Six Sigma
Prof. Jitesh J Thakkar
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 43
Frictional Factorial Design

Hello, friends. We are step by step advancing in our six sigma journey and right now we are in lecture 43, Fractional Factorial Design. And, I would like to remind you that we are basically at present discussing the improved phase of our DMAIC cycle of six sigma. So, we have talked about design of experiment and factorial design and we also seen the Minitab application of this, we have also seen randomized complete blog design and now, we want to advance with fractional factorial design.

So, you may have a question that when factorial is there; what is the need of fraction we will go to that, but let us try to appreciate this quote – a theory can be proved by experiment, but no path leads from experiments to the birth of a theory, Albert Einstein.

(Refer Slide Time: 01:23)

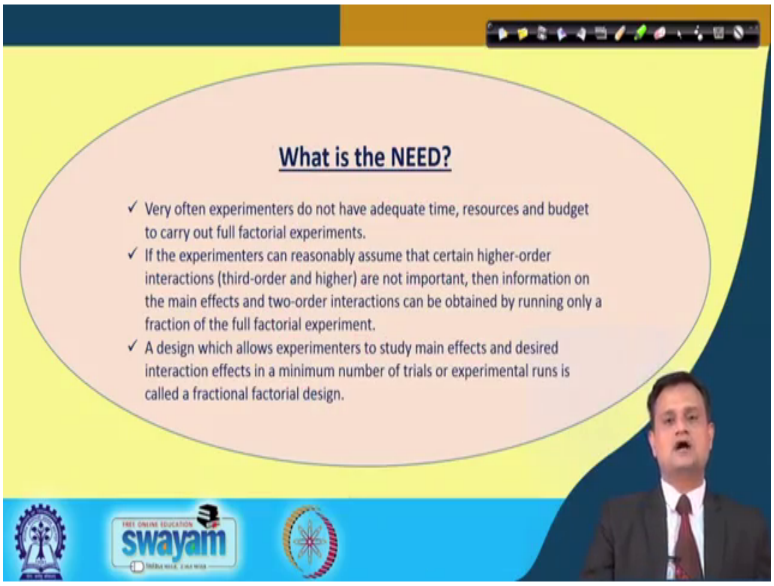
We had seen as a recap that Minitab application of factorial design in the previous lecture.

(Refer Slide Time: 01:28)

And, now here we will talk about the concept of the fractional factorial design, alias structure, resolution, one-half 2 raised to k design. There could be one fourth 2 raised to k design, but the procedure will remain same we will just discuss the one-half 2 raised to k design. Once you understand the design the computation part calculation part will be as it is that we would not like to go into say discussion.

So, we will basically try to appreciate the concept of fractional factorial design and what to do what not to do and remaining computation part will be as usual.

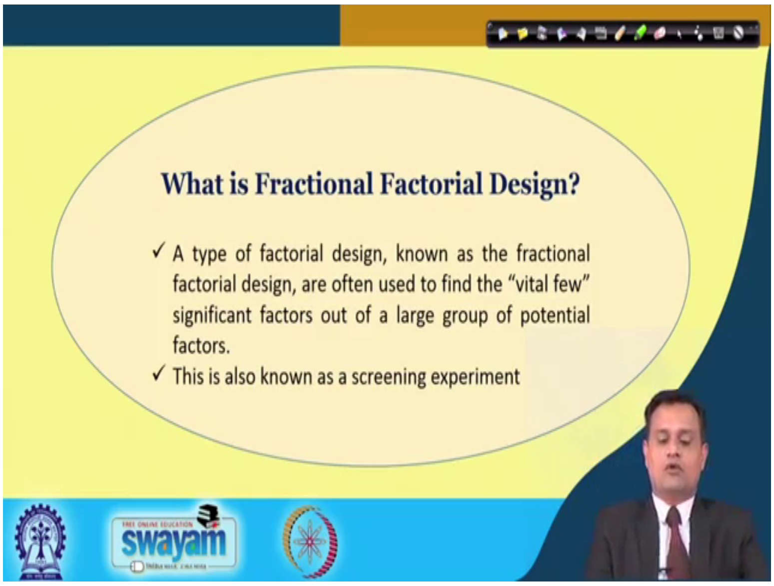
(Refer Slide Time: 02:08)



So, now the question is that what is the need? When we have fractional factorial concept which allows us to extract as much information as possible about the main factor and the interaction factor interaction of the factor, what is the need of going for a fraction when full gives me the lot of information. So, many a times you do not have adequate time, resource, budget. Now, we want to see that it is not only about conducting the experimentation we talked about of that one factor at a time, then we talked about the RCBD, then we are talking about the factorial design. So, it is not only important to conduct the experiment design, it is also important to conduct the experiment design in a smarter way.

So, smarter way means the experimentation strategy which can help me to consume less amount of resources, whether time, material, manpower and without compromising much with the inferences that I can draw from the experimentation and analysis, if I can do this then I would be conducting my experiment design in a smarter way. So, here my issue is that I want to save upon my resources and in what way I can conduct the experimentation using fractional factorial concept that can help me to achieve this less amount of utilization of resources without compromising much with the quality of the results.

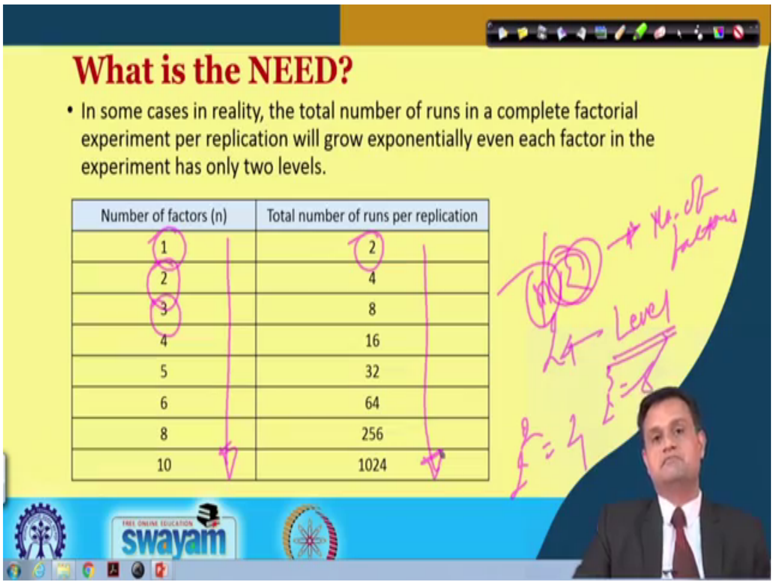
(Refer Slide Time: 03:48)



So, what is fractional factorial design? It is a type of factorial design first thing is that known as the fractional factorial because we are focusing on to find or focusing on vital few significant factors out of a large group of potential factors.

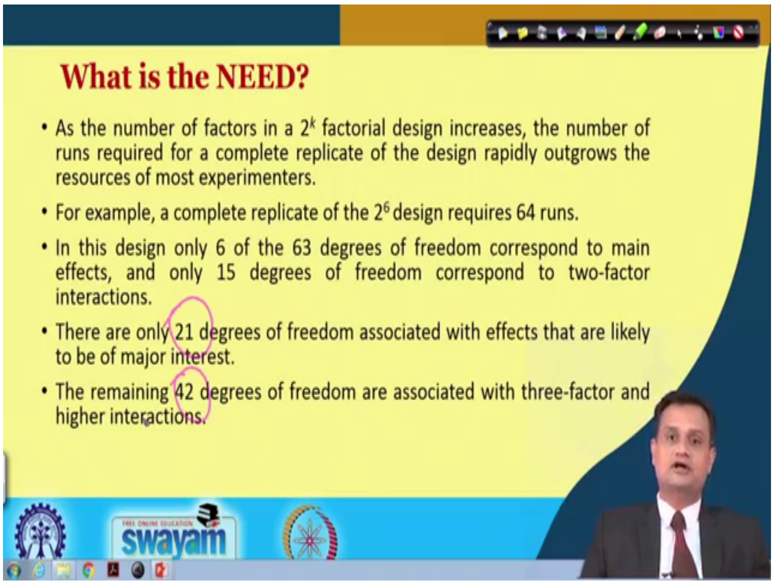
So, many times this is also called a screening experiment. I would like to focus on only few factors and I would like to screen it out. So, I do not have to say experiment with all the factors and all the possible interactions, but I can easily get a detail that what are the important factors that really needs to be considered for the investigation.

(Refer Slide Time: 04:35)



Now, just see this and you will be extremely clear that why we are discussing this topic. So, let us say our factor 1 and total number of runs per replication is 2. So, how it is? So, let us say 2 raised to k. Here 2 is the level, I am considering 2 levels here let us say temperature 10 degree and 50 degree. So, I am considering to level k is the number of factors.

(Refer Slide Time: 05:50)



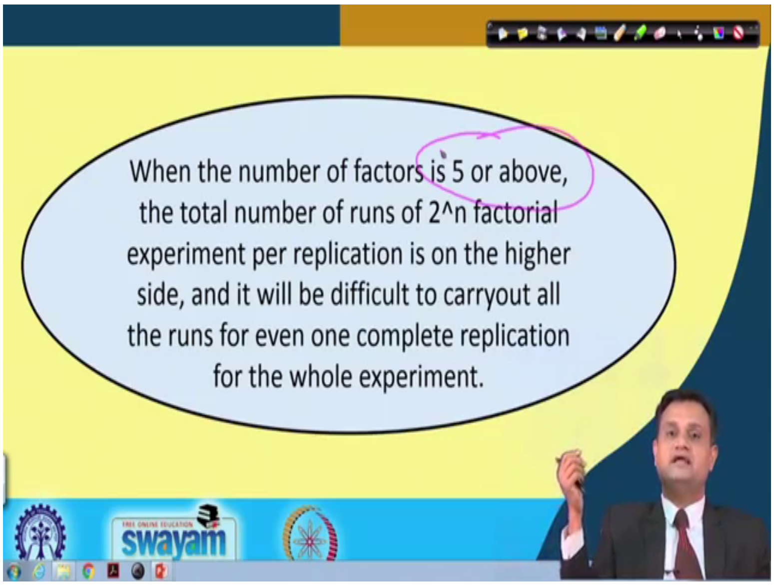
So, I am keeping the level 2 only. Now, let us see if I change the number of factor you can say n or k whatever you like to say. So, if I change the number of factor n or k then if it is 2 then 2 raised 2 is 4, if it is 3 then 2 raised 3 is 8. Now, you just see as I increase the number of factor my n value exponentially my total number of runs required per replication increases. So, I would be investing a huge in conducting full factorial experiment with 2 raised to n or 2 raised to k strategy.

Now, if you look at little deeper than the number of factors in 2 raised to n or 2 raised to k where n or k refers to number of factors increases the number of runs required for a complete replicate of the design rapidly out grows and resources are limited. Now, if you look at the complete replicate of the 2 raised to 6 then 2 raised to 6 is 64 runs. It means I have to conduct the 64 runs to complete my full factorial design.

Now, in this design 6 of the 63 degree of freedom correspond to main effects. So, basically your total 64 runs if I just do minus 1 I will have total 63 degree of freedom. Out of this only 6 are mainly specific to main effect, only 15 degree are specific to two-factor interaction. 21 degree are likely to be degree of freedom associated with the effect that are likely to be of major interest and 42 degree of freedom are associated with three-factor or the higher order interaction.

So, typically this 21 means I say 15 plus 6; 6 is the main 15 is the 2 level and 42 would be three-factor and higher order interaction. Now, here the question is that is it really necessary to analyze all the higher order interactions?

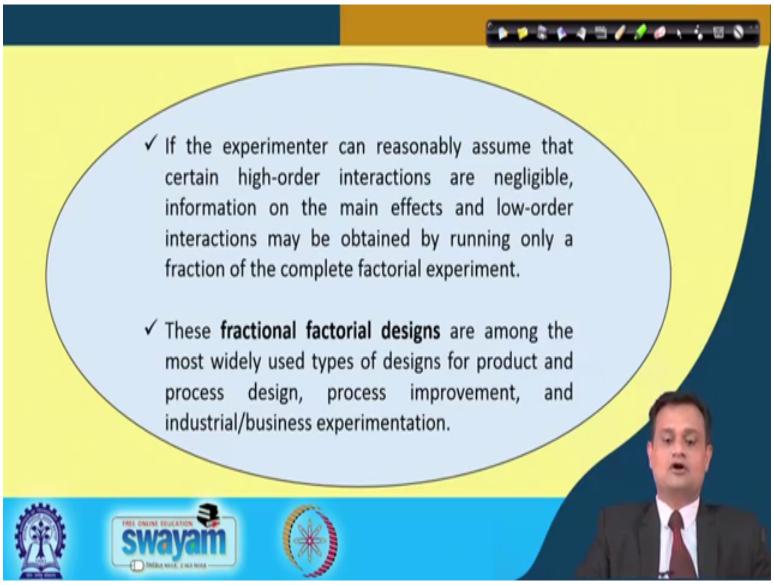
(Refer Slide Time: 07:28)



And, if yes, then obviously, there is no other option you have to go for full factorial design. But, yes if you can figure out that no higher order interactions are not that much significant or that much important by some way through some logic then you can reduce your experimental runs and hence the resources is required for conducting the experimentation.

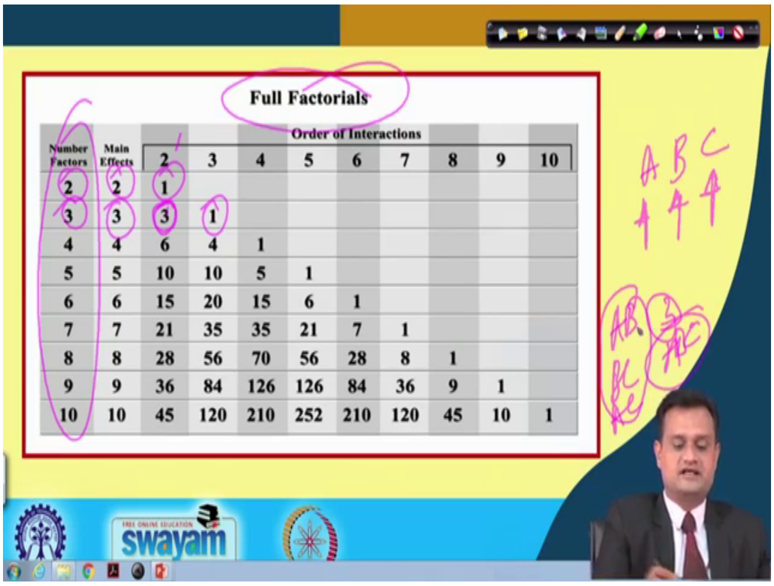
So, when typically the number of factors are more than 5 the total number of run 2 raised to n factorial experiment per replication is on the higher side and it will be difficult to carry out all the runs for even one complete replication forget about multiple replication in order to achieve better accuracy of the experimentation and this is where that I am concerned about the usage of the resources in conducting the experimentation.

(Refer Slide Time: 08:40)



So, here I would like to look at some other strategy like fractional factorial and if fractional factorial can really help me to reduce my number of say experimental runs then that would help me to save a lot on my consumption of resources.

(Refer Slide Time: 09:02)



So, just try to see why so many treatments. This table is very important and it would be really interesting that you just see I am considering full factorial. It means I will analyze all the main and interact main effects as well as interaction effects, just see the number of factors. Now, suppose my number of factors are 2, then obviously, main effects are 2;

interaction effect is only 1 because there are 2 factors. Suppose, number of factors are 3 main effect is 3 and you will have three 2 level interaction and one 3 level interaction; very simple. Suppose you say A, B and C these are the hree factors. So, I will have effect of factor A, I will effect A, factor B, effect of factor C. I can have the interaction effect of AB 2 level means 2 factor, BC and AC. So, this is basically 3 so these 3 is 2 level interaction effect.

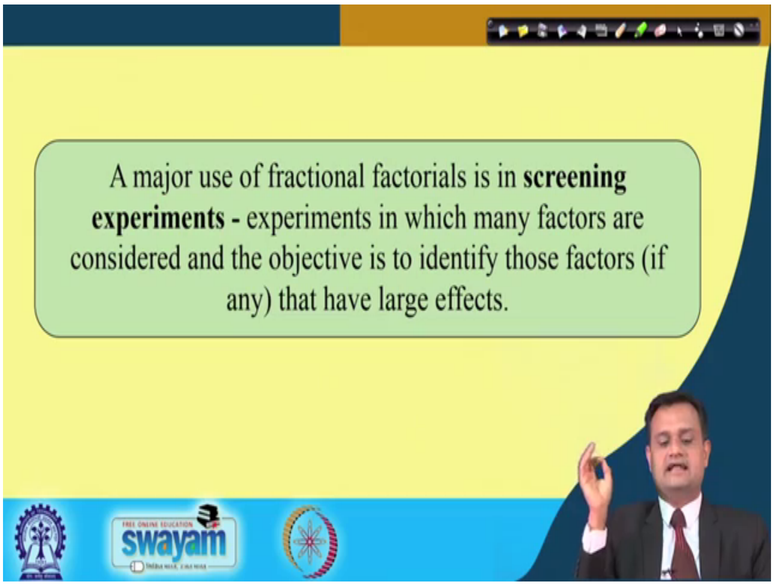
And, obviously, ABC is the only 3 level interactions. So, there is one now likewise you just try to analyze that what is happening? Suppose, I go for more than 5, 6 factor there would be 6 main effects to be analyzed or estimated and then 15 2-level, 20 3-level, 15 4-level 6 5-level means 5 interaction order interaction and one is 6 because there are 6 factor. If you look at 10 you just see how many effects you will try to estimate. It out grows exponentially and it is extremely difficult to handle such an experiment where huge amount of resources, time, machine over operators, material is required and you would not be able to really conduct your experimentations smartly, effectively.

(Refer Slide Time: 11:13)

So, now, I have to do something to see that I can get rid of too much of the experimentations, trials, runs and I can only focus on vital few which are really important to be analyzed. So, that tends to be a redundancy in the full factorial design. So, now the only solution is that you figure out what is the redundancy? If you can figure out this redundancy you will automatically come down to estimating only those effects main effect or interaction effect which are really important vital few significant and then you will just exclude all other interactions or effects from your list of estimation and your number of trials number of experimentation runs will drastically go down.

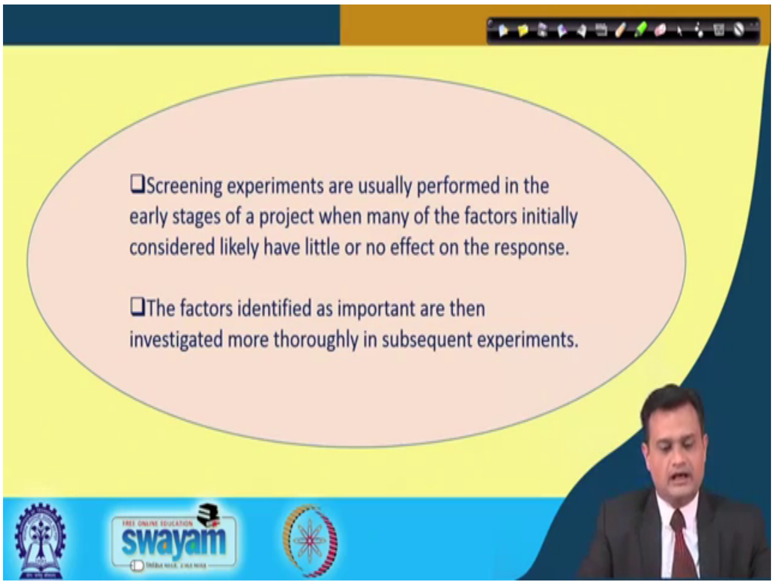
So, redundancy in terms of excess number of interaction that can be estimated and fractional factorial design basically exploits this redundancy. So, try to get rid of redundancy your experimentation size will come down and you would be more effective in conducting your experimentation. But, it is not just say terminating some of the effect estimation without any logic. So, I want to teach you what is that logic by which you can really be reasonable, logical, scientifically true in reducing the size of the experimentation and can really focus on vital few. So, how to figure out this redundancy that is the issue that we want to address.

(Refer Slide Time: 12:51)



Now, as I mentioned that a major use of fractional factorial is in screening experiment and experiment in which many factors are considered and the objective is to identify those factors if any that have large effects.

(Refer Slide Time: 13:10)



So, this is this screening experiments basically and usually performed in the early stage of the project when many of the factors initially considered likely have little or no effect on the response. So, for example, I will just say that I want to consider response called productivity. Now, I am just trying to put couple of factors as the main factors. Let us say factor A – skill level of the worker, factor B – state of the art technology or the efficiency of the machine, factor 3 – layout of your plant, factor 4 – moral of the worker factor file top management policies in providing necessary resources.

Now, these are the factors now let us say these are five factors; then 2 raised to 5 there would be 32 experimentations trials you have to conduct. Now, let me try to see that all the interactions are really interesting to analyze. For example, moral of the worker and top management policy, yes, there could be a relationship, but efficiency of the machine and maybe let us say your worker moral it may have any effect, it may not have an effects or very very not significant effect or you can say a plant layout and maybe the state of the art technology.

So, technology is a different issue, plant layout is a different issue, interaction may not have any real value. So, if you can really figure out that what are the interactions that are not really important then this redundancy can be avoided and you can equip yourself in a smarter way for conducting the experimentation.

(Refer Slide Time: 15:10)

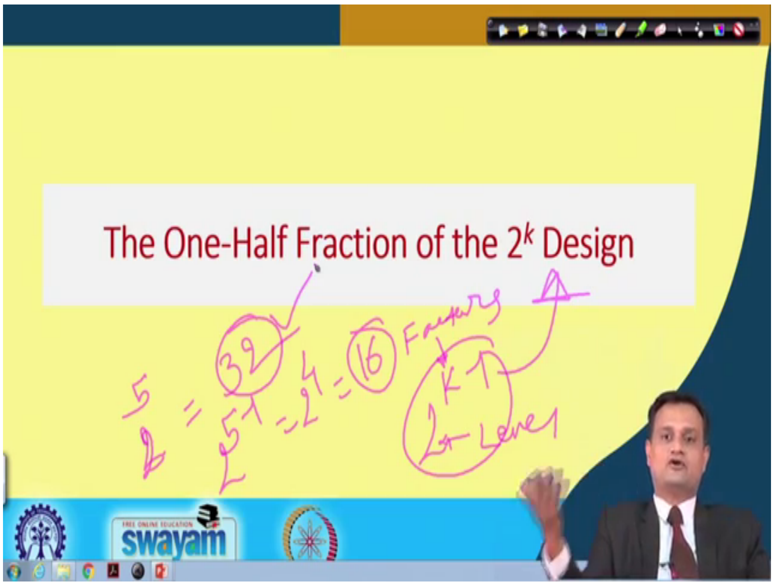
So, there are three important principles to be appreciated and that creates the basis for fractional factorial design. So, number 1, the sparsity of effect principle. So, when there are several variables the system or product is likely to be driven primarily by say some of the main effects and low-order interactions. So, many higher order interactions maybe 3-level, 4-level you may say that they are not very much impacting my response and this is the principle number 1.

Principle 2, the projects and property; so fraction factorial design can be projected into a stronger, larger design in this subset of the significant factors, so, not that your manufacture so your design is robust or design is stronger. Just think in a very simple way I can set the analogy, you are purchasing a particular product like mobile. Now, do you think that this mobile will have thousands of features and that is why you would be happy?

No, it is not necessary because many features are redundant you may not like it or many times if the features they will interact with each other they may not have any real effect on really your usage of the mobile and then you would not like to pay for it. Same applies here that stronger design does not mean I should have many factors many interactions to be analyzed, stronger design means getting rid of redundancy having the relevant.

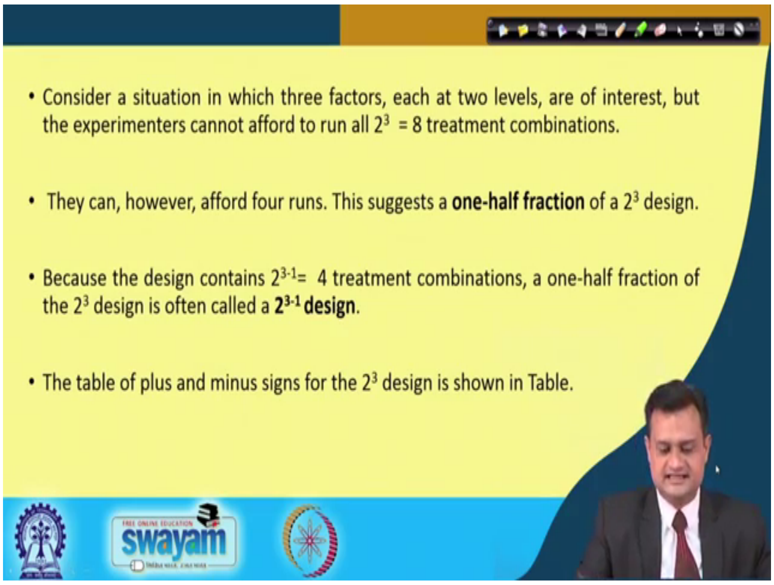
The third one is third principle is sequential experimentation it is possible to combine the runs of two fractional factorials to construct the sequentially a large design to estimate the factory effects and the interaction effect. So, you can partition your design sequentially run it and then you can better estimate defects rather than conducting a full factorial design in one go.

(Refer Slide Time: 17:17)



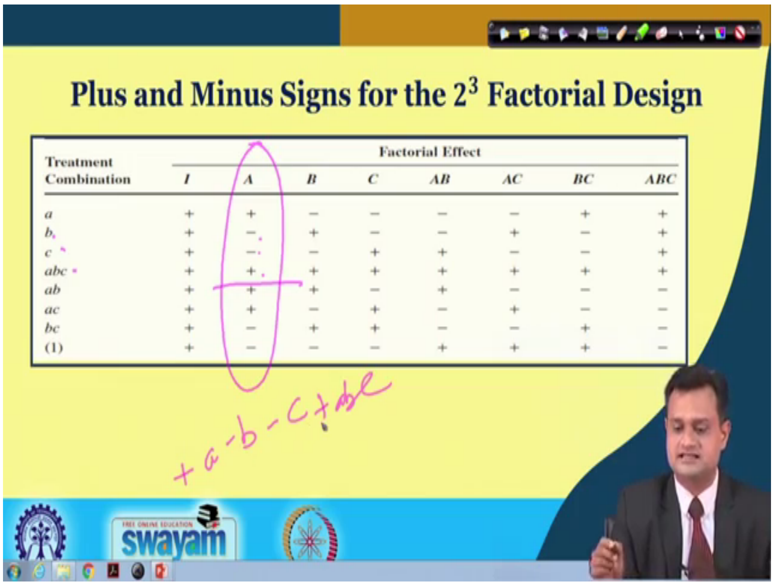
So, one-half fractional factory design, you can also express it as 2 raised to k minus 1. So, just see with the example suppose if I say 2 is the level this is the level, these are the factors and if I say this is my 2 level 5 factor full factorial I will have 32. Now, if I say 2 raised to k minus 1 which refers to one-half fractional factorial design. So, 2 raised to 5 minus 1 is equal to 2 raised to 4 is a 16. So, this is exactly half of 32 and that why it is called one half fraction of the 2 raised to k design. It means I would be actually conducting only 16 experimental runs to reach to more or less the same conclusion which otherwise I would reach through full factorial design by conducting all 32.

(Refer Slide Time: 18:21)



So, this is what I am trying to do as a part of my fraction factorial and this is exactly what I explained here.

(Refer Slide Time: 18:24)



Now, let us see that how your experimentation look like; so this is the treatment combination. So, you have a, b, c, abc, ac, bc and 1. You have something called I, so, this is my identity column and just I am introducing it. I have A, I have B, I have C, AB, AC, BC and ABC. Now, let us try to appreciate that how we will read this table. So, when I say that my factor A you just see initially it is set at plus means the high level and then two times it is low. Then it is set plus plus plus then again it is low low. I have set B at low level, then I have set plus then I have set low then again I am setting is twice high level low high low you can see here low low high and then this is high.

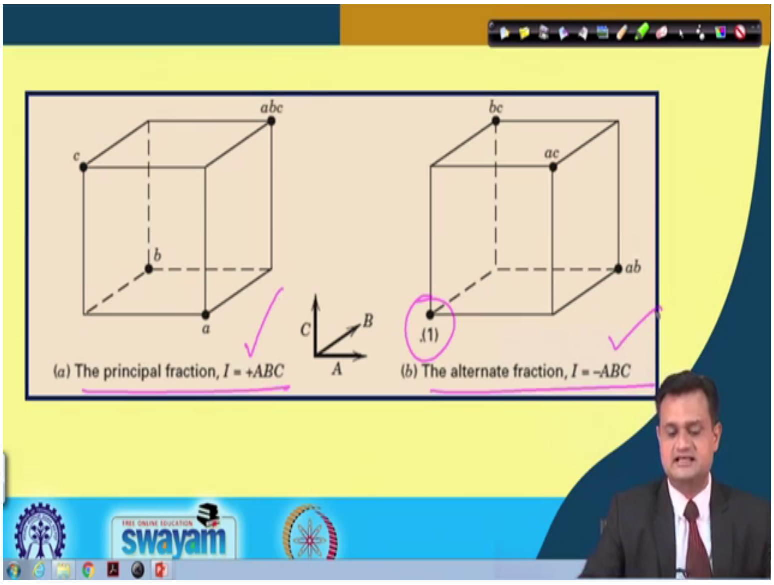
So, now, what exactly you can see that when you try to develop the table you must see I am telling very important point it is not about plus and minus. When you have three factors A, B and C and you are setting it at high level or low level and then changing it in the next experimental run, please try to understand that it is not only about changing the factors changing the levels of the factors there is a cost associated. For example, a factor which is very difficult to set or change if your are changing it frequently it will have a great impact on your response or the setup time would be very large and this kind of say approach would not be good.

So, let us say once again if I see A is changed first plus and then it is kept minus and minus. Now, you see there is only one time change. If you see this is set minus then again change and again change, two times change. This is only one time change if I just refer up to here. So, you can say that the factor which is most difficult to change should be changed less and hence you do not frequently change from low to high and high to low.

So, abc is just the generic term alphabet you need to assign the factors and assign the factors in such a way that the factor which are easy to change may accept frequent change the factor which are difficult to change or has a negative or adverse effect on your response they should be assigned to the column which has less frequent changes in the level; so I hope it is clear.

Now, you can see here that I have AB. So, this is nothing, but the multiplication of plus and minus; so this is AB, AC. So, this A plus minus this is minus BC both or minus this is plus and this is how you construct this particular table. So, this table basically is about the strategy in what way I will conduct the experimentation and this is the table of 2 raised to 3 full factorial. So, 2 raised to 3 is 8. So, it is full factorial; I am trying to estimate all the main effect and the interaction effect you can see here main effect a, main effect b, c, abc, ab, bc and ac, ac and bc and this is the initial setting so, where all the factors are set at the low level. So, this is what you can appreciate about the.

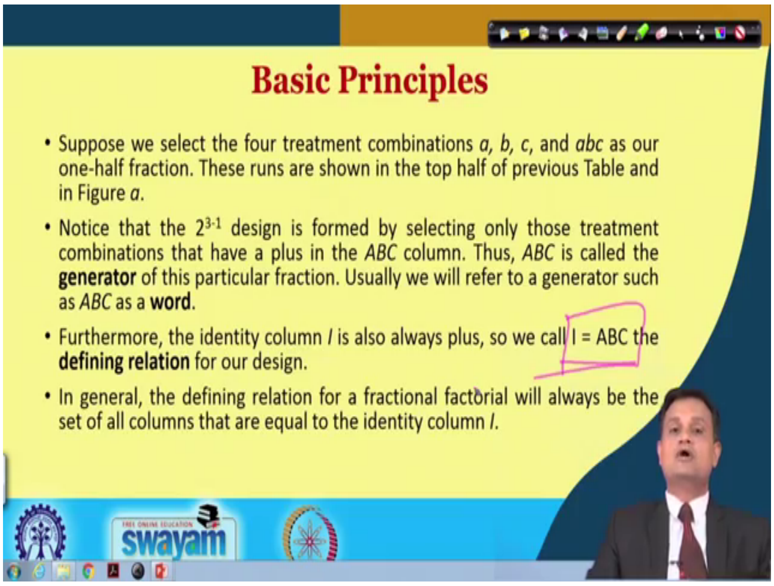
(Refer Slide Time: 22:29)



Now, you just see here in a particular form that this is the principal fraction is equal to abc plus abc and this is the alternate fraction that I is equal to minus abc, you have the initial condition here. So, the design which I had shown in the previous table now could be divided into two fraction, that is, positive plus abc and minus abc. So, I can go back to help you to appreciate this that you just see this you have plus plus plus you have minus minus minus. So, you have two say separate fractions to be analyzed.

Now, let us see that what is actually happening.

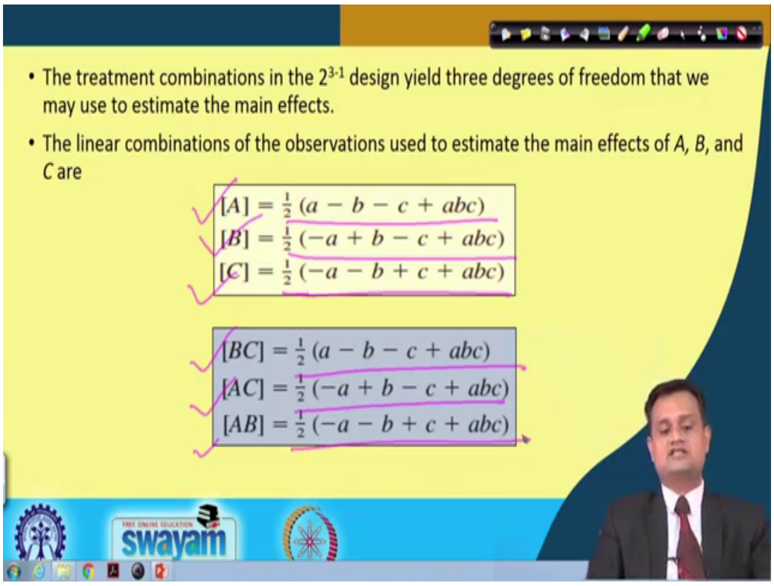
(Refer Slide Time: 23:20)



Now, there are some basic principles when we want to appreciate the fractional factorial design and how to get off the redundancy. So, typically say you have the treatment combination a, b, c and abc as our one half fraction and this runs are shown in the top half of the table as I was shown with plus plus plus. Now, 2 raised to 3 minus 1 design is formed by selecting only those treatment combination let us say they have plus plus plus plus sign in abc column and abc typically called is a generator. So, I am just trying to introduce a term technical term it is a generator.

Now, when I try to equate I is equal to ABC, then I am calling these as a defining relationship and this defining relationship will help me to identify the redundancy in conducting my full factorial experimentation and hence to reduce the size of my experimentation as a part of fractional factorial. So, remember that I am defining the relationship in terms of say generator and identity column I is equal to ABC because I free factor factorial design now let us see how we use it for identifying the redundancy.

(Refer Slide Time: 24:42)



Now, if I estimate effect A, B and C this is really interesting to see. What is coming here? a minus b plus c, a minus b minus c plus abc; let us see how it is.

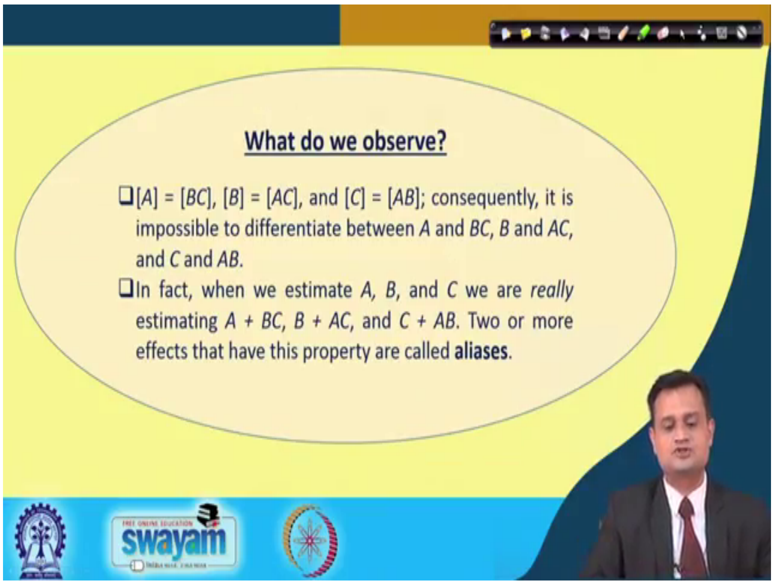
So, I will just go back just see here that I am estimating A and you are referring this particular r table you are referring the half. So, just see a into plus is plus a, b into minus is minus b, c into minus is minus c and abc into plus is plus abc. So, a minus b minus c plus abc you got I am just try to verify whether it is true or not? Yes, it is there and this is for averaging out one-half; a minus b minus c plus abc, same way you can estimate B and C.

Now, just try to see BC, a minus b minus c plus abc. Let me go back again to help you. If you look at this particular then suppose if I look at this then minus a minus b plus minus a minus b to estimate this interaction plus c plus abc. If you look at BC then it is plus a minus b minus c and plus abc. So, this is how I am just trying to estimate the effects.

Now, what you can see here something interesting that if you see the A and BC then their expressions are same; if you see B and AC the expressions are same; if you see C and AB their expressions are same. So, this means that when you are estimating A actually you are not expressions A, but you are estimating A and BC; when you are estimating B main effect you are actually not estimating B you are estimating B and AC and similarly, C you are estimating C and AB.

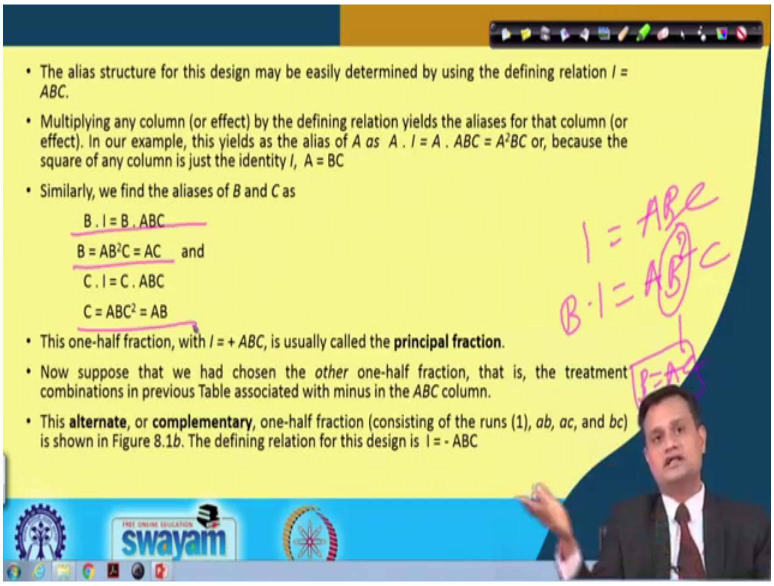
So, this is typically called aliasing of the effects or confounding of the effects it means that some of the effects are aliased or confounded with some of the effects and higher order interactions are confounded with either the main effect or lower level interaction and hence this is the redundancy which can be avoided in conducting the experimentation. So, this is the key point and I hope you appreciated.

(Refer Slide Time: 27:24)



So, exactly this is what I have explained that A is equal to BC, B is equal to AC, C is equal to AB.

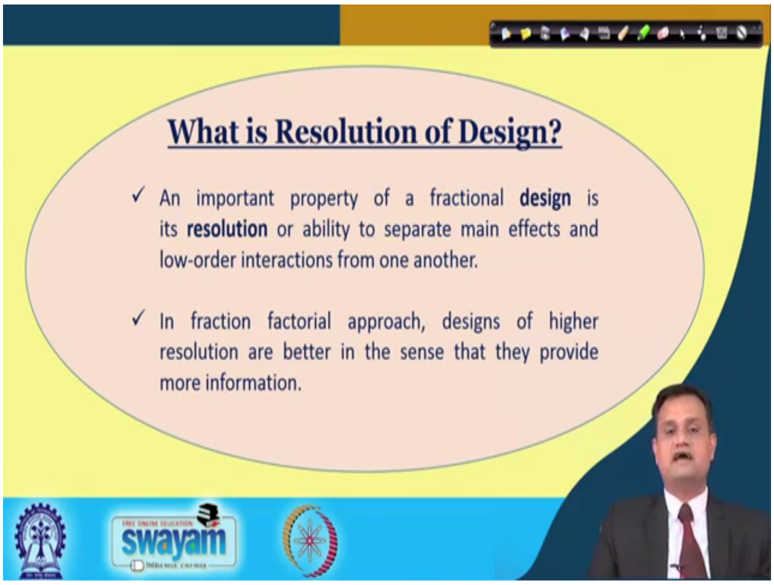
(Refer Slide Time: 27:31)



And, I can use my defining relationship also to conduct the same analysis. So, let us say I have defining relationship I is equal to ABC for three factor A, B and C let me multiply both the sides with B. So, it is A B square C. So, B square is nothing, but entire column is squared, so, this become identity. So, basically you have B is equal to AC and this is how exactly what we have seen B is equal to AC. Similar way C is equal to ABC.

So, these effects are confounded with the main effects AC and AB and really when I am estimating the main effect within that the estimation of this effect is included and there is no need to separately estimate these effects. So, this is the redundancy in full factorial design that we should try to avoid.

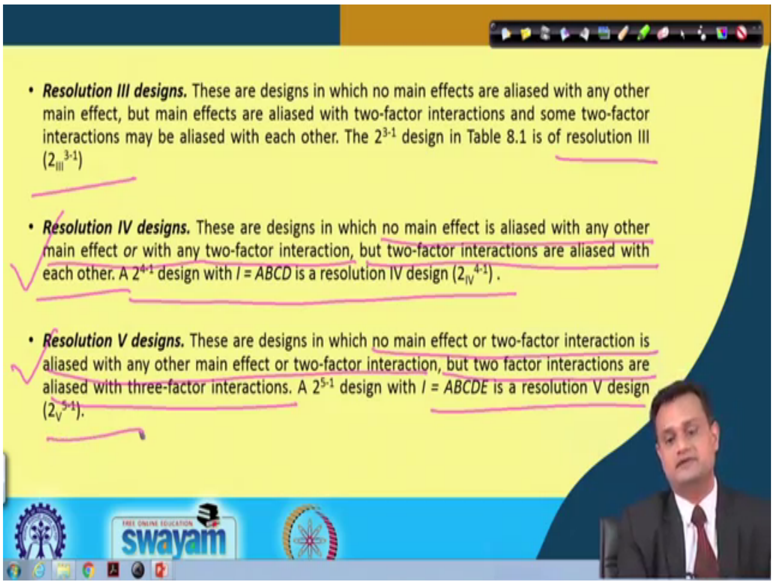
(Refer Slide Time: 28:24)



Now, there is another concept which is very important in fraction factorial design when I am only working with the fraction of the trial number of runs by getting rid of the redundancy by identifying the aliasing or confounding effects I also need to look at the resolution. Resolution means the accuracy in judging the effects. So, this is an important property and basically it indicates the ability to separate main effects and the lower order interaction from one another.

So, typically in fraction factorial approach design of higher resolutions are better and that can provide more information.

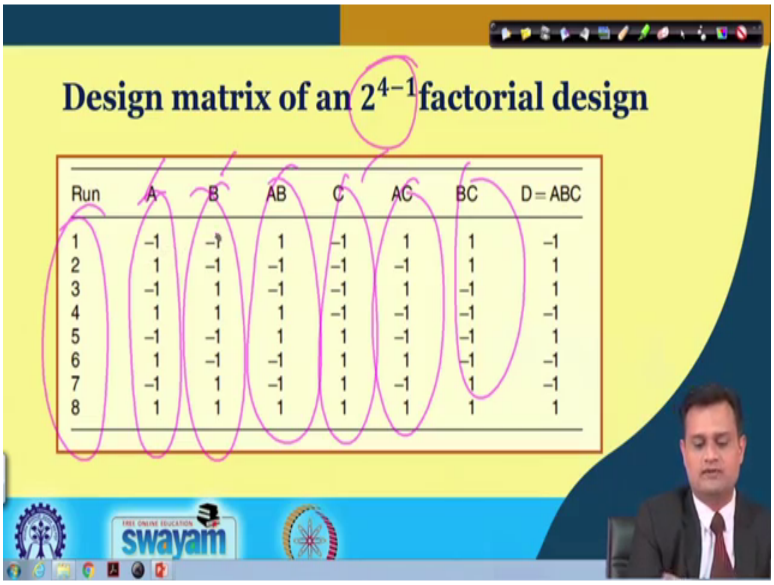
(Refer Slide Time: 29:08)



So, there are three different types of resolution typically we refer resolution I and II do not add much value. So, typically we refer resolution III, IV and V. So, resolution III design means it is a design in which main effects are aliased or confounded with any other main effect no main effects rather or confounded with the main effects. So, there is no confounding of the main effect, but main effects are aliased with two-factor interaction and some two-factor interactions maybe aliased with each other. So, this is typically a resolution III design and express as resolution III design.

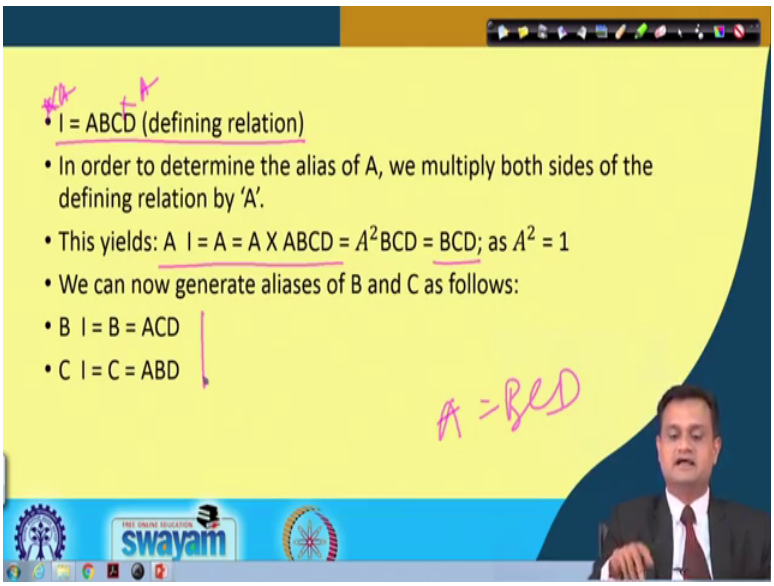
Similar way resolution IV design, the condition is that no main effect aliased with any other main effect or with any two-factor interaction even two-factor interactions are not aliased, but two-factor interactions are aliased with each other this is expressed as resolution IV design. For resolution V design, no main effects or two-level interaction is aliased with any other main effect or two-level interaction, but two-factor interaction, but two-factor interactions are aliased with three-factor and this is called resolution V design.

(Refer Slide Time: 30:34)



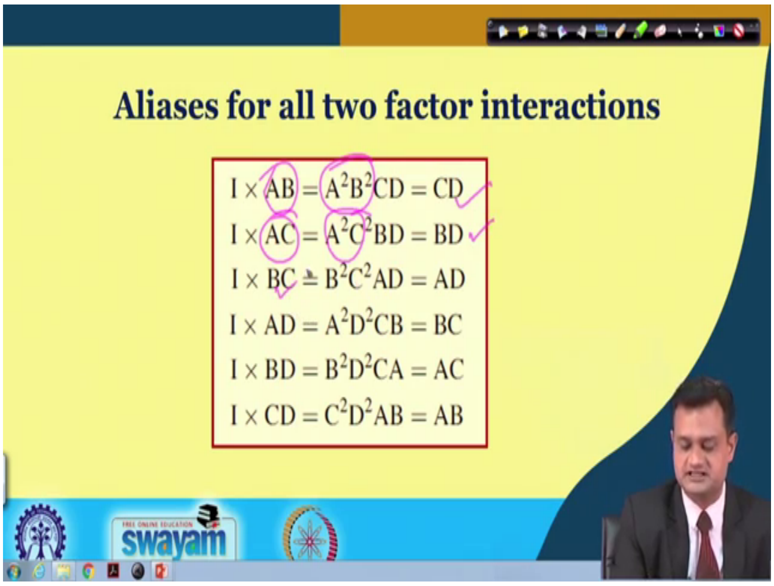
So, please keep in mind that there are various resolutions when you talk about fractional factorial design and you can analyze the design metrics just another example for 2 raised to 4 minus one factorial design fractional factorial design. If I conduct the 2 raised to 4 then I have to conduct 16, when I go for 2 raised to 4 minus one I have to conduct basically only 8 experimental runs. So, here it is 8 experimental runs. See the setting of A, B, AB, C, AC, BC; basically once you are set A, B, C other are just derived by the multiplication. So, minus 1 into minus 1 is plus 1 minus 1 plus 1 into minus 1 is minus 1 and so on.

(Refer Slide Time: 31:25)



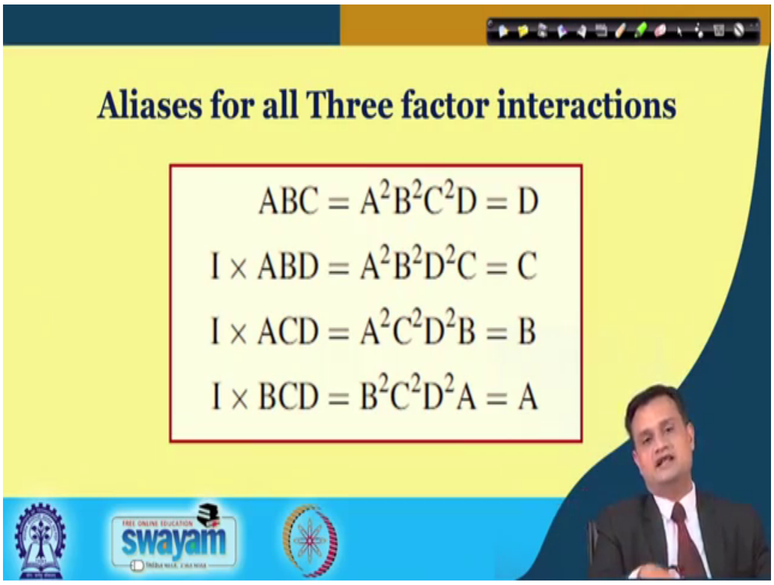
So, this is the table initial table. Now, I am just trying to analyze the aliasing. So, I have defining relation I is equal to ABCD. I can multiply both sides by A multiply A multiply A and what do you will get here is this. So, you get BCD. So, you have basically A is equal to BCD, similar way B is equal to ACD, C is equal to ABD. It means your aliasing confounding of these three interaction effect three-factor interaction effect is taking place with the main effect and this is where the redundancy exist.

(Refer Slide Time: 32:06)



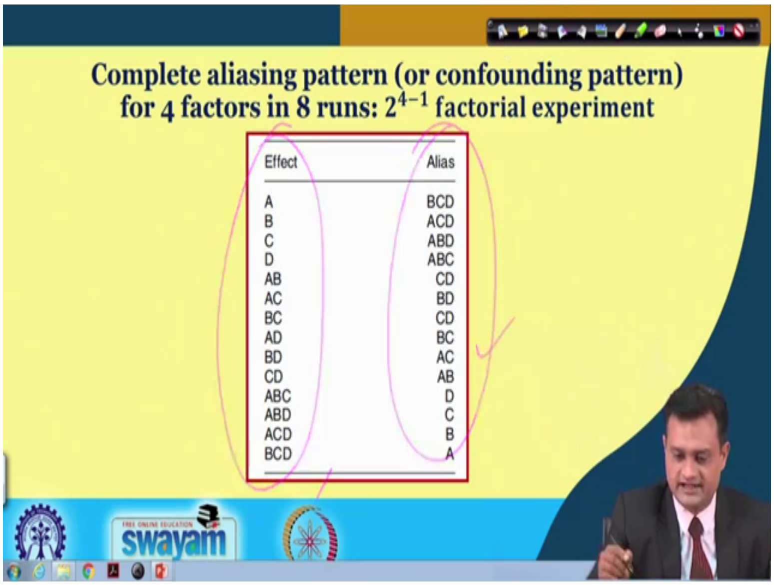
So, similar way you can do it aliasing structure for all two factor interaction. You define the relationship multiply them by the same two level interaction. So, you will I into AB multiplied also by AB, so, this will become your I. So, you have basically CD; you multiply this by AC, A square C square this becomes I. So, AC is equal to BD, BC is equal BD and so on.

(Refer Slide Time: 32:38)



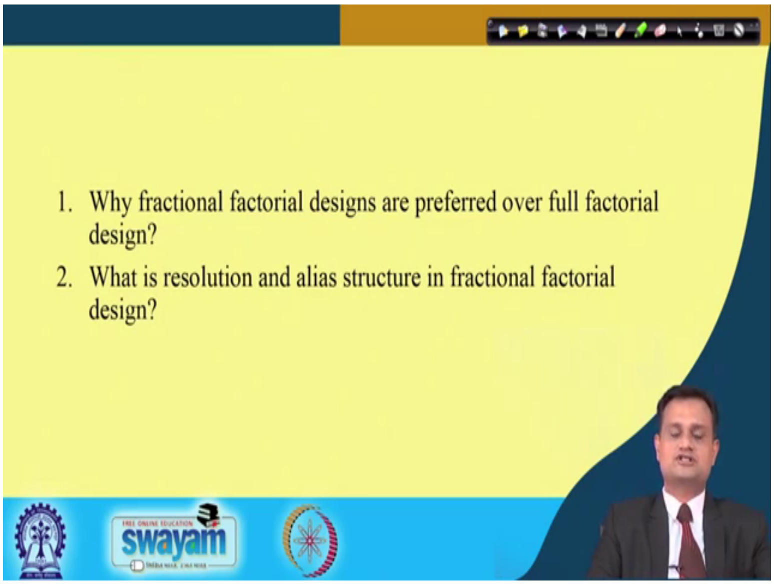
So, I hope the idea is clear similar way for three factor multiply your defining relationship is equal to ABCD by three-factor ABC, ABD, ACD, BCD and you will get the structure.

(Refer Slide Time: 32:52)



So, in summary you can say that you have the effects you have the aliases and typically these alias is are basically confounded or aliased with these effects and there is no point in separately estimating them it just creates the redundancy. So, with this I would like to float couple of think it for your introspection and to go deeper into the concept.

(Refer Slide Time: 33:17)

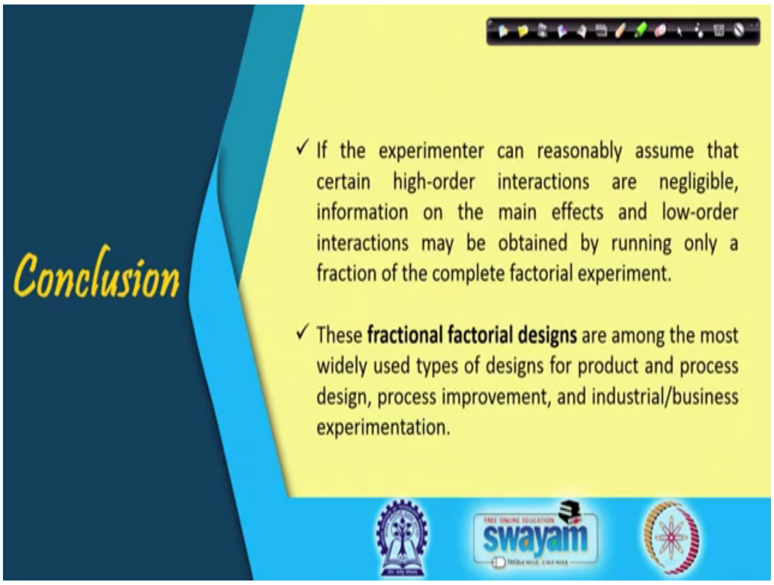


So, why fractional factorial designs are preferred over full factorial design? What is resolution and alias structure in fractional factorial design? Just give a thought and the idea would be clear. The rest of the computation will follow the same procedure as we did for the factorial design and you need not to say bother much about the calculation part. You can also use the Minitab and you can follow the calculation.

(Refer Slide Time: 33:47)

So, these are the references basically typically you refer Douglas Montgomery for design of experiment and that would really help you to appreciate the concept.

(Refer Slide Time: 33:56)



So, fractional factorial helps us to conduct the experimentation with an effective way, smarter way by saving our resources and some more practical feasible approach for conducting the experimentation.

So, thank you very much for your interest in learning the concept of fractional factorial and I hope this video will definitely help you to appreciate the importance of fractional factorial and how to get rid of some of the redundancy present in the factorial design. So, please keep revising, solve couple of examples to gain deeper inside.

Thank you very much. Be with me, enjoy.