Six Sigma

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Lecture – 29
Hypothesis Testing: Single Population Test

Hello friends, I welcome you to our ongoing six sigma journey. And I would like to remind you that we are in the analysed space of DMAIC cycle. We have completed define, we have completed measure phase, and now we have has started the analysed space wherewith hypothesis testing concepts and the examples.

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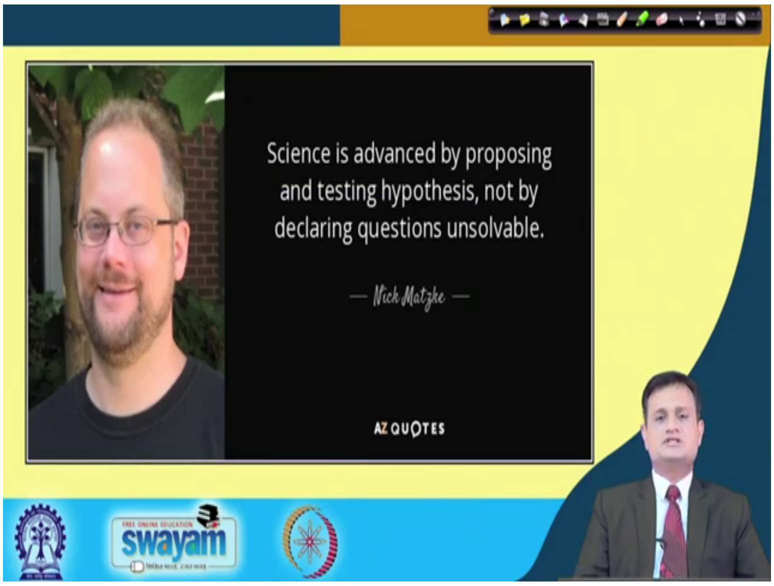

So, if we just revisit our inspiration, that science is advanced by proposing and testing hypothesis not by declaring questions unsolvable.

So, here we must appreciate that any organisation company if they want to say test a theory about a market, about a consumer, about a product or they want to propose a new theory they cannot just do it on a trial an error basis arbitrary, they need to test it scientifically and the scientific way is to check your claim which is the hypothesis through a statistical analysis.

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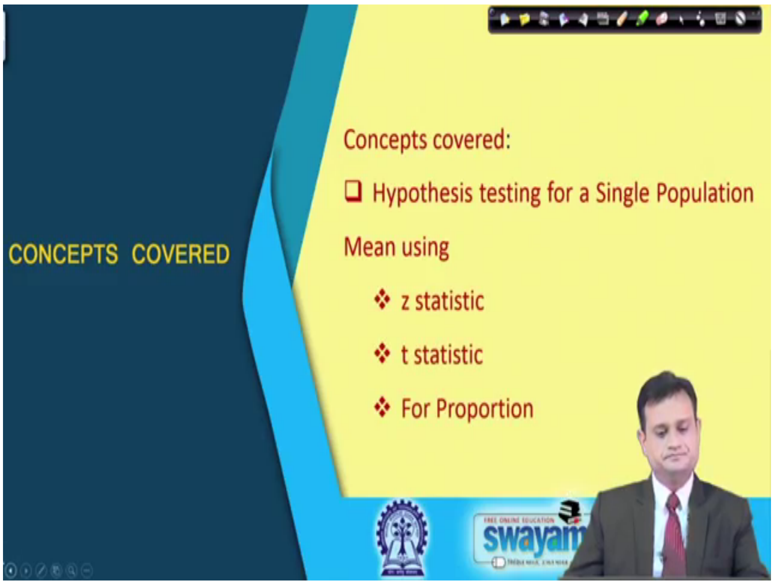

So, in this particular lecture 29, we will apply the concepts we have studied in the previous lecture on hypothesis testing and we will solve couple of interesting examples on hypothesis testing single population test.

So, this will give you a feel of real life application of the hypothesis testing and how the concepts we have seen in the previous lecture could be applied for a real life situation.

So, in this lecture, we will focus only on single population test. It means, you are basically interested to verify your claim for a single population and this population could be a tertiary, could be a gender, could be a kind of particular product and its production and so on.

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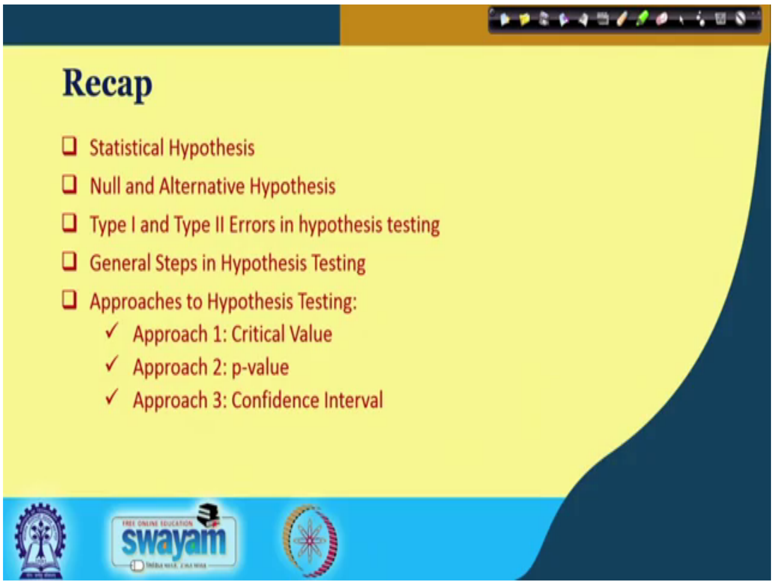

So, we have I have basically talked about some of the very very important concepts in the previous lecture what is statistical hypothesis? It is basically your claim. What is null and alternative hypothesis? So, null hypothesis basically dictates the status quo, it means this will happen. And alternate hypothesis says, that this will not be true this will not happen. So, we had seen this concept then there was a very very important concept type 1 and type 2 error.

I would like to remind you that, all this statistical analysis basically we try to draw the inferences based on sample and ultimately there is a chance phenomenon, there is a random phenomenon, and the probability associated with this chance or random phenomenon cannot be ignored. So, I cannot say that I am hundred percent sure that this claim is accepted or rejected, I can only say that at some level of significance alpha and the type 2 error beta I would accept this claim or I would reject this claim.

So, we had a good discussion on type 1 error and type 2 error and as I mentioned that selection of the type 1 error and type 2 error this are inevitable, you have to accept. But you choose the appropriate level depending upon the situation and its impact on making the wrong decision through inferential analysis. So, you have general steps in testing the hypothesis and there are basically 3 approaches for hypothesis testing. Approach number 1 critical value approach, you get the value from a statistical table this is the critical value which we call it as a theoretical or tabulated value and all the statistical tables are developed using the probability distribution function of a particular say distribution and for the convenience for the ease. We have this readymade tables available that we can refer to get the critical values so that we can make a decision whether accept or reject about our claim which is null hypothesis and alternate hypothesis.

You have approach 2 this is p value approach, we had seen the merits of this approach. It is more intuitive to the managers because it expresses the probability of rejecting or accepting the null hypothesis and how far we are our observed level of significance which the p value how far we are from the accepted level of significance for testing the hypothesis that is alpha.

And approach 3 is the confidence interval. I want to check that on an average suppose I have 95 percent confidence interval then my particular value or my null hypothesis will be true for which particular interval if I have to check it for 95 percent, 99 percent, or 90 confidence interval.

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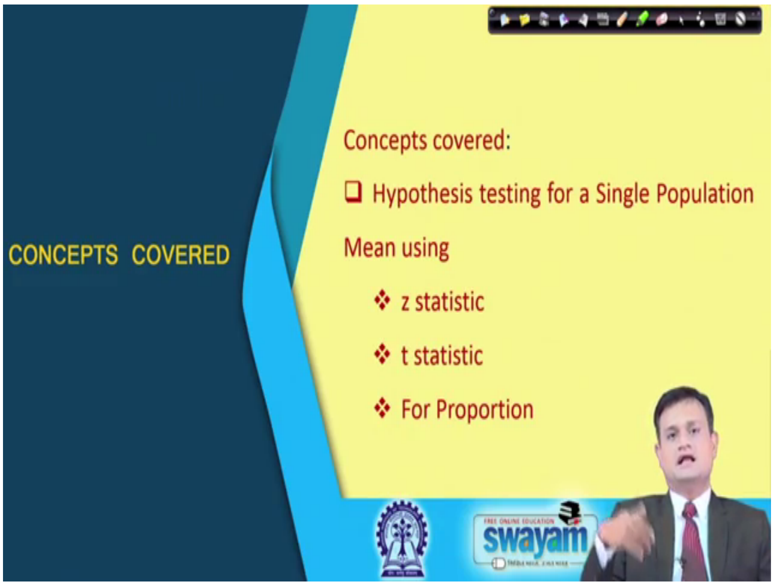

So, we had cleared our basic concepts. Now this lecture typically will devoted is devoted on solving couple of real life examples to help you to appreciate the application of hypothesis testing typically for a single population. So, we will see hypothesis testing for a single population using z statistic, using t statistics, and for proportion.

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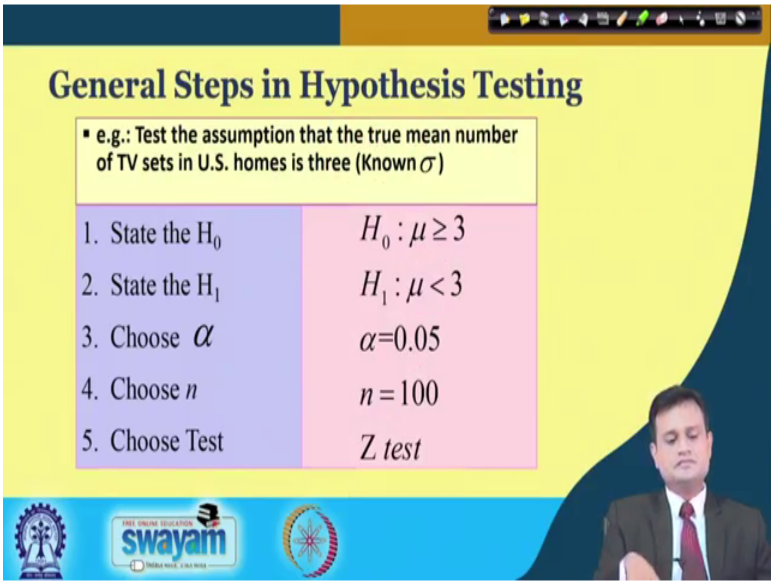

So, I would like to repeat the general steps we follow in hypothesis testing. So, you state the null hypothesis typically called H 0. Suppose in the pink box right hand side I put it as H 0 mu is greater than or equal to 3. State alternative hypothesis so I will say mu is less than 3.

So, here it could be 3 could be 3 kg. You want to fill the material in the bag; it could be the average of this core. Whatever may be the problem, you say that my mu is greater than or equal to 3 or alternatively mu is less than 3, so my alternate is H 1. Now you choose the alpha value depending upon the severity of the say decision or consequences looking to the particular situation you are investigating. To remind you, suppose if I punish an innocent person then the alpha value chosen if it is too high and if I punish the innocent person there would be a great societal loss as well as lot of consequences.

But its ok, if I by chance make a wrong decision and declare a guilty person innocent fine again there is an error, but the societal consequences are less and later on because this person has certain traits definitely I can triangulate, I can verify, or this person will make another mistake and there is a probability that he will be caught.

So, if nothing is given or you do not have much confidence in understanding the impact of making wrong decision then, as a rule of thumb we feel comfortable with alpha is equal to 0.05. Choose n, n is equal to 100 just it may be 100 or 50 or 150, choose test I am choosing here Z test. Let us say, this is just the description of the general steps of hypothesis testing.

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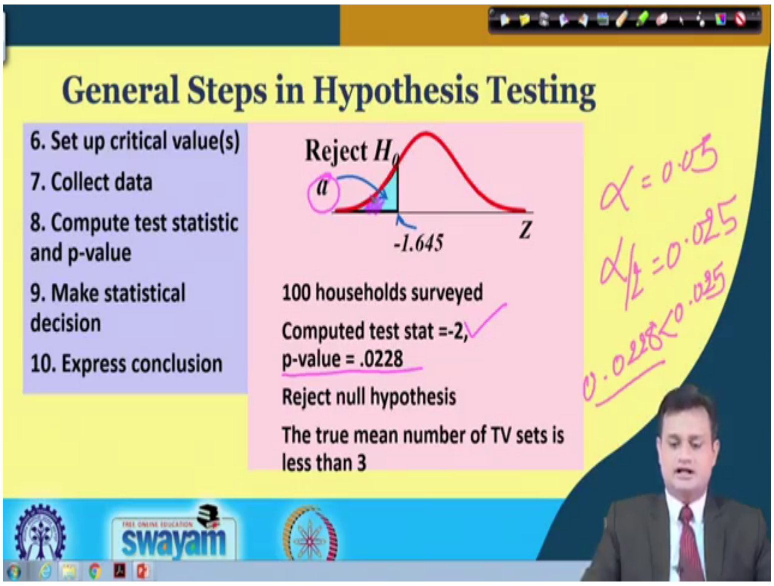

Then number 6, set up the critical value. So, once you have the alpha you refer the statistical table and for the given alpha you will have this particular critical value. Here it is minus 1.645 for your alpha is equal to 2.05. So, then you collect the data and compute the test statistic p value make the statistical decision and express the conclusion.

So, here for this particular example you can see that my alpha basically is alpha is equal to 0.05. And if I conduct the two-tail test, it means I consider the both the sides suppose then alpha by 2 would be 0.025. Now if you see my p value corresponding to computed test statistic minus 2, then it comes out to the 0.0228. So, this value is basically falling in the rejection region because it is 0.0228. So, 0.0228 is less than 0.025, so this is the probability value. So, you can say that some region here will be your p value and as it falls in the rejection region, you will reject the null hypothesis. So, this was the example about the true mean number of number of TV sets less than 3, so I accept the alternate hypothesis.

Now we will see more examples to make the idea clear.

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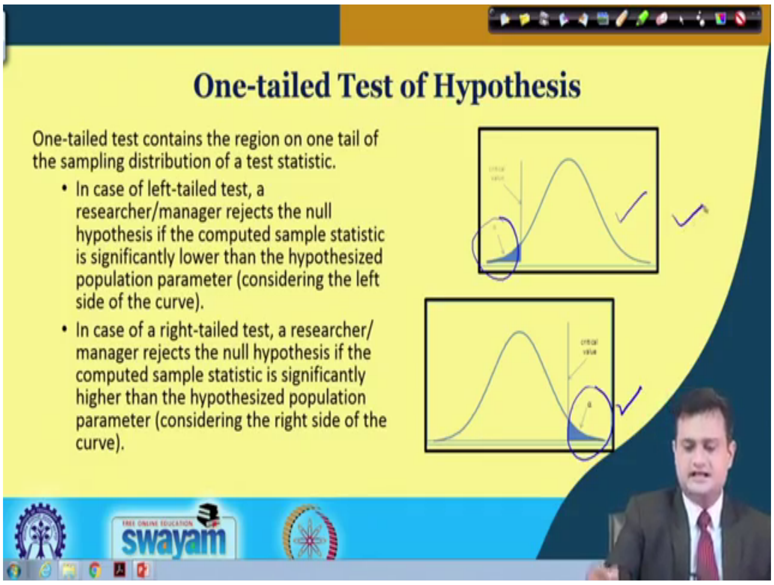

So, just see as I mentioned that you can have one-tail test as well as you can have two-tail test. When I say one-tail test, you are only interested either in the lower side or lower tail of the distribution it is a normal distribution and if I say higher or value then if I am interested in the right side tail higher value then this is again a one-tail test, this is again a one-tail test. So, may be my interest would be either on the left side or right side and I may conduct only the one-tail test.

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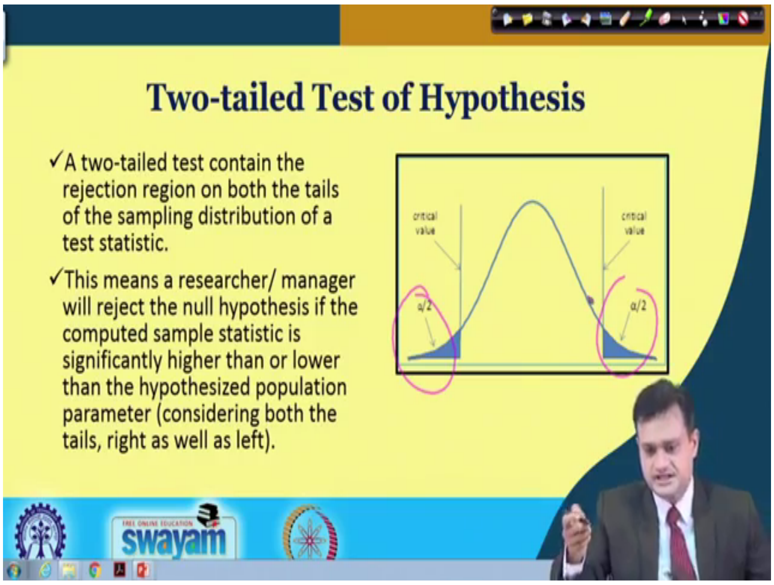

Alternatively I can go for say two-tail test like this.

So, here I am interested in both side alpha by 2 and alpha by 2 and I will compare my computed statistics. And the p value with this level of significance alpha by 2 and if it falls within then, I will say my null hypothesis is basically in the rejecter is reject because my value p value or critical value is falling in the rejection region. But always remember we reject or accept the null hypothesis or accept the alternate hypothesis at a given level of significance and we have to accept of commuting type 1 and type 2 error because inferential statistics is for a chance phenomenon or random phenomenon.

And when I deal with the probability then, I have to accept of making a wrong decision the probability of making a wrong decision as well as say probability of rejecting the null hypothesis or accepting the alternate hypothesis.

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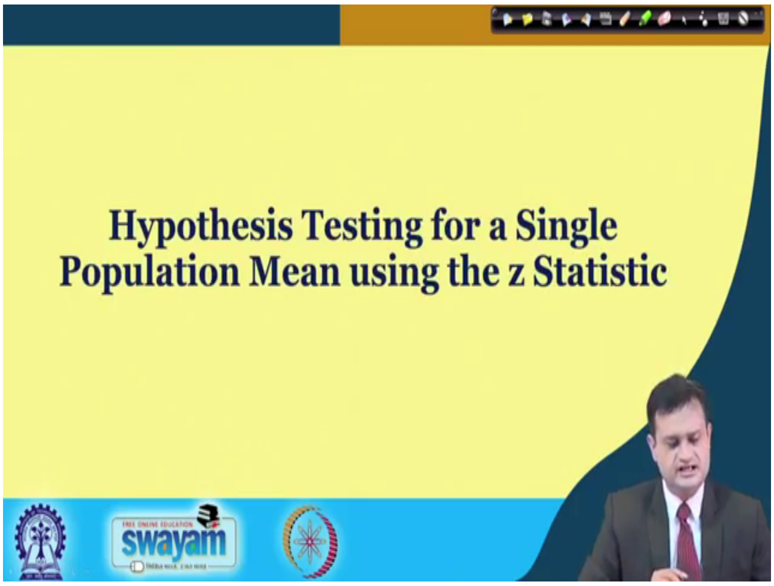

So, this is something that we try to do in hypothesis testing. Now, let us try to consider hypothesis testing for a single population mean using z statistics. So, we had seen it previously that there are various types of distributions and you have the associated probability distribution function. This probability distribution function is used to develop the statistical table and for each particular distribution you have the statistical table for ease in computation getting the values corresponding to your computed statistic.

So, you have normal distribution and for standard normal we refer the z statistic when I say standard normal then my mean is 0 my standard deviation is 1 and I refer the z statistics and the z table.

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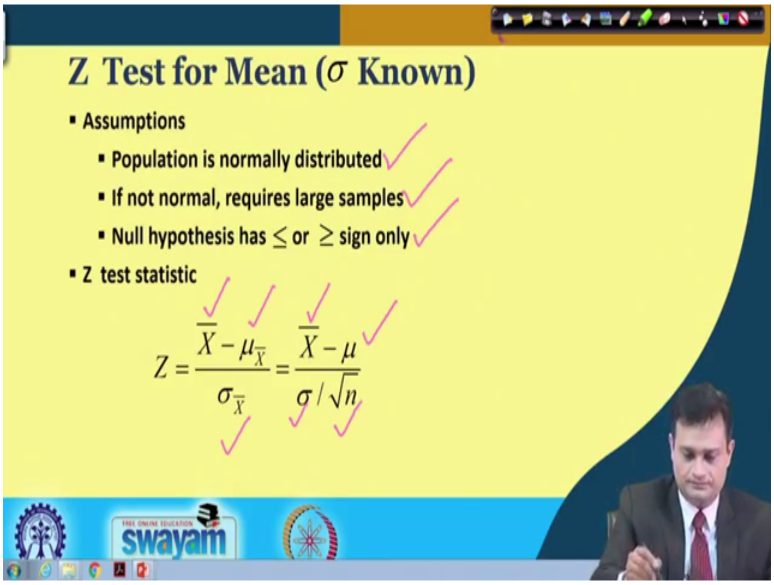

So, here let us say we want to do the analysis with z statistics for mean and my assumption here is that sigma is known. It means, sigma is for population and I know that what is the population standard deviation, so population is normally distributed. This is my say first assumption population is normally distributed then if not normal requires large samples and null hypothesis has less than or equal to or greater than or equal to sign only.

So, my z statistic basically is X bar that is your sample mean mu X bar this is your population and sigma X bar this is your sample standard deviation. So, here I will use the central limit theorem, so I will say X bar minus mu divided by sigma square root n as I making use of central limit theorem.

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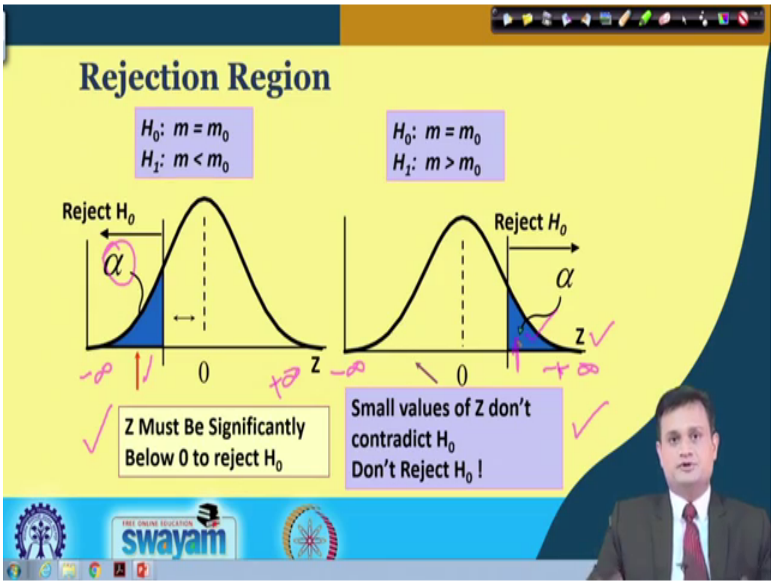

So, this is the expression that I can conveniently use for computing my values and comparing the values with the critical values obtained from the table. So, just see that I have typically a rejection region. And here, let us say I am talking about one-tail test. So, I can conduct the one tail test either for the left tail or for the right tail just see this is the left tail and this is my alpha which is level of significance and also type 1 error and what I say that Z must be significantly below 0 to reject null hypothesis.

So, it should fall in this particular region as shown by the arrow and then I will reject the null hypothesis for this one-tail test then my region of interest is on left tail. Now here you can see that you have this Z value and again your distribution this ranges from minus infinity to infinity plus infinity. Here also it is minus infinity to plus infinity and you have this particular region that is of interest and I would say that small values of Z do not contradict H 0, do not reject H 0 or else you should have the value of Z which falls in this particular region. So that, you can reject your claim status quo that is the null hypothesis and you can accept the alternate hypothesis.

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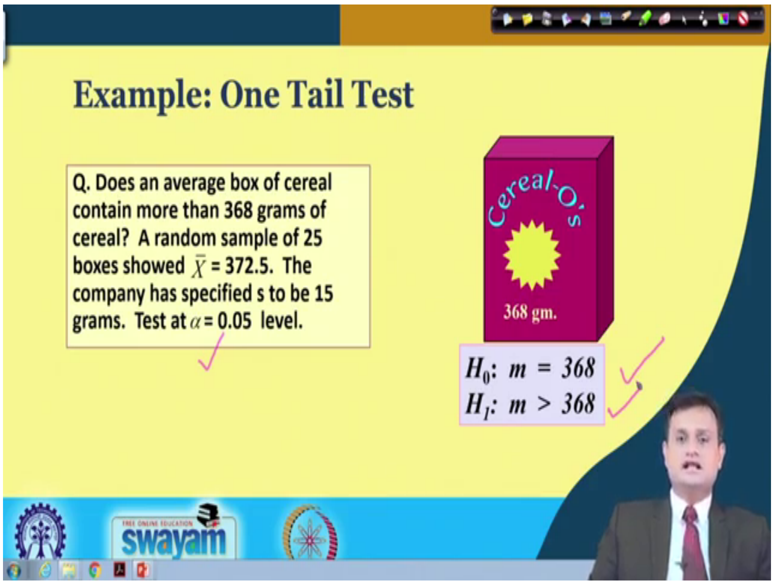

So, here you have this single tail.

So, let us try to see the example for the one-tailed test and the question here is that does an average box of cereal. You are purchasing various kind of corn flakes, oats, and other things. So, cereal contain more than three 68 gram of cereal. So, fine there could be variation in packaging, but; obviously, company would not like to fill the excess or too less which will basically either disadvantageous to the company or to the consumer. So, they want to check that on an average suppose if we fill this much then rather we are significantly lower compared to our value or whether we are in the acceptable zone.

So, a random sample of 25 boxes showed that 378.5 is the average and 368 gram is the actual content that should be there. So, the company has specified s to be 15 grams and s is your basically sample standard deviation and they want to check that claim H alpha is equal to 0.05. So, now, my null hypothesis here it is m is equal to 368 that on an average I am filling up 360 grams 8 grams of cereal in a box and alternate hypothesis is that, I am filling up 360. So, now, here you would say that it is very much visible that 372.5 is greater than 368, what is the need of conducting this analysis?

So, I would like to remind that there is descriptive analysis there is inferential analysis. Descriptive analysis yes I get an idea that 372.5 is greater than 368, but this is a chance phenomenon, this is random phenomenon. I have taken the sample and I want to check at the given level of significance alpha is equal to 0.05 that to what extent my claim that I am exactly feeling 368 or greater than equal to greater than 368 is really true.

So, inference in statistic provides more scientific basis when you are not actually becoming clear just with the comparison on a values that you do through comparison of various measures of central tendency or measures of dispersion. In descriptive statistics, we try to check our claim with more rigorous process scientifically and that is where I make use of inferential statistics. So, here you have this particular say hypothesis.

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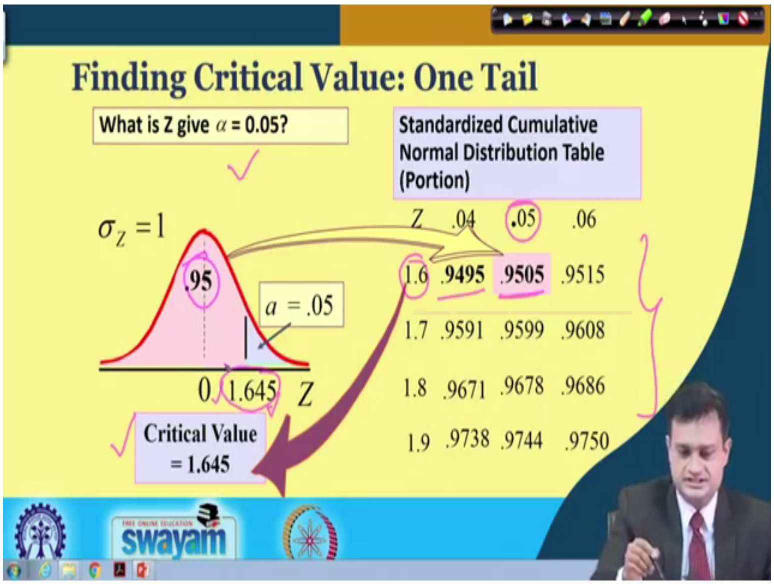

Now, I want to check it. So, procedure is very simple and what you need to do? Just see, you have alpha is equal to 0.05 and first I need to identify this particular value. So, for this level of significance what is the tabulated value in z table statistical table or theoretical value and this value is called typically the critical value 1.645. I can just give you the little snap shot of the table here and your full table you can see in the statistical book suggested. Here, it is just a small portion and you can see that I have z value 1.6 and here it is expressed in decimal.

So, I want to find the alpha is equal to 0.05, so you have alpha is equal to 0.05. So, now, for alpha is equal to 0.05 you want to find out the value of this and it could be one point say or 65 or little bit you can average it out so, it would be say 1.645 averaging is required because my level of significance is 5 percent and hence this value is 0.95. So, if you see here it is 0.9505, it is 0.9495. So, if you little bit do averaging then it would come out to be 1.645.

So, this is my critical value. I want to test my claim null hypothesis and alternate hypothesis against this critical value to check that to what extent, I am right in my claim or I should reject the null hypothesis.

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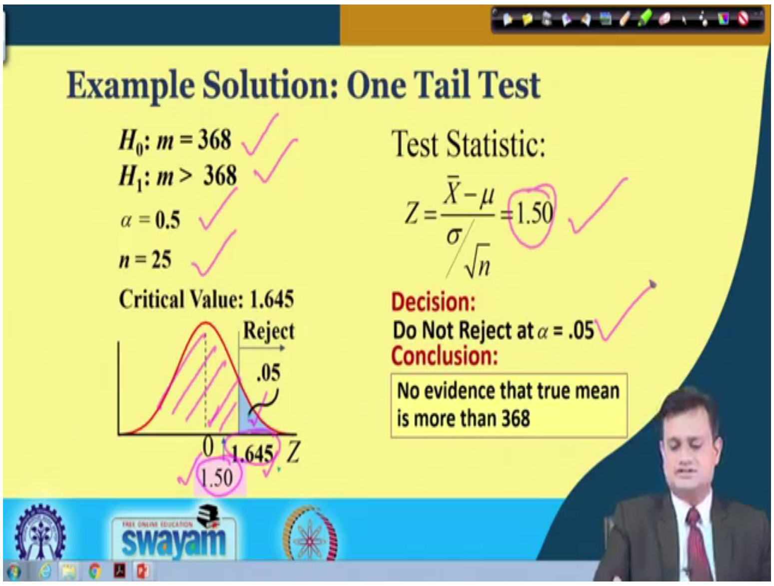

So, this is how to read the statistical table already we discussed previously. So, my null hypothesis is H n is equal to 368, and n is greater than 368, and alpha is equal to 0.5, n is 25. So, now, this is my critical value and this is my computed value.

So, for computed value you can just make use of this particular expression X bar minus mu divided by sigma divided by square root n. You just plug in the values, you have all the values given in your data set and you can compute this 1.5. So, 1.5 because I am say computing with reference to the 0, 1.5 is in this region, this is the region and this region is basically my accept region if it would fall in this region it is my reject. So, I would say that when I compare 1.5 with 1.645, 1.5 is less so it would fall in the accept region not in reject region and I do not have enough evidence to say that mean is more than 368.

So, the conclusion is that do not reject at alpha is equal to 0.05. So, is that not interesting? Visibly if you will say then because it is 372.5.

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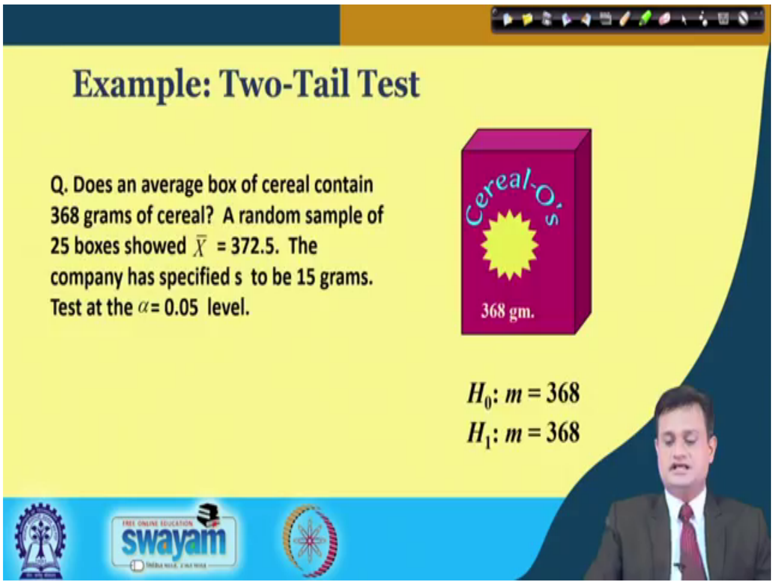

So, if you just see this then you have 372.5 as the mean of your 25 units its the sample. And by observation without statistical analysis you say that 372.5 is greater than 368, but when you check it with scientific basis hypothesis testing.

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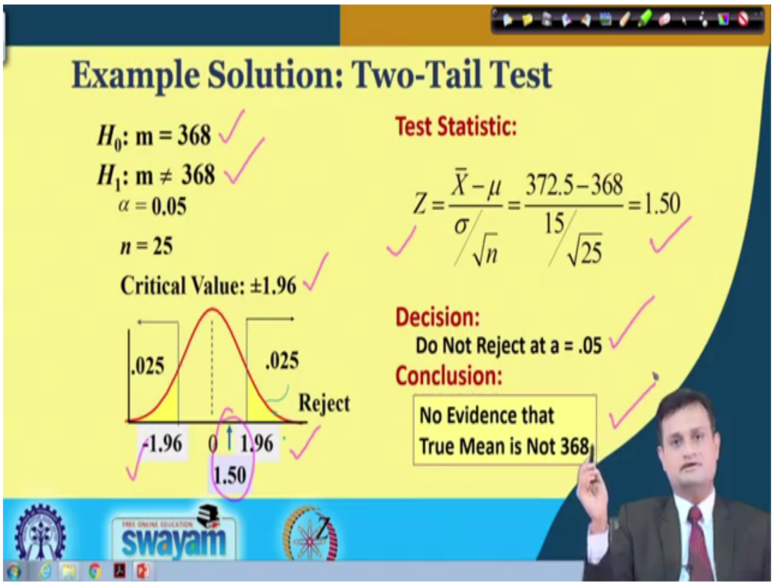

You come to the conclusion that do not reject at alpha is equal to 0.05.

So, my conclusion is there is not much evidence significant evidence available that true mean is not 368 ok. So, this is something that we have seen about two-tail we can also do. So, this is we have done for one tail test initially and now we would like to also do it for the two-tail. So, initially I just explain you this particular part, so do not reject my I will just go back, this was the table. So, do not reject at alpha 0.05.

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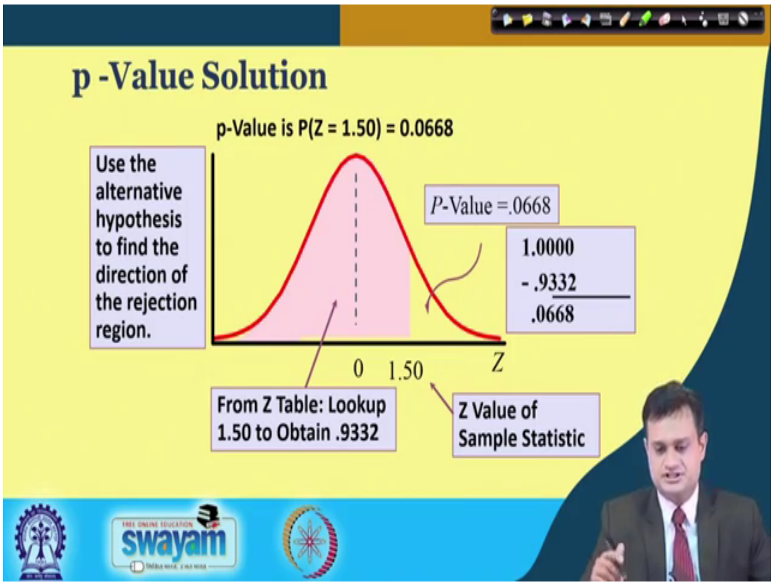

Now, p value is 0.0668 and if I go by even the p value I would get this same result that do not reject null hypothesis.

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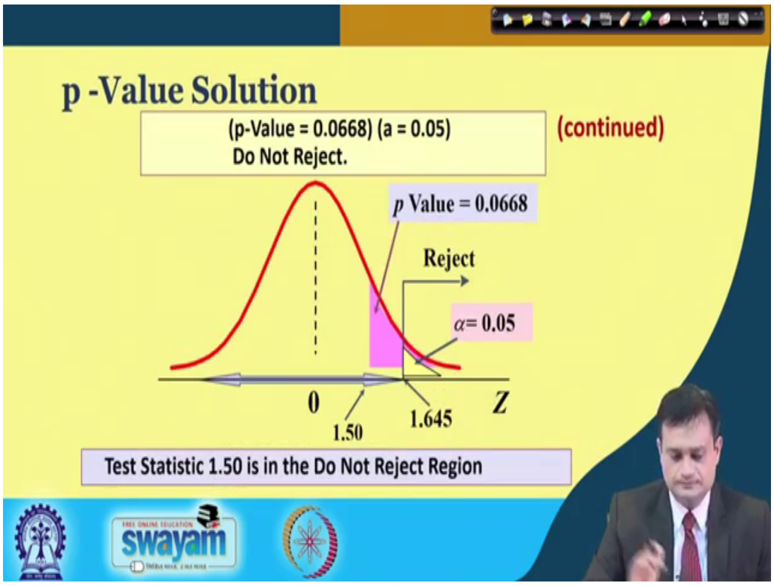

Now, this is exactly what that your p value is 0.068. So, 0.0668 is greater than your 0.05, so basically your total probability region is given by this and it is larger than the alpha, so I will say that do not reject null hypothesis. Now let us go ahead for the same example I want to do a two-tail test. So, I have let us say an average box of cereal contain 368 gram and again I took the random 372.5 the company has specified s sample standard deviation to be 15 gram and test the hypothesis and alpha is equal to 0.05.

So, I am setting here say n is equal to 368 and alternate hypothesis you just put this not equal to 368. So, just you put here n is not equal to 368. So now, I want to test my claim for this particular situation. So, let us say H 0 is m is equal to 368, m is not equal to 368 and my critical value is 1.96. I am referring to two-tail test so, this is minus 1.96, plus 1.96, we just plug in the value we have already seen Z is equal to 1.5. Now again, I will end up with the same decision because minus 1.5 is less than this and I will say that do not reject at alpha or a is equal to 0.05 same thing level of significance and I do not have the enough evidence available.

So, this is two-tail test which also confirms the same result and if you go by the p value approach you will see that this is the region which is larger than my significance say assume significance level which is alpha and it falls in the acceptance region. So, I will not reject the null hypothesis.

So, sometimes it may sound little bit confusing, but the logic is very simple. By chance, if you have a confusion just revisit the steps proposed right at the beginning of this lecture and also in the previous lecture, various steps to be followed in hypothesis testing and you will have the very clear idea.

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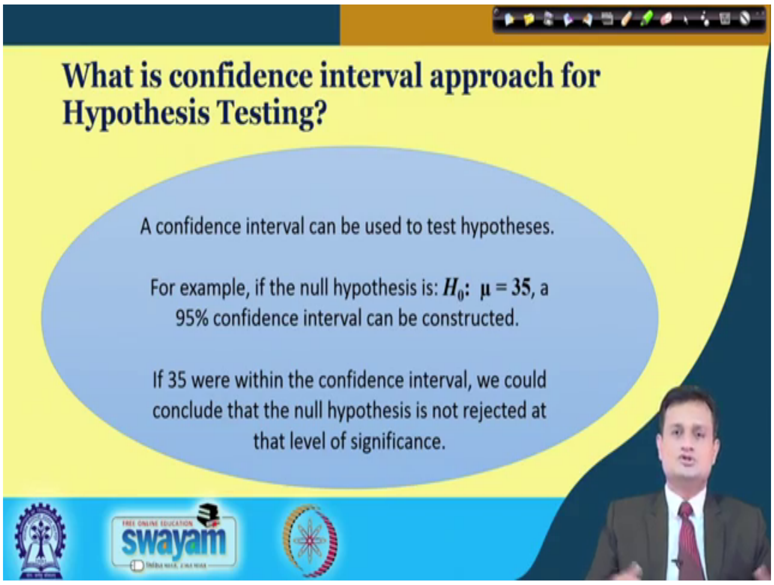

So, what is now the confidence interval approach? We have already seen. Suppose if I say null hypothesis mu is equal to 35, 95 percent confidence interval. So, I want to see that if 35 were within the confidence interval, we could conclude that the null hypothesis is not rejected at the given level of significance.

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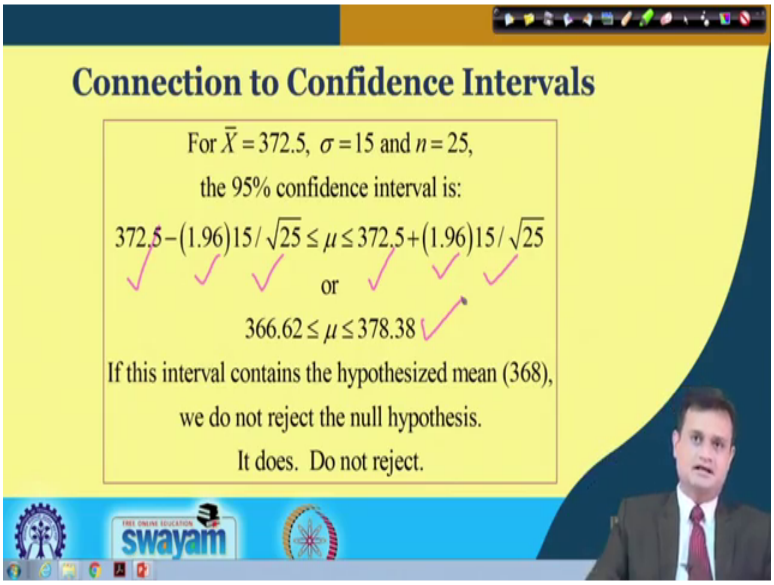

So, this is another approach as I mentioned for checking or testing my hypothesis and here for the example which we are discussing that how much gram of cereal I fill in a particular box. So, X bar is equal to 372.5 sigma is equal to 15, or s is equal to 15, n is equal to 25. So, 95 percent confidence interval if I constitute then it would be X bar plus or minus K sigma. So, here my K is 1.96.

I have already discussed this part and here sigma by root n sigma by root n because I am doing it for sample, I am going by this central limit theorem and this is my X bar. So, my confidence interval comes out to be 368.62 less than or equal to mu less than or equal to 378.38.

So, if this interval contains the hypothesised mean, this is 368 which is true we do not reject null hypothesis it does do not reject. So, 368 within this interval and at a given level of alpha I will not reject my null hypothesis.

So, this is something that we can do either through critical value through p value through confidence interval and we have seen this.

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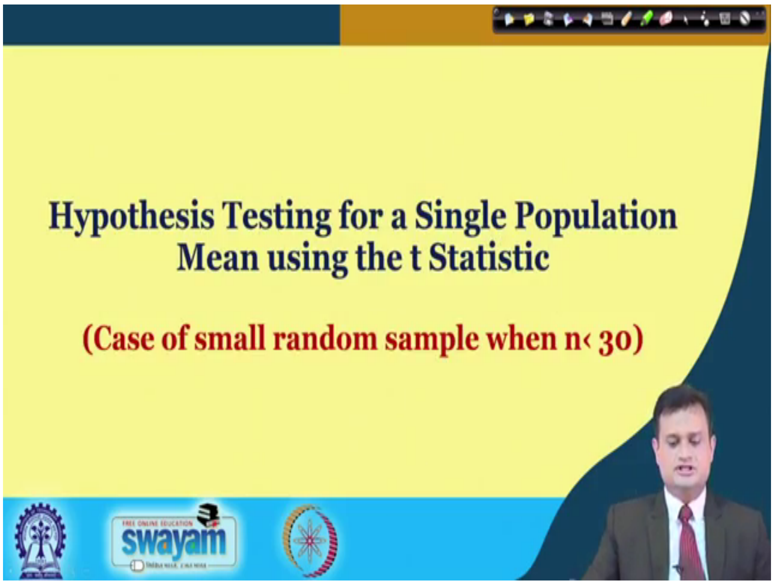

Now, we have a situation when hypothesis testing to be done for a single mean using t statistics.

So, you have t distribution and same way you have t statistic. Typically this kind of test we conduct for a small random sample having less than 30.

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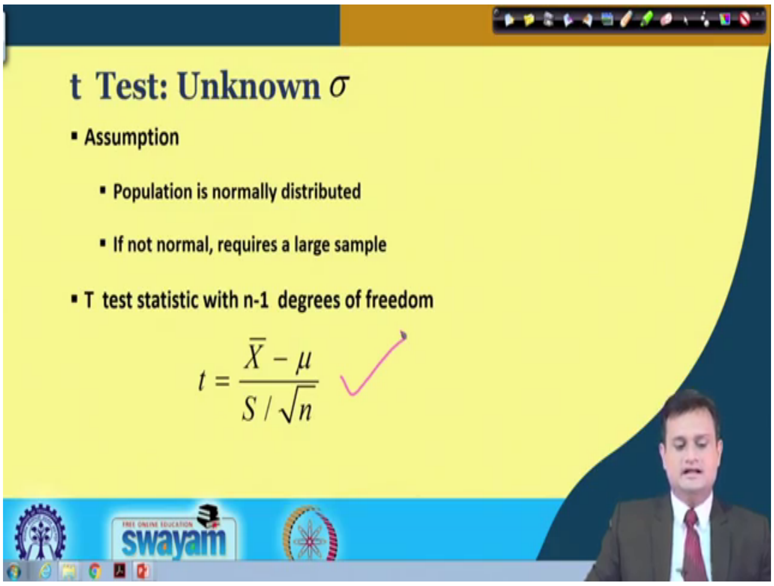

So, let us say the t statistics is X bar minus mu divided by your s divided by square root n and population is normally distributed and if not normal requires a larger sample.

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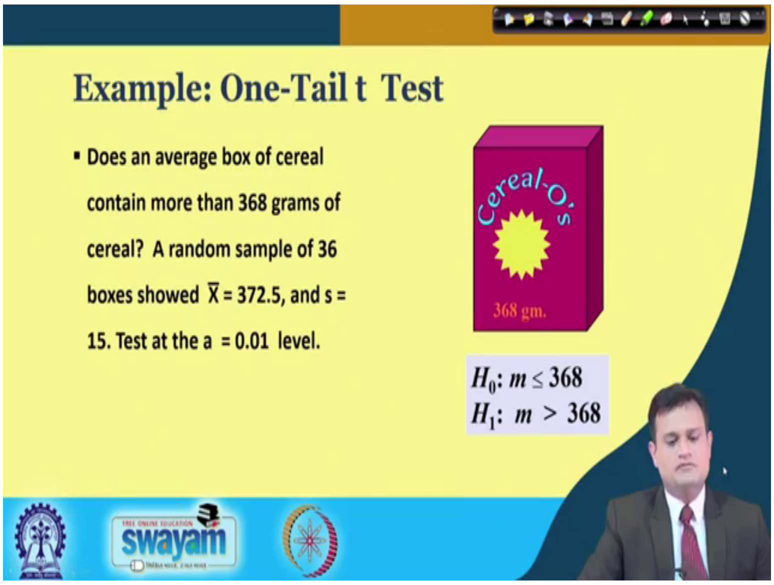

So we have the example, same example I may continue. And here, what I say that does an average box of cereal contain more than 368 grams of cereal, a random sample of 36 boxes showed X bar 372.5. S is equal to 15 and you test the hypothesis as 0.01 level of significance. So, these are my alternate and null hypothesis and I want to test my claim against this.

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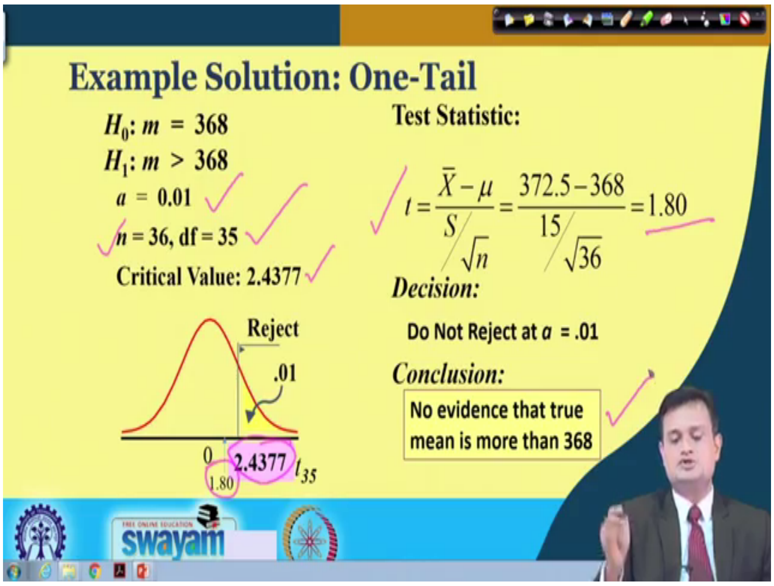

So, I am just repeating the same procedure, but here I would be using the t statistics and you can see very well that my alpha is 0.01, n is 36, degree of freedom is 35, n minus 1 I explained that I have 1 less degree of freedom to maneuver. Critical value is 2.4377 and my computed value, critical value where respect to with respect to critical value is less which is 1.8. So, my conclusion is that no evidence that true mean is more than 368.

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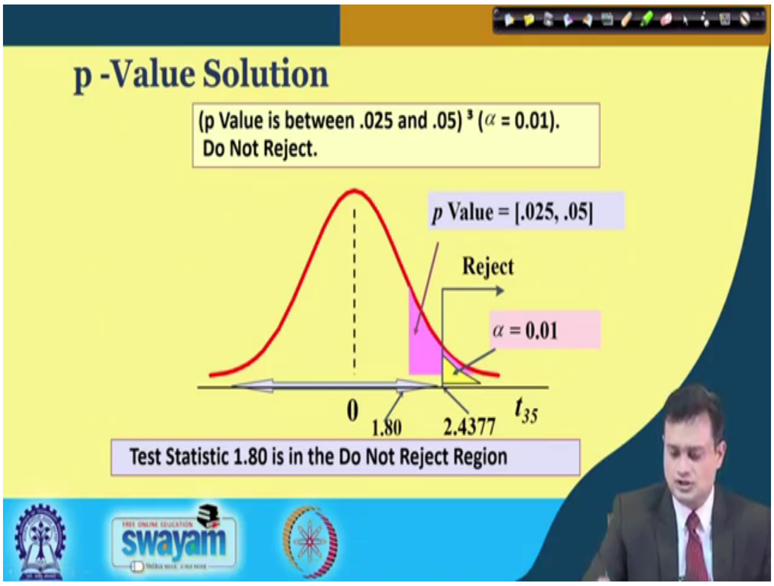

So, procedure is same only thing here I demonstrated the application of p statistics and you can say conveniently use it. Similar way, you can apply the p value approach for even the t distribution and check. So, here again it falls within this particular range larger than my yellow region larger than my this yellow region so, it is in the acceptance region and I cannot reject my null hypothesis.

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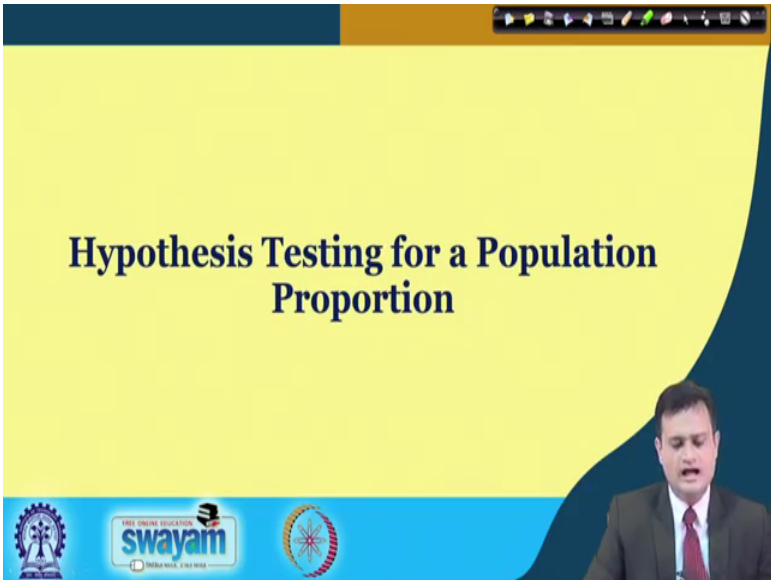

So, hypothesis testing we can also do for population proportion.

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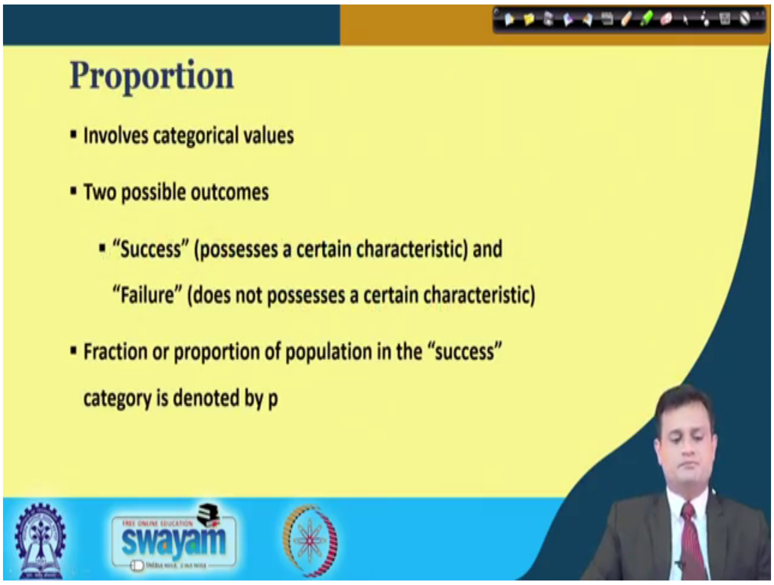

Typically, when I say this kind of test it mainly includes the categorical values. So, you have 2 possible outcomes typically, success or failure. And I would like to express it in terms of the fraction or proportion of success and typically category is denoted by p.

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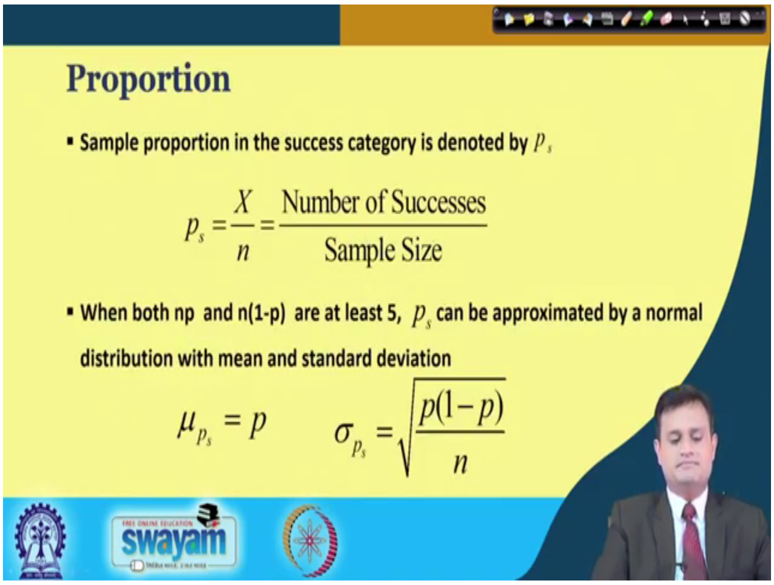

So, p s is equal to X by n number of success divided by sample size and mu p s is basically p and sigma p s these are the things that basically you need in order to compute the test statistic.

So, sigma p s for sample is equal to square root of p 1 minus p divided by n.

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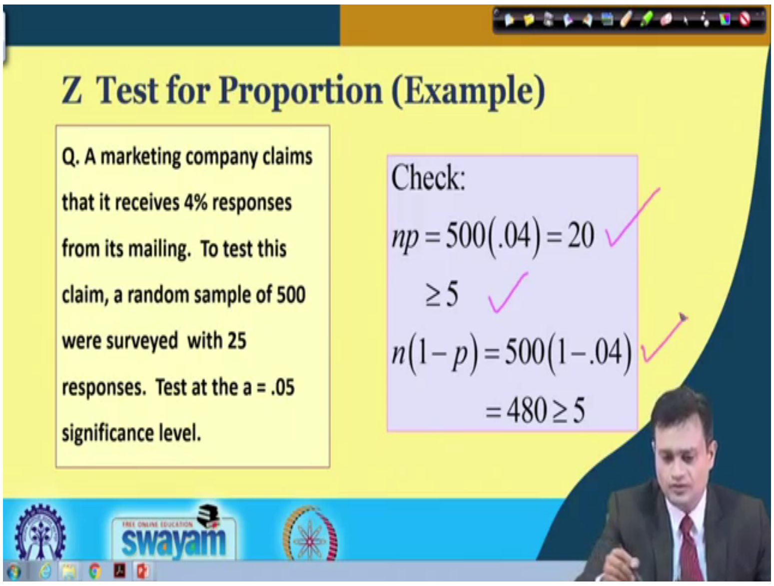

So, the question is like this. A marketing company claims that it receives 4 percent responses from its mailing to test this claim random sample of 500, where surveyed with 25 responses, check this claim at alpha or a is equal to 0.05 significance level. So, you just need to compute couple of things n p which is 500 into 0.05, so 20 greater than or equal to 5, n 1 minus p is 480 greater than equal to 5.

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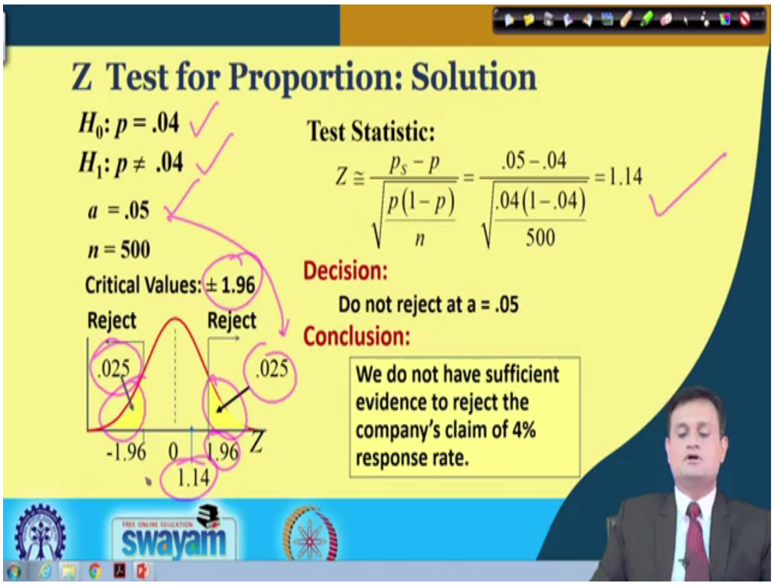

And now, say I would like to test the hypothesis. So, just see this I have the region available here p is equal to 0.04 that is the claim of the company p is not equal to 0.04 my alternate claim, hypothesis alpha is equal to 0.05, I conduct two-tail test. So, it is divided this is divided into 0.025 and 0.025 the corresponding critical value is 1.96 and this is my region which is basically the reject region which is 0.025. So, when I compute the z statistic for proportion I get the value 1.14 and this 1.14 is less than 1.96 so, it is in the acceptance region.

So, I will say we do not have sufficient evidence to reject the company claim of four percent response range. So, this is for proportion.

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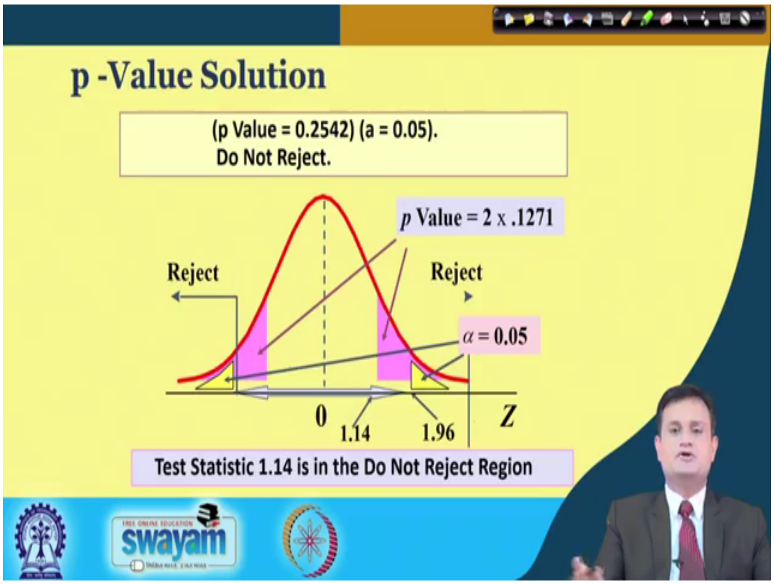

And again same way I am not going into detail, but you can do it for p value.

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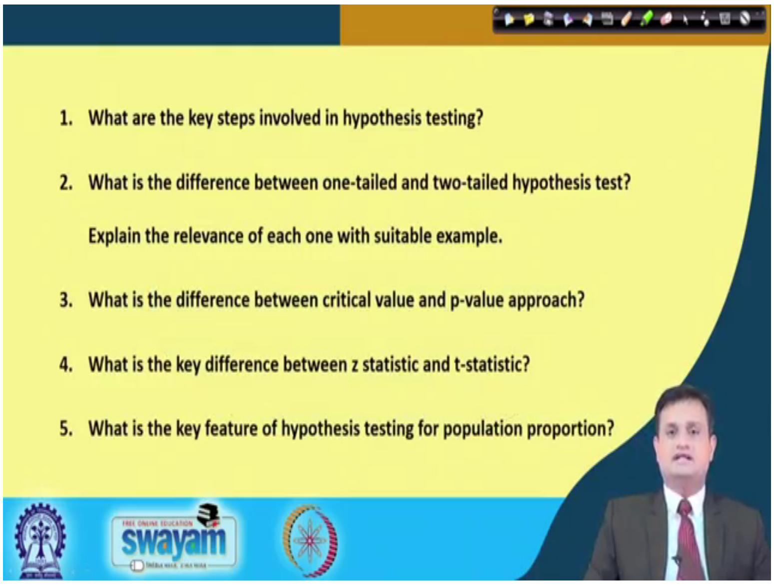

So, let us try to end this session with some think it, so that you can introspect revise. What are the key steps involved in hypothesis testing? This is very important. Once you understand this and whatever test statistic you need to follow for a given situation you can execute hypothesis testing very easily. What is the difference between one-tail and two-tail hypothesis test? Explain the relevance of each with suitable example? What is the difference between critical value and p value approach? What is the key difference between z statistics and t statistics? And what is the key feature of hypothesis testing for population proportion?

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Please use this book Aczel and Levine. I in fact, refer couple of examples from this book and that will help you to enrich your concept and understanding.

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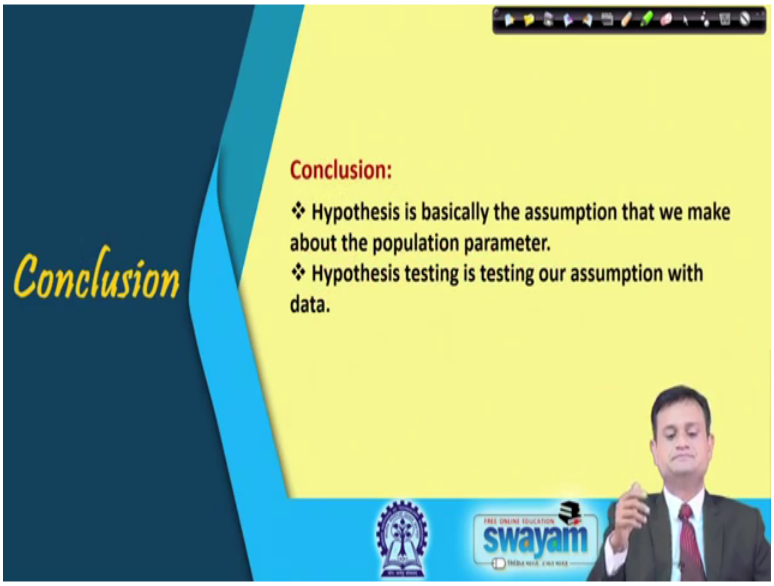

So, basically hypothesis testing is the assumption that we make about the population parameter and scientifically we try to check it at a given level of significance alpha with the consideration that I may commit type 1 and type 2 error.

So, thank you very much for your interest in learning hypothesis testing. Keep revising the various topics we are going one after another in our six sigma journey. Be with me enjoy.