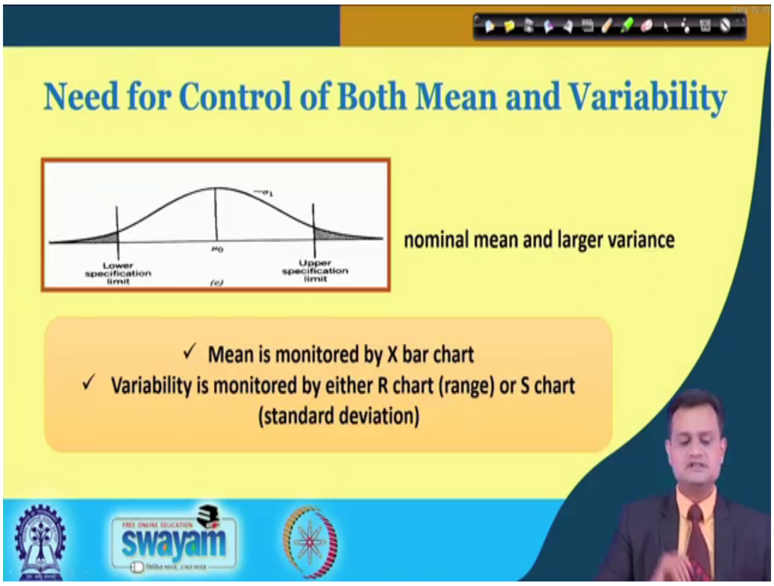

Now, if you see the third case, then in this case you have say phenomenal mean and larger variance. So, you will see that this variance is large and you are producing defective items on both the sides lower specification, upper specification and this is something is the third case where variability is really significant, but mean is mu 0 there is no shift.

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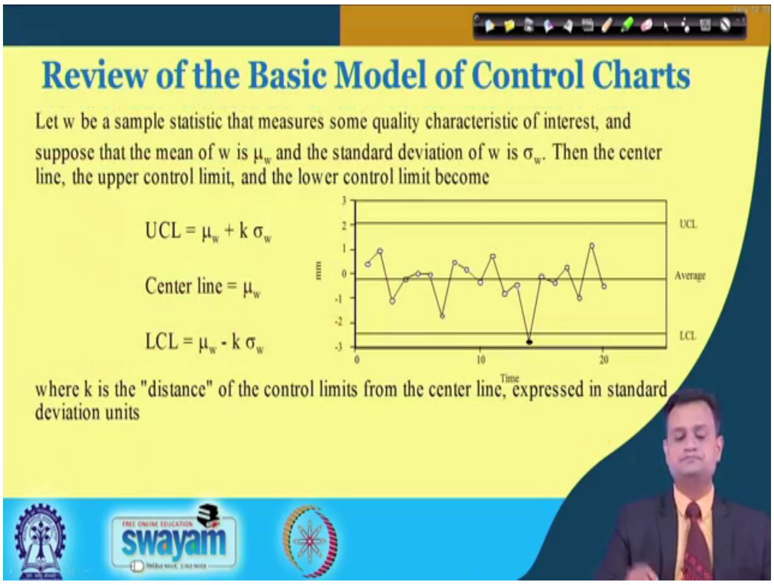

So, the different cases they can convince us that yes there is a need to monitor of both mean and variance and if you just see the review the basic model of control chart then I have the upper control limit. So, upper control limit basically is mu w plus k sigma w and typically you have the quality characteristics, some quality characteristics maybe piston diameter or the inner diameter of the bearing or the thickness whatever you have some quality characteristic of interest and suppose that mean of w; w is your some quality characteristic and sample statistics. So, w is the mean of w is mu w and the standard deviation of w is sigma w.

So, then I can is this, so mu plus or minus k sigma is my standard expression to set my upper control limit and lower control limit. I would also like to remind you that as a rule of thumb we go by plus or minus 3 sigma, the reason is it gives me the best (Refer Time: 11:32) of between type I and type II error. So, this part we had seen in the previous lecture.

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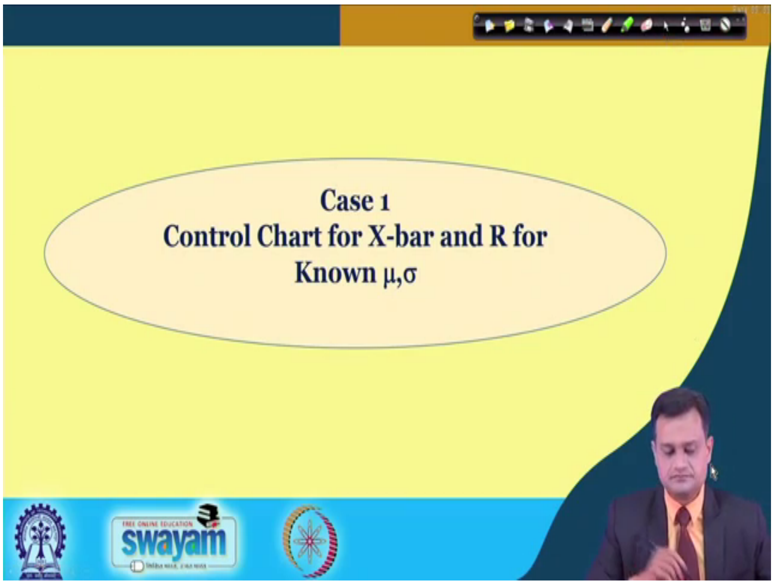

So, now we are moving ahead with the design of control chart for variables. So, before that let me try to analyse couple of things and I have 2 different cases to be considered. Case number 1 you have X bar and R chart for known mu and sigma. So, your population mean and standard deviation are known and under this situation I want to develop, I want to design my X bar and R chart. So, let us try to see what are the consideration here.

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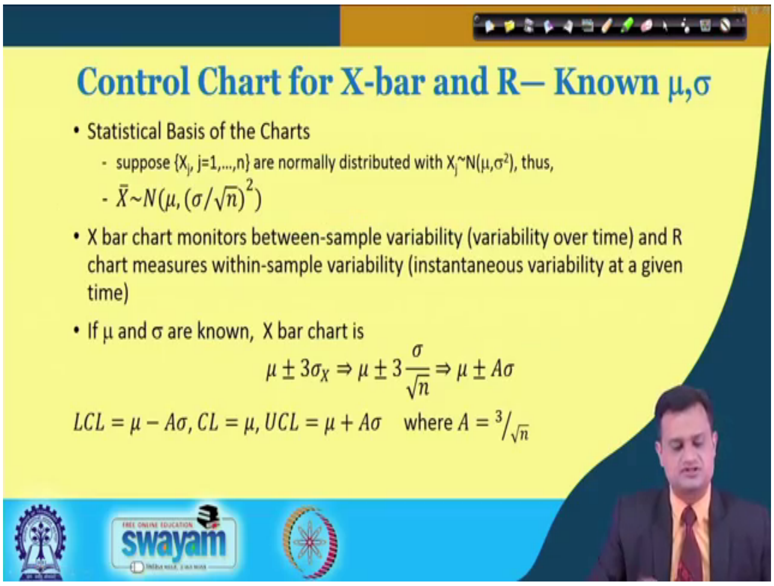

So, you have let us say X j is equal to X j is equal to 1 to n and this is normally distributed with X j n mu is the mean and sigma square is the variance and the distribution sample distribution is X bar which is also normally distributed and you can use this central limit theorem. So, mu and sigma by square root n whole square, so this is also normally distributed. Now X bar chart monitors between sample variability.

So, variability over at time and R chart measures within sample variability. So, please try to appreciate this part which is very important that when I say X bar chart my interest would be to see that whether there is any significant shift in the mean or not and when I say R then I am interested in capturing the variability, but when it is X bar it is between sample variability or subgroup variability between subgroup and when I say R these basically within particular sample or subgroup variability.

So, mu and sigma are known, so X bar chart basically mu plus or minus k sigma I am taking k is equal to 3 sigma X and this I can replace by sigma by square n as we have seen in mu plus or minus A sigma. So, now you will say what is this A, so we will use A, d2, d3, c2 many such kind of constants and this constants are basically standardized values for control charts and they are available in the appendix of the suggested textbook. So, mu minus A sigma mu plus A sigma and A is basically nothing, but 3 by square root n.

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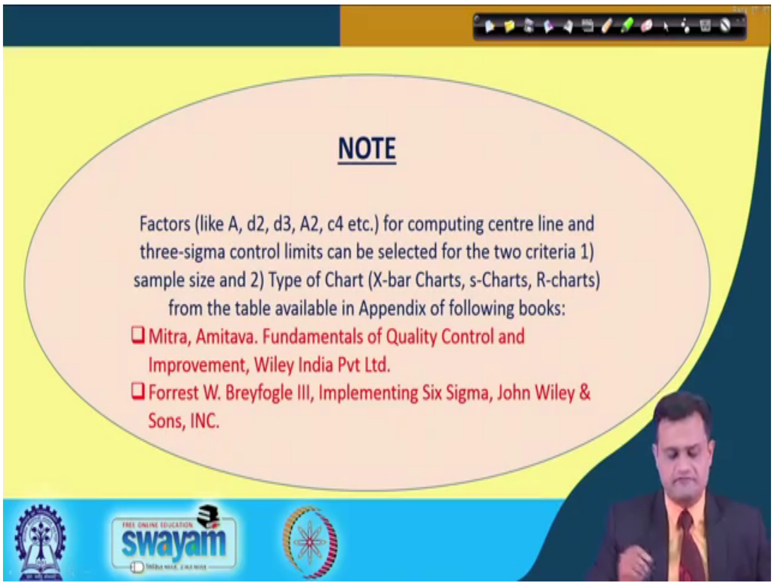

So, typically you need to refer couple of say standardise values. So, factor likes A, d2, d3, A2, c4 etcetera, for computing the central line and three sigma control limits, they can be selected from the table based on 2 criteria number 1 sample size and number 2 type of control chart X bar chart, S chart or R chart you are using.

And this you can find very easily in the following books like Mitra Amitava which we are mainly referring fundamentals of quality control and for rest implementing six sigma. So, you will find this table readily available and for a given say sample size and the control chart whether it is X bar or R or S or sigma, you want to set the control limit, you can find the values of this constants.

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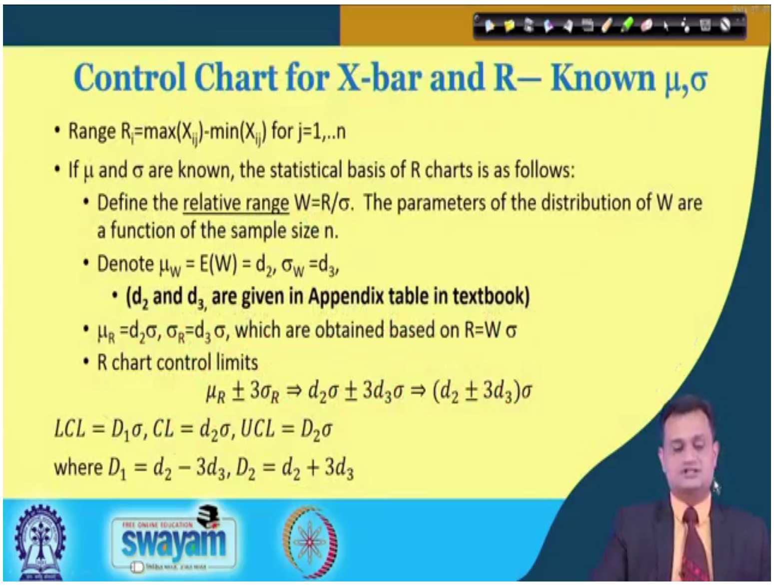

Now, let us say range. So, my R i is basically nothing, but maximum of X ij minus minimum of X ij. So, you have a particular sample and in this sample you have the maximum value, minimum value take the difference this is your range and mu and sigma are basically known. So, the statistical chart for R is like this. So, define relative range and I would say W is equal to R by sigma and the parameters of the distribution of W are a function of n.

So, typically it changes with respect to n and I will have mu W is equal to expected value of the W is equal to d 2 again a constant and sigma is d3 again a particular constant you will get from the statistical standard statistical table for control chart. So, mu R is basically d 2 sigma and sigma R is d 3 sigma, so based on R is equal to W sigma.

So, typically what you get here is the control limit with respect to the mean value of the ranges that is mu R plus or minus 3 sigma R and sigma R you have already determined this is d3 sigma. So, I am just putting the value here d 3 sigma, so you can just simplify this that d2 plus or minus 3 d 3 sigma. So, now, further simplification is also possible that D 2 is equal to d 2 plus 3 d 3 and D1 is equal to d 2 minus 3 d 3. So, if I just replace it, lower control limit will be D 1 sigma and central line will be d 2 sigma upper control limit will be D 2 sigma. So, this is something when I have the known mu and sigma.

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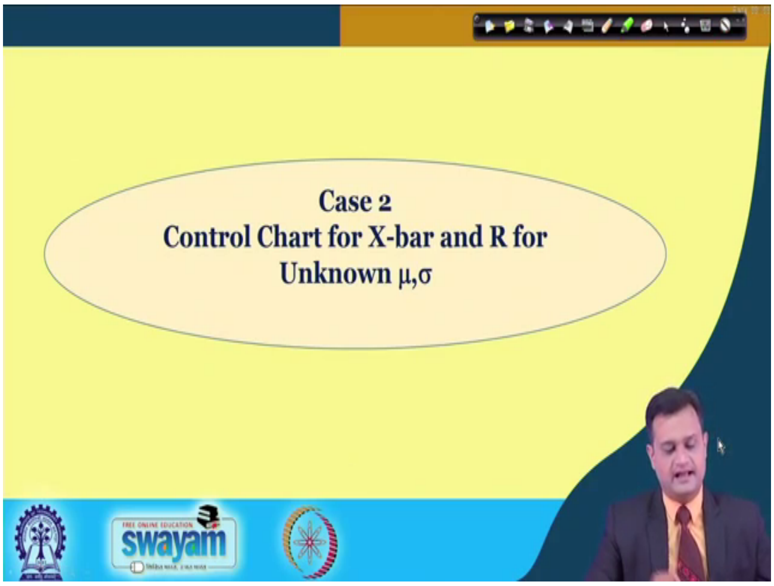

And now let us see the case 2, so case 2 pertains to again X bar and R chart for unknown mu and sigma. So, I am not aware of population mean and standard deviation.

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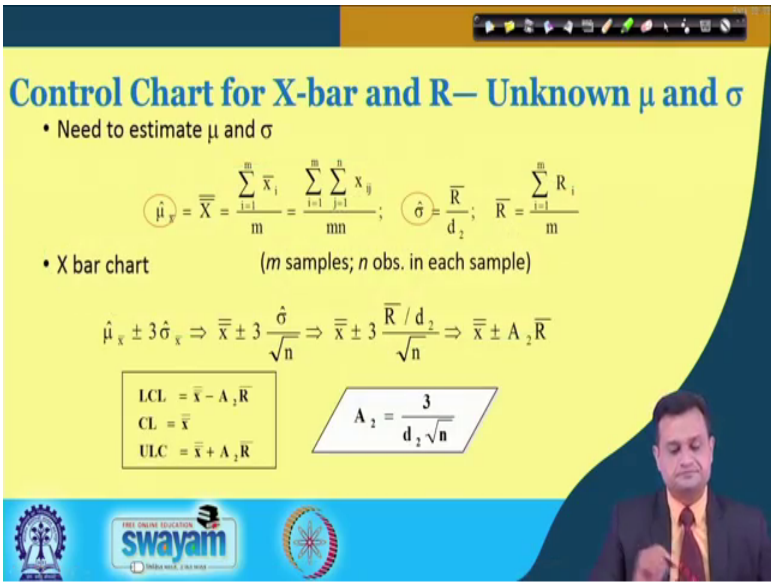

So, in this case you have to estimate mu hat X bar is equal to X double bar and you have sigma X i bar i is equal to 1 to m divided by m; m is basically samples and n is the observation in a sample. So, for example, you make it 25 sample each sample may have 5 as the sample size or it could be different.

So, this is where m into n is my total number of observations m sample let us say 25 within the sample there are 5 reading, so 25 into 5 is 125 and this is the summation of my all the observed value or say measured value for the quality characteristics and you have sigma hat this estimate you get by dividing your R bar by d 2 and R bar is nothing, but sigma is equal to 1 to m R i up to divided by m.

So, you have total m sample for each sample you have one particular range, so within that particular sample you have maximum value, minimum value find the R 1, R 2, R, R i, take the summation divided by number of say sample let us say 25 you will get the value of R bar. So, now, you have the mu X bar hat plus or minus 3 sigma X bar hat and this could be simplified as X double bar plus or minus 3 sigma hat divided by square root n and X plus or minus 3 R bar by d 2 because we had seen that the estimated value of sigma hat can be given by R bar by d 2.

So, this particular say expression can be simply be re written as X double bar plus or minus A 2 R bar, so A 2 is 3 divided by d 2 square root n. So, here you may have little bit say query that why all these values? So, basically this tables are developed for various constants for developing the control chart, so that very easily you can pick up the values from this table and easily you can compute your upper control limit and lower control limit for the given sample size and the kind of control chart you want to basically design and developed.

So, you have lower control limit X double bar A 2 R bar central line is X double bar and upper control limit this. So, any control chart you take you need to have the central line you need to have the upper control, limit lower control limit and that will basically help you to device or design a particular tool called control chart to check that whether your process is in control or not. So, this is I think I explain in detail.

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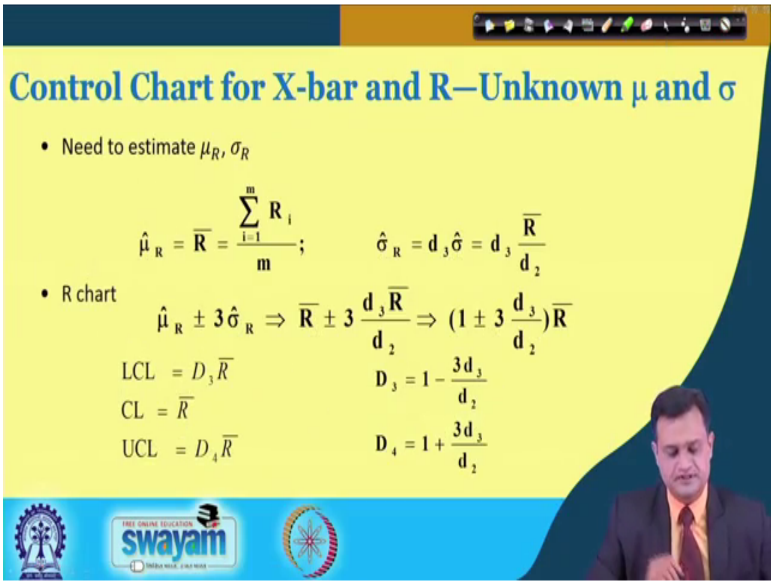

So, now further computations for unknown mu and sigma. So, now, for R mu R hat this is called hat. So, it would be R bar and you can take the average of all the R i and sigma R hat is basically d 3 sigma hat and sigma hat is R bar by d 2. So, some of the things you need to get familiarized it would be used say invariably and you will have the lower control limit D 3 R bar because your D 3 is this and D 4 is basically plus and minus. So, you will have D 3 D 4 again I will tell that value of D 3 and D 4 you can take from the standard table and this will really make your task very simple, so this is for unknown mu and sigma.

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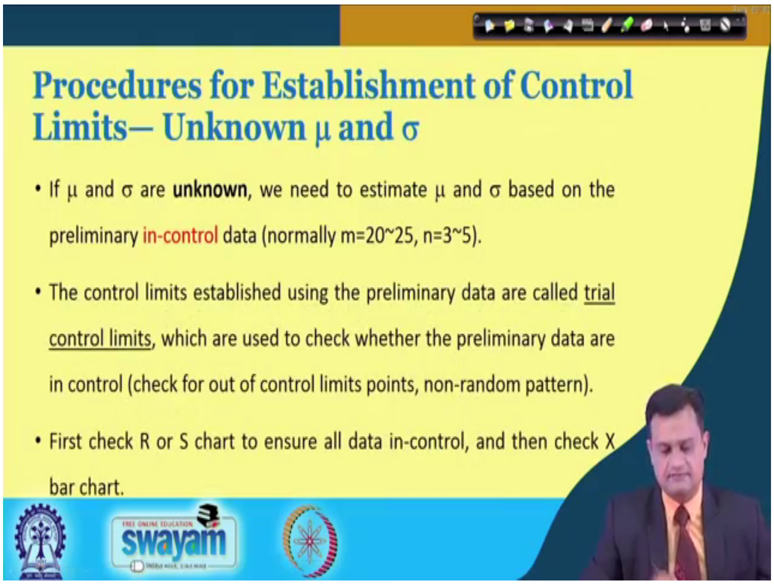

Now, let us try to see the procedure for establishing of control limit, typically for unknown mu and sigma. Now when you are not actually aware of mu and sigma, then what we need to do? We need to say estimate mu and sigma I was using the hat value. So, based on preliminary in control data, so normally say your sample maybe number of sample m is 20 to 25 and let the sample size maybe 3 to 5.

So, you will collect this much of data and let us say you have the process which is in control and what is the measure that the process you whatever operating is not producing say defective you say that my process is in control. Now for such a process typically I am analysing that I do not know mu and sigma, for such a process I am taking 20 to 25 sample each sample is of 3 to 5.

So, now you control limit are established using the preliminary data and typically these are known as trial control limits. So, initially mu and sigma are not known I will set the trial control limit and then which are typically are used to check whether the preliminary data are in control or not. So, I am not just leaving like that I will now check it with this trial control limit and first check R and S chart to ensure all data in control, first try to target the variability and then you check for the X bar chart.

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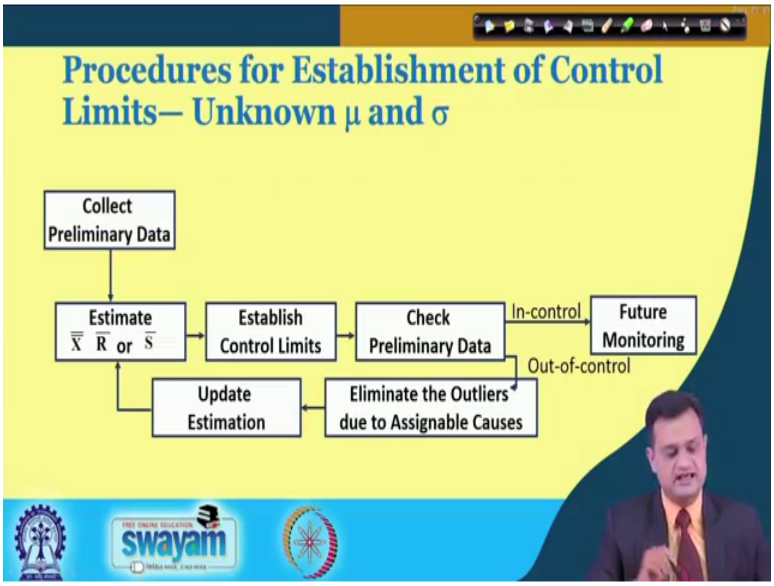

So, once you are done with this, then I have simplified everything through a simple flowchart and it goes like this collectively preliminary data then estimate X double bar R bar S bar establish control limit this will be called trial control limits. Check preliminary data if it is in control, then use this control chart your tool for the future monitoring because your tool is ok.

Now if it is showing out of control, eliminate the outliers due to assignable cause, update the estimation and then once again you operate with this unless you get the all the points falling within without any typical patterns and then you will have the final control chart for the implementation available.

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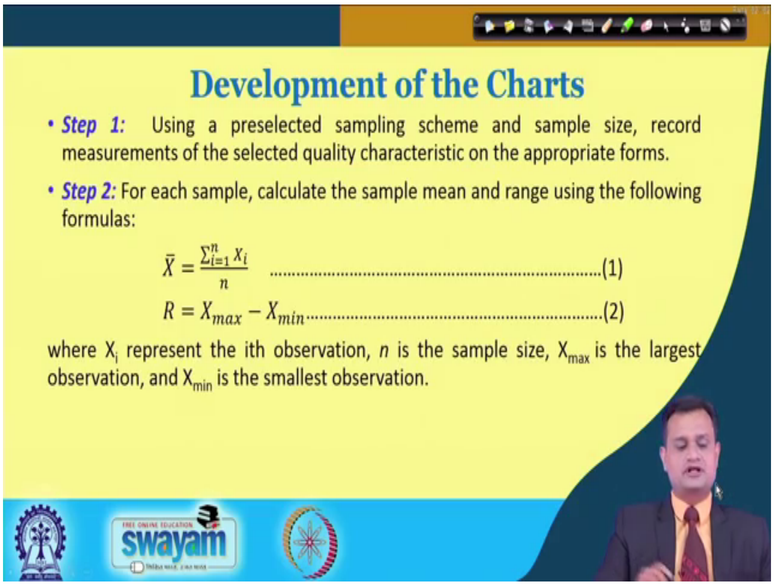

So, this is exactly what we do, so if we quickly go through the general steps for constructing a control chart then step 1 using a pre selected sampling scheme sample size, record measurements of the selected critical quality characteristic on appropriate form. Step 2 compute X bar R which is the X bar is the average of your all the measure the readings for the given quality characteristics, R bar is within sample maximum minus minimum.

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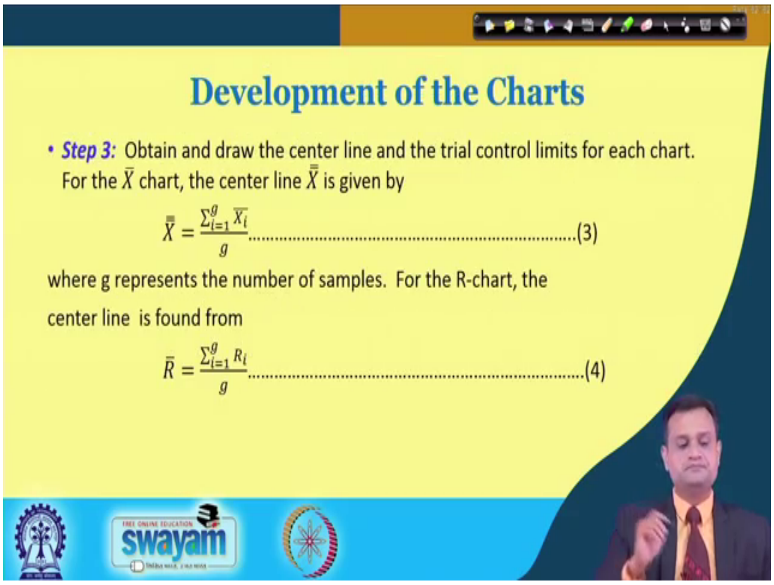

Then step 3 compute X double bar because you need the central line also. So, this is basically the when I say X i bar it is specific to a particular sample, so average of the sample and when I say take the average of the average it is my X double bar and then I have R bar which is the average of all the ranges.

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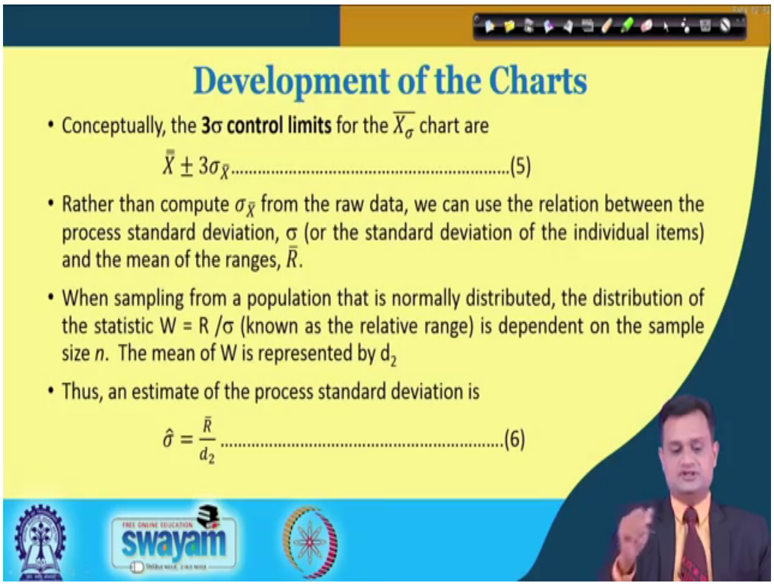

Then you compute the 3 sigma control limit X double bar which is the central line plus or minus 3 sigma X bar and sigma X sigma hat basically is R bar by d 2. So, this I can very easily find d 2 I can find from the table.

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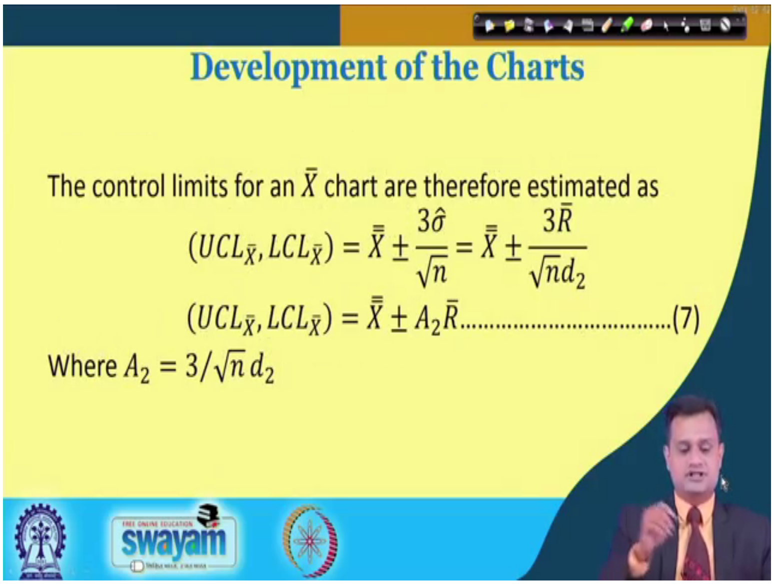

So, then you will have the upper control limit and lower control limit for X bar, so it would be X double bar plus or minus 3 sigma hat divided by square root n and you can put sigma hat is R bar by d 2 and this word give X double bar plus or minus A 2 R bar because A 2 is 3 sigma root and d 2. So, 3 sigma route n d 2 is my basically A 2 and I can just simplify the value of A 2 can also be read from the table.

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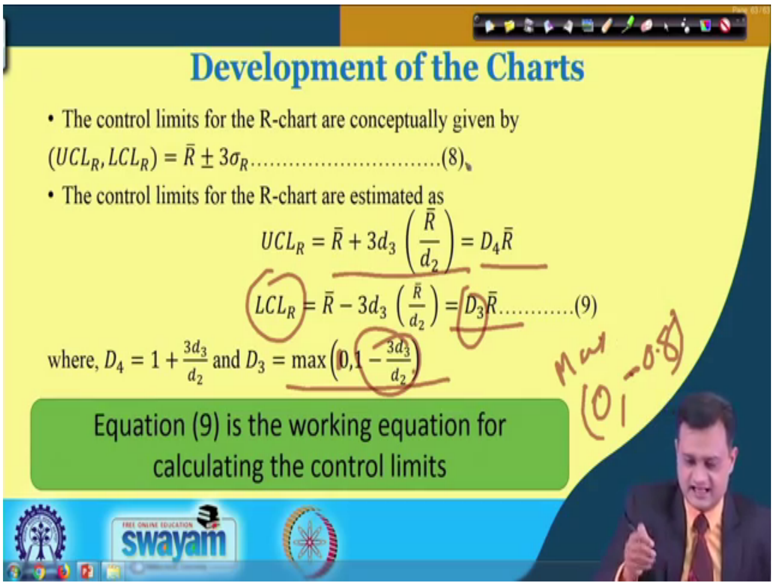

Similar way you said the control limit for R chart, so upper control limit would be say this we have already seen it would be D 4 R bar, D 3 R bar and here you just say that I say that D 3 which is specific to lower control limit, it should be either 0 or this maximum of this it means suppose this is negative.

Let us say I have this value negative 0 and let us say minus 0.8, now I will take the maximum of these can you just saying what could be the reason? Please understand that I cannot have lower control limit negative it can maximum go up to 0. Point number 2, that do not randomly say that when the point is falling out side upper or lower control limit you need to take the corrective action, give a thought I am explaining you a very very critical point.

If a point is falling outside the lower control limit on R chat what does it indicate? It indicates that my variability is going down and this is a positive sign, so you need to take the sustaining action and not the corrective action. Suppose you are going to a doctor with a very good blood survey report or maybe the blood pressure report or maybe your other parameters will doctor give you additional medicine or he will ask you to maintain the status compliment you, congratulate you and give you the same medication or reduced medication.

So, same applies over here please remember it, there are 2 types of actions corrective action, sustaining action when my process is improving I must investigate the reason behind improvement and I must take the sustaining action. Corrective action; obviously, I will check where it is going other outside the upper control limit lower control limit on X bar chart or outside the upper control limit on my R chat then it is a concern and there could be an assignable cause.

So, my two points I am repeating R chat the lower control limit maximum it could be 0 it cannot be negative and second thing if the point is falling outside the lower control limit on R chart it is not an indication of any assignable cause or corrective action, it is a matter of say sustaining action and you need to investigate that what is that something which is really improving the process. So, everyday if you are doing a exercise and you are becoming fit, it is a good symptom you need not to spoil it.

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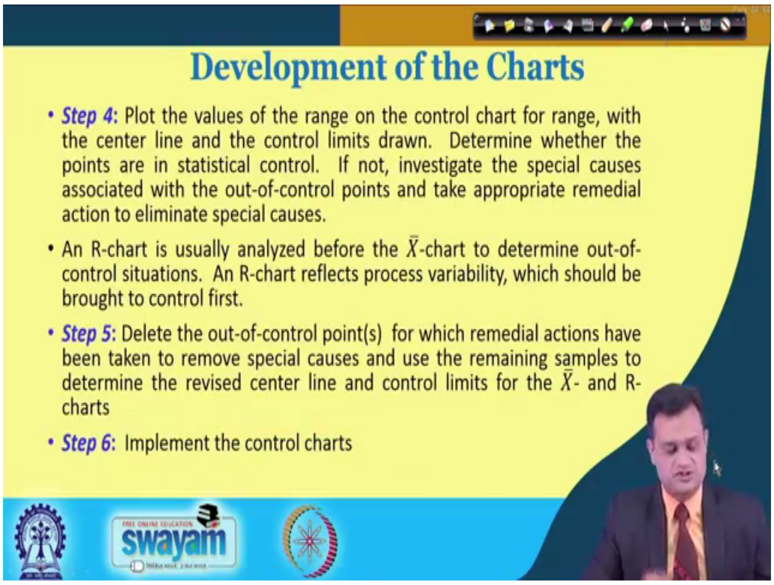

So, this is something which many a times people make the mistake and I just would like to remind you again and again. Step 4 plot the values try to see whether they are within the control limit or going outside. Step 5 delete the out of control point and once again calculate your upper control limit, lower control limit central line. Once you say that your process is within control then accept this control chart limits for the implementation and now you are gauge is perfect, I will say your control chart is like a gauge you are measuring the process it is perfect you can rely on it and then you go for the implementation.

So, you cannot just implement the control chart just like that you need to compute the trial control limit, stabilize the process and then once you have the confidence that your control limit for well stabilized, then use it as an instrument to keep a control over the process. So, I think we have discuss in detail now let us quickly go through couple of examples. So, computation is very simple once you have the formula, but let us try to go introduction part.

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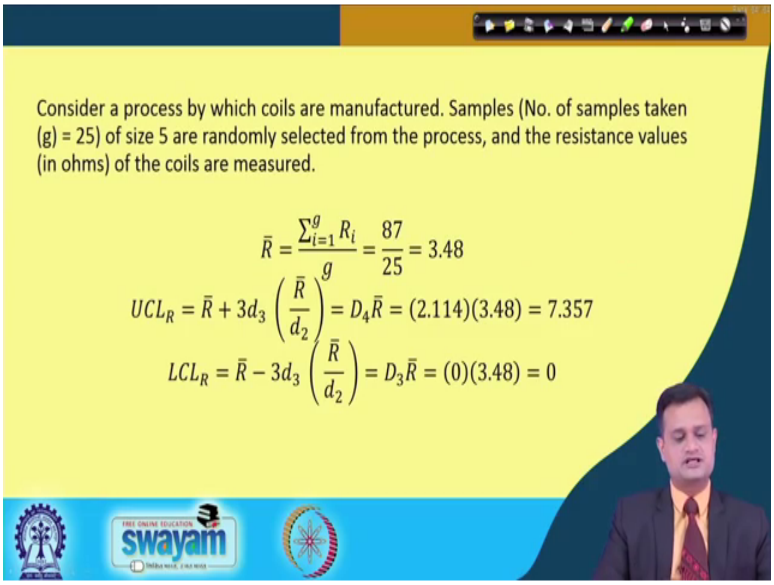

So development of X bar and R chart for coil manufacturing process, the data is like this you have a process which manufactures the coil and samples number of sample taken at 25 size 5 are randomly selected from the process and resistance value in ohm are measured. So, R bar you can compute, you have UCL R, so D 4 R bar D 4 is from table you get this LCL R this is 0 say maximum of this so by chance if it comes out to be negative then you will take 0 as I discussed.

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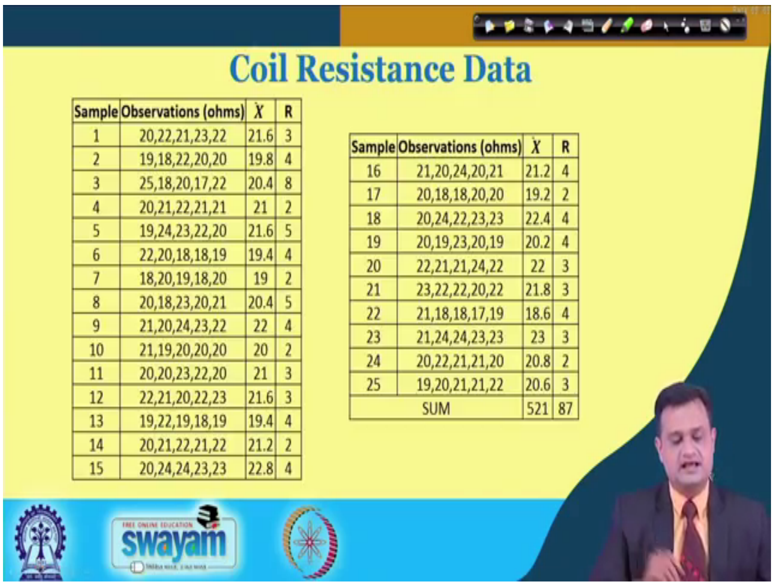

And then this is the data you have, so you have the sample 25 sample each sample has 1, 2, 3, 4 1, 2, 3, 4, 5 reading. So, your sample size is 5, you compute the X bar for each particular sample I will call it as X i bar and then R, a total is 521 that is the X bar value R is 87. So once you understand, so that I need not to explain for every example.

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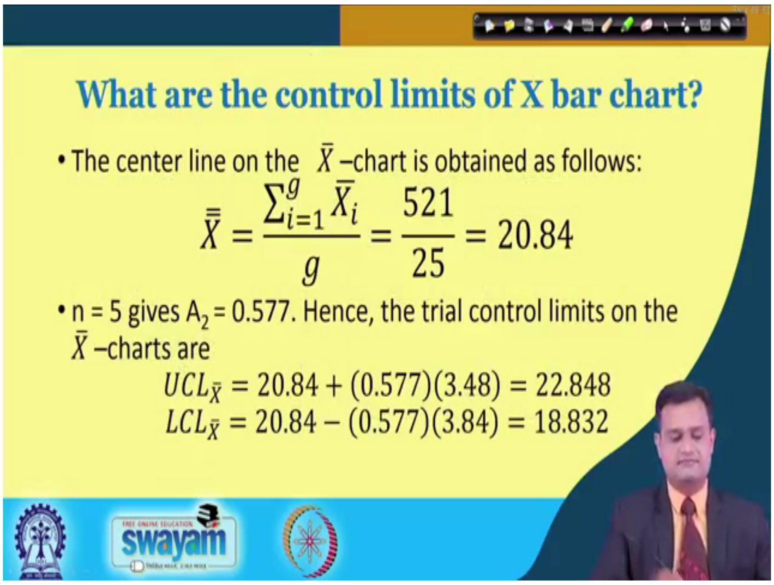

You compute the X double bar and then you compute the UCL X bar and LCL X bar. So, this A 2 is from the table for n is equal to 5 and the X bar chart. So, only two thing sample size and the kind of chart you want to plot you will get the value of the constant from the appendix of the suggested textbook standardised table.

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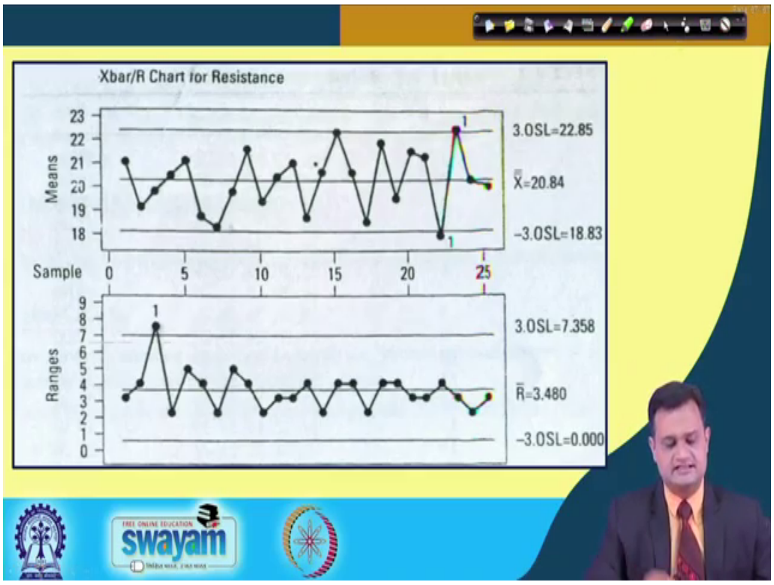

So, now once you have done this you can plot the data and data means what? You will plot the values of X i bar, it means you will plot the values of average of each particular sample and you have X double bar is a centre these are the means which is X i bar and this is X i bar and here you will plot the values of R i and this is your basically R bar.

So, you can see here there is something wrong here; there is something wrong here; there is something wrong here. So, on X bar chart typically I can see that somewhere around 22 23 sample there is some issue and here I can see that some issues specific to may be sample 1 or 2.

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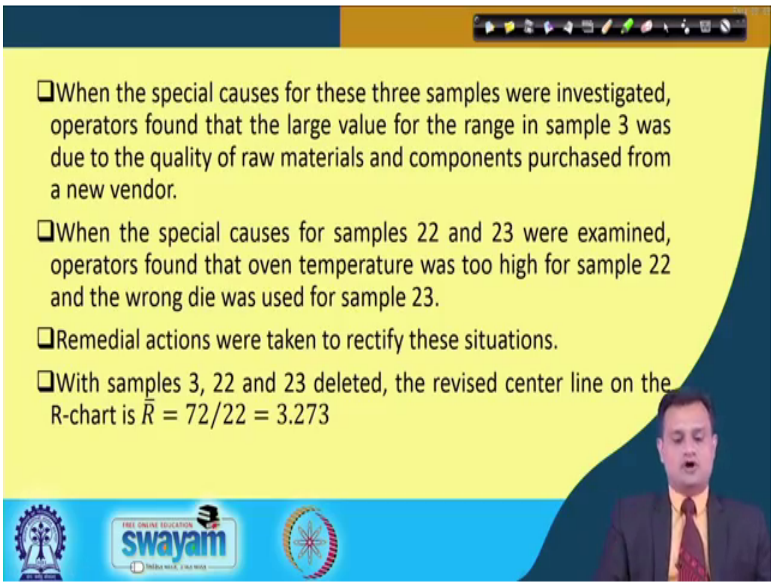

So, based on this some say conclusions were drawn that. When the special cause for sample 22 and 23 on X bar chart where examined operator found that temperature was too high for sample 22 and the wrong die was used for sample 23. So, this you have to investigate you may have to do brainstorm, you must have a process knowledge and typically remedial actions were taken.

But with simple 3 say typically 22 and 23 deleted and then the revised centre lines where say developed, but just see here that in simple 3 was there was a 3 sample where investigated and operator found the large values for range in simple 3 and due to the quality of the raw material.

So, here range chart gave you the idea that there is something wrong with the quality of raw material and then X bar chart gave you the idea for 22 and 23 sample, that there is something wrong with the temperature and other settings. So, this is something that you can really appreciate.

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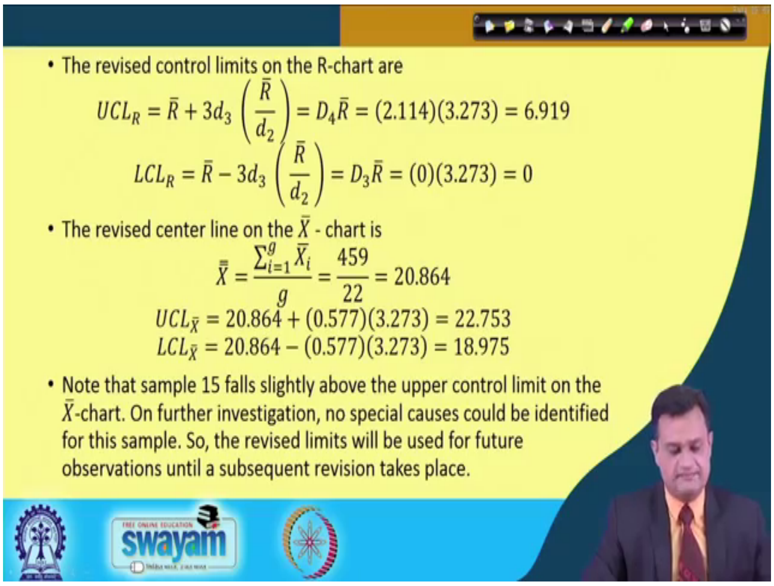

Now you compute the revised control limits by excluding those point, so I am not explaining as usual you compute the UCL R, LCL R, UCL X bar, LCL X bar and now once you have you can further plot and see that whether all the points are falling within or not if yes, then accept this gauge statistical control chart implemented.

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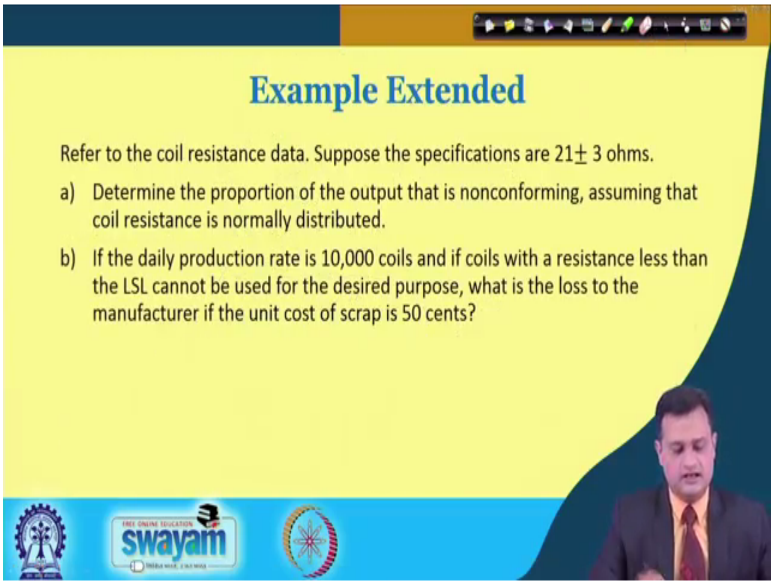

Now let us say I just extend the example and suppose there are specification 21 plus or minus 3 ohm. So, determine the proportion of the output that is non conforming assuming that coil resistance is normally distributed. And suppose that is a daily production is 10,000 and specifically you cannot accept the product which is less than LCL lower specification limit and if there is a cost of scrap that is 50 cent then what would be the loss to the company?

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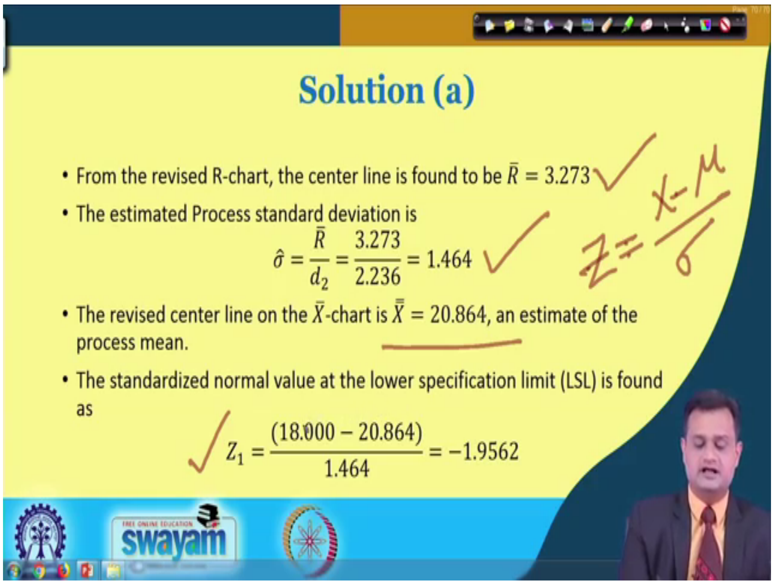

So, I have the value of R bar that is 3.273 and I can estimate the sigma hat, so I have X double bar. Now I am using Z is equal to you know the expression Z is equal to X minus mu divided by sigma and I am putting the values here.

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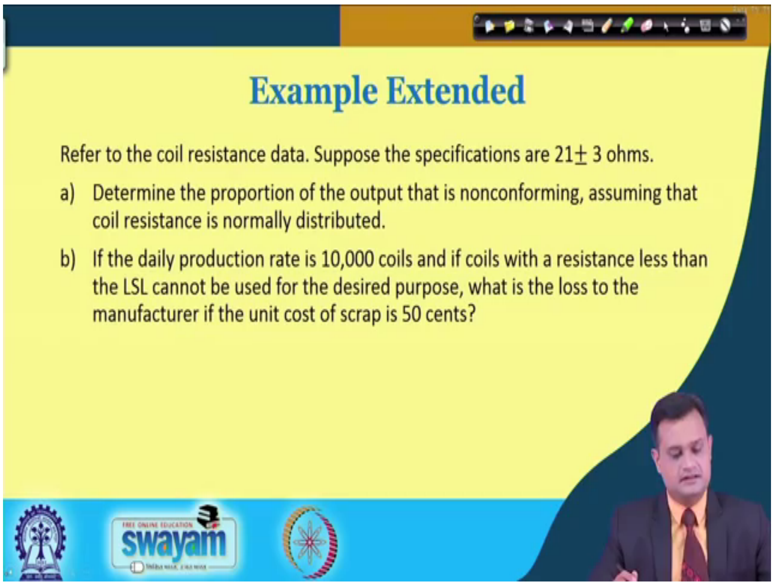

So I have basically the specification 21 plus or minus 3, so lower specification is 21 minus 3 is 18 so I am putting here 18 this is my 20.864. So, it is basically this value 20.864 and I get this Z 1 minus 1.9562 by referring the normal table.

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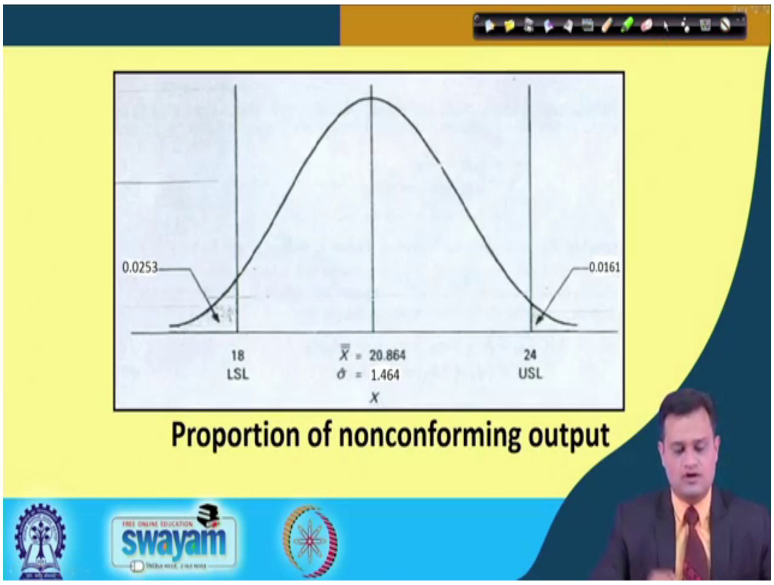

So, now once this is done you can see very well what is happening here, you have some rejections below LSL, you have some rejections above USL and total you can do by just by adding this two probabilities that what is the percent that is going to be rejected. So you have Z 2 and you can total it, so you will get 0.04145.

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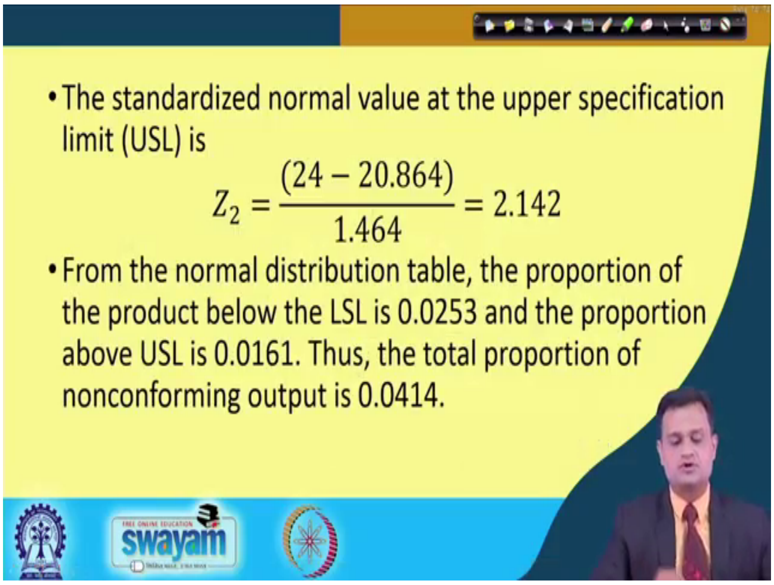

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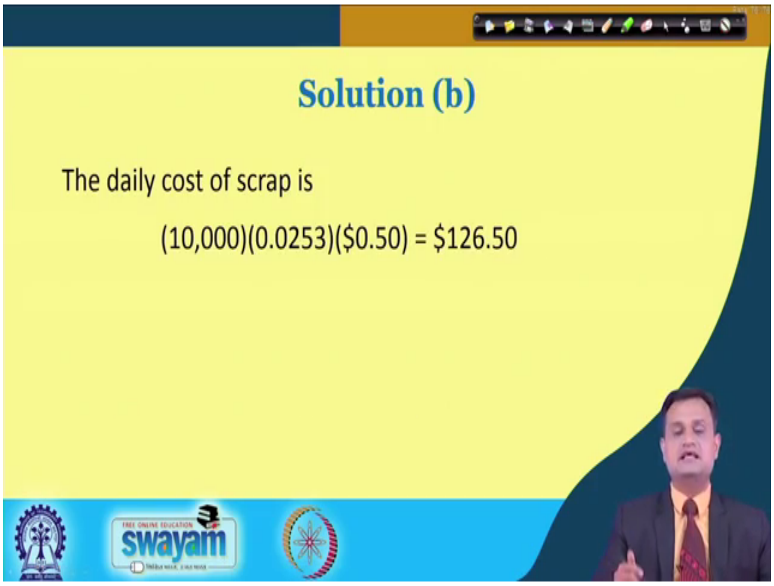

Now, here because I am only interested to find the value of scrap, it means the components which are below LSL, so I am only considering 0.0253 I am producing 10,000 and the cost is 0.5 dollar. So, my total cost is 126.50 on scrap, so this much money I will lose because of wrong production. We can have illustrative example 2 for X bar and R chart for bearing manufacturing.

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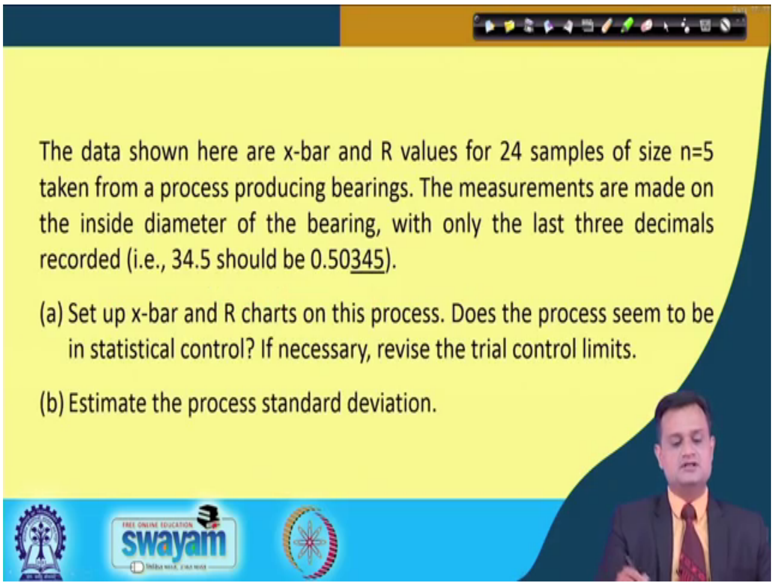

So, you are given the data for x bar and R chart and you can record up to 3 decimal. So setup x bar and R chart and estimate the process standard deviation. So, this is the data and directly I am giving you the values of X bar for each particular sample as well as the R, so you have this data available.

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X bar	34.5	34.2	31.6	31.5	35.0	34.1	32.6	33.8	34.8	33.6	31.9	38.6
R	3	4	4	4	5	6	4	3	7	8	3	9
X bar	35.4	34.0	37.1	34.9	33.5	31.7	34.0	35.1	33.7	32.8	33.5	34.2
R	8	6	5	7	4	3	8	4	2	1	3	2

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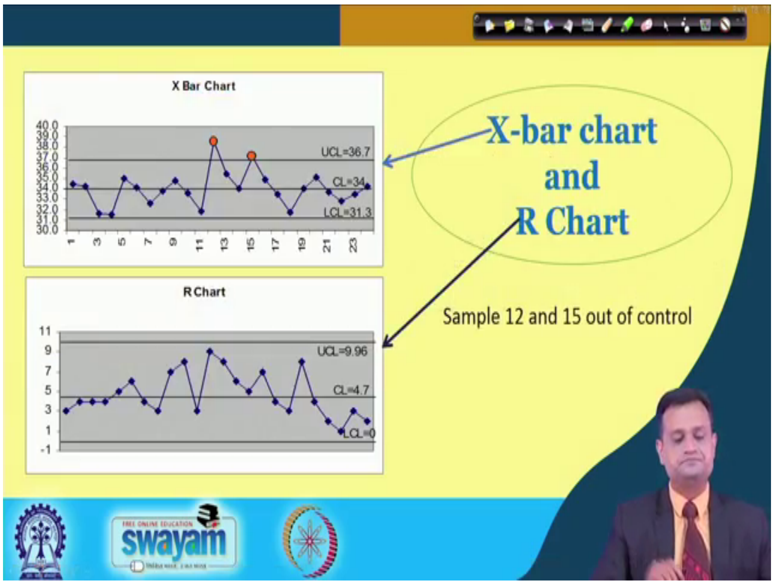

Now, I will just plotted this particular graph and you can easily see that couple of points indicated in orange are going outside the control limit of my X bar chart, more or less there is no issue on the R that it is ok, but this is the problem this is the problem.

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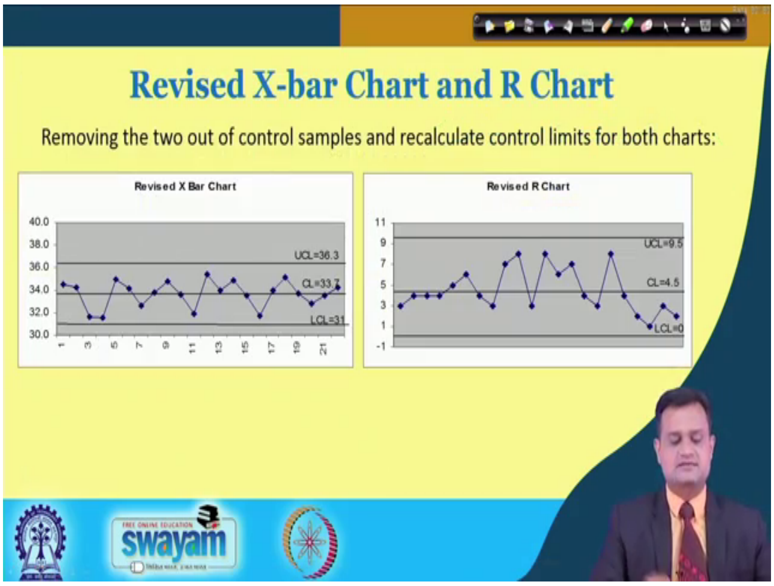

So, I will exclude this points once again I will calculate the control limits and then once I feel that my control limits are stabilized I will go for. So, this is the recalculated control limits more or less it is falling within the control limits I will accept this my gauge is ready for implementation gauge is my control chart.

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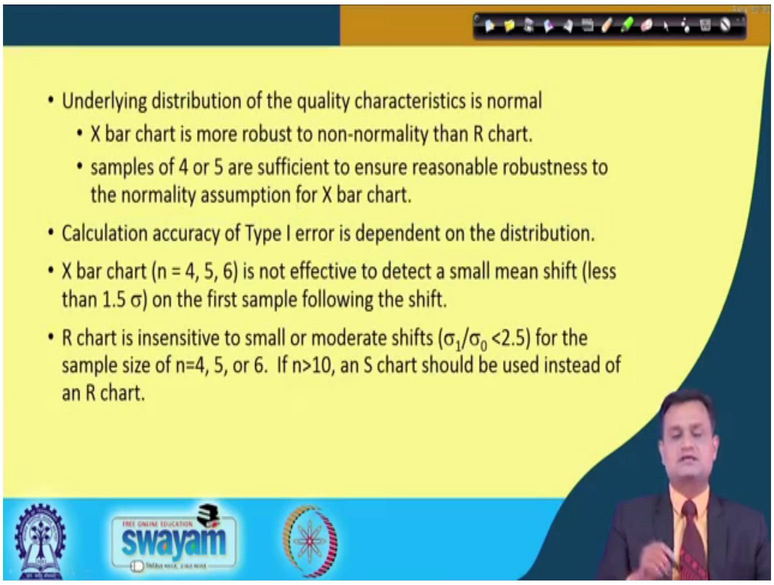

So, there are some important assumptions and properties of the X bar and R chart. So, X bar chart is more robust to non normality than R chart. So, there is an assumption that my data must be normally distributed, but this X bar chart is less sensitivity more robust to any kind of say non normality if it is present, but because normal non normality is directly related to your variability and shape, so your R chart is more sensitive.

Sample of 4 to 5 are sufficient to ensure reasonable robustness and calculation accuracy of type I error is dependent on the distribution X bar chart 4, 5, 6 is not effective to detect a small means shape less than 1.56 on the first sample following the shift. So, maybe you have to go for larger let us say sample size, if there is a shift in mean which is less than 1.5 sigma. And R chart is insensitive to small or moderate shape sigma 1 by sigma o, so original versus the shift less than 2.5 and for the sample size 4, 5, 6.

So, you have to go for any greater than 10 and the preferred chart would be the S chart, standard deviation chart and not the R chart. So, third example is specific to X bar and R chart. So, as I mentioned that when the variability is of concern typically R chart because it is only taking the maximum minus minimum, it is not capable enough to capture some small change in the variability the ratio which we have seen here sigma 1 divided by sigma o less than 2.5 in this case I would like to compliment my X bar chart with S chart instead of R chart. So, these are the expressions for your X bar and S chart.

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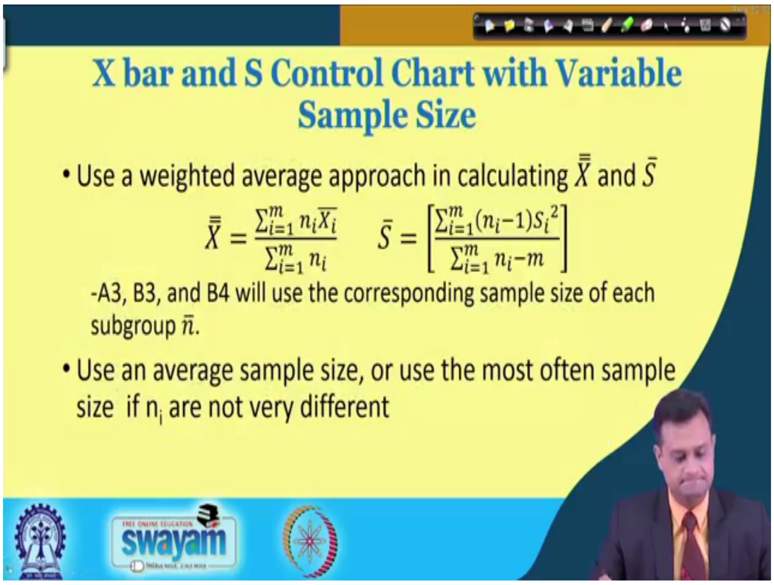

So, now you can easily appreciate it just make use of this is the number of sample here I have assumed n i node n because I may have different number of units in a particular sample. So, my sample maybe of varying size and that is why I have put n i.

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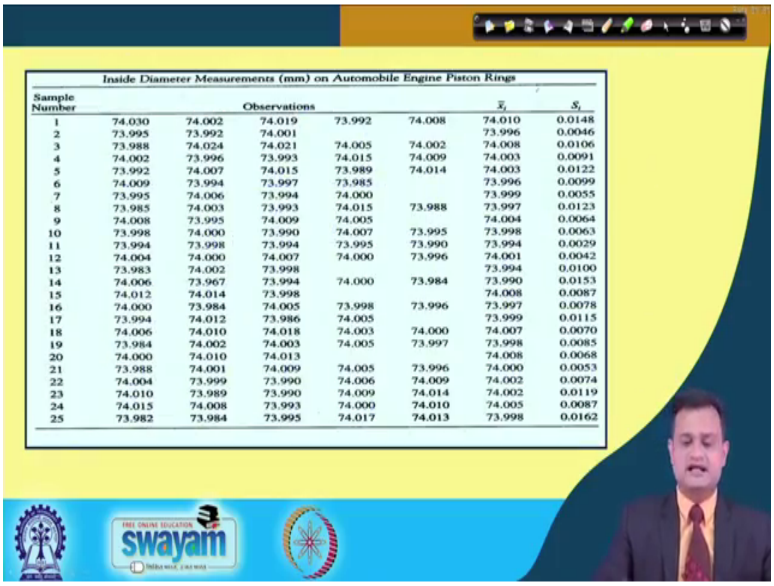
Inside Diameter Measurements (mm) on Automobile Engine Piston Rings

Sample Number	Observations					$\bar{x}_i$	S_i
1	74.030	74.002	74.019	73.992	74.008	74.010	0.0148
2	73.995	73.992	74.001			73.996	0.0046
3	73.988	74.024	74.021	74.005	74.002	74.008	0.0106
4	74.002	73.996	73.993	74.015	74.009	74.003	0.0091
5	73.992	74.007	74.015	73.989	74.014	74.003	0.0122
6	74.009	73.994	73.997	73.985		73.996	0.0099
7	73.995	74.006	73.994	74.000		73.999	0.0055
8	73.985	74.003	73.993	74.015	73.988	73.997	0.0123
9	74.008	73.995	74.009	74.005		74.004	0.0064
10	73.998	74.000	73.990	74.007	73.995	73.998	0.0063
11	73.994	73.998	73.994	73.995	73.990	73.994	0.0029
12	74.004	74.000	74.007	74.000	73.996	74.001	0.0042
13	73.983	74.002	73.998			73.994	0.0100
14	74.006	73.967	73.994	74.000	73.984	73.990	0.0153
15	74.012	74.014	73.998			74.008	0.0087
16	74.000	73.984	74.005	73.998	73.996	73.997	0.0078
17	73.994	74.012	73.986	74.005		73.999	0.0115
18	74.006	74.010	74.018	74.003	74.000	74.007	0.0070
19	73.984	74.002	74.003	74.005	73.997	73.998	0.0085
20	74.000	74.010	74.013			74.008	0.0068
21	73.988	74.001	74.009	74.005	73.996	74.000	0.0053
22	74.004	73.999	73.990	74.006	74.009	74.002	0.0074
23	74.010	73.989	73.990	74.009	74.014	74.002	0.0119
24	74.015	74.008	73.993	74.000	74.010	74.005	0.0087
25	73.982	73.984	73.995	74.017	74.013	73.998	0.0162

So, now just see the data and how you see the data? Sample number in this sample I have 1, 2, 3, 4, 5 total 5 units measured, but in second if you see my sample size is 1, 2, 3 only 3. Same way you can see here it is 1, 2, 3, so I have basically n i varying sample size I computed the Xi bar, if there are 5 units I will divide the total by 5, if there are 3 I will divide by 3 and I computed the S i.

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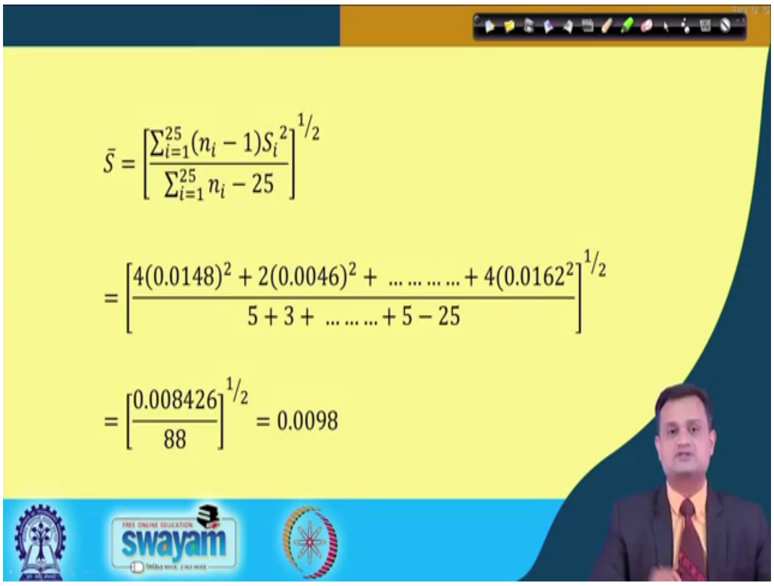

So, this is my initial computation for X bar and S chart I computed the S bar by using the expression I have given and once this is done, then for n 1 is equal to 5 X bar chart you can compute UCL as well as LCL.

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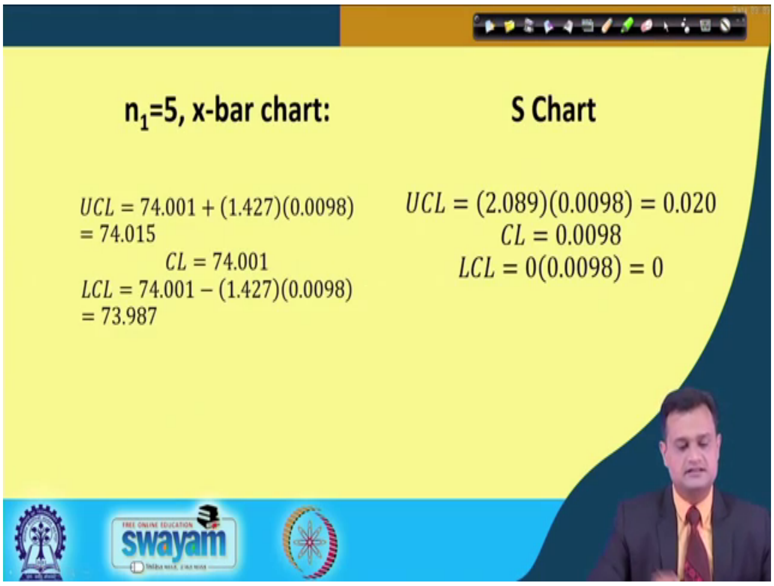

So, now, this is 74.001 and this value is basically the value of A 2 that you get from the table for the given sample size. Let us check from our particular table just see this is the value 74.010. So, this value is basically the average of this n I will use this value as well as the value of A 2 I got from the table in order to find my upper control limit and the lower control limit.

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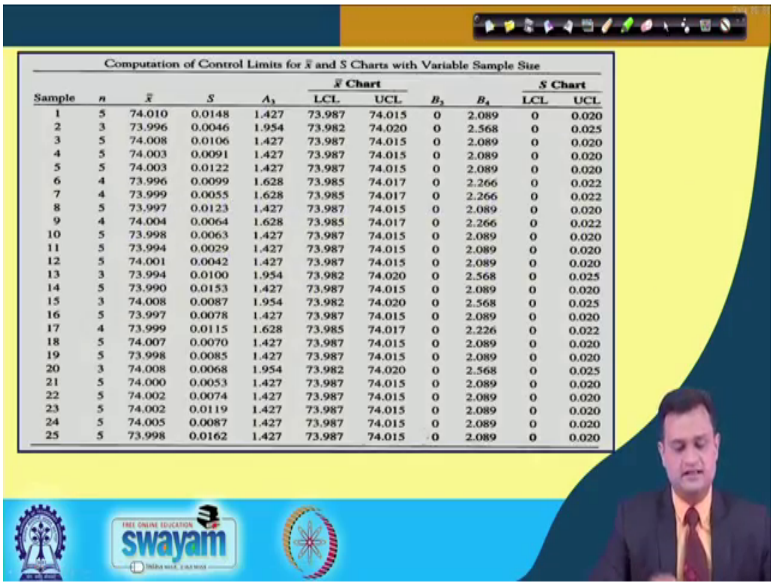

Computation of Control Limits for $\bar{x}$ and S Charts with Variable Sample Size

Sample	n	$\bar{x}$	S	A_3	$\bar{x}$ Chart LCL	$\bar{x}$ Chart UCL	B_3	B_4	S Chart LCL	S Chart UCL
1	5	74.010	0.0148	1.427	73.987	74.015	0	2.089	0	0.020
2	3	73.996	0.0046	1.954	73.982	74.020	0	2.568	0	0.025
3	5	74.008	0.0106	1.427	73.987	74.015	0	2.089	0	0.020
4	5	74.003	0.0091	1.427	73.987	74.015	0	2.089	0	0.020
5	5	74.003	0.0122	1.427	73.987	74.015	0	2.089	0	0.020
6	4	73.996	0.0099	1.628	73.985	74.017	0	2.266	0	0.022
7	4	73.999	0.0055	1.628	73.985	74.017	0	2.266	0	0.022
8	5	73.997	0.0123	1.427	73.987	74.015	0	2.089	0	0.020
9	4	74.004	0.0064	1.628	73.985	74.017	0	2.266	0	0.022
10	5	73.998	0.0063	1.427	73.987	74.015	0	2.089	0	0.020
11	5	73.994	0.0029	1.427	73.987	74.015	0	2.089	0	0.020
12	5	74.001	0.0042	1.427	73.987	74.015	0	2.089	0	0.020
13	3	73.994	0.0100	1.954	73.982	74.020	0	2.568	0	0.025
14	5	73.990	0.0153	1.427	73.987	74.015	0	2.089	0	0.020
15	3	74.008	0.0087	1.954	73.982	74.020	0	2.568	0	0.025
16	5	73.997	0.0078	1.427	73.987	74.015	0	2.089	0	0.020
17	4	73.999	0.0115	1.628	73.985	74.017	0	2.226	0	0.022
18	5	74.007	0.0070	1.427	73.987	74.015	0	2.089	0	0.020
19	5	73.998	0.0085	1.427	73.987	74.015	0	2.089	0	0.020
20	3	74.008	0.0068	1.954	73.982	74.020	0	2.568	0	0.025
21	5	74.000	0.0053	1.427	73.987	74.015	0	2.089	0	0.020
22	5	74.002	0.0074	1.427	73.987	74.015	0	2.089	0	0.020
23	5	74.002	0.0119	1.427	73.987	74.015	0	2.089	0	0.020
24	5	74.005	0.0087	1.427	73.987	74.015	0	2.089	0	0.020
25	5	73.998	0.0162	1.427	73.987	74.015	0	2.089	0	0.020

So, you find the upper control, lower control for X bar chart and S chart once this is done, then you can see this table also, that I have put all the values of X bar S you will see the value of A 2 LCL, UCL which is basically which has used the value of A 2, B 3, B 4 and the S chart.

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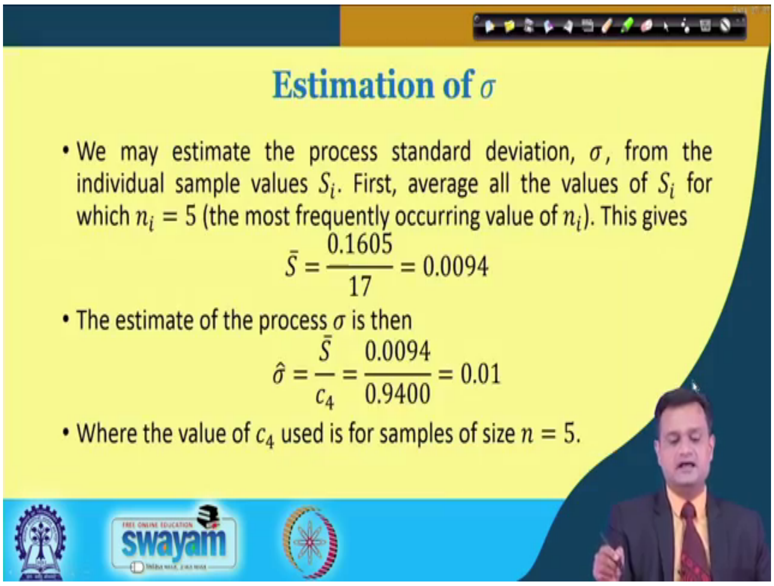

So, we have computed the upper control limit, lower control limit for X bar chart as well as S chart and I can also estimate the sigma S bar is equal to 0.1605 divided by 17. I will just consider the samples which has the sample size 5 and then I will take the submission of all the S i of this number of sample here it is 17 sample which has sample size 5. So, I will get the S bar 0.0094 and sigma hat is this for n is equal to 5.

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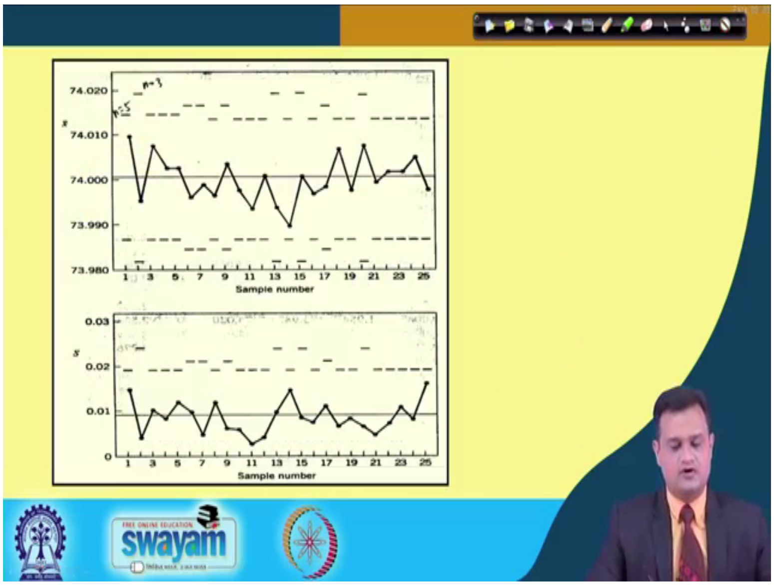

So, I hope this is clear what you can observe here is that see this is my control limit; this is my control limit. So, for each particular sample I have different control limit because my n i is different, n i means my sample size is different. So, here you do not get 2 lines one is upper control line and another is lower control line, what you get here is like this dash; dash; dash. So, my control limits are varying and this is when my n i is different for different samples, so then I can just check the status of my process.

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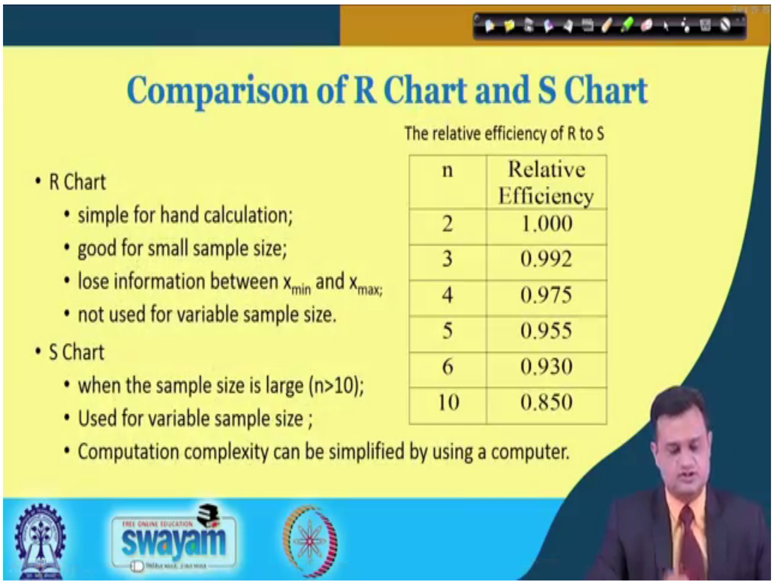

Now before we conclude I would like to compare the relative efficiency of R n sigma and R when it is simple to calculate because just maximum minus minimum this is good for the small sample size, but when you are large sample size it cannot capture the over all variability within the sample accurately and this is reflected exactly here that for n is equal to 2 my sample size is only 2, it hardly makes any difference my efficiency of R to S is 1, but when sample size is 3 my efficiency of R is reducing, 4 it is reducing, for 10 it is 0.85.

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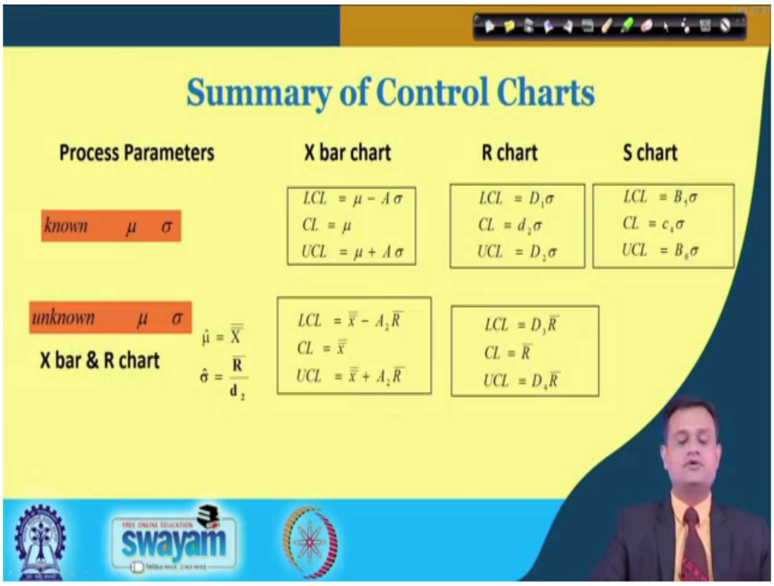

So, for larger sample size R chart is not effective I must use the S chart, so you can see the summary if known mu and sigma X bar chart R chart S chart, if unknown these are the expressions for my upper control lower control limit for X bar and R chart.

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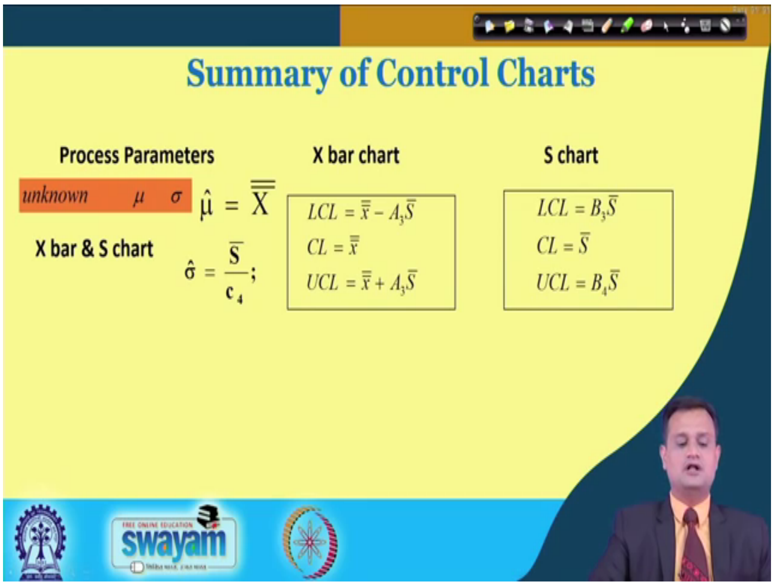

This is again for unknown mu and sigma X bar and S chart, so you can easily make use of this control limits and plot the data, set the trial control unit plot the data, get rid of the points outside the control limit make your control chart gauge perfect and then implement.

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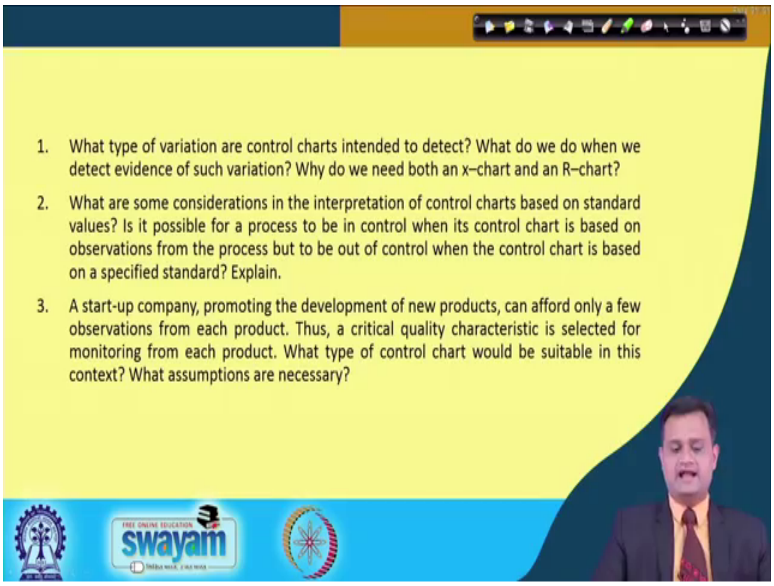

So, before we end as usual practise I want to float couple of questions. What type of variations are control charts intended to detect? What do we do when we detect evidence of such variation? Why do we need both X bar and R chart? As I mentioned, I want to have the hold on mean and variability both, so they are complementary to each other.

What are some considerations in the interpretation of control charts based on the standard values? And a start up company promoting the development of new product can afford only a few observations from each product. Thus a critical quality characteristics is selected for monitoring from each part product. What type of control chart would you like to use?

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So, many I have followed the book Mitra examples and also referred from that you can go through it Montgomery is also a very good book another regular book of our Six Sigma you can refer.

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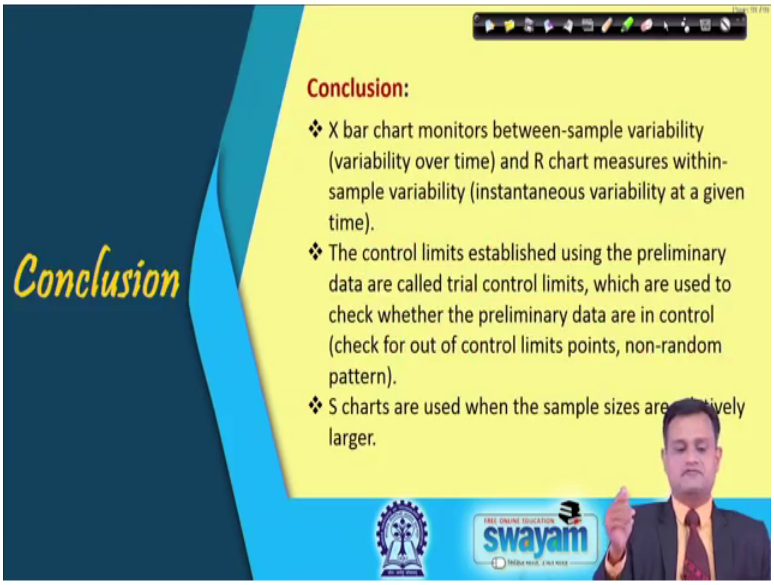

So, X bar and R are complementary, mean and variability both needs to be controlled where the sample size is larger than I must use the S chart which has the better efficiency.

So, thank you very much for your interest in learning the design of control charts for variables, I hope this video would have provided you the enough and adequate knowledge on development of the control chart at least collect some data or use the hypothetical data and develop the control chart X bar and R or X bar and S and try to gain the confidence, till that time keep revising introspecting applying the concept stock be with me, enjoy.