

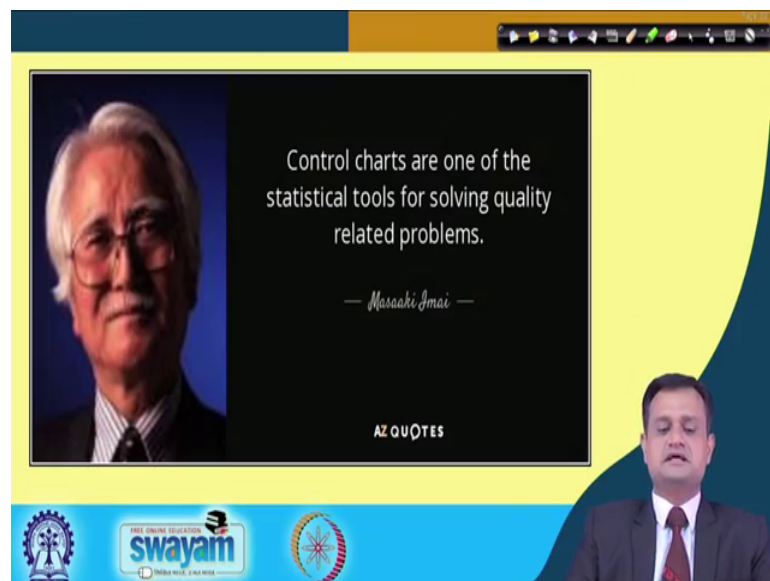
Six Sigma
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Lecture – 48
Statistical Process Control: Key Concepts

Hello friends, once again I welcome you to our ongoing exciting journey of Six Sigma and now you are little bit relaxed because we are discussing the final phase of our DMAIC cycle that is the control phase. As a part of control phase we initiated our discussion in the last lecture and we talked about various 7 QC tools. Now I would like to devote some time on Statistical Process Control which is the very very important concept and majority of the quality gurus they have shown their faith in the statistical quality control.

So, here as a part of lecture 48 we would like to appreciate some of the key and basic concepts. So, that later on when we talk about the design of the control chart we use this concepts easily. So, please try to be attentive digest the concepts, so that subsequently we can, that control charts are one of the statistical tools for solving quality related problems.

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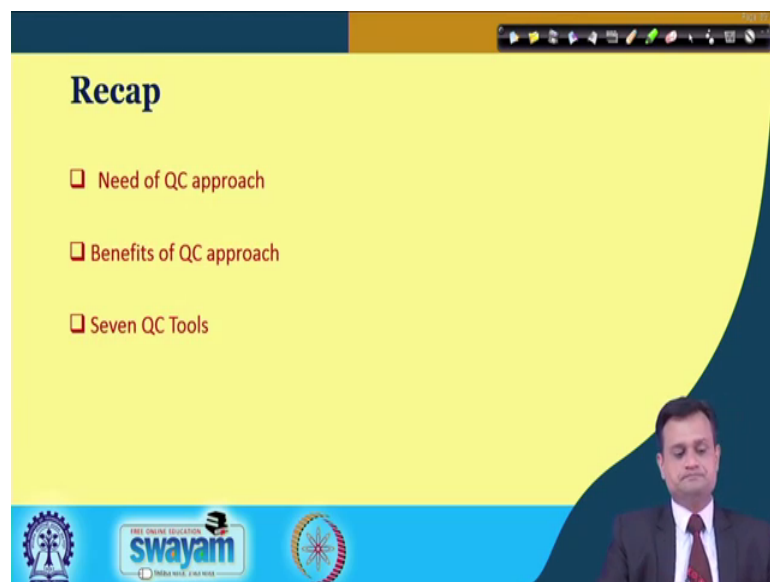


So, you have the control charts and you can see that these are statistical control charts quality control, I am trying to make use of my statistical knowledge to set a control on

the process. Now just think what do you mean by control? The key here is control and achieving this control statistically. So, let us say you are visiting a doctor with a problem and now you have been advised to go through couple of test you got your diagnostic report, once again you went back to the doctor and doctor after referring the test is finding that what is the cause of the problem? And giving you some medication.

So, basically it is the rectification of the problem and once the problem is rectified you try to control it by taking the necessary action. Same thing applies over here that I have a process and if I can set a control over it I can have the diagnostic tool which is typically called as control chart, then I can set a better control over my process and take the necessary action at an appropriate time.

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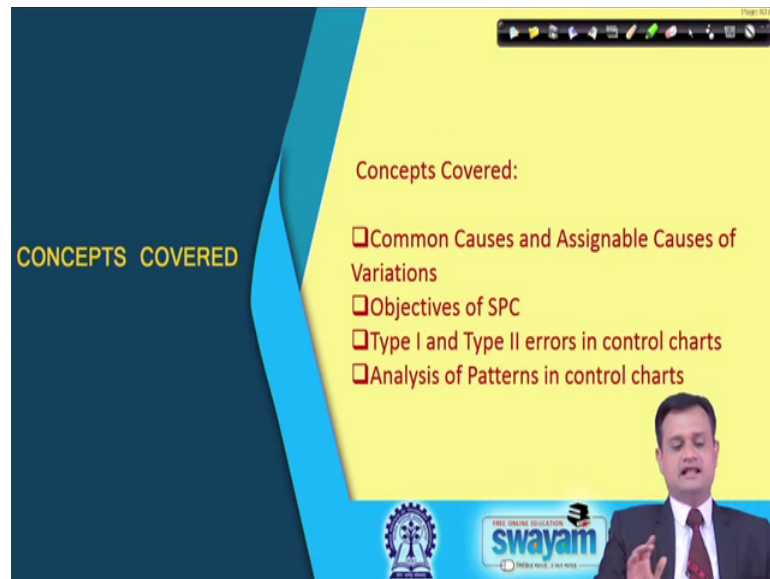
The image shows a presentation slide titled "Recap" on a yellow background. The slide lists three items, each preceded by a red square icon: "Need of QC approach", "Benefits of QC approach", and "Seven QC Tools". At the bottom of the slide, there is a blue banner containing the logos of the Ministry of Education, Government of India, and the Swayam initiative. A small video inset in the bottom right corner shows a man in a suit and tie.

Recap

- ❑ Need of QC approach
- ❑ Benefits of QC approach
- ❑ Seven QC Tools

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CONCEPTS COVERED

Concepts Covered:

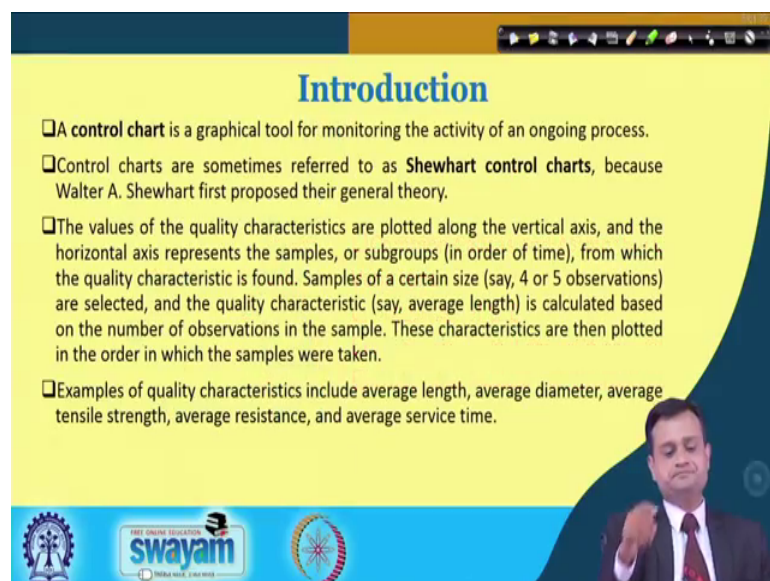
- ❑ Common Causes and Assignable Causes of Variations
- ❑ Objectives of SPC
- ❑ Type I and Type II errors in control charts
- ❑ Analysis of Patterns in control charts

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So, we have talked about various QC tools and its importance and now in this particular lecture we want to discuss common causes assignable causes of variation, objectives of SPC, type I and type II error in control chart and analysis of the patterns in control chart.

So, now here type I and type II error again we will discuss when we talked about hypothesis testing they have already discussed this particular topic in detail, but type I and type II error is the heart of designing the control chart. So, once again we will try to appreciate it.

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Introduction

- ❑ A **control chart** is a graphical tool for monitoring the activity of an ongoing process.
- ❑ Control charts are sometimes referred to as **Shewhart control charts**, because Walter A. Shewhart first proposed their general theory.
- ❑ The values of the quality characteristics are plotted along the vertical axis, and the horizontal axis represents the samples, or subgroups (in order of time), from which the quality characteristic is found. Samples of a certain size (say, 4 or 5 observations) are selected, and the quality characteristic (say, average length) is calculated based on the number of observations in the sample. These characteristics are then plotted in the order in which the samples were taken.
- ❑ Examples of quality characteristics include average length, average diameter, average tensile strength, average resistance, and average service time.

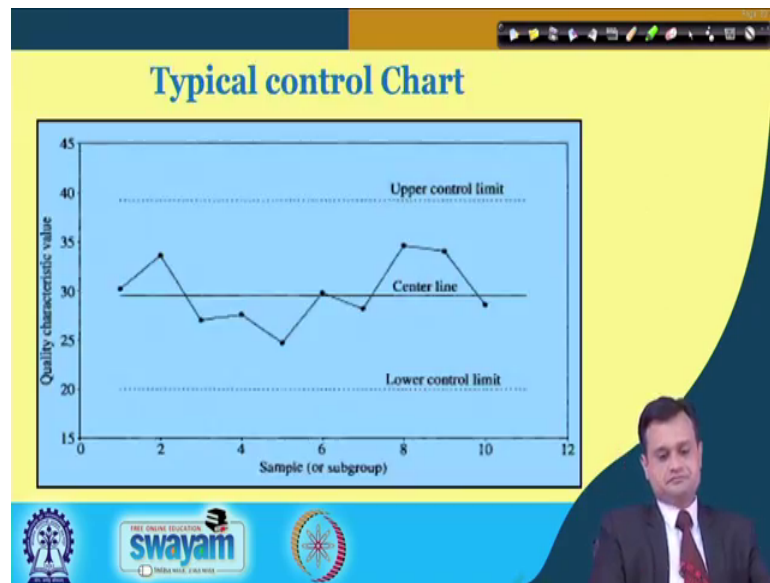
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Now, before I begin with my slides and discussion let me just ask you a very simple question is variability good or bad? So, you will say that; obviously, because there is a variability we are doing all this six sigma, if the variability is reduced I will achieve the six sigma my process is more centric towards the target and it would be fantastic. So, you will say reduce the variability just to cut a joke let me ask you if I give you everyday only rasagulla or only let us say [FL], only one item everyday for let us say 1 year would you like you will not like.

So, there you would like to have the variety some variability in terms of your dishes in terms of your menu card. So, please understand variability is part and parcel of our life whether it is a process, whether it is a product, whether it is the life we cannot get rid of variability as I said measure your blood pressure in the morning it would be 110 by 70, afternoon 120 by 80 may be evening 130 by 85 all is ok. So, long variability is within control I need not to worry when the variability shoots a particular threshold value then there is a matter of concern and I need to take the corrective action and doctor like doctor will prescribe the medicine.

So, with this simple logic I would say that control chart basically is a graphical tool for monitoring the process advocated and design by is Shewhart, so also known as Shewhart control chart and typically you try to draw the inferences or try to say make the conclusions, comments about the control of the process by collecting the sample maybe 4 to 5 observations and over a period of time.

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So, just see this a typical control chart how it look like and this will give you a bigger picture. So, I have the sample or subgroup here, I have the quality characteristic value this quality characteristic maybe your blood pressure, it may be sugar level, it may be specific diameter, it may be your customer waiting time. So, whatever case you may take this is your quality characteristic value, you are the upper control limit, you are the lower control limit and there is a central line.

So, I would like to see that let there be a variability, but this variability should remain within upper control and lower control limit, so long it is well within this prescribed limit I need not to worry, I need not to disturb my process let it continue if it crosses this boundary, then I must think about some corrective action I will not say take the corrective action I must think I must express my concern.

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Benefits Realized by Using Control Charts

Such charts indicate the following:

- ☐ When to take corrective action
- ☐ Type of remedial action necessary
- ☐ When to leave a process alone
- ☐ Process capability
- ☐ Possible means of quality improvement

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So, you have the various benefits to be realized through control chart, when to take corrective action, type of remedial measure you can also conduct the process capability analysis we have done and possible means of quality improvement. If you recall you can only conduct the process capability analysis, if your process is in the state of statistical control.

Now if your process is not in the state of control you cannot conduct the process capability and you cannot calculate your CP CPK CPM PP PPK PPM and other measures. So, here also you need the tool to put your process in the statistical control, so, that the process capability analysis and other subsequent inferences can be drawn.

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Common Cause & Assignable Cause

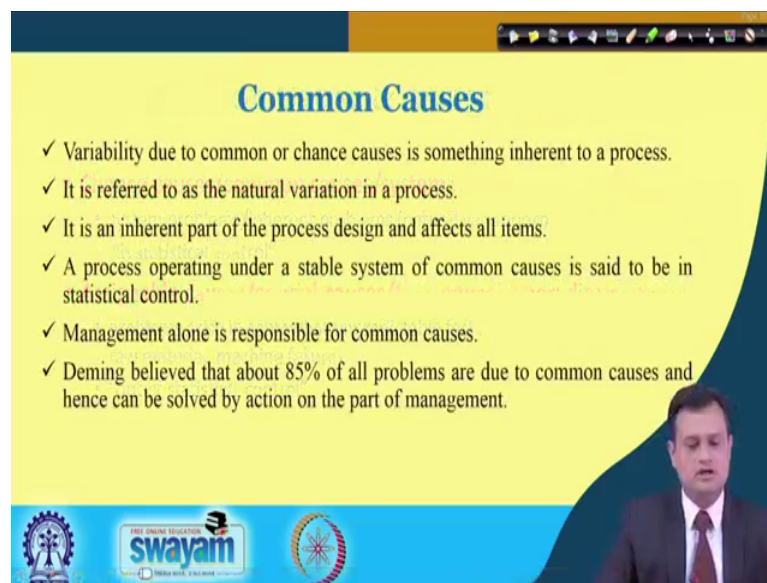
- **Chance causes/common causes/system faults/chronic problems**
 - system problems/inherent problems (natural variation/background noise)
 - "in statistical control"
- **Assignable causes/special causes/local causes/sporadic problems**
 - problems arise in somewhat unpredictable fashion (operator error, defective raw material, machine failure)
 - "out of statistical control"

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So, just see that broadly you have common or chance or random. So, I put here that it could be chance, it could be common, it could be system fault something like that, so you have chance or assignable cause assignable or special. So, please remember that if the variability is just because of some randomness, its by chance, its random, it is a common cause you need not to worry. For example, you are receiving the steel plate and this steel plate; obviously, we will have some variation in the thickness, but this variation is well within the limit, so its this variability is accepted.

Assignable cause suddenly, you see that your particular value of your quality characteristic is going something too high or in terms of control chart crossing your upper control limit let us say then there could be an assignable cause maybe because your process got disturb, you are bearing is worn out, your operator is unskilled or that are other reason because of which your assignable cause is detected. So, chance cause, random, cause common, cause I am not worried assignable cause yes.

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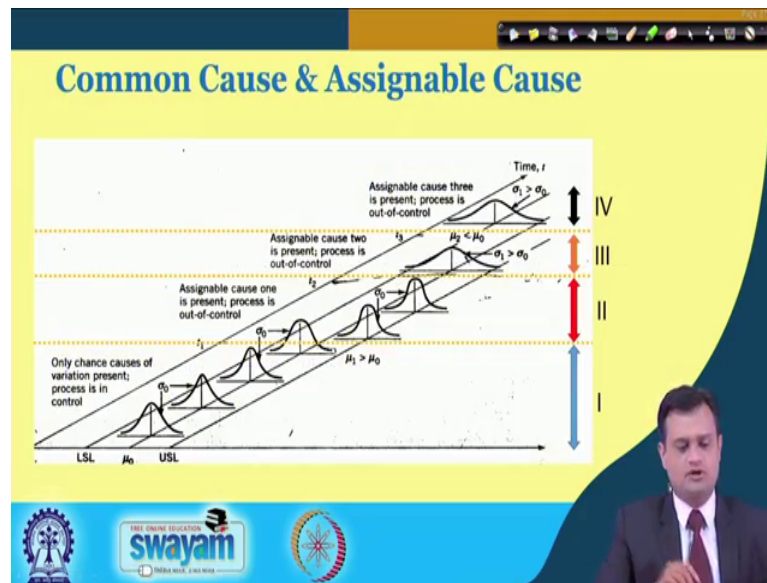
Common Causes

- ✓ Variability due to common or chance causes is something inherent to a process.
- ✓ It is referred to as the natural variation in a process.
- ✓ It is an inherent part of the process design and affects all items.
- ✓ A process operating under a stable system of common causes is said to be in statistical control.
- ✓ Management alone is responsible for common causes.
- ✓ Deming believed that about 85% of all problems are due to common causes and hence can be solved by action on the part of management.

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So these are the couple of issue specific to common cause. So, something inherent to process, natural variation in the process and inherent part of the process and typically 85 percent of the problem are due to common causes and can be solved by drawing the attention of the management.

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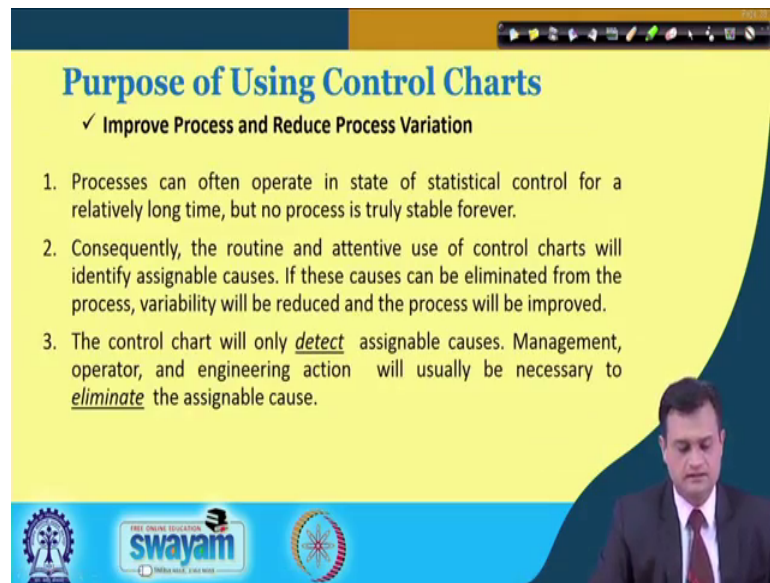
So, they can help you to address the common causes, now just see here this is something interesting you have the μ_0 . Now you have the μ_0 line, this is the initial set value of your process you have LSL and you have USL these are your specification limits.

Now, if you just see here then in this particular region only chance causes of variation present in your process is well control process is well controlled. Now what is happening? You just see that there is some assignable cause and process is out of control. So, you were within the control up to this, now if you just go ahead you will see that your process has gone out of control.

When you see this assignable cause 2 is present process is out of control, so maybe you have more assignable causes present 3 is present and just see that your process is not only going in this direction with respect to mean value you have μ_1 greater than μ_0 , you have μ_2 less than μ_0 may be on the other side, but also here you can see the variability is also increase. So, σ_1 is greater than σ_0 ; σ_0 is your initial process variability.

So, you have a very clear cut picture that you do not have any assignable cause randomness or variability is only because of chance here you have assignable cause 1, cause 2 and cause 3 and you can see that the variability change is your mean is shifted and this kind of issues demands immediate attention.

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Purpose of Using Control Charts

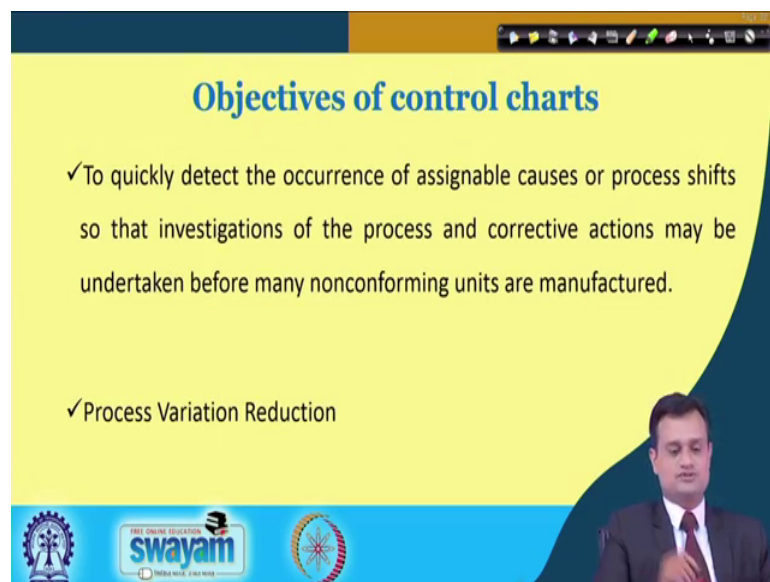
- ✓ Improve Process and Reduce Process Variation

1. Processes can often operate in state of statistical control for a relatively long time, but no process is truly stable forever.
2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved.
3. The control chart will only detect assignable causes. Management, operator, and engineering action will usually be necessary to eliminate the assignable cause.

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So, purpose of using control chart is to keep my process within the control and identify the assignable causes separate them out from the chance causes and to see that I can really figure out an appropriate option to address my assignable cause.

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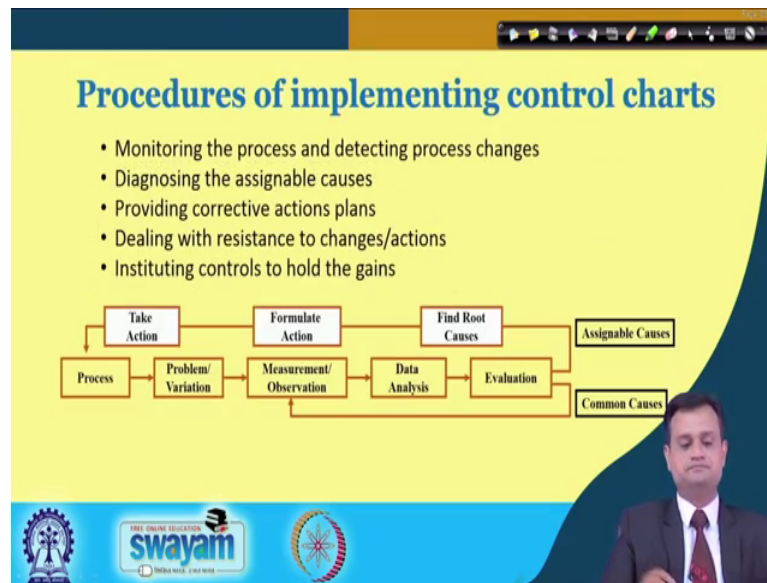
Objectives of control charts

- ✓ To quickly detect the occurrence of assignable causes or process shifts so that investigations of the process and corrective actions may be undertaken before many nonconforming units are manufactured.
- ✓ Process Variation Reduction

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So, objective is very clear is to reduce the process variability or keep the process variability within the control.

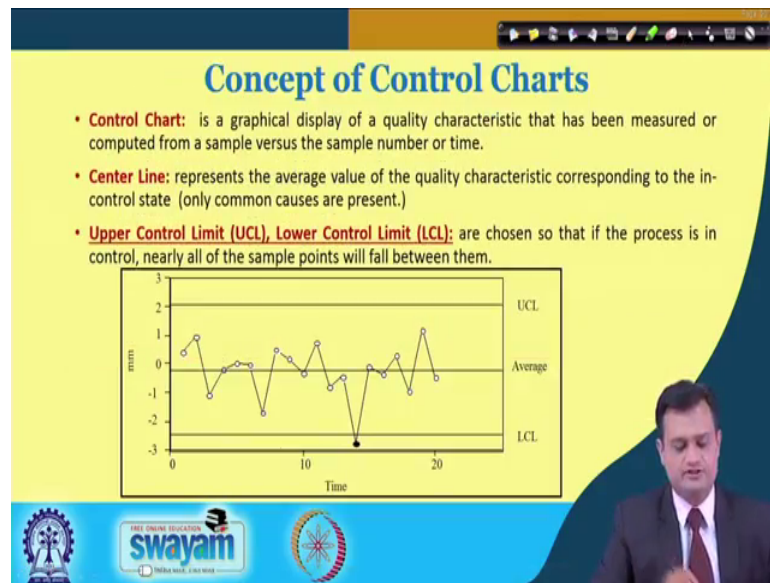
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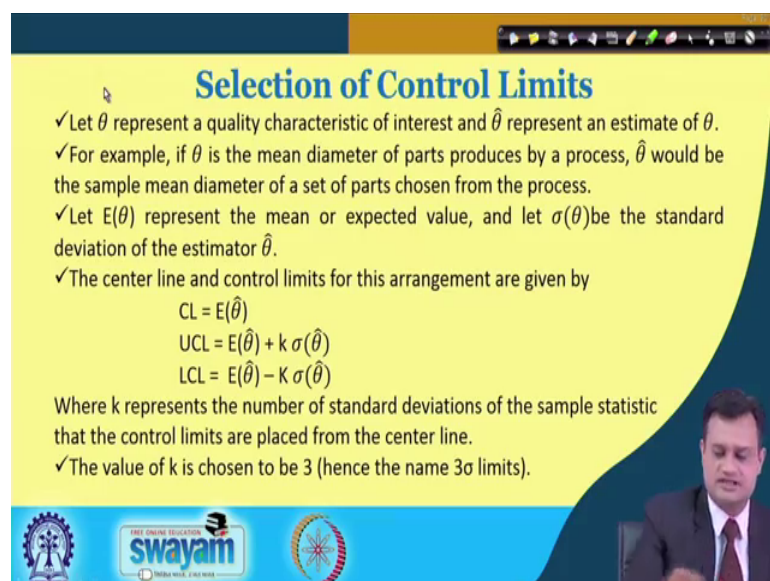
Just see this what is the procedure of implementing control chart? So, you have the process, you have the problem variation or variation, you have the measurement or observation, you have the data analysis, you have the evaluation, then find root cause, formulate action take action. Now this is the process which is after intervention, once again go for these keep doing this if you encounter assignable cause. If it is only because of common cause take a turn from these and just go to measurement of observation and keep plotting your readings.

So, once again see that if my process is going in this direction and if there is an assignable cause point is going out of control limit then add up this path have the set correction and then after treatment after correction or post say intervention rather I will say once again check this. If it is only common cause keep collecting data plotting it and keep an eye on the health of the process.

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So, this is what exactly we try to do. So, you have the control chart graphical display, you have the central line, you have the central line control chart centre line, this is the central line average value your upper control limit and lower control limit, so this is your upper control this is your lower control limit and this is exactly what I want as a part of my control chart.

So, you have to select the control limit a little statistics have presented here, that let us say θ is the mean diameter of the part you are interested in θ hat would be the

sample mean diameter that is the estimate, so your population, your sample we have studied this concepts in detail and extensively used.

Now, I can use this expression that what is the expected value of my mean diameter $\bar{E}$ of θ that will be my central line, then I put my control limit at plus or minus 3 sigma the reason is for all the manufacturing industry 3 sigma limit gives me the best tradeoff between type I and type II error. So, always remember that why we usually choose plus or minus 3 sigma limit as upper control limit and lower control limit, it gives me the best tradeoff between type I and type III error.

So, type I error and type II error you all know that say if I am punishing the innocent or let us say if I am accepting the guilty, then I will be encountering either type I or type II error. So, I have the estimated value of θ that is the mean value plus k ; k I will use generally 3. So, plus or minus 3 sigma apart from the central line and the reason I told it gives me the best (Refer Time: 16:58) of between type I and type II error and this is the LCL. So, I will set my tool at upper control limit, lower control limit with respect to centre line.

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What Can a Control Chart Show?

- ☐ Is the process in control?
- ☐ Is the pattern of variation stable or unstable?
- ☐ Is there a shift in the average?
- ☐ Does the output from the process meet specifications?

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So, now what control chart show is the process in control, is the pattern of variation stable or unstable and is there a shift in the average or does the output from the process meet specification.

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Difference between Control Limit and Specification Limit

- ✓ Control Limits are used to determine if the process is in a state of statistical control (i.e., is producing consistent output).
- ✓ Specification Limits are used to determine if the product will function in the intended fashion. (conforming/non-conforming)

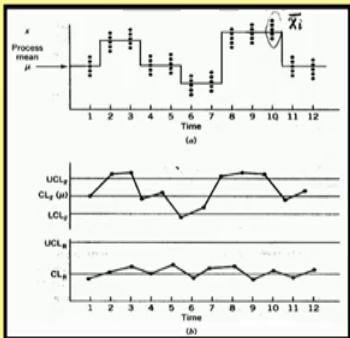


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So, there is a difference between control limits and the specification limit remember very very clearly. Control limits are determined are used to determine if the process in a state of statistical control or not and specification limit are used to determine if the product with function in the intended fashion or not. And a specification limits basically you get from the customer by capturing the voice of customer, control limits they are set based on the statistical analysis to check that whether my process is in control or not.

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Samples (Subgroups) in Control Charts



A point on a control chart is not necessarily a single measurement of the quality characteristic.

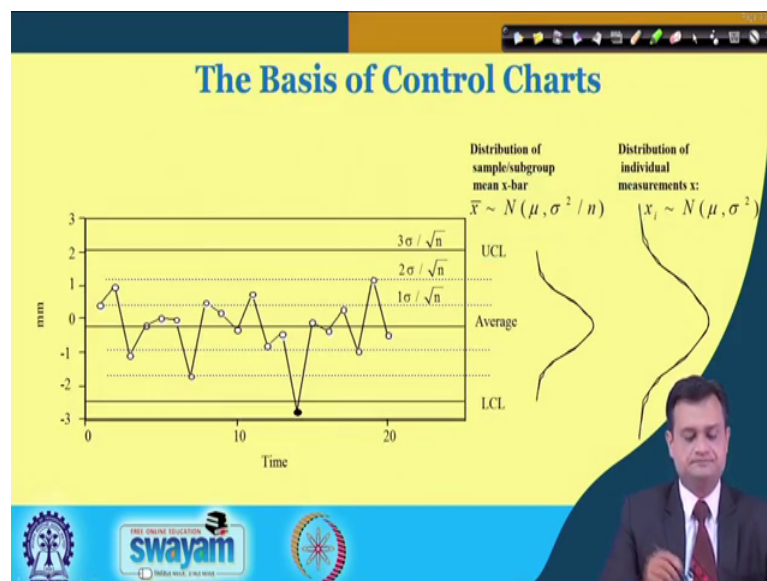
(a) Behaviour of the process mean, (b) corresponding $\bar{x}$ -bar and R control charts



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So, please do not confuse between these two and what you can see here? That a control chart typically plotted like this you have the time, you have the central line upper control limit, the another one upper control limit $\bar{x}$ bar CL $\bar{x}$ bar. So, you have a point on a control chart is not necessarily a single measurement of the quality characteristics. You can plot you on the multiple measurement just see here that you have this readings and this is my process mean. So, you can have subgroup size variation also and that way you can consider it and plot the control chart.

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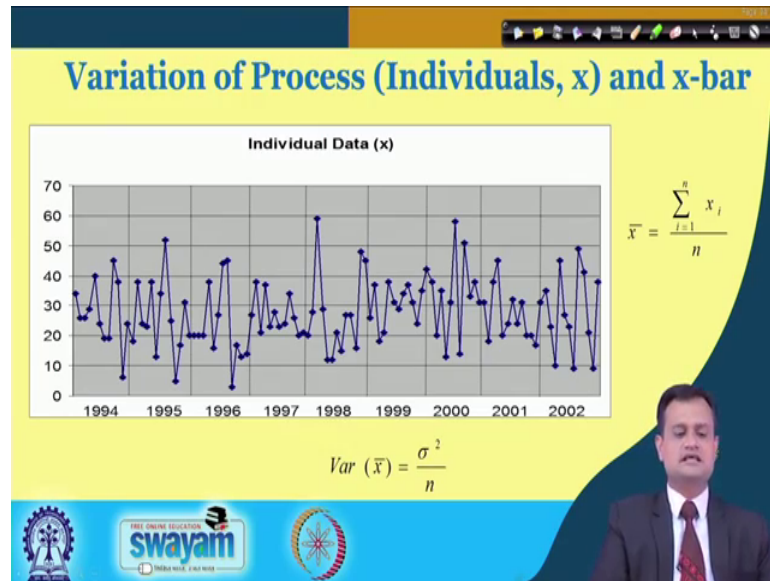


So, now just see here that usually I deal with the sampling distribution and I try to say plot the values of $\bar{x}$ that is the average of sample particular sample. So, suppose in a sample or subgroup you have some 5 readings of the productivity 100, 20, 40, 50, 70, 80 you just check the average of this summit of divide by 5, this will be your sample average.

So, when you are dealing with the distribution of sample of subgroup $\bar{x}$ bar, then your $\bar{x}$ bar is normally distributed with mean μ and obviously, as per central limit theorem say variability or variation σ^2 divided by, when you are dealing with the individual distribution instead of plotting my $\bar{x}$ value I plot individual observation then my x_i is normally distributed with say μ and σ^2 mean and variance and this is my time and mm.

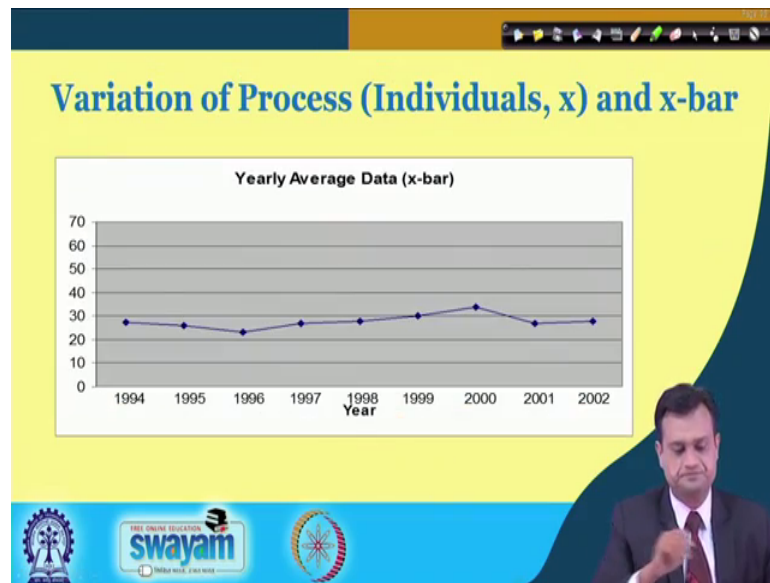
Now, you just see here this upper control limit is at a distance of 3 sigma by root m or 3 standard deviation this is another limit 2, this is another limit 1, we will see the significance of these two limit, but mainly you are concerned with the upper control limit and lower control limit which are placed at 3 sigma for analyzing the health of your process or whether your process is in control or not statistically.

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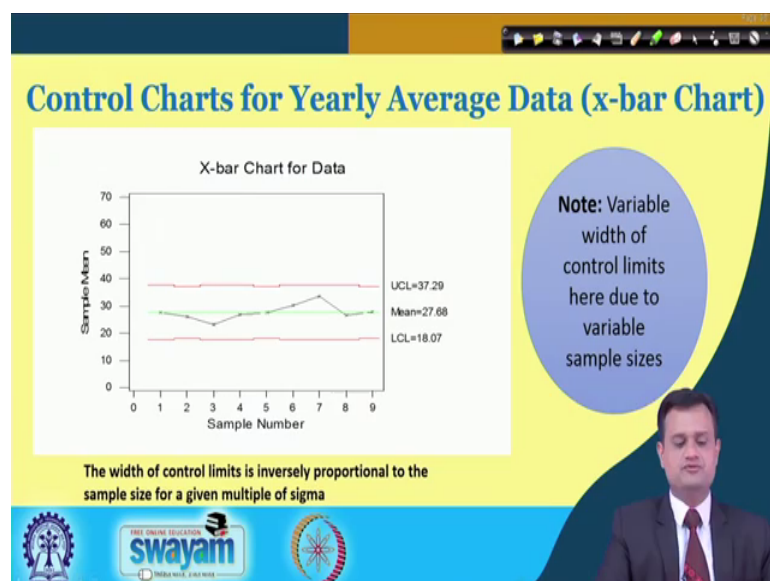


So, this is the variation of the process individual x, as a mentioned you can plot the values of x bar or you can put the x. So, here you are putting basically the values of x for particular year let us say individual data x and x bar is basically sigma x i divided by n. So, you can find out some x bar value or put an central line here and this is your plotting of the individual value.

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Yearly average data again you are putting the individual value, this is your $\bar{x}$ bar chart because I am putting the average values of the sample and you have the upper control limit, lower control limit and the mean. So, fine I think this would have at least give you given you some idea that how your particular say control chart looks like.

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Control Charts vs. Hypothesis Test

$H_0: \mu = \mu_0$
 $H_1: \mu \neq \mu_0$

Reject H_0 if $\left| \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \right| > Z(\alpha/2)$

- Hypothesis Testing
 - Hypothesis testing has a rejection region
 - H_0 is rejected if the data fall in the rejection region

☐ Control chart has UCL & LCL
☐ The process is out of control if the data beyond the control limits

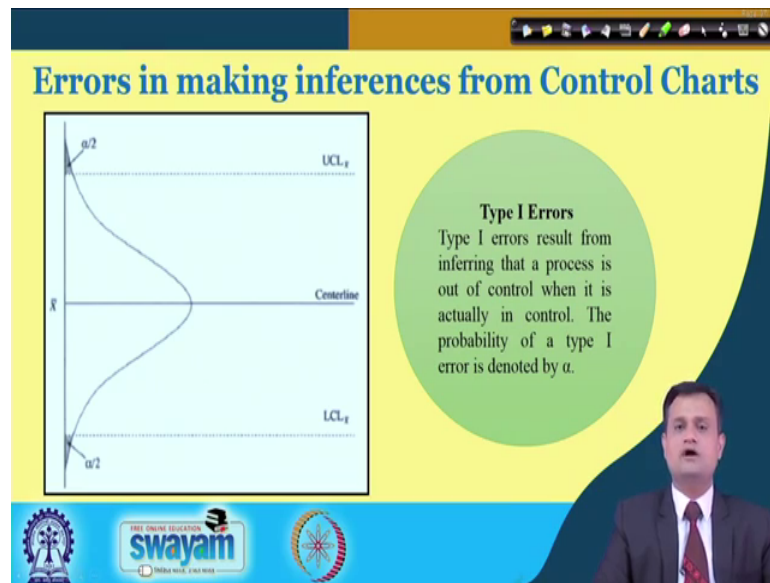
The slide also features a normal distribution curve with the mean μ_0 and standard deviation $\sigma/\sqrt{n}$. The rejection regions are shaded in the tails, each with an area of $\alpha/2$. The upper control limit (UCL) and lower control limit (LCL) are marked on the x-axis. The Swamy logo is visible at the bottom.

Now I am not going away from my inferential statistics and hypothesis testing, here also it is valid and what I say that my null hypothesis is true if μ is equal to μ_0 . And if there is a shift in the mean, it means there is some deviation because of which my process may go out of control and I can encounter the points out of the control limit. Then there is the reason to say that some assignable cause is present.

So, hypothesis testing as a rejection region you can see $\alpha/2$; $\alpha/2$ even with respect to your upper control limit and lower control limit and H_0 is rejected, if the data fall in the rejection region. So, extensively we have done this hypothesis testing, but if you recall that time we were referring the critical value from the table Z value t value and we have the calculated value, we were trying to see either the critical value or p value, that whether it falls in the rejection region or acceptance region and based on that we were taking the decision.

Here also you have this critical value, but this critical value is in terms of upper control limit and lower control limit and if you find that point is falling outside this; outside this then it is in the rejection region. It means you reject null hypothesis status q it means that there is no shift in mean, actually there is sub shift in the mean of the process.

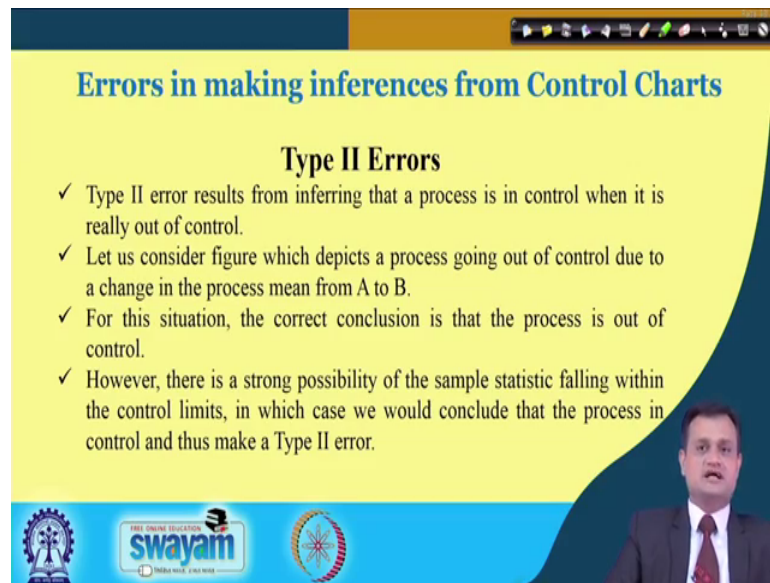
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So, you have 2 errors just too little bit repeat again type I error and type I error results from inferring that a process is out of control where it is actually in control, the probability of type I error is denoted by this is 2 tails α by 2 plus α by 2 is α . Let me give you a very simple example, that today you went to a school or your college or those who are married people say you went to home after an extremely tiresome day and let us say you have some conflict with your teacher or with your wife or husband I am not gender biased.

So, now, after having such conflict you will check your blood pressure and you will find that it has shot up. Now, it is in the rejection region and I will say that yes I have got a blood pressure problem, but it is not. So, you would understand that I may commit a type I error and say that the process is out of control, blood pressure has increased it means I have become a permanent the patient of blood pressure, but it is not. So, your process you say that out of control, but it is within the control this is my type I error.

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Errors in making inferences from Control Charts

Type II Errors

- ✓ Type II error results from inferring that a process is in control when it is really out of control.
- ✓ Let us consider figure which depicts a process going out of control due to a change in the process mean from A to B.
- ✓ For this situation, the correct conclusion is that the process is out of control.
- ✓ However, there is a strong possibility of the sample statistic falling within the control limits, in which case we would conclude that the process is in control and thus make a Type II error.

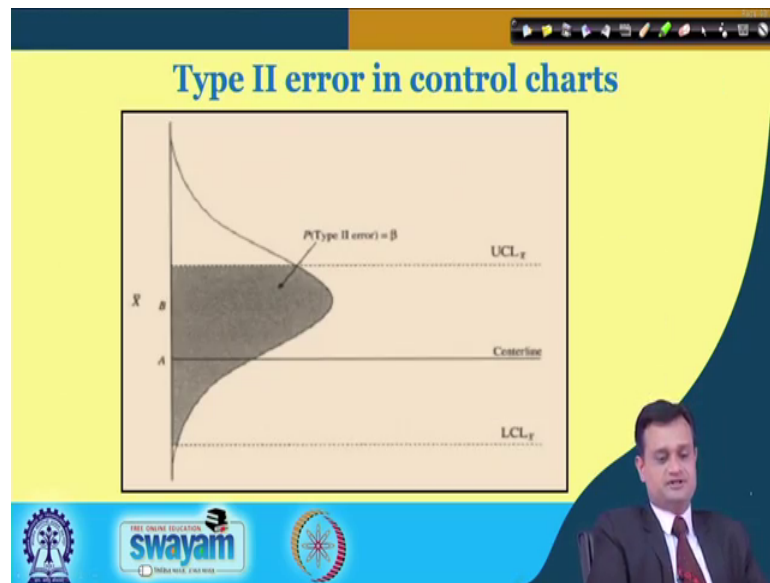
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So, now, same way you can interpret the type II error that my process I say that it is within the control, but actually it is out of control. Just see the another example, that many people they ignore the health issue and some days suddenly they experience the heart attack. So, you do not encounter the heart attack just like that you might have been exposed to very high blood pressure, sugar level and many other factors and then 1 day you may realize that your heart has gone weaker and you encounter the heart attack.

So, you are just ignoring, so here I say that my process is in control, but actually it is out of control, so this is my type II error. So, both are dangerous if you are type doing type I error you will unnecessarily take corrective action when actually your process is in control, if you are having very high type II error you will say that I am actually in control, but the process is producing defective items.

So, both type I and type II you have to balance it out have a trade off and this is basically ultimately say when I said to my limits plus or minus 3 sigma, I get the proper idea about the health of my process whether it is in control or not and as I mention 3 sigma limits give me the better tradeoff between type I and type II error and that is why usually I go for plus or minus 3 sigma limit.

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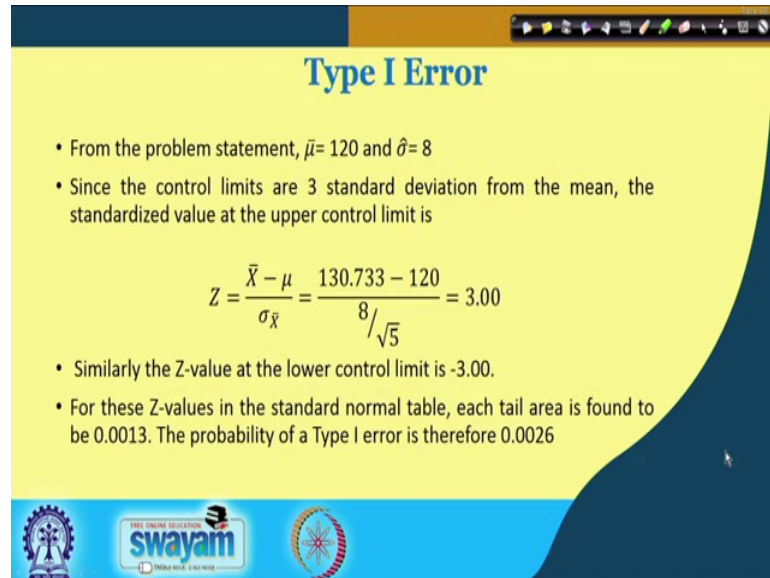
A control chart is to be constructed for the average breaking strength of nylon fibers. Samples of size 5 are randomly chosen from the process. The process mean and standard deviation are estimated to be 120kg and 8kg respectively.

- If the control limits are placed 3 standard deviations from the process mean, what is the probability of a Type I error?
- If the process mean shifts to 125 kg. what is the probability of concluding that the process is in control and hence making a Type II error on the first sample plotted after the shift?
- What is the probability of detecting the shift by the second sample plotted after the shift if the samples are chosen independently?

So, this is exactly your type II error you can have an illustrative example, that a control chart is to be constructed for the average breaking strength of nylon fibers. Sample of size 5 or randomly chosen and process mean standard deviation are estimated to be 120 kg and 8 kg. If the control limits are placed at 3 standard deviation, then what is the probability of type I error?

Now, when I talk about type II error there is a shift in mean. So, let us say there is a shift in process mean 225 what will be the type II error? What is the probability of detecting the shift by the second sample? And this is what I would like to analyse.

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Type I Error

- From the problem statement, $\bar{\mu} = 120$ and $\hat{\sigma} = 8$
- Since the control limits are 3 standard deviation from the mean, the standardized value at the upper control limit is

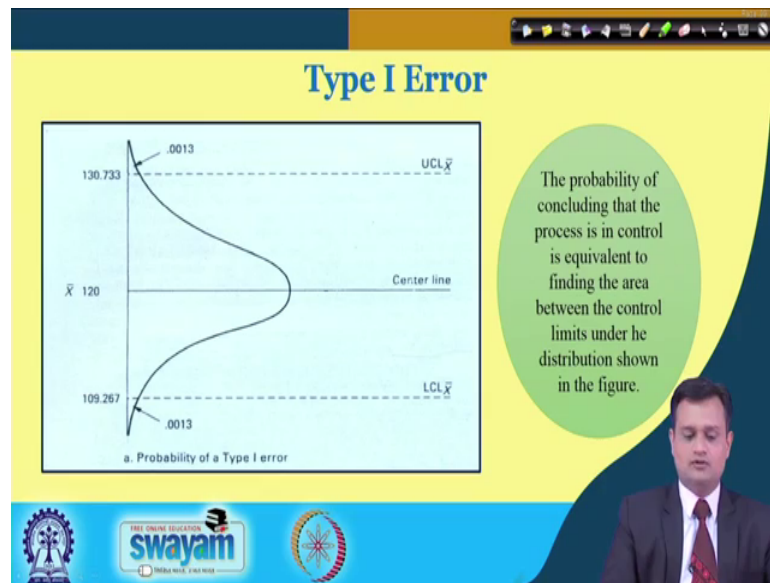
$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{130.733 - 120}{8/\sqrt{5}} = 3.00$$

- Similarly the Z-value at the lower control limit is -3.00.
- For these Z-values in the standard normal table, each tail area is found to be 0.0013. The probability of a Type I error is therefore 0.0026

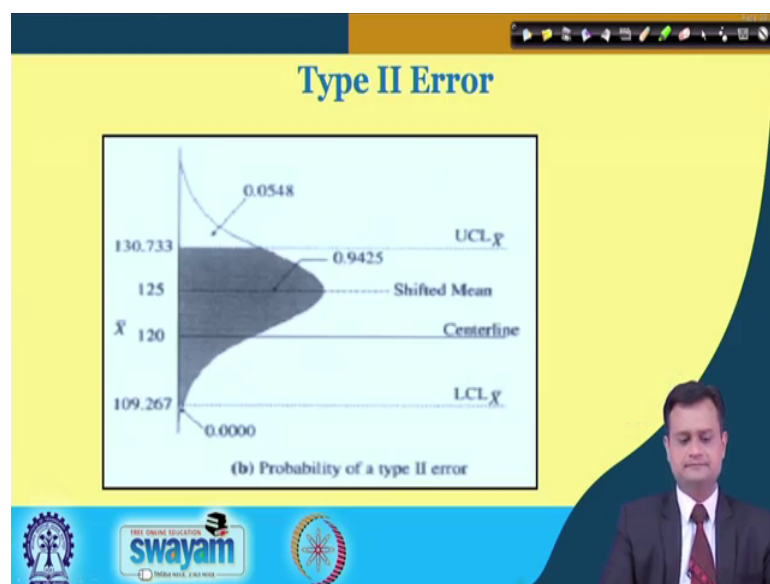
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So, type I error this is my μ 120 and 8, this equation is well known to you $\bar{X}$ minus μ by σ . So, I am plugging the values and I get the minus 3, so will say my type I error is 0.0026. So, this is the probability of my type I error, that I would commit a mistake saying that my process is out of control in this case my mean is typically say this is my $\bar{\mu}$ my process is out of control when actually it is in the control.

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Now, this is a type I, type II error you can compute very easily just see that I want to find Z_1 I want to find Z_2 .

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From the standard normal table, the tail area above the upper control limit is 0.0548 and the tail area below the lower control limit is 0.0000. The area between the control limits is $1 - (0.0548 + 0.0000) = 0.9452$. Hence, the probability of concluding that the process is in control and making a Type II error is 0.9452 or 94.52%.

$$Z_1 = \frac{130.733 - 125}{8/\sqrt{5}} = 1.60$$
$$Z_2 = \frac{109.267 - 125}{8/\sqrt{5}} = -4.40$$


So, 130.733 minus 125 this is the process mean, 109.267 minus 125, so I will find this particular Z 1 and Z 2 and corresponding probability I will subtract it from 1, so that will basically give me the region because it is 2 tailed. So, I have found this two Z 1 and Z 2 you can see here that it is 109 and 130. So, I will have this is the 120, that is a shifted mean 125. So, I use this value I can find out the 0.000 plus 0.0548 is basically my type I error when is subtract it from the 1 I get 0.9425 that is my type II error.

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Type II Error

- The probability of detecting the shift by the second sample is P (Detecting the shift on sample 1) + P (Not detecting shift in sample 1 and detecting shift in sample 2)
- The first probability = 0.0548
- The second probability = $(1 - 0.0548)(0.0548) = 0.0518$, assuming independence of the two samples
- The total probability is $0.0548 + 0.0518 = 0.1066$



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Measures of Performance of Control Chart

1. Operating Characteristic (OC) Curve
2. Average Run Length (ARL)

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So, with this I can have the type II error, so exactly that is what I have done here. So, you have type II error and total probability is this, so there are 2 broad methods of checking the performance of the control chart. Again going with the example of blood pressure you are checking your blood pressure, but the instrument itself is faulty and you are getting 180 blood pressure you are making and actually it is 120, so making huge type I error.

So, when there is an error there must be some mechanism by which I can check the performance of my control chart, my tool I can calibrate it. So, here you have 2 methods one is OC curve, other is average run length.

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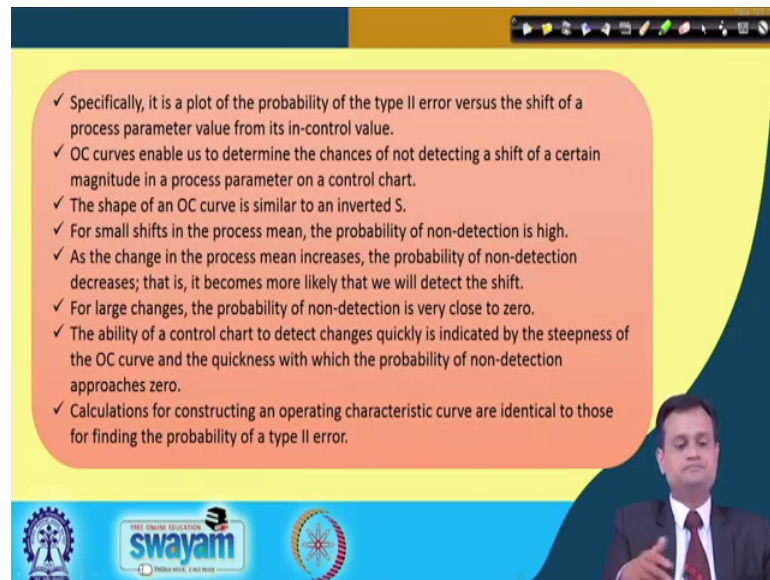
What is Operating Characteristic Curve?

An operating characteristic (OC) curve is a measure of goodness of a control charts ability to detect changes in process parameters.

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So, the first is OC curve, so typically your OC curve basically indicates what is the discriminatory power of your control chart? It means measure of goodness of the control chart. Will it really identify the bad point or the assignable cause separate it from the common cause or not and this is the role of my typical OC curve.

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- ✓ Specifically, it is a plot of the probability of the type II error versus the shift of a process parameter value from its in-control value.
- ✓ OC curves enable us to determine the chances of not detecting a shift of a certain magnitude in a process parameter on a control chart.
- ✓ The shape of an OC curve is similar to an inverted S.
- ✓ For small shifts in the process mean, the probability of non-detection is high.
- ✓ As the change in the process mean increases, the probability of non-detection decreases; that is, it becomes more likely that we will detect the shift.
- ✓ For large changes, the probability of non-detection is very close to zero.
- ✓ The ability of a control chart to detect changes quickly is indicated by the steepness of the OC curve and the quickness with which the probability of non-detection approaches zero.
- ✓ Calculations for constructing an operating characteristic curve are identical to those for finding the probability of a type II error.

So, I have just put various characteristics of the OC curve that it plots the probability of type II error versus the shift of a process parameter. Enables us to determine the chances of not detecting a shift of certain magnitude, similar to an inverted S curve and this typically OC curve as an ability to detect change quickly. So, if there is a small shift in the process, then the probability of non deduction must be high it means immediately it should not trouble you and if there is a large shift in the process mean then probability of non deduction should be low, so that assignable cause can be rectified.

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A control chart is to be constructed for the average breaking strength of nylon fibers. Samples of size 5 are randomly chosen from the process. The process mean and standard deviation are estimated to be 120kg and 8kg respectively. Construct the operating characteristic curve for increases in the process mean from 120 kg.



So, you can just see the example that a control chart it is to be constructed for the average breaking strength same example 5 are randomly chosen samples and estimated say mean and standard deviation 120 kg and 80 kg, so construct the operating characteristic curve.

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Solution

Probabilities for OC Curve

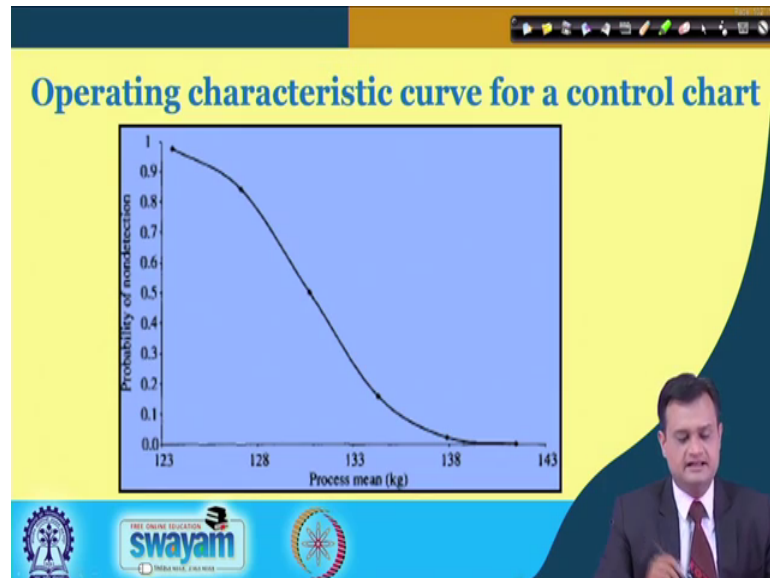
Process Mean	Z-value at UCL, Z_1	Area Above UCL	Z-value at LCL, Z_2	Area Below LCL	Probability of Nondetection, β (Type II Error)
123.578	2.00	0.0228	-4.00	0.0000	0.9772
127.156	1.00	0.1587	-5.00	0.0000	0.8413
130.733	0.00	0.5000	-6.00	0.0000	0.5000
134.311	-1.00	0.8413	-7.00	0.0000	0.1587
137.888	-2.00	0.9772	-8.00	0.0000	0.0228
141.466	-3.00	0.9987	-9.00	0.0000	0.0013



So, here I am just showing you the table that this is what is the shift in the process mean and these are the Z values at UCL Z 1 area above UCL, so you can easily find from the Z table the corresponding probability for a given Z value; Z value and LCL, area below

LCL and probability this of non detection beta that is type II. So, you can easily compute this and once you have compute all this, then you can put it on the graph.

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So, process mean; process mean and the probability of non detection, so this will basically help you to see that to what extent it is discriminatory. So, let us say if there is a shift in the mean and suppose if this is the probability, then and suppose if I just plot another OC curve and then just see that the magnitude is very high.

So, stiffness of the OC curve basically decide the discriminatory power, I do not want to say that my OC curve should be extremely discriminatory if it is then I will be making a huge type I error, that even I do not want I must have a reasonable OC curve which can give me the balance between type I and type II error. So, it can detect the assignable causes and simultaneously do not miss guide me when I do not have to take the corrective action.

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What is ARL?

ARL denotes the number of samples on average required to detect and out of control signal.

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Now there is another measure which is useful in checking the performance of my say control chart and this is called average run length. So, on an average how many samples you need to take to detect? That there is a change in the process or there is sub shift in the mean. Again I am repeating on an average how many sample you need to collect to detect a change in the process?

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Average Run Length (ARL)

- An alternative measure of the performance of a control chart, in addition to the OC curve, is the **average run length (ARL)**.
- This denotes the number of samples, on average, required to detect an out-of-control signal.
- Suppose that the rule used to detect an out-of-control condition is a point plotting outside the control limits. Let P_d denote the probability of an observation plotting outside the control limits. Then the run length is 1 with a probability of P_d , 2 with a probability of $(1 - P_d) P_d$, 3 with a probability of $(1 - P_d)^2 P_d$, and so on. average run length is given by

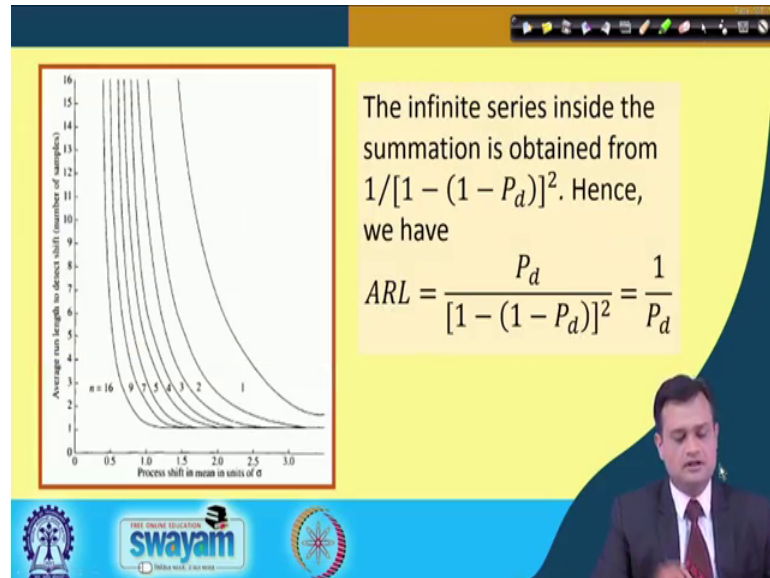
$$ARL = \sum_{j=1}^{\infty} j(1 - P_d)^{j-1} P_d$$

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And this is typically called average run length and it is expressed as P_d denote the probability of an observation plotting outside the control limit. The run length is 1 the

probability is P_d , if it is $1 - P_d$ and P_d you have the probability knowledge, 3 with probability $1 - P_d$ square into P_d and typically this series can be captured as j is equal to 1 to infinity $j - 1$ minus P_d raise to $j - 1$ into P_d .

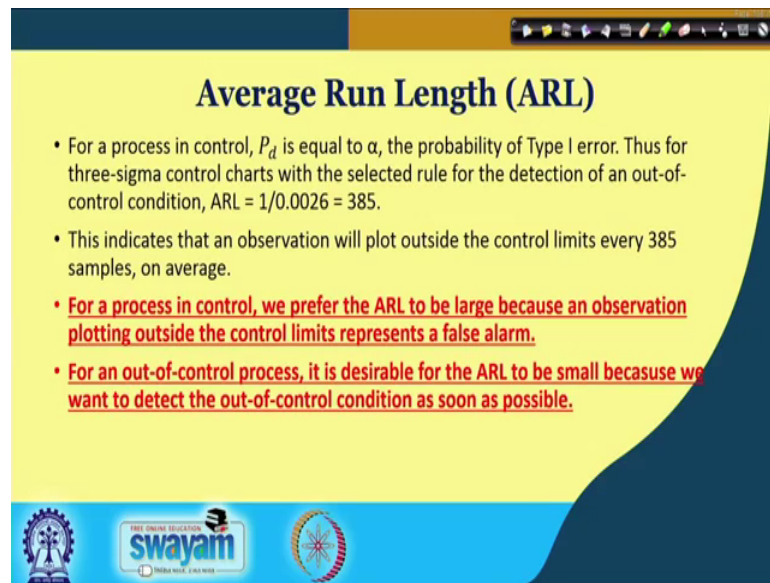
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So, you have typically the expression when you put the series in a simplified manner and you can see here that what exactly I am doing? Process safety mean in units of sigma. So, some sigma there is shift in mean and there is the average run length to detect the shift.

Now, suppose you are using let us say sample 2 and let us say there is some shift in mean, then you can say that on an average how many sample you will need to detect this particular shift?

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Average Run Length (ARL)

- For a process in control, P_d is equal to α , the probability of Type I error. Thus for three-sigma control charts with the selected rule for the detection of an out-of-control condition, $ARL = 1/0.0026 = 385$.
- This indicates that an observation will plot outside the control limits every 385 samples, on average.
- For a process in control, we prefer the ARL to be large because an observation plotting outside the control limits represents a false alarm.
- For an out-of-control process, it is desirable for the ARL to be small because we want to detect the out-of-control condition as soon as possible.

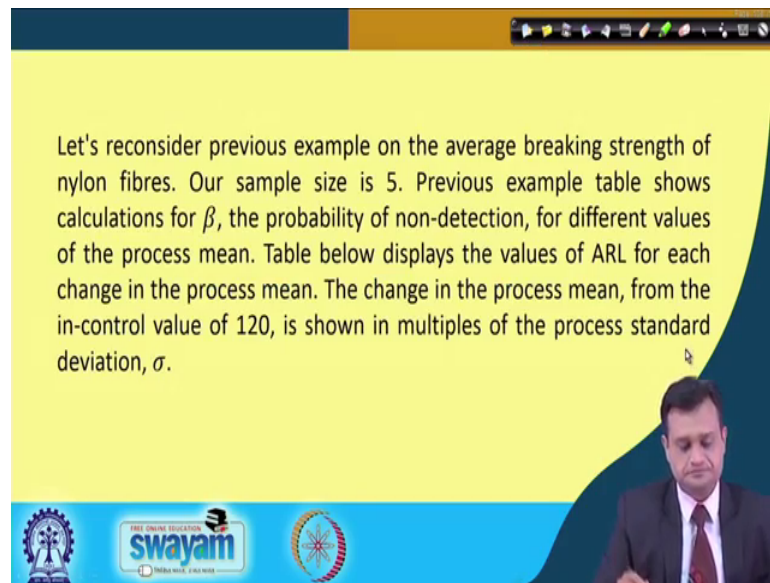
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So, this is your another measure and you would be happy to see some analysis the process in control we prefer the ARL to be large because an observation prodding outside the control limit represent a false alarm [FL], I do not want to improve upon my detect ability because my process is in control.

So, I would be happy with the ARL to be large it means on an average more number of samples let it be there before I really detect the change, but for an out of control process it is desirable for the ARL to be small means the length of my ARL must be small. So, that has assignable cause can be detected as early as possible.

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Let's reconsider previous example on the average breaking strength of nylon fibres. Our sample size is 5. Previous example table shows calculations for β , the probability of non-detection, for different values of the process mean. Table below displays the values of ARL for each change in the process mean. The change in the process mean, from the in-control value of 120, is shown in multiples of the process standard deviation, σ .

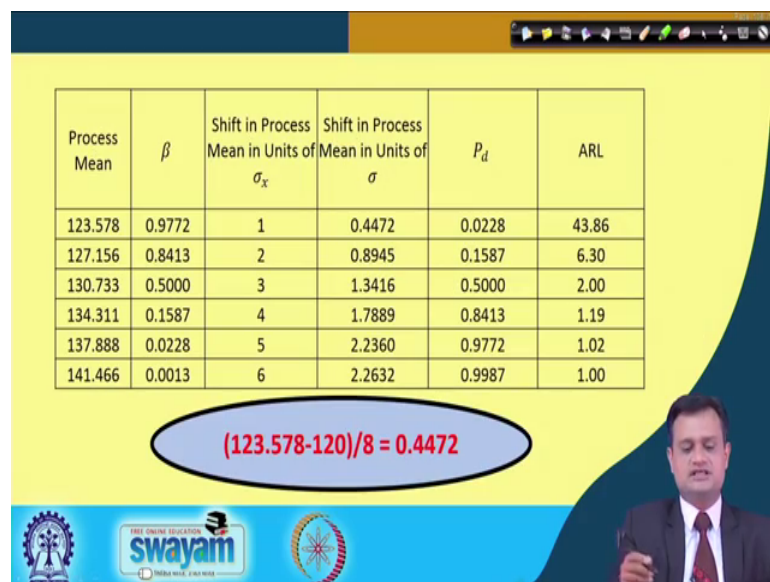


This is just the example, that average breaking strength of nylon fiber, sample size is 5 and you have the calculated beta value that is the probability of non detection type II error and you have computed the ARL, so just see this table and what I have done?

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Process Mean	β	Shift in Process Mean in Units of σ_x	Shift in Process Mean in Units of σ	P_d	ARL
123.578	0.9772	1	0.4472	0.0228	43.86
127.156	0.8413	2	0.8945	0.1587	6.30
130.733	0.5000	3	1.3416	0.5000	2.00
134.311	0.1587	4	1.7889	0.8413	1.19
137.888	0.0228	5	2.2360	0.9772	1.02
141.466	0.0013	6	2.2632	0.9987	1.00

$(123.578-120)/8 = 0.4472$

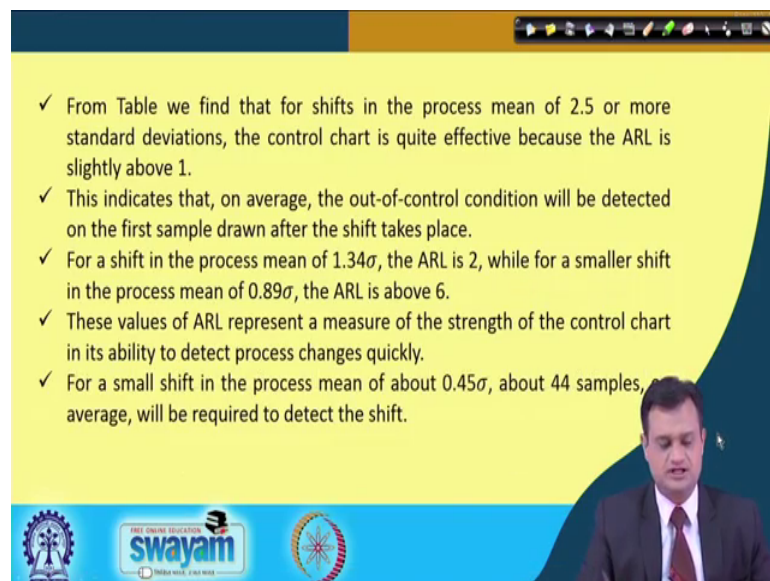


These are the mean this is the same table we have used, these are the corresponding beta value which we have already computed, this is the shift in the process mean in unit of six sigma $\bar{x}$. So, let us say I am saying that my mean is shifting by 1 sigma, 2 sigma, 3 sigma, 4 sigma.

Now, this is shift in process mean in units of sigma, just for your understanding I have put here how this value has come 0.4472. So, my process mean or set value target value is 120, there is some shift let us say 123.578. So, I am subtracting 123.578 minus 120 dividing by the standard deviation that is 8, so I will get 0.4472, so this is the shift in process been in units of sigma. Once I have done this I can compute my P d value, this is the probability of detection and once I have done this I can compute my ARL, so Average Run Length.

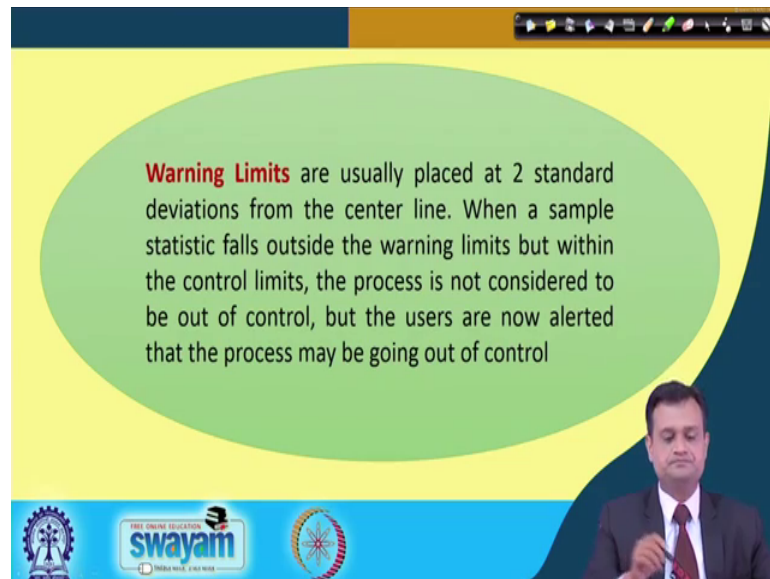
Now what do you see here? You see that when there is a small change shift in the mean in terms of sigma my average run length is larger means, I will not end up with a false alarm that detection will not be very fast. Similar way when the change is higher my run length reduces for 4 it is 1.19, 5 it is 1.02 6 means on an average at every sample if there is a six sigma shift in the mean every sample will detect this change, give me the alarm that is an assignable cause I need to correct it in order to bring my process back into statistical control.

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- ✓ From Table we find that for shifts in the process mean of 2.5 or more standard deviations, the control chart is quite effective because the ARL is slightly above 1.
- ✓ This indicates that, on average, the out-of-control condition will be detected on the first sample drawn after the shift takes place.
- ✓ For a shift in the process mean of 1.34σ , the ARL is 2, while for a smaller shift in the process mean of 0.89σ , the ARL is above 6.
- ✓ These values of ARL represent a measure of the strength of the control chart in its ability to detect process changes quickly.
- ✓ For a small shift in the process mean of about 0.45σ , about 44 samples, average, will be required to detect the shift.

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Warning Limits are usually placed at 2 standard deviations from the center line. When a sample statistic falls outside the warning limits but within the control limits, the process is not considered to be out of control, but the users are now alerted that the process may be going out of control

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There are warning limits also and if you recall I say that you have upper control limit, lower control limit and you are also setting some limit say 2 sigma, so these are warning limits. So, here my logic is very simple why should I wait till I get the heart attack? Should I not have some warning or should I not have some indication that yes, there is an opportunity to correct instead I stop the machine and say entire shut up and go for the corrective action.

So, this will yield a bigger cause revert rejection, so warning limits are extremely useful in this condition. When I understand that my process is new, it is variable, it has lot of ups and downs then I can even said the warming limit to get hold on the situation at the early stage and before my process goes out of control.

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Selection of Rational Samples

- ❑ Shewhart described the fundamental criteria for the selection of rational subgroups, or rational samples.
- ❑ The premise is that a rational sample is chosen in such a manner that the variation within it is considered to be due only to common causes.
- ❑ Samples are selected such that if special causes are present they will occur between the samples. Therefore, the differences between samples will be maximized, and differences within samples will be minimized.

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So, there are certain rules for the selection of the rational sample and it says that the within sample variability it should be chosen in such a way that variation within is considered to be only due to common causes and if special causes are present they will occur between the sample.

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Sample Size and Sample Frequency

- ❑ Strategies:
 - Small samples at short intervals (favorable in high volume or “problem-prone” processes)
 - larger samples at longer intervals
 - Adaptive or variable sampling interval
- ❑ Average run length and average time to signal ($ATS = ARL \times \text{sampling interval } h$) are considered in design and then check the detection power.

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So, these are some of the strategies small samples at short interval, problem prone processes, if processes are good then longer interval can be chosen and likewise.

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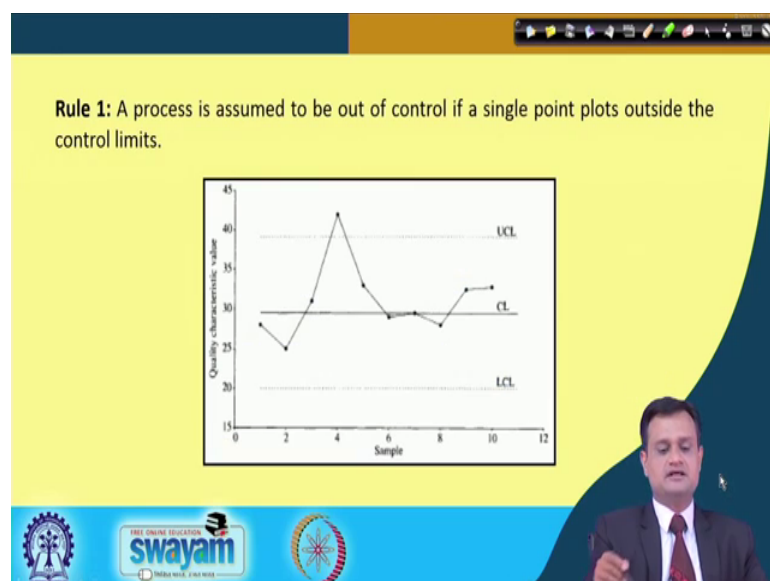
Analysis of Patterns in Control Charts

- ✓ One of the main objectives of using control charts is to determine when a process is out of control so that necessary actions may be taken.
- ✓ Criteria other than a plotted point falling outside the control limits are also used to determine whether a process is “out of control”.



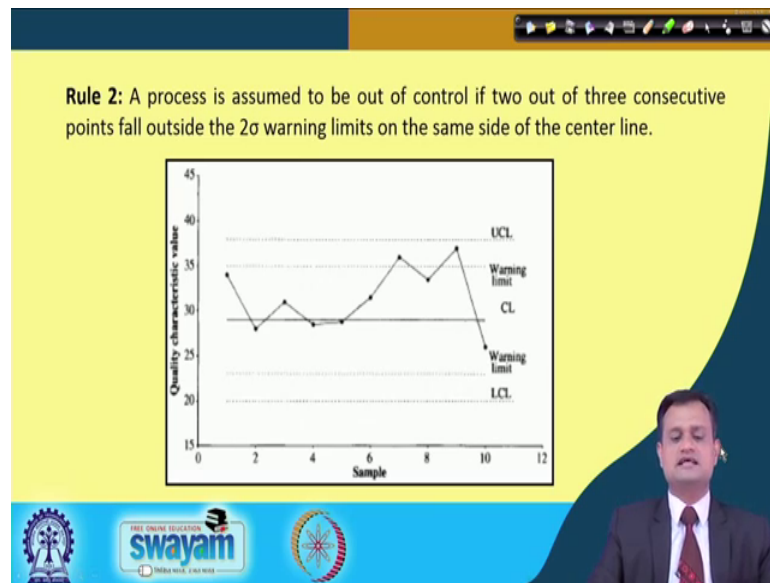
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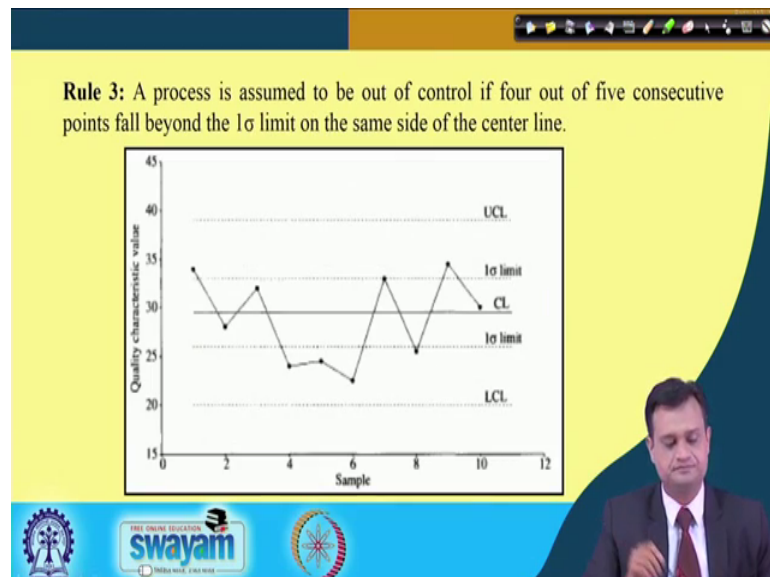
So, typically you can also do the analysis of pattern in control chart. So, just see this process is assume to be in the control and if everything is within, but here the point is falling out side. So, it is out of control and I have to take some corrective action.

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We can see the other, process is assume to be out of control if two or the three consecutive points fall outside the 2 sigma and you can see it very well here. So, three out of two they are falling outside the 2 sigma warning limit there is something wrong with my process.

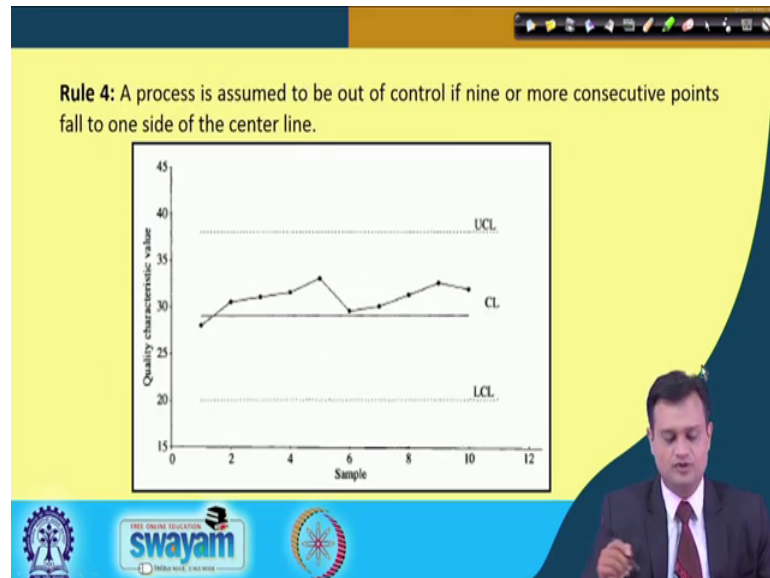
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You can see that process is assume to be out of control if, four out of five consecutive points fall beyond 1 sigma limit, this is another limit that is 1 sigma and this is the case

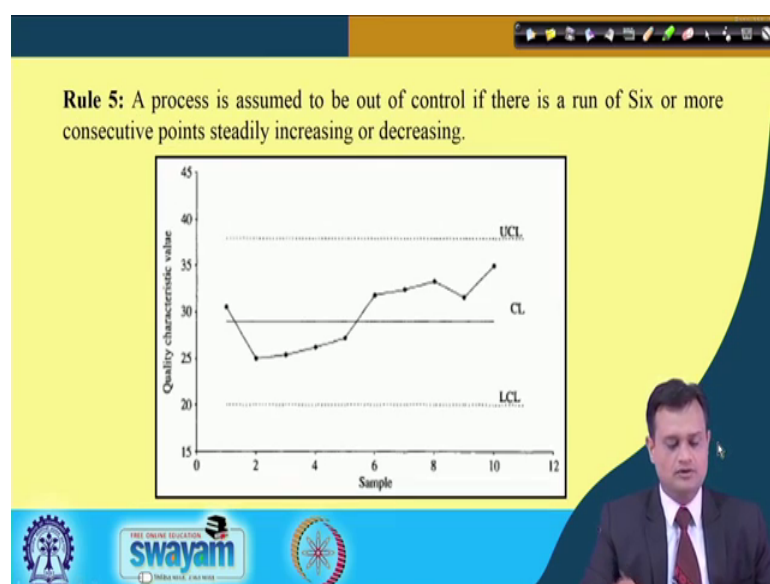
that five consecutive points, so you can very well see here that they fall within the 1 sigma limit sorry outside the 1 sigma limit this is the region.

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Similar way, you can see that this is again the indication of out of control if nine or more consecutive points fall to one side of the central line then you can see this is the central line and you have the points, this is the central line you have the points falling just on the upper side of the centre line.

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So another case if there is the run of six or more consecutive points steadily increasing or decreasing, again all this case you see they are within the upper control and lower control limit, but still the process is out of control there is some assignable cause because some peculiarity is observed.

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1. What are the rules for describing an out of control process?

2. What are the benefits of using control charts? What is the benefits of OC Curve in Six Sigma Processes?

3. Explain the difference between common causes and special causes. Give examples of each.

4. Define and explain type I and type II errors in the context of control charts. Are they related? How does the choice of control limits influence these two errors?

5. What are warning limits, and what purpose do they serve?

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So, just think it what are the rules for describing an out of control process? How do you detect the type I and type II error? How do you calculate the ARL? What is the importance operator and characteristic curve? And so on.

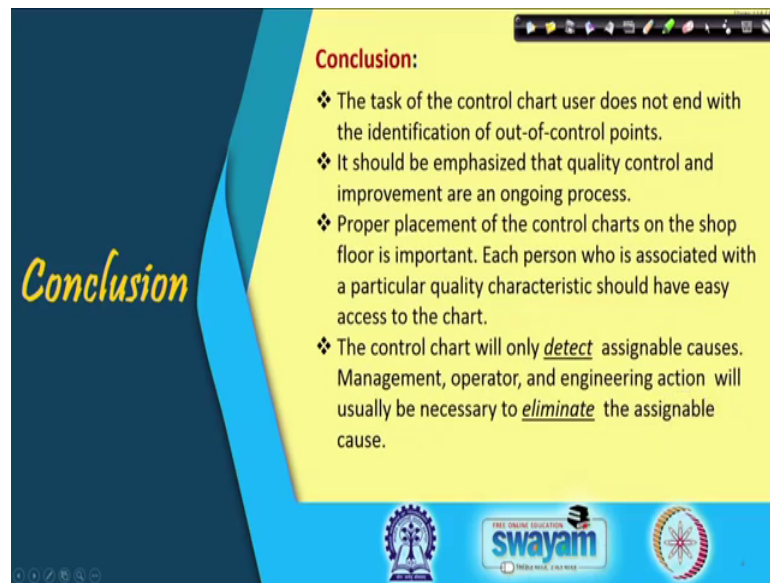
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- ❑ Montgomery, D C. Statistical Quality Control: A modern introduction, Wiley.
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The slide is titled "Conclusion" in a large, stylized font. It contains a list of four points, each preceded by a diamond symbol. The points discuss the ongoing nature of quality control, the importance of chart placement, and the distinction between detecting and eliminating assignable causes. The slide also features a logo for "swayam" and a circular emblem at the bottom.

Conclusion:

- ❖ The task of the control chart user does not end with the identification of out-of-control points.
- ❖ It should be emphasized that quality control and improvement are an ongoing process.
- ❖ Proper placement of the control charts on the shop floor is important. Each person who is associated with a particular quality characteristic should have easy access to the chart.
- ❖ The control chart will only detect assignable causes. Management, operator, and engineering action will usually be necessary to eliminate the assignable cause.

These are the references you can use. So, with this we can conclude that control chart will only detect assignable causes management, operator, engineer action will usually be necessary to eliminate the assignable causes. And you must have a sound control chart mechanism to see that you can take the action when it is really required and you allow the process to continue without interruption when this is actually not required.

So, you need to have an appropriate tradeoff between type I and type II error this actually you get at plus or minus 3 sigma, in order to be more precautionary you said the plus or minus 1 sigma limit you said the warning limit plus or minus 2 sigma and take the corrective action.

You even do not ignore some of the patterns emerging when the points are well within the control limits and that may give you the early indication that your process has some problem. So, thank you very much for your interest in learning the key concepts of control chart, now we will go ahead with our discussion in this topic in detail.

Thank you very much till the time keep revising be with me enjoy.