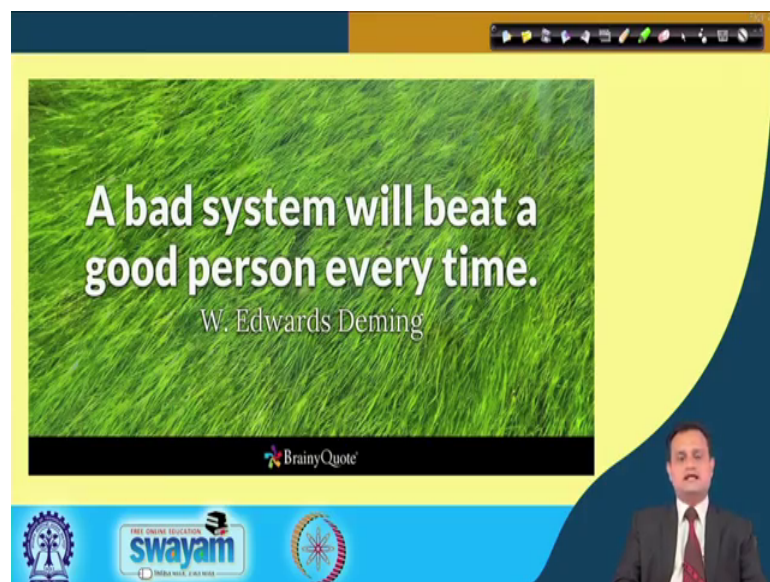


Six Sigma
Prof. Jitesh J Thakkar
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture - 25
Process Capability Analysis : Measures and Indices

Hello friends, I invite you to the lecture 25, Process Capability Analysis: Measures and Indices and I would like to remind you that we are in the major phase of our DMAIC Six sigma journey. And last time we have talked about various concepts key concepts related to process capability analysis. This time we will go deeper and try to appreciate the importance of various measures and indices.

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So once again I would like to emphasize that a bad system will beat a good person every time and it is necessary to see that, my processes are really capable enough to meet the customer expectations or not. So, processes which are not meeting the customer expectations, specifications, then these processes first needs to be corrected. And once you achieve the desired capability of the process, then only you can really produce the good quality products.

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Recap

- ❑ Need of process capability analysis
- ❑ Natural Process Limits, Specification Limits, Control Limits, Natural Tolerance Limits
- ❑ Short-term and Long-term process capability
- ❑ Difference between Process Performance Metrics and Process Capability Indices

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So, if you just see the small recap, then we talked about need of process capability analysis, natural process limit, specification limit, control limits, natural tolerance limits, short term and long term process capability, differences between process performance metrics and process capability indices. So, many a times you get confused, we had seen DPMO DPU, how to find sigma level, all these are process performance metrics. when we talk about process capability indices, these are the aggregate measures basically represents the capability of the process and C_p , C_{pm} , C_{pk} . These are the various measures that we will see in this particular lecture

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CONCEPTS COVERED

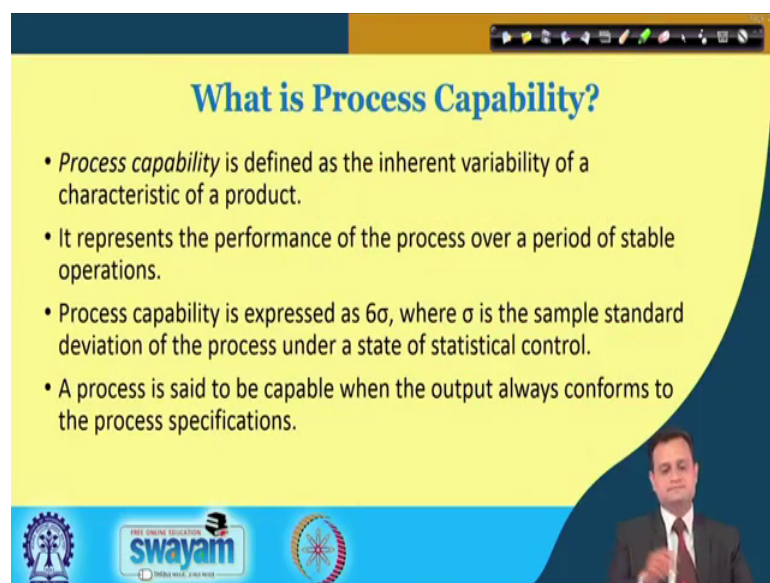
Concepts Covered:

- ❑ Process Capability and Capability Indices
- ❑ Process Performance and Performance Indices
- ❑ Short-term and Long-term Capabilities
- ❑ Capability Related Assumptions
- ❑ Conducting a Process Capability Study
- ❑ Natural Process Limits and Specification Limits
- ❑ Process Performance Metrics

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So, we would also keep revising couple of things which we discussed last time, but in addition to that, we will mainly focus on process capability and capability indices in this lecture. Process performance and performance indices, short term long term capabilities and related indices, capability related assumptions. Then conducting process capability study, natural process limit, specification limit and process performance metrics. So, these are the broad topics that we would like to see and simultaneously I would like to give you some recap of the last lecture, so that you can strengthen your understanding on process capability indices.

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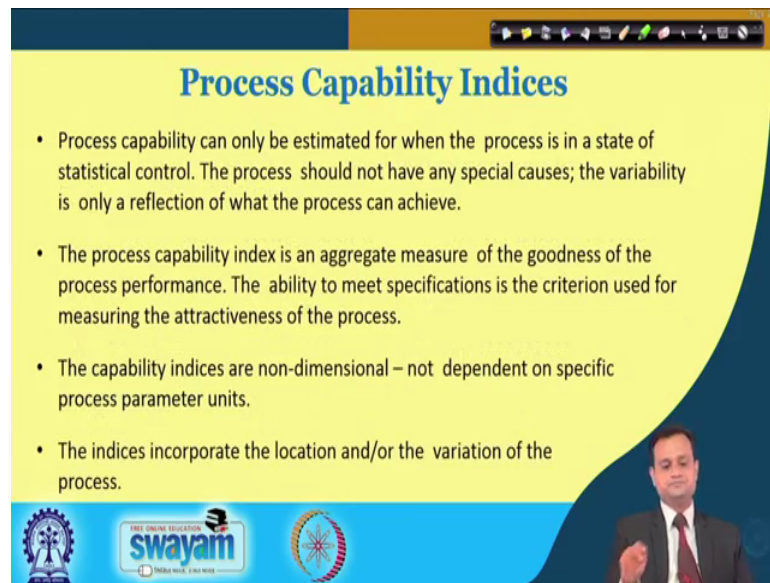
What is Process Capability?

- *Process capability* is defined as the inherent variability of a characteristic of a product.
- It represents the performance of the process over a period of stable operations.
- Process capability is expressed as 6σ , where σ is the sample standard deviation of the process under a state of statistical control.
- A process is said to be capable when the output always conforms to the process specifications.

Logos at the bottom: IIT Bombay, SWAYAM, and a circular logo. A small video inset of a man in a suit is visible in the bottom right corner.

So, we have seen what is process capability? So, typically it expresses the capability of the process to meet customer specifications and to what extent my process is capable enough to meet the customer requirement. So, typically it is expressed as 6 sigma and where sigma is the sample standard deviation of the process which is under a state of control. So, please be reminded that I can only conduct the process capability study if my process is under the state of statistical control.

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Process Capability Indices

- Process capability can only be estimated for when the process is in a state of statistical control. The process should not have any special causes; the variability is only a reflection of what the process can achieve.
- The process capability index is an aggregate measure of the goodness of the process performance. The ability to meet specifications is the criterion used for measuring the attractiveness of the process.
- The capability indices are non-dimensional – not dependent on specific process parameter units.
- The indices incorporate the location and/or the variation of the process.

The slide features a presenter in the bottom right corner and logos for IIT Bombay and Swayam at the bottom.

We have various process capability indices, but basically these are aggregate measures, and they typically, say check the goodness of the process performance and ability to meet the specifications. So, indices in corporate locations as well as variability both, and we will try to see it through different indices.

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Process Capability Indices

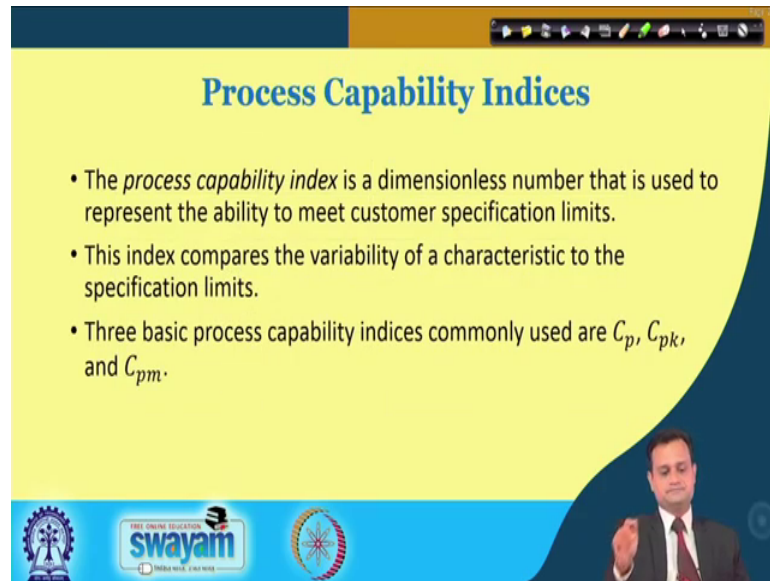
Short Term	Long Term
C_p	P_p
C_{pk}	P_{pk}
C_{pm}	P_{pm}
CPU	PPU
CPL	PPL

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So, broadly I can classify the indices as short term indice, long term indice, C_p C_{pk} C_{pm} , CPU, CPL. These are the various short term indices and long term indices P_p , P_{pk} P_{pm} .

So, when I says short term, I am specific on a particular sub group or a limited period of time, I am trying to capture the status of process. When I say long term, then for the entire horizon, overall a say working period, long span I am trying to absorb the process and then I am trying to say calculate the process capability to comment about the process performance.

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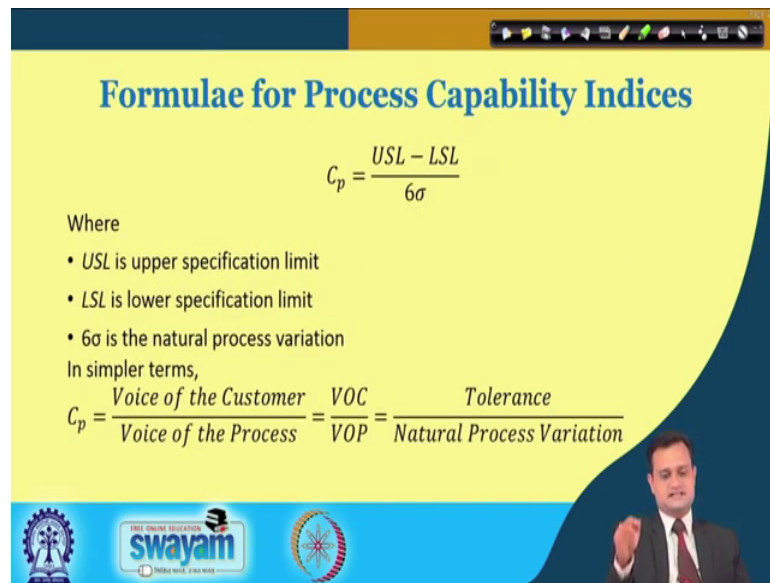


The slide is titled "Process Capability Indices" in blue text on a yellow background. It contains three bullet points: "The process capability index is a dimensionless number that is used to represent the ability to meet customer specification limits.", "This index compares the variability of a characteristic to the specification limits.", and "Three basic process capability indices commonly used are C_p , C_{pk} , and C_{pm} ." The slide is part of a presentation, as evidenced by the navigation icons at the top and the logos at the bottom, including the Indian Institute of Technology (IIT) logo and the "swayam" logo. A small video inset in the bottom right corner shows a man in a suit gesturing.

- The *process capability index* is a dimensionless number that is used to represent the ability to meet customer specification limits.
- This index compares the variability of a characteristic to the specification limits.
- Three basic process capability indices commonly used are C_p , C_{pk} , and C_{pm} .

So, basically it is a dimensionless number and is used to represent the ability to meet customer specification limit and typically for short term as we have seen C_p , C_{pk} , C_{pm} are the widely used process capability indices.

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Formulae for Process Capability Indices

$$C_p = \frac{USL - LSL}{6\sigma}$$

Where

- *USL* is upper specification limit
- *LSL* is lower specification limit
- 6σ is the natural process variation

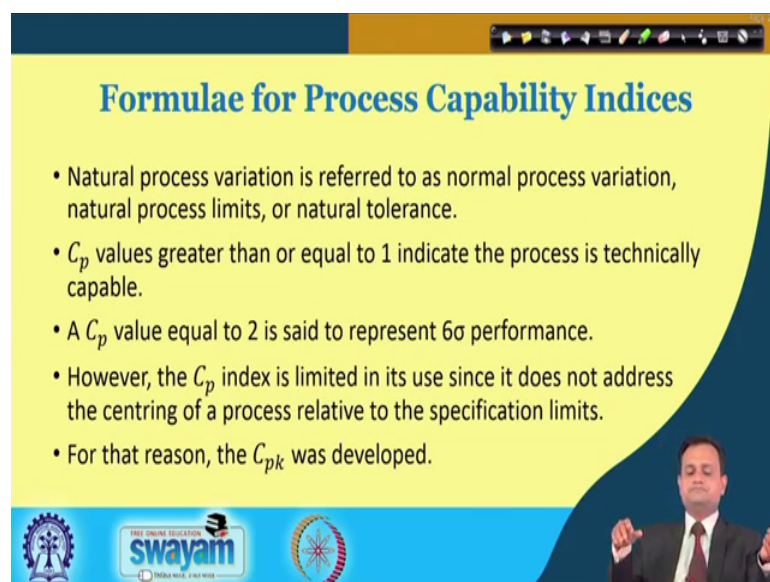
In simpler terms,

$$C_p = \frac{\text{Voice of the Customer}}{\text{Voice of the Process}} = \frac{VOC}{VOP} = \frac{\text{Tolerance}}{\text{Natural Process Variation}}$$

The slide includes logos for Swamyam and other educational institutions at the bottom, and a small video inset of a presenter in the bottom right corner.

So, just see the expression for C_p . I am considering the overall band USL minus LSL divided by 6 sigma; that is the variability associated. So, when I say USL minus LSL by 6 sigma typically it tries to capture the voice of the customer and voice of the process. So, VOC divided by voice of process VOP , tolerances divided by natural process variation.

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Formulae for Process Capability Indices

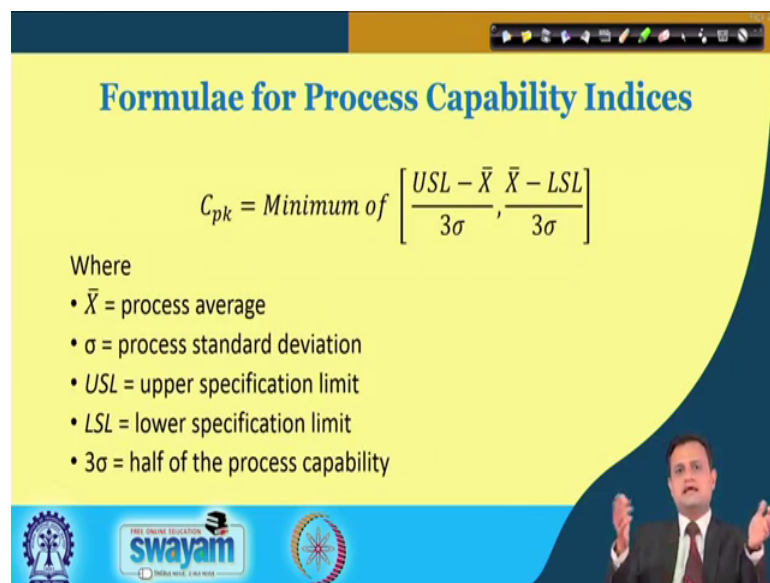
- Natural process variation is referred to as normal process variation, natural process limits, or natural tolerance.
- C_p values greater than or equal to 1 indicate the process is technically capable.
- A C_p value equal to 2 is said to represent 6σ performance.
- However, the C_p index is limited in its use since it does not address the centring of a process relative to the specification limits.
- For that reason, the C_{pk} was developed.

The slide includes logos for Swamyam and other educational institutions at the bottom, and a small video inset of a presenter in the bottom right corner.

So, we have certain guidelines that C_p value is greater than equal to 1, indicates the process is technically capable. When I say C_p is equal to 2, then it represents the world

class performance like 6 sigma performance, and C p value equal to 2 is said to represent typically your 6 sigma performance which we are aiming for and C p index is limited to use, since it does not address the center of a process. So, process has a center mean as well as the variability, but typically C p does not take into account the center or shift in the mean, because over a period of time there may be change in the setting and process may get shifted to the new mean. This part is not considered and that is why the new measure is developed that is C pk.

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Formulae for Process Capability Indices

$$C_{pk} = \text{Minimum of } \left[\frac{USL - \bar{X}}{3\sigma}, \frac{\bar{X} - LSL}{3\sigma} \right]$$

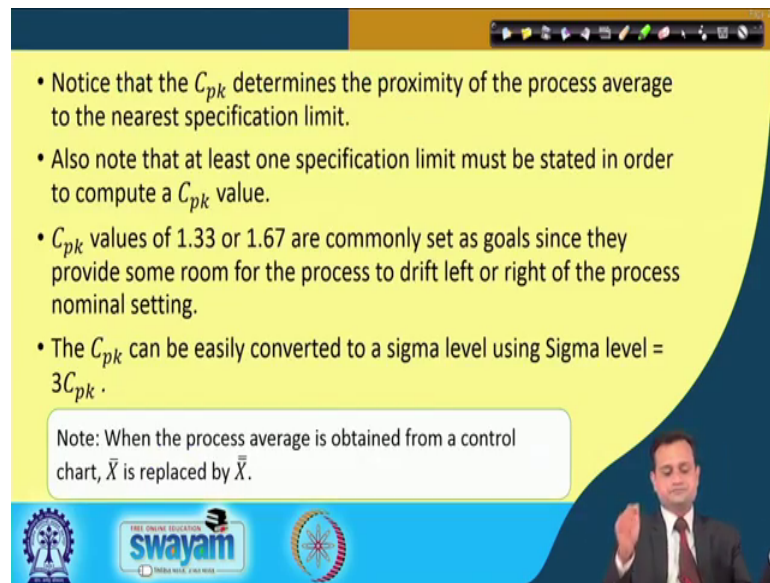
Where

- \bar{X} = process average
- σ = process standard deviation
- USL = upper specification limit
- LSL = lower specification limit
- 3σ = half of the process capability

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So, C pk is basically minimum of USL minus X bar divided by. So, X bar is your process average, USL is upper specification limit divided by 3 sigma and another term that we calculate is X bar minus LSL divided by 3 sigma. So, I will take minimum of these two, to check whether my process also meets the center requirement or not or it is now performing poorly, because there is shift in say center or mean value.

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• Notice that the C_{pk} determines the proximity of the process average to the nearest specification limit.

• Also note that at least one specification limit must be stated in order to compute a C_{pk} value.

• C_{pk} values of 1.33 or 1.67 are commonly set as goals since they provide some room for the process to drift left or right of the process nominal setting.

• The C_{pk} can be easily converted to a sigma level using Sigma level = $3C_{pk}$.

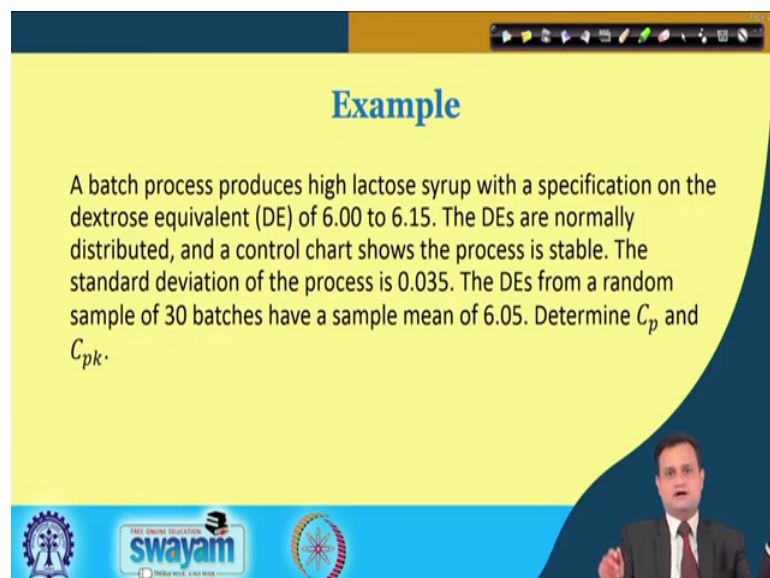
Note: When the process average is obtained from a control chart, \bar{X} is replaced by $\bar{\bar{X}}$.

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So, typically C_{pk} determines the process, proximity of the process average to the nearest specification either lower or upper, and we can note that one specification limit must be stated in order to compute the C_{pk} . Sometimes some products they have only one specification limit, so it is ok, but at least you should have one specification limit.

So, C_{pk} can be easily converted to a sigma level using sigma level is equal to 3 into C_{pk} and when the process average is obtained from control chart, \bar{X} is typically replaced by $\bar{\bar{X}}$.

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Example

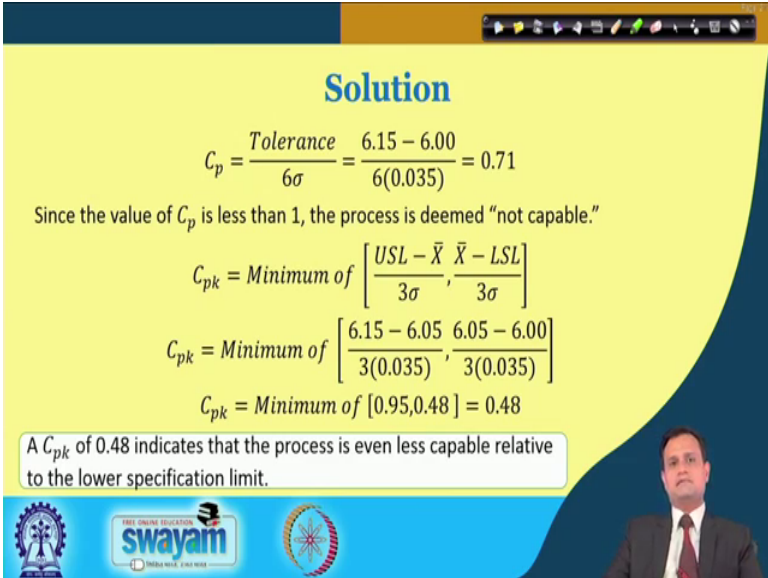
A batch process produces high lactose syrup with a specification on the dextrose equivalent (DE) of 6.00 to 6.15. The DEs are normally distributed, and a control chart shows the process is stable. The standard deviation of the process is 0.035. The DEs from a random sample of 30 batches have a sample mean of 6.05. Determine C_p and C_{pk} .

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So, let us see the example to make the idea clear ah. Suppose you have a batch process, typically produces high lactose syrup with a specification on the dextrose equivalent; that is to 6 to 6.15 and the DEs are normally distributed. So, I assume that my production is normally distributed and a control chart shows the processes stable.

So, the first requirement for going for process capability is satisfied. Their standard deviation of the process is 0.035, and the DE is from the random sample of 30 batches have a sample mean of 6.05. Now I want to comment on the capability of this process, so let me find C_p and C_{pk} . So, these two measures they do not have any conflict with each other. In fact, they provide the more information, complemented to each other and I would like to find both C_p and C_{pk} .

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Solution

$$C_p = \frac{\text{Tolerance}}{6\sigma} = \frac{6.15 - 6.00}{6(0.035)} = 0.71$$

Since the value of C_p is less than 1, the process is deemed "not capable."

$$C_{pk} = \text{Minimum of } \left[\frac{USL - \bar{X}}{3\sigma}, \frac{\bar{X} - LSL}{3\sigma} \right]$$

$$C_{pk} = \text{Minimum of } \left[\frac{6.15 - 6.05}{3(0.035)}, \frac{6.05 - 6.00}{3(0.035)} \right]$$

$$C_{pk} = \text{Minimum of } [0.95, 0.48] = 0.48$$

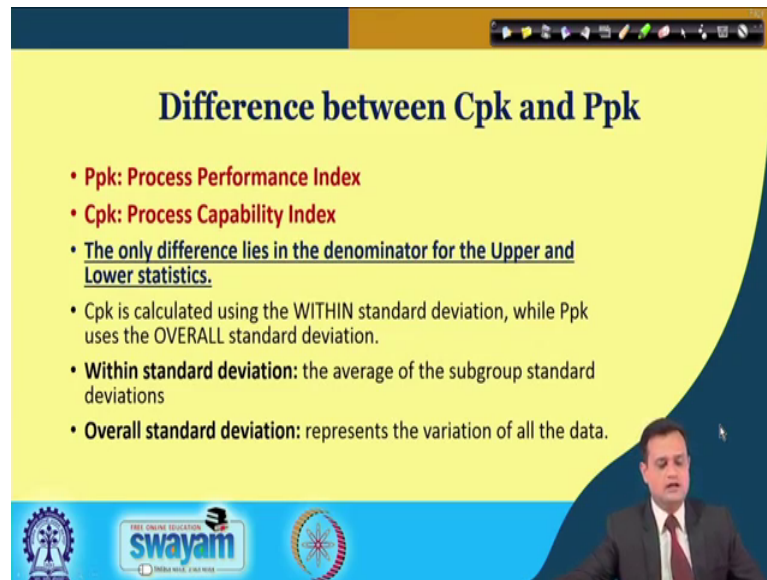
A C_{pk} of 0.48 indicates that the process is even less capable relative to the lower specification limit.

So, here my C_p is tolerance divided by 6 sigma, you say it is voice of customer divided by voice of process or sometimes you say USL minus LSL divided by 6 sigma. So, here it comes out to be 0.71. So, this is the value I am referring. This is 0.71. Now you can see that C_p is less than 1. So, when the C_p is less than 1, my say threshold limit says that if it is equal to 1 or more than 1, technically my process is capable enough, but here my process is not capable.

Now, if I investigate further and find out the C_{pk} , then I will take the minimum of these two with respect to, say upper and lower specification limit compared with the mean value and I will get 0.48. So, C_{pk} 0.48 typically indicates that process is even less



capable relative to the lower specification limit. So, number 1 conclusion; processes not capable and second process is even very much less capable compared to relative to the lower specification limit.

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Difference between Cpk and Ppk

- **Ppk: Process Performance Index**
- **Cpk: Process Capability Index**
- The only difference lies in the denominator for the Upper and Lower statistics.
- Cpk is calculated using the WITHIN standard deviation, while Ppk uses the OVERALL standard deviation.
- **Within standard deviation:** the average of the subgroup standard deviations
- **Overall standard deviation:** represents the variation of all the data.

So, you have other measures like P_p , P_{pk} , P_{pm} . So, you would be wondering what is the difference. I have already highlighted that when I say C_p , C_{pk} , C_{pm} , typically these particular measures they go by the subgroup standard deviation, within standard deviation and when I go by P_p , P_{pk} , P_{pm} , this is more pertaining to the long term process capability and it goes with, say variability of all the data or the process which is being absorbed for a long period of time and this is where C_p , C_{pk} , C_{pm} are short term measures to investigate the process and P_p , P_{pk} , P_{pm} are long term measures for investigating the capability of the process.

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Process Capability Indices

- C_p and P_p do not take into account location of the process (i.e., the process mean). They only consider process variability.
- C_{pk} , CPU, CPL, and P_{pk} , PPU, PPL incorporate both process location and process variability.
- C_p - measures process potential - what the process capability could be if there were no drifts in the process location.
- C_{pk} - measure of the process performance.
- Given a target value (ideal process mean).

C_{pm} - Taguchi capability index.

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So, you have C_p and P_p , this do not take into account the shift in mean or location of the process. You have C_{pk} , CPU, CPL, P_{pk} , PPU, PPL short term and long term discovery respectively, they do consider the center as well as location of the process. And if my process, because of say usage wear and tear of some of the components or problem in the setting has shifted to the other mean, then that would be captured by this C_{pk} P_{pk} CPU CPL measures. C_p is basically measures the process potential.

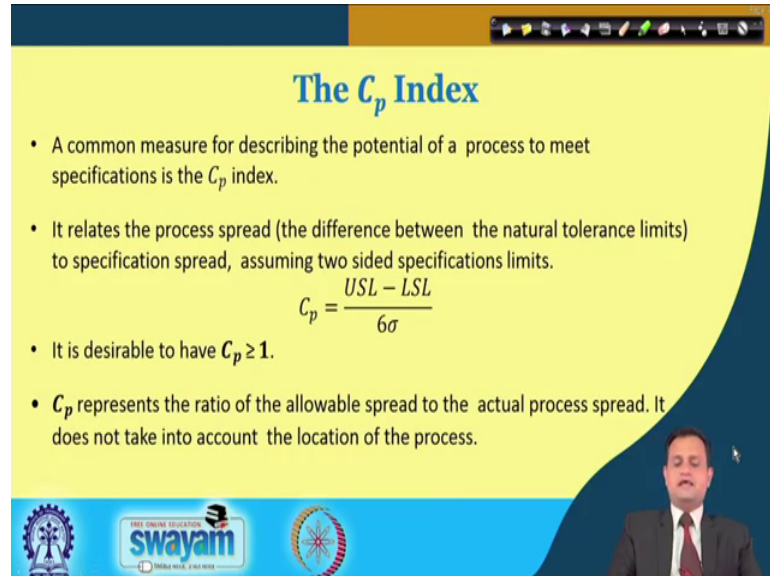
So, please try to focus on this particular word, that C_p measure the process potential. So, what the process capability could be, if there were no drips in the process location. There is no shaped disturbance in the setting in that case, what is the capability of my process, what is the potential of my process. When you say C_{pk} , typically it talks about the measures of, measure of the process performance. So, even if there is a shift, there is a change in settings, shift in the mean to what extent my process would be capable enough to produce the products which can meet the customer service specification.

When I talked about it, it is about process performance. And you can see the third one that is C_{pm} . So, this is typically called Taguchi capability index. And here instead of I would say some central value or mean value, I will talk about the target value.

So, there is a target value which I am trying to heat as much as possible and to what extent my process is capable enough to meet this target value, when my interest of

analysis is this, then I would make use of C_{pm} ; that is Taguchi capability index. So C_p is $USL - LSL$ by 6σ , desirable is greater than equal to 1.

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The C_p Index

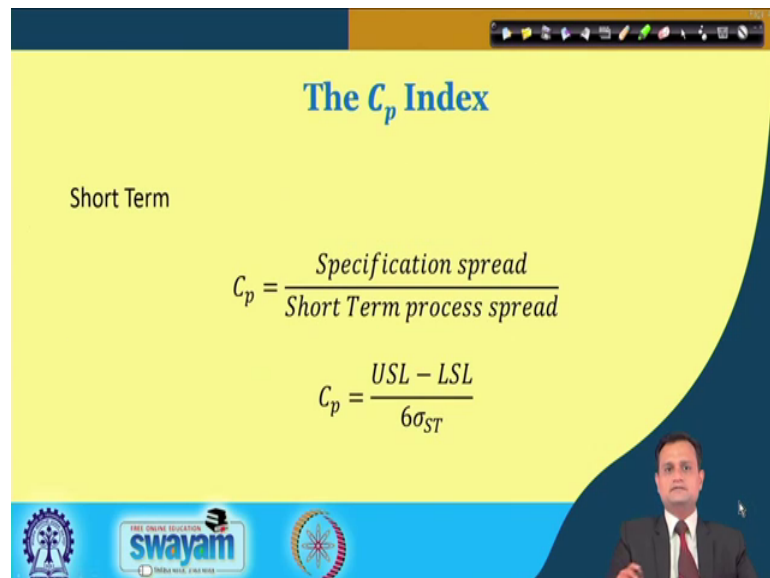
- A common measure for describing the potential of a process to meet specifications is the C_p index.
- It relates the process spread (the difference between the natural tolerance limits) to specification spread, assuming two sided specifications limits.

$$C_p = \frac{USL - LSL}{6\sigma}$$

- It is desirable to have $C_p \geq 1$.
- C_p represents the ratio of the allowable spread to the actual process spread. It does not take into account the location of the process.

The slide includes logos for IIT Bombay, Swayam, and the Ministry of Education, India, along with a video feed of a presenter in the bottom right corner.

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The C_p Index

Short Term

$$C_p = \frac{\text{Specification spread}}{\text{Short Term process spread}}$$

$$C_p = \frac{USL - LSL}{6\sigma_{ST}}$$

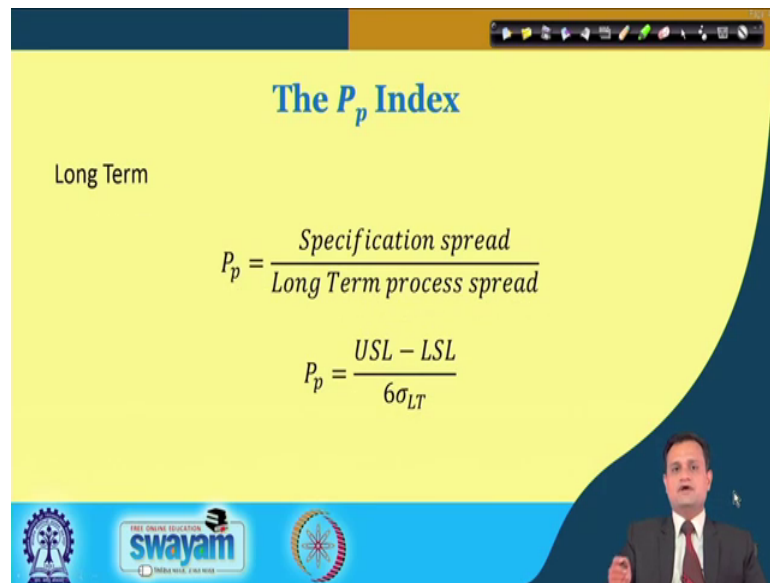
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C_p when you talk about long term, you have to consider σ . You talk about the short term, you have to consider the short term process spreads σ_{ST} .

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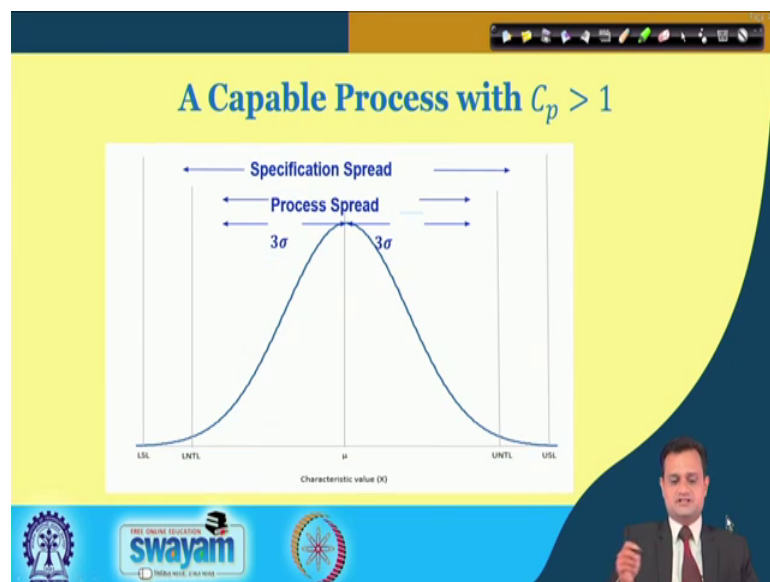
The P_p Index

Long Term

$$P_p = \frac{\text{Specification spread}}{\text{Long Term process spread}}$$
$$P_p = \frac{USL - LSL}{6\sigma_{LT}}$$


When you talk about the long term, you will consider the long term process spread 6 sigma LT; you are allowing the process, observing the process for a longer or extended period of time and then computing the standard deviation.

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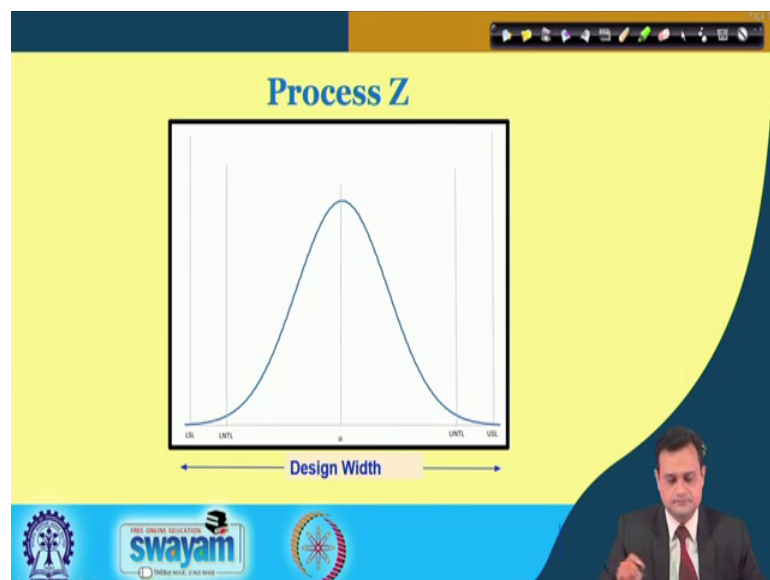


So, just see this and you will get an idea that here C_p is greater than one, and now you can relate these with the understanding we evolved in the previous class. So, that time we just commented about the health of the process capability of the process in terms of

upper and lower specification limit and tolerance limit, natural tolerance limit. Here I we are also relating that with the C_p value.

So, to remind you, tolerance limits basically decided by the process variability, capability of the process and specification limits are given by the customer. So, this is a process which is capable and we can very well see, that my upper natural tolerance limit and lower natural of tolerance limit, they are well within the upper and lower specification limit. So, my process is capable enough to meet or produce the products as per the specifications.

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So, this is my design width, I have to design my product process within this particular range, and this is typically my process Z that. I call it Z because we are referring to the normal distribution.

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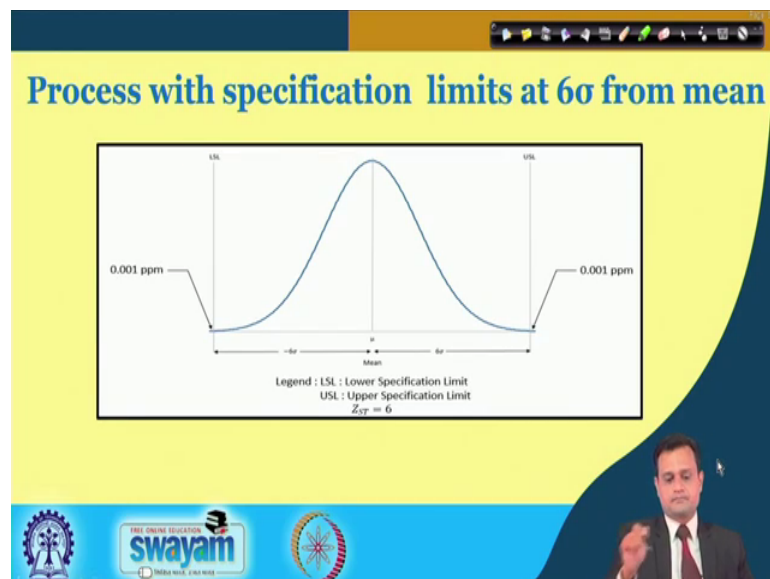
Process Z

- Process Z = Number of standard deviations in half of the design width:
$$= \frac{USL - LSL}{2\sigma}$$
- Short Term
- $Z_{ST} = \frac{USL - LSL}{2\sigma_{ST}}$
- $C_p = \frac{Z_{ST}}{3}$
- In a Six Sigma Process, $Z_{ST} = 6$



So, you can find process Z as USL minus LSL divided by 2 sigma. So, it is the number of standard deviation in half of the design width, and you can find out ZST, it is short term. So, USL minus LSL divided by 2 sigma ST short term and Cp is equal to Z ST divided by 3. It is a standard expression, because half of the width I am considering. So, in a 6 sigma process; obviously, my ZT ST would be 6.

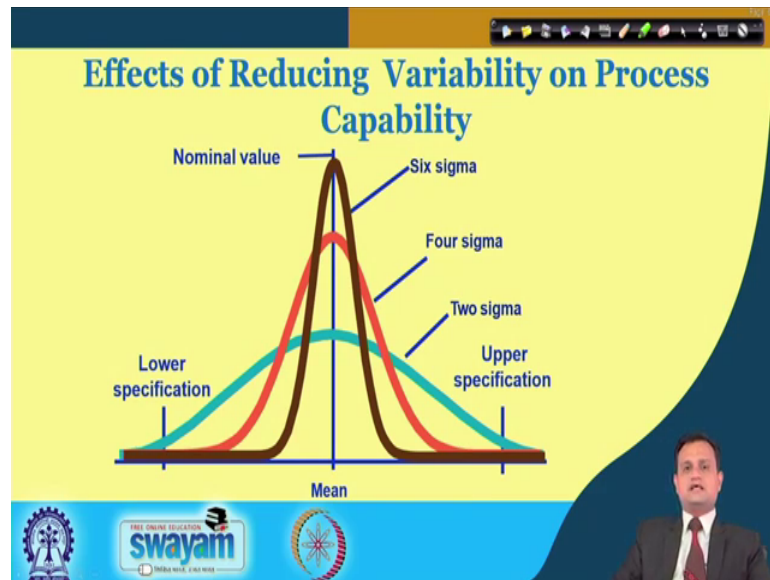
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So, process with specification as a limit at 6 sigma, you can see here that my particular process has plus 6 sigma minus 6 sigma and the number of components I would be

producing, which is defective is only 0.001 P pm on either side. And again I would say that 6 sigma means not larger, 6 sigma means a process which has extremely less variability, and hence it can give you the 6 sigma outcome; that is 3.4 say defectives per million, or here you can say 0.001 parts per million.

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So, just see to remind you and as I mentioned that 6 sigma. This is my nominal value, this is my 6 sigma and this is my 6 sigma curve. So, I have extremely less variability. This is my upper specification, this is my lower specification, and you can see that this process, because it is part of, say well within the upper and lower, the probability that I would produce the defect you part is very very less. As you move to 4 sigma, then your quality level goes down 2 sigma, it further goes down and so on.

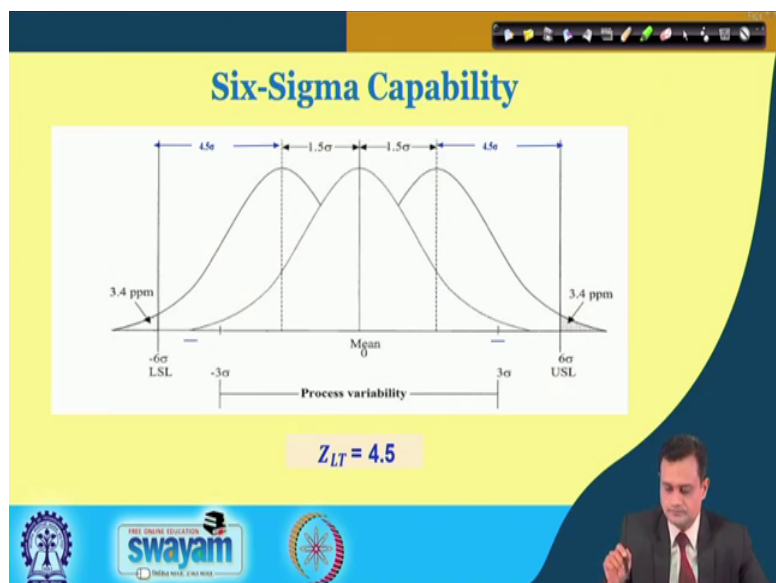
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What it means to operate at 6-sigma

Range	Population in range	Expected frequency outside range	Approx. frequency for daily event
$\mu \pm 1\sigma$	0.682689492137	1 in 3	Twice a week
$\mu \pm 2\sigma$	0.954499736104	1 in 22	Every three weeks
$\mu \pm 3\sigma$	0.997300203937	1 in 370	Yearly
$\mu \pm 4\sigma$	0.999936657516	1 in 15,787	Every 43 years (twice in a lifetime)
$\mu \pm 5\sigma$	0.999999426697	1 in 1,744,278	Every 5,000 years (once in history)
$\mu \pm 6\sigma$	0.999999998027	1 in 506,842,372	Every 1.5 million years

So, this is what we have seen, I would like to remind you again, that if my process is controlled within plus or minus 1 sigma, then expected frequency outside range 1 in 3, very very high, 1 in 3. If you see 2 sigma 1 in 22, if you see 3 sigma 1 in 370 and if you see 4 sigma 1 in 15787; if you see 5 sigma 1 in, say 1744278 and if you see 6 sigma, then the probability of producing defective component is very very less.

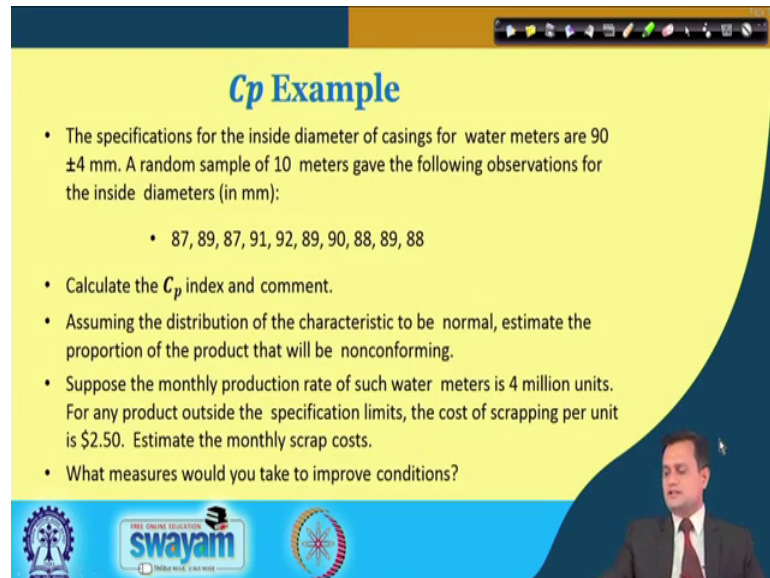
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You can once again see that when I talk about the process capability, this is my process which is centered at mean, this is my process which is centered at mean and you can see

that there could be a shift of 1.5 sigma, this side there could be a shift of 1.5 sigma this side, and even if there is a one 5 sigma shift I would only be producing 3.4 P pm on the either side. So, this is where I am trying to achieve the reduction in the process variability, but in order to do so, first I must have an idea what is the capability of my process and what are the measures necessary to improve the capability.

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C_p Example

- The specifications for the inside diameter of casings for water meters are 90 ± 4 mm. A random sample of 10 meters gave the following observations for the inside diameters (in mm):
 - 87, 89, 87, 91, 92, 89, 90, 88, 89, 88
- Calculate the C_p index and comment.
- Assuming the distribution of the characteristic to be normal, estimate the proportion of the product that will be nonconforming.
- Suppose the monthly production rate of such water meters is 4 million units. For any product outside the specification limits, the cost of scrapping per unit is \$2.50. Estimate the monthly scrap costs.
- What measures would you take to improve conditions?

The slide also features logos for 'swayam' and 'All India Open University' at the bottom, and a small video inset of a man in a suit in the bottom right corner.

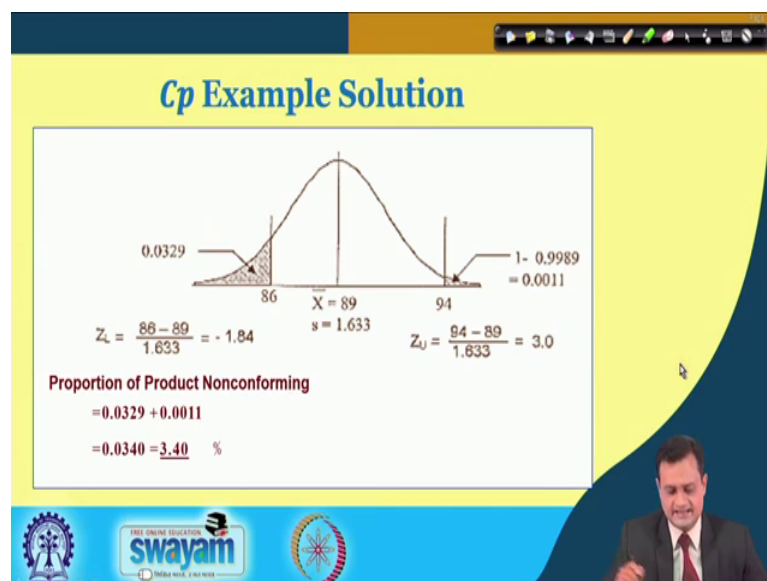
So, there is an example, a couple of examples I am including to help you to appreciate the concept in detail. So, let us say specification of the inside diameter of casing for water meter are 90 plus or minus 4 mm in a random sample of 10 meters gave the following observations. So, there is 87 89. So, diameter is measured in mm 87, 91 and so on. We are expected to compute the C_p index and comment, assuming the distribution is normal, and suppose the monthly production rate of such water meter is 4 million units. For any product outside the specification limit, the cost of scrapping this particular meter water meter, is let us dollar 2.5, estimate the monthly scrap costs and what measures would you like to recommend to improve the condition.

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		Cp Example
X	X ²	
87	7569	<ul style="list-style-type: none"> Sample Mean (\bar{X}): $\bar{X} = \frac{890}{10} = 89$ Sample Variance (s^2): $s^2 = \frac{79234 - (890)^2/10}{9}$ $s^2 = \frac{79234 - 79210}{9} = 2.667$ Sample standard Deviation (s): $s = \sqrt{2.667} = 1.663$ $C_p = \frac{94-86}{6(1.663)} = \frac{8}{9.798} = 0.816$ Not Desirable
89	7921	
87	7569	
91	8281	
92	8464	
89	7921	
90	8100	
88	7744	
89	7921	
88	7744	
890	79234	

So, this is just the computation which can be easily followed. So, I have the X bar, I have taken the X square, I can find out the X bar. So, this is the total 890 divided by 10 is 89. I can find the sample variants, so I have 79234 then this is the value 890 whole square divided by 10 and divided by 9. So, my S square is 2.667. I can find the standard deviation as the square root and C p comes out to be 0.816. So, I would; obviously, say C p is less than one, so it is not desirable and my process is actually not capable.

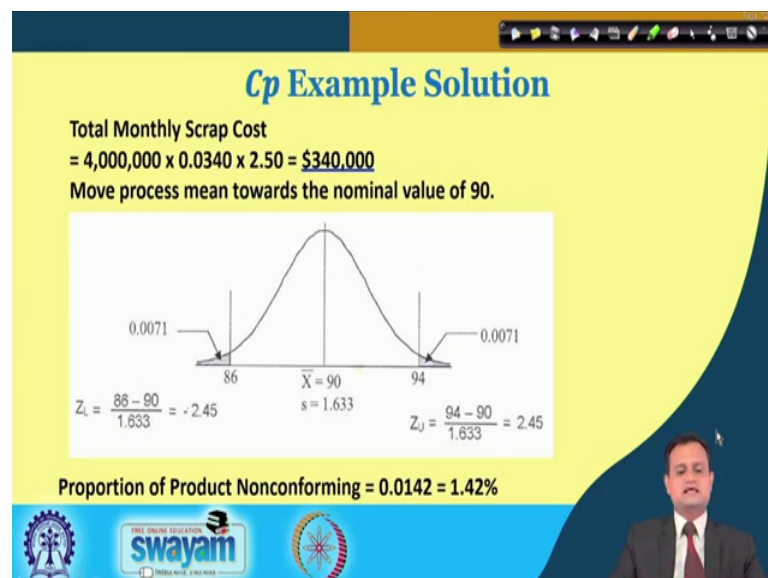
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So, now, the issue is that, if it is not capable then how much amount of scrap I would be producing, non-conforming items I would be producing. So, here you can see that you have 86 and 94, because 90 plus or minus. If you just see the specifications then it is, this is 90 plus or minus 4 mm.

So, 86 to 94 is the specification range. And if I consider this, then this side is the region which shows me that 0.0329 will go out of the specification limit and this side says that 0.0011 will go out of the specification limit. So, typically I would have 0.0329 plus 0.0011 equal to 3, 0.0340 34 percent, as the defective product reject product.

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So, if I just convert this into the number, then you can say that 400 into, 04000; that is 4,000,000 into 0.0340 into 2.5 is the cost dollar. So, I would be almost losing dollar 340,000. So, it is not a small amount. So, proportion of product non-conforming, if you just move the process mean to, towards the value of 190, then again you will have some non-conforming, but this goes down and the proportion of product non-conforming is 0.0142. So, typically it is 0.0071 plus 0.0071. And if you do a little bit resetting, you will get 1.42 percent.

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C_p Example Solution

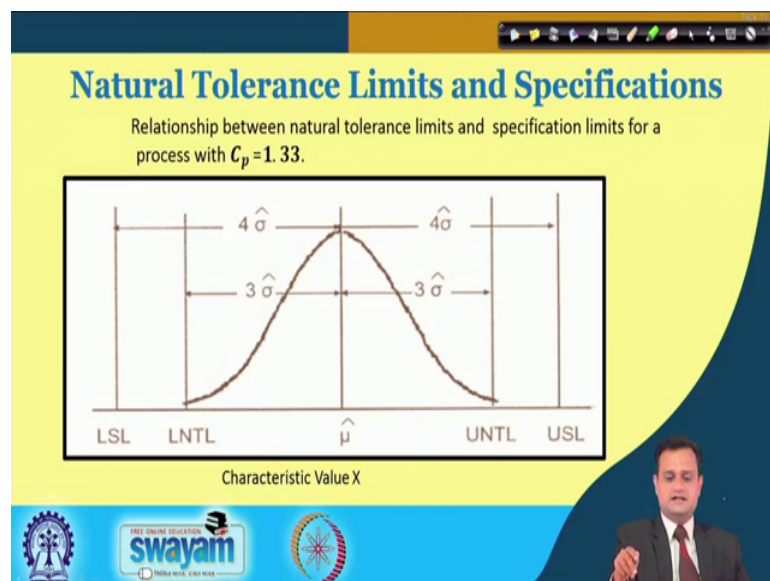
Total Monthly Scrap Cost
 $= 4,000,000 \times 0.0142 \times 2.50$
 $= \$142,000$

Monthly Savings = \$198,000



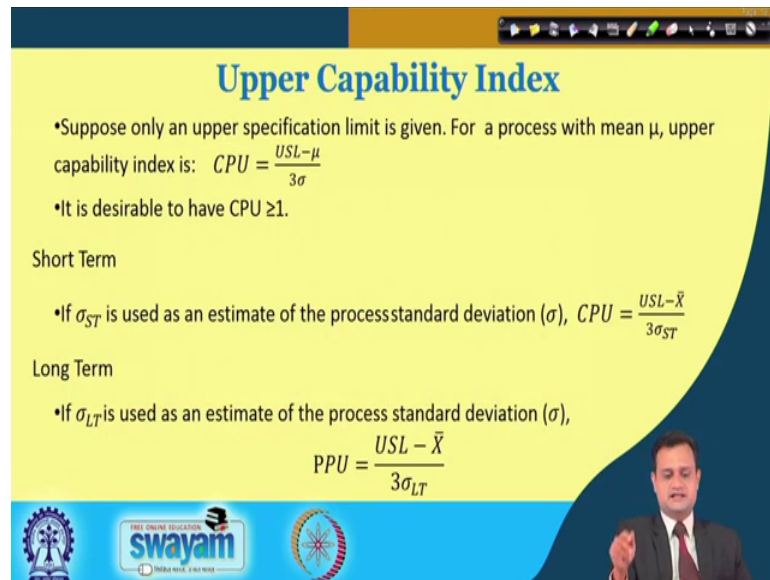
So if I consider this 1.42 percent then total monthly scrap will come down and it would be 142,000. So, compared to the previous one, compared to the previous one which was 340,000 Dollar, it has come down to the 142,000, so saving would be almost 198,000. So, just see the importance of, say knowing in the process capability and doing a little bit setting to the my process, and setting it at the right mean can really help me to reduce my say scrap and hence that could be huge saving in my, say rejecting rejected products scrap products and this is where it is beneficial.

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So, you have natural tolerance limit and specification limit just again to remind you, that natural tolerance limits are pertaining to the process capability and variability of the process where in your specification limit are basically given by the customer and we try to produce a product within this particular specification limits.

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Upper Capability Index

- Suppose only an upper specification limit is given. For a process with mean μ , upper capability index is: $CPU = \frac{USL - \mu}{3\sigma}$
- It is desirable to have $CPU \geq 1$.

Short Term

- If σ_{ST} is used as an estimate of the process standard deviation (σ), $CPU = \frac{USL - \bar{X}}{3\sigma_{ST}}$

Long Term

- If σ_{LT} is used as an estimate of the process standard deviation (σ),

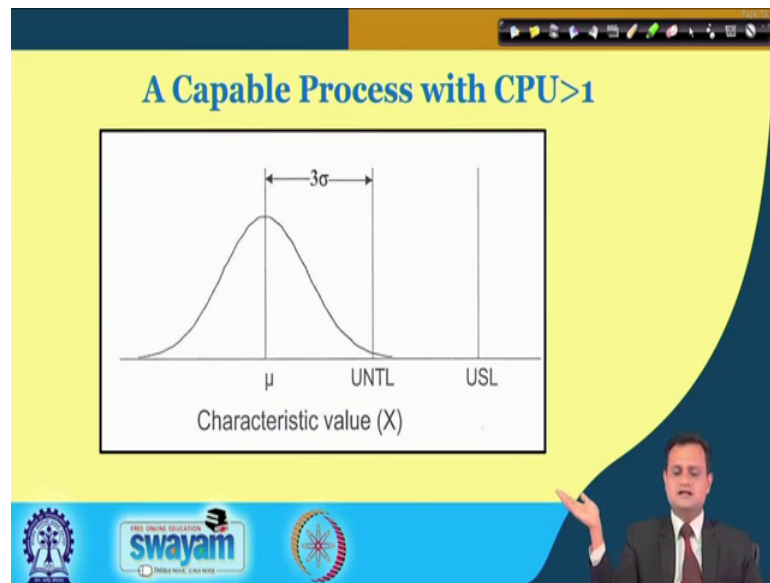
$$PPU = \frac{USL - \bar{X}}{3\sigma_{LT}}$$

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So, you have a couple of other indices derived indices; like upper capability index. So, CPU is basically USL, upper specification limit, I just consider half minus mu divided by 3 sigma, because it is half and the desirable is that CPU should be greater than or equal to 1.

Similar way short term I can find my CPU just by dividing it by sigma ST; that is short term and I can find long term PPU dividing it by sigma LT.

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Lower Capability Index

- Suppose only a lower specification limit is given. For a process with mean μ , lower capability index is: $CPL = \frac{\mu - LSL}{3\sigma}$
- It is desirable to have $CPL \geq 1$.

Short Term

- If σ_{ST} is used as an estimate of the process standard deviation (σ), $CPL = \frac{\bar{X} - LSL}{3\sigma_{ST}}$

Long Term

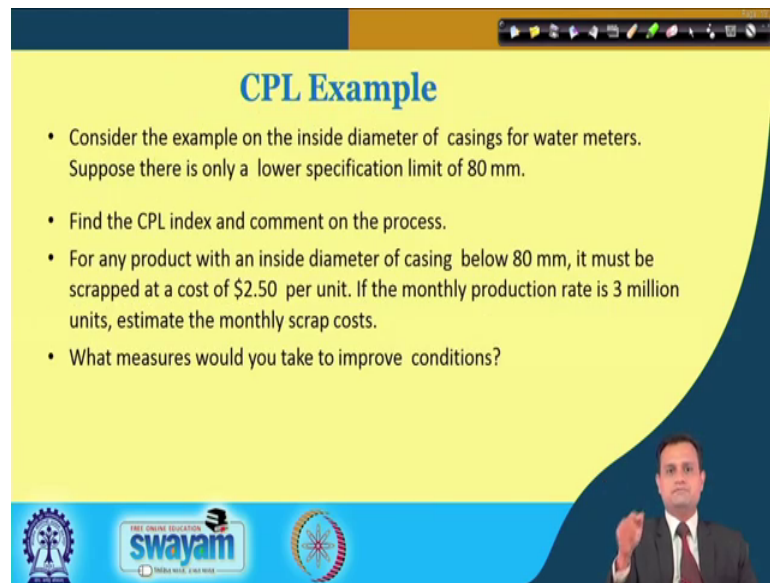
- If σ_{LT} is used as an estimate of the process standard deviation (σ),

$$PPL = \frac{\bar{X} - LSL}{3\sigma_{LT}}$$

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So, this is the process capable with respect to upper specification, CPU is greater than 1. And same way you define the lower, and you lower is for short term CPL, for long term it would be PPL.

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A presentation slide titled "CPL Example" with a yellow background and a blue header. It contains four bullet points. At the bottom, there is a blue banner with logos for "swayam" and "INDIA WISE, CHINA WISE". A small video inset of a man in a suit is in the bottom right corner.

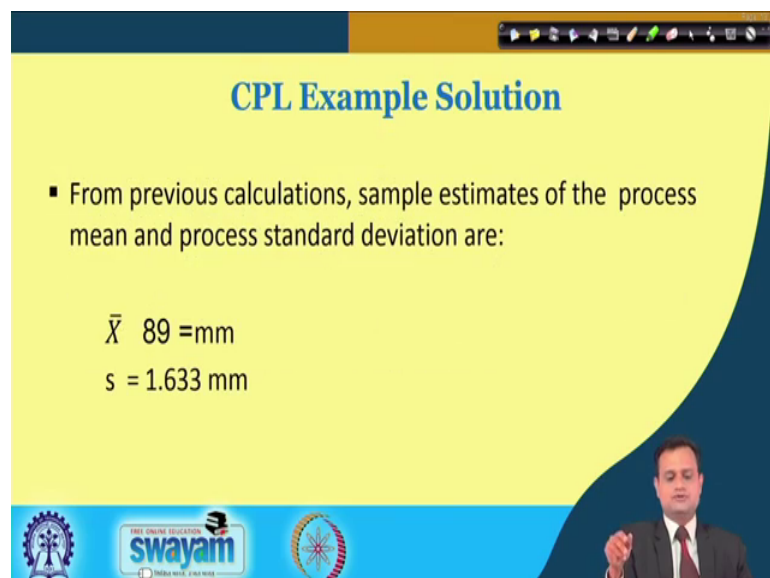
CPL Example

- Consider the example on the inside diameter of casings for water meters. Suppose there is only a lower specification limit of 80 mm.
- Find the CPL index and comment on the process.
- For any product with an inside diameter of casing below 80 mm, it must be scrapped at a cost of \$2.50 per unit. If the monthly production rate is 3 million units, estimate the monthly scrap costs.
- What measures would you take to improve conditions?

So, we can just see the example of a CPL; that is with respect to one specification limit, lower specification. So, consider the example on the inside diameter of casing of water meter, the same example. And suppose there is only one lower specification limit of 80 mm.

So, now find the CPL index and comment on the process. So, for any product within inside diameter of casing below 80 mm, it must be scrapped at a cost dollar 2.5 as we had considered previously and what measures would you like to consider.

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A presentation slide titled "CPL Example Solution" with a yellow background and a blue header. It contains a bullet point and two equations. At the bottom, there is a blue banner with logos for "swayam" and "INDIA WISE, CHINA WISE". A small video inset of a man in a suit is in the bottom right corner.

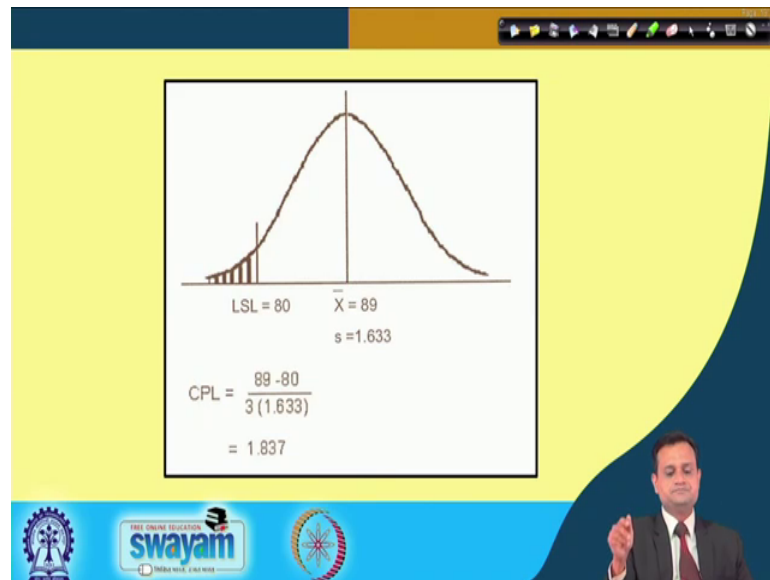
CPL Example Solution

- From previous calculations, sample estimates of the process mean and process standard deviation are:

$$\bar{X} = 89 \text{ mm}$$
$$s = 1.633 \text{ mm}$$

So, \bar{X} is 89 mm and s is 1.633. From the previous calculation I can consider. And just by putting this it is represented on a normal distribution, and I can put these values in the CPL expression, so it comes out to be 1.837.

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CPL Example Solution

- ✓ Since $CPL > 1$, a very small proportion is below the LSL.

$$Z_L = \frac{80 - 89}{1.663} = -5.51$$

- ✓ Proportion below LSL is less than $(1 - 0.999996289)$

$$✓ < 0.00000372$$

- ✓ Monthly scrap costs $< 3,000,000 * 0.00000372 * 2.50$

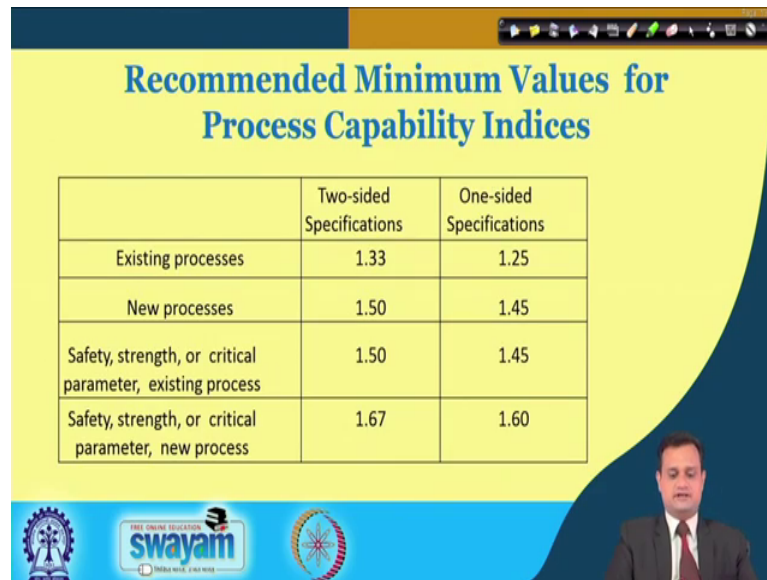
$$✓ < \$2.79$$

- ✓ Move process mean further up, if feasible.

So, you have CPL greater than one. So, a very small proportion is below LSL lower limit, lower specification limit, and if you find a Z_L . So, this comes out to be minus 5.51. So, proportion below your LSL is less than 0.00000372 and your monthly scrap would just be less than 2.79 dollar. So, I can say that more process mean further up if

feasible. there is absolutely no problem so far the lower part is concerned, you have the flexibility to move the process up to adjust the process, in order to improve upon your scrap production and the cost saving.

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The slide features a yellow background with a blue header and footer. The title 'Recommended Minimum Values for Process Capability Indices' is centered in blue. Below it is a table with three columns: a blank header, 'Two-sided Specifications', and 'One-sided Specifications'. The table contains four rows of data. In the bottom right corner, there is a small video feed of a man in a suit and tie.


	Two-sided Specifications	One-sided Specifications
Existing processes	1.33	1.25
New processes	1.50	1.45
Safety, strength, or critical parameter, existing process	1.50	1.45
Safety, strength, or critical parameter, new process	1.67	1.60

So, just see that recommended minimum value for process capability indices, when you consider two specification when you consider one specification. If you are talking about the existing process where the knowledge is available, about the process variability 1.33 for two sided and for one sided 1.25 is ok. When you say new process, where the process knowledge is less process is not stable, you may go for 1.5, 1.45. And similar way you can see the safety strength or critical parameter existing process 1.5 and when there is a concern of safety strength critical for new process, we are just putting the criteria a little bit tougher 1.67 and 1.6. So, these are the recommended, I must have greater than this values in order to accept my process.

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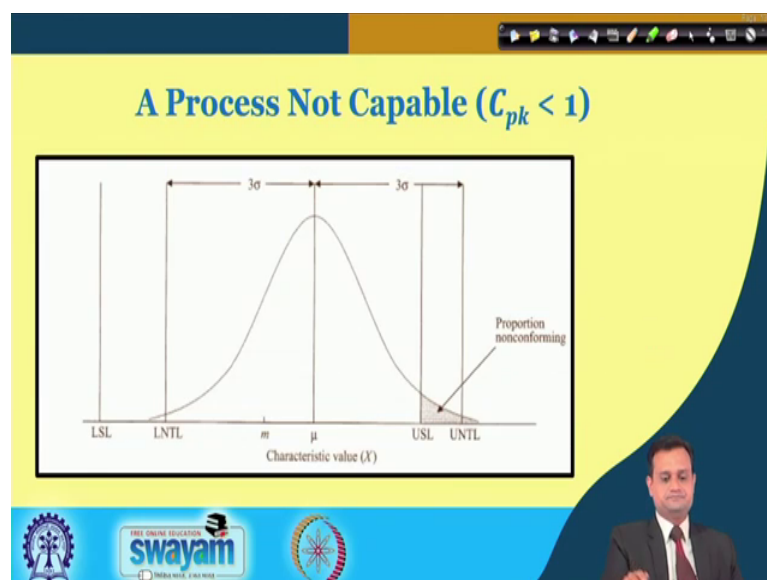
The C_{pk} Index

- $C_{pk} = \min \left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right] = \min\{CPU, CPL\}$
- C_{pk} represents the scaled distance, relative to half the process spread or 3 standard deviations, between process mean and closest specification limit.
- Desirable values are $C_{pk} \geq 1$.
- C_p index represents the process potential and the C_{pk} value represents the actual capability of the process with the existing parameter values - it measures process performance.
- If a process is centered, $C_{pk} = C_p$



So, you have C_{pk} index, already I express that it takes into account the mean value, your location part and tries to say that what is the performance of the process even if there is a shift in my process and also it considers the process variability through sigma. So, if a process is centered perfectly at the mean value, your C_p is equal to C_{pk} . So, a process is not capable.

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Now I am talking about C_{pk} which is less than 1, and you can see that you are producing proportion non-conforming on this side, and this is where your C_{pk} is less than 1.

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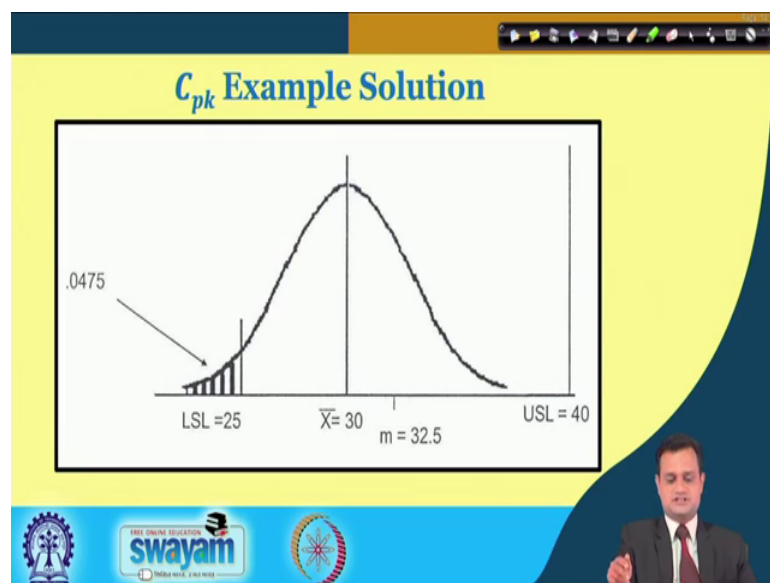
C_{pk} Example

- In an electrical circuit, the capacitance of the component should be between 25 and 40 picofarads (pF). A sample of 25 components yields a mean of 30 pF and a standard deviation of 3 pF.
- Calculate the process capability index C_{pk} , and comment on the process performance.
- If the process is not capable, what proportion of the product is nonconforming, assuming a normal distribution of the characteristic?



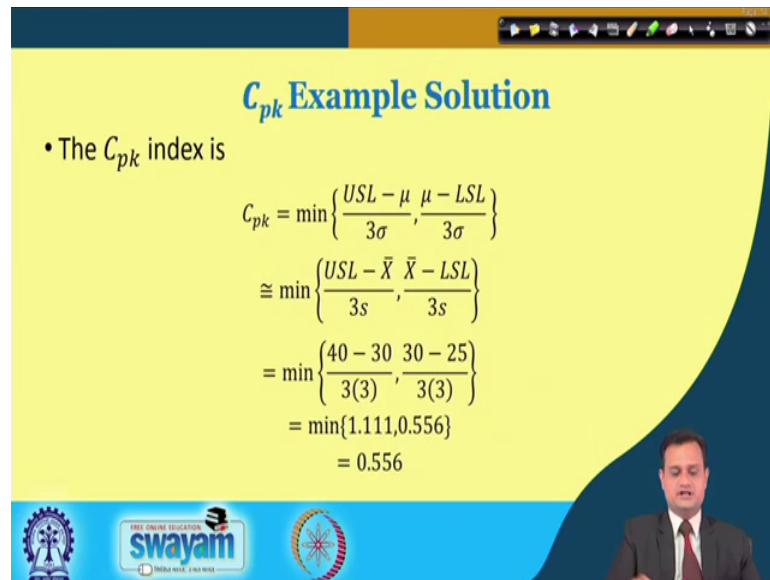
We can see the C_{pk} example that you have an electric circuits. The capacitance of the components should be between 25 to 40 and a sample of 25 component yields a mean of 30 pF, standard deviation is 3 pF. Try to find out the C_{pk} and comment on the process.

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So, this C_{pk} example I can just figure out through calculation; that LSL is equal to 25 USL is equal to 40, X bar is equal to 30. My m comes out to be 32.5, so 0.0475 is the rejection which I can show in the shaded region. So, C_{pk} is my minimum of these two and 0.556.

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C_{pk} Example Solution

- The C_{pk} index is

$$C_{pk} = \min \left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\}$$

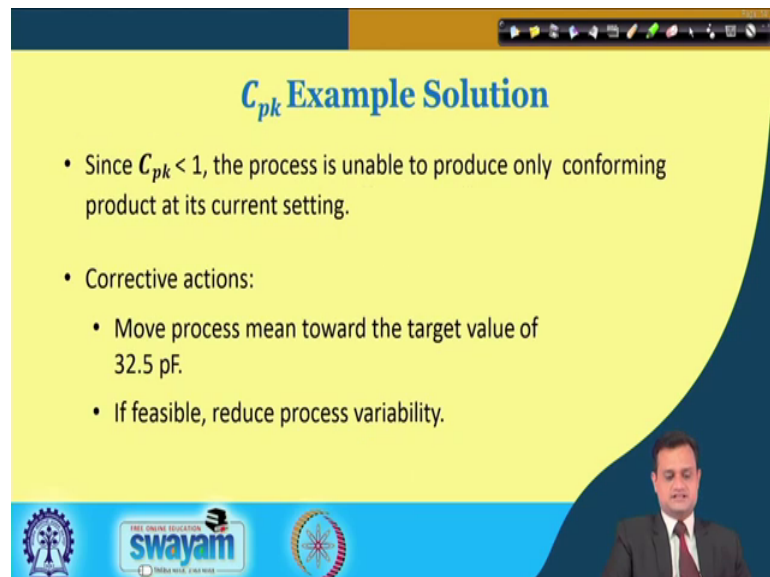
$$\approx \min \left\{ \frac{USL - \bar{X}}{3s}, \frac{\bar{X} - LSL}{3s} \right\}$$

$$= \min \left\{ \frac{40 - 30}{3(3)}, \frac{30 - 25}{3(3)} \right\}$$

$$= \min \{1.111, 0.556\}$$

$$= 0.556$$

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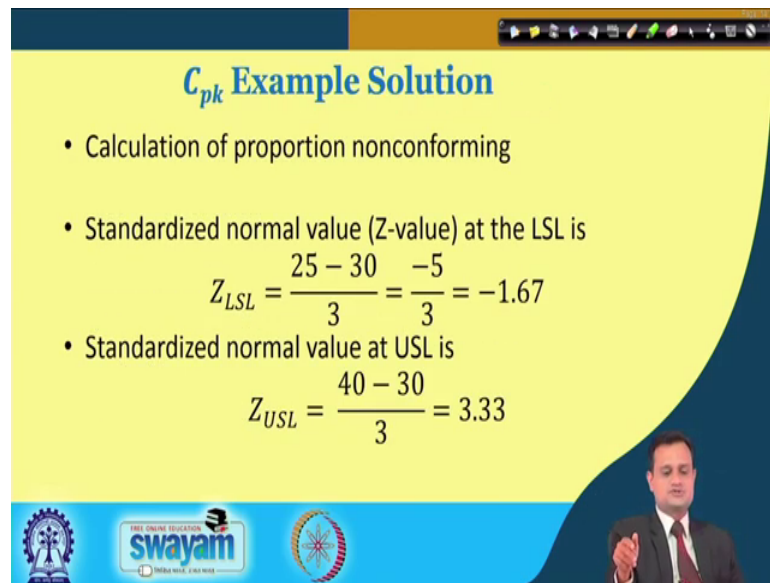


C_{pk} Example Solution

- Since $C_{pk} < 1$, the process is unable to produce only conforming product at its current setting.
- Corrective actions:
 - Move process mean toward the target value of 32.5 pF.
 - If feasible, reduce process variability.

So, C_{pk} is less than one, the process is unable to produce any conforming product and typically the corrective action is, move process mean towards target value; that is 32.5 and if feasible try to also reduce the process variability.

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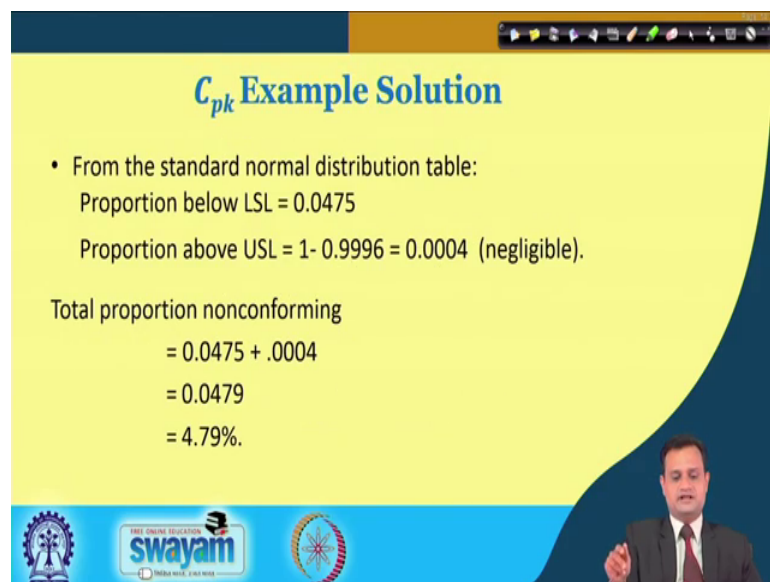
C_{pk} Example Solution

- Calculation of proportion nonconforming
- Standardized normal value (Z-value) at the LSL is
$$Z_{LSL} = \frac{25 - 30}{3} = \frac{-5}{3} = -1.67$$
- Standardized normal value at USL is
$$Z_{USL} = \frac{40 - 30}{3} = 3.33$$

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. So, you can further extend the calculation, to find the proportion non conforming; so, Z LSL 25 minus 30 divided by 3. This will give you minus 5 divided by 3 minus 1.67. Similar way you can find the Z USL 40 minus 30 by 3, this will give you 3.33.

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C_{pk} Example Solution

- From the standard normal distribution table:
Proportion below LSL = 0.0475
Proportion above USL = 1 - 0.9996 = 0.0004 (negligible).

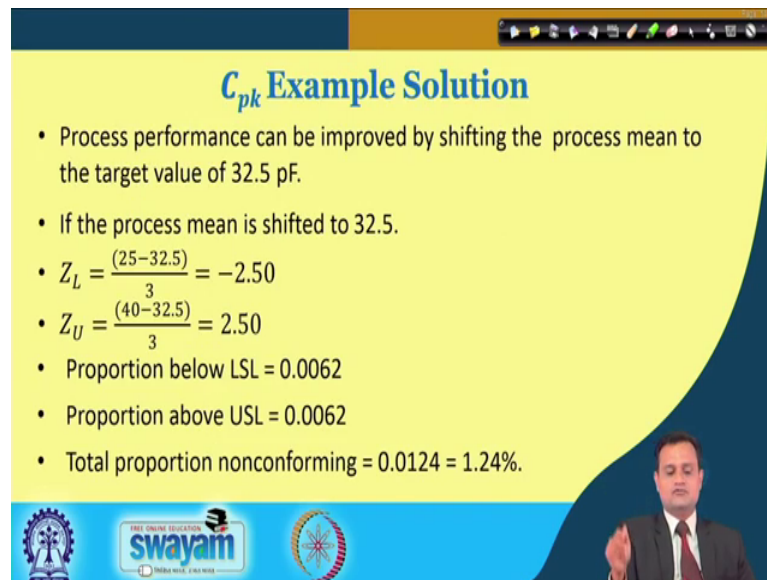
Total proportion nonconforming

$$\begin{aligned} &= 0.0475 + .0004 \\ &= 0.0479 \\ &= 4.79\%. \end{aligned}$$

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So, USL is the proportion below your USL is comes out to be 0.0475, when you use this Z table and refer the normal table, and USL is 0.0004 which is quite negligible. So, total proportion non-conforming on both the side would be 4.79 percent.

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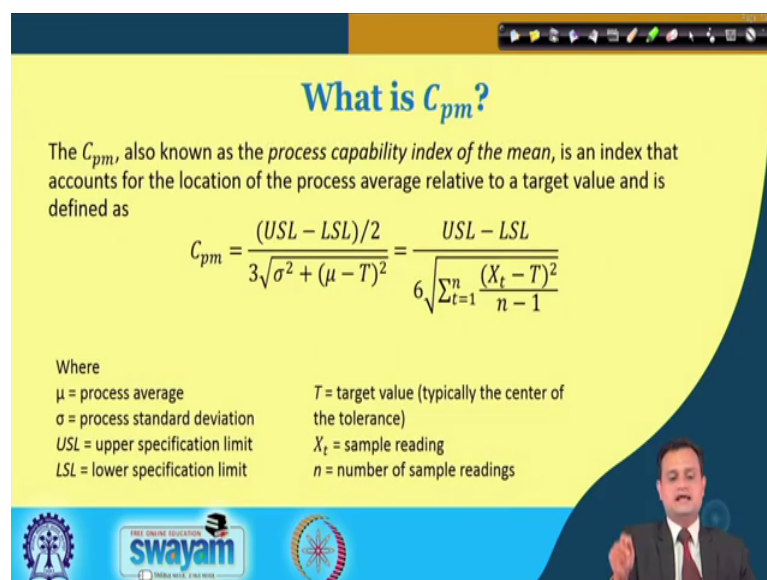
C_{pk} Example Solution

- Process performance can be improved by shifting the process mean to the target value of 32.5 pF.
- If the process mean is shifted to 32.5.
- $Z_L = \frac{(25-32.5)}{3} = -2.50$
- $Z_U = \frac{(40-32.5)}{3} = 2.50$
- Proportion below LSL = 0.0062
- Proportion above USL = 0.0062
- Total proportion nonconforming = 0.0124 = 1.24%.

Logos: IIT Bombay, Swayam, IIT Madras

Now, process performance you can improve, by shifting the process as I mentioned and suppose I shift the process, let us into the target value 32.5 pF. So, in this case you once again calculate the Z_L and Z_U , you will get the proportion below LSL is 0.0062, above USL is 0.0062. So, proportion nonconforming would be only 1.24 percent and this would really improve your situation.

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What is C_{pm} ?

The C_{pm} , also known as the *process capability index of the mean*, is an index that accounts for the location of the process average relative to a target value and is defined as

$$C_{pm} = \frac{(USL - LSL)/2}{3\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{USL - LSL}{6\sqrt{\sum_{t=1}^n \frac{(X_t - T)^2}{n-1}}}$$

Where

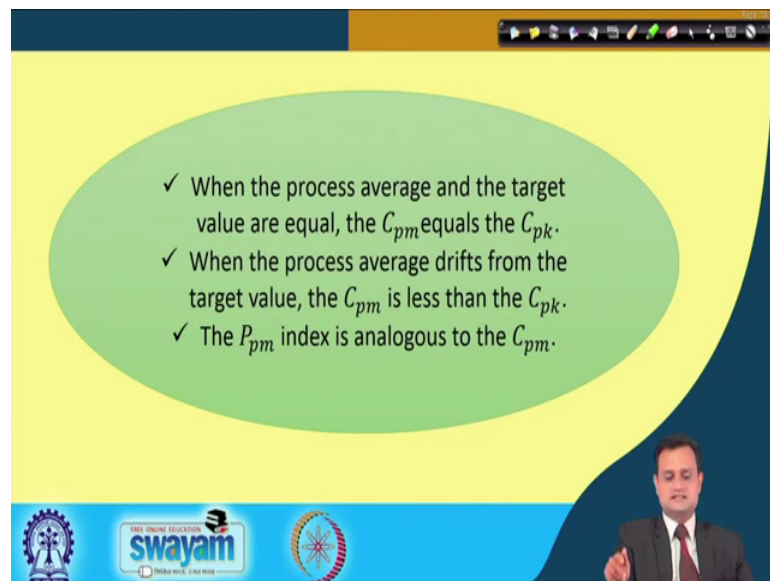
μ = process average	T = target value (typically the center of the tolerance)
σ = process standard deviation	X_t = sample reading
USL = upper specification limit	n = number of sample readings
LSL = lower specification limit	

Logos: IIT Bombay, Swayam, IIT Madras

Now, you have another measure, this is C_{pm} . So, typically if this measure is into say process capability index of the mean and that accounts for the process location target

value and it is also called the Taguchi process capability index. So, I am just giving you the expression for your reference and you can plug in the values for a particular case we just did for one example; $\frac{USL - LSL}{2.3 \sqrt{\sigma^2 + \mu^2 - T^2}}$, T is the target value square and this will give you another measure that is C_{pm} .

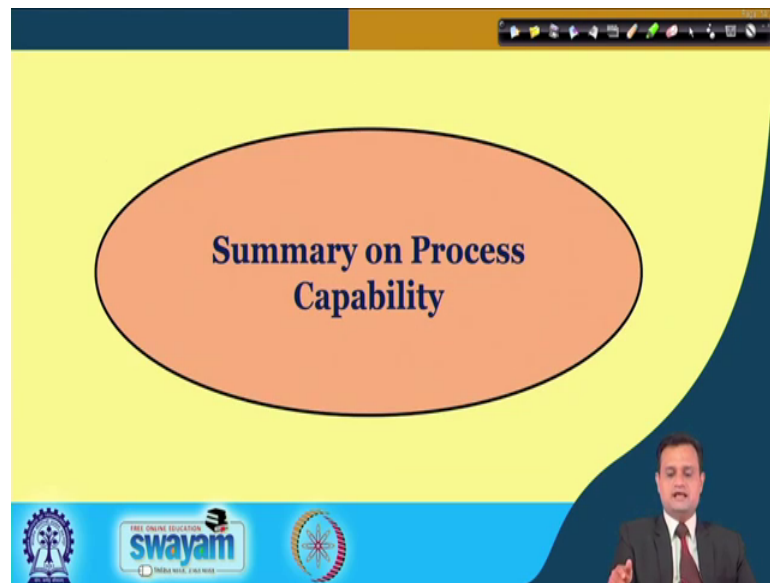
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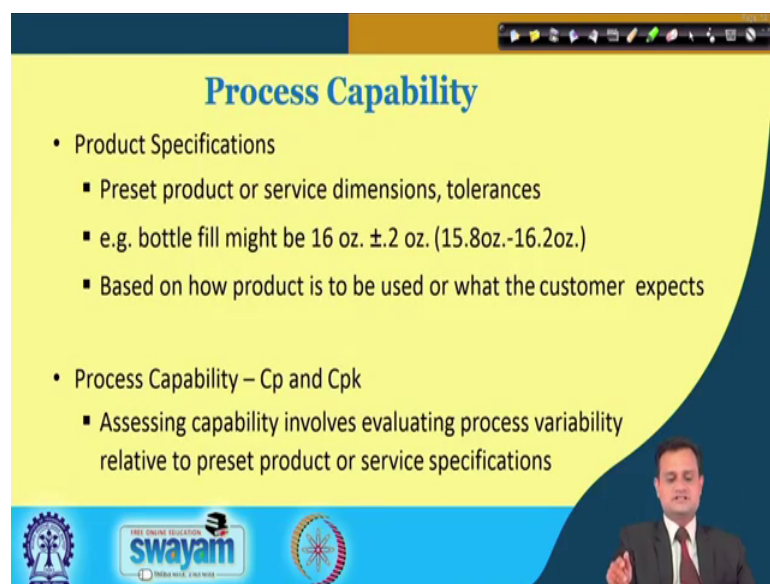
- ✓ When the process average and the target value are equal, the C_{pm} equals the C_{pk} .
- ✓ When the process average drifts from the target value, the C_{pm} is less than the C_{pk} .
- ✓ The P_{pm} index is analogous to the C_{pm} .

So, in summary we can say that when process average and target value are equal, C_{pm} equal C_{pk} . When the process average drips from the target value, pm is less than C_{pk} and P_{pm} index is analogous to C_{pm} , only long term and short term difference is there

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So, my summary on process capability is that, preset product or service dimensions tolerance; for example, bottle fill might be 16 ounce plus or minus 0.2, based on how product is to be used, based on the customer expectation, capture this data and find the C_p and C_{pk} .


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Process Capability

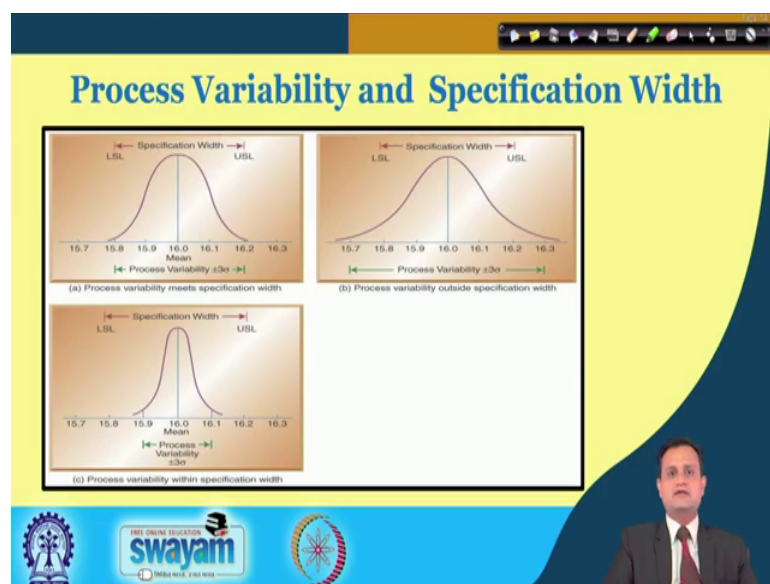
- **Process Capability – C_p and C_{pk}**
 - Assessing capability involves evaluating process variability relative to preset product or service specifications

$$C_p = \frac{\text{Specification width}}{\text{process width}} = \frac{USL - LSL}{6\sigma}$$

- C_p assumes that the process is centered in the specification range
- C_{pk} helps to address a possible lack of centering of the process

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$


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C_p is the total span USL minus LSL , C_{pk} considers the mean value, and you can further see the three situations, three cases; a, your process variability meets the specifications and b, processed variability outside the specification and c the variability is extremely less process variability within the specification limit.

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Process Variability and Specification Width

- Three possible ranges for C_p
 - $C_p = 1$, as in Fig. (a), process variability just meets specifications
 - $C_p \leq 1$, as in Fig. (b), process not capable of producing within specifications
 - $C_p \geq 1$, as in Fig. (c), process is capable of producing within specifications
- One shortcoming, C_p assumes that the process is centered on the specification range
- $C_p = C_{pk}$ when process is centered




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C_p and C_{pk} for Six Sigma

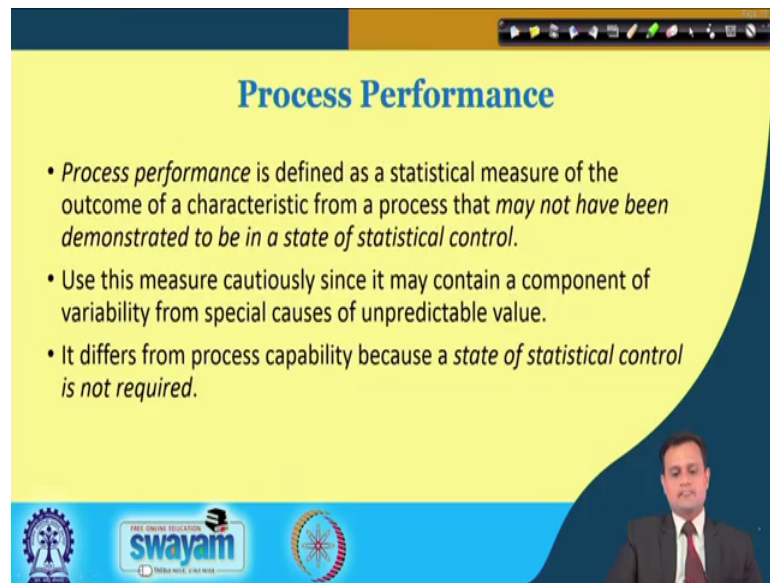
$C_{pk} = 1$	Process probably needs attention
$C_{pk} = 1.33$	process is desirable
$C_{pk} = 1.5$	process is excellent
$C_{pk} = 2$	Six sigma process – the process is outstanding



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So, we have process variability and specification limit and we try to capture the health of the process. So, C_p and C_{pk} for 6 sigma, C_{pk} should be targeted to one, it needs attention and $C_{pk} 1.33$, it is a desirable; 1.5 excellent, 2 is basically 6 sigma process. So, we are targeting $C_{pk} 2$ for 6 sigma 1 or less than 1 not capable, 1 demands immediate attention.

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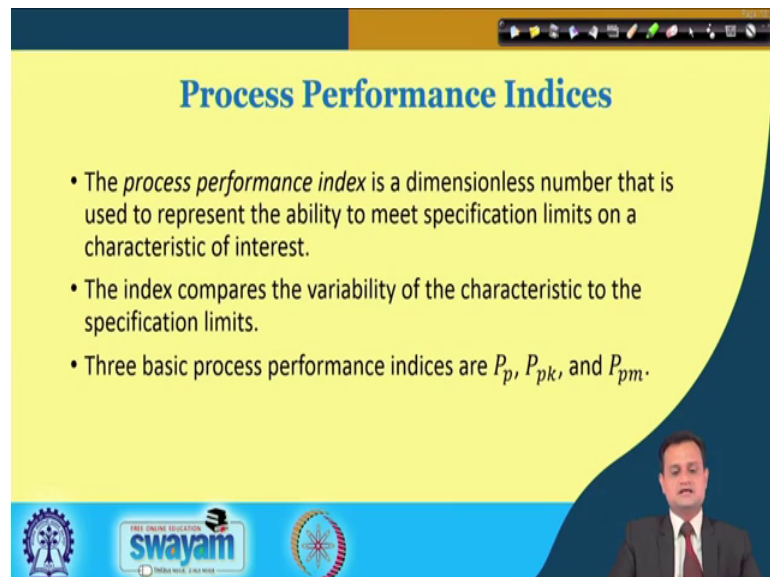
Process Performance

- *Process performance* is defined as a statistical measure of the outcome of a characteristic from a process that *may not have been demonstrated to be in a state of statistical control*.
- Use this measure cautiously since it may contain a component of variability from special causes of unpredictable value.
- It differs from process capability because a *state of statistical control is not required*.

Logos: IIT Bombay, swayam, and a circular emblem.

Now, to conclude with we have process performance and typically you try to capture the process performance of the process which is under statistical control for a long period of time, not by focusing on the sub group, as we did for C_p , C_{pk} and other measures. So, P_p , P_{pk} , P_{pm} are the measures which basically talks about process performance indices.

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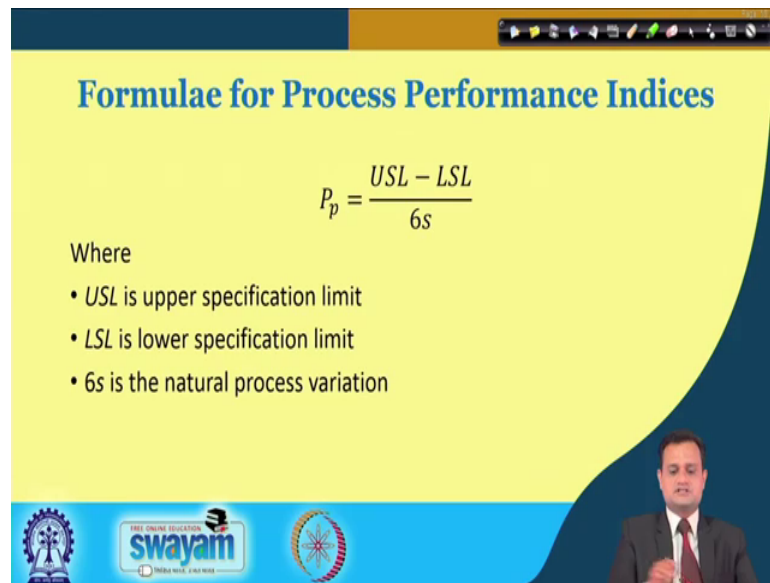


Process Performance Indices

- The *process performance index* is a dimensionless number that is used to represent the ability to meet specification limits on a characteristic of interest.
- The index compares the variability of the characteristic to the specification limits.
- Three basic process performance indices are P_p , P_{pk} , and P_{pm} .

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Formulae for Process Performance Indices

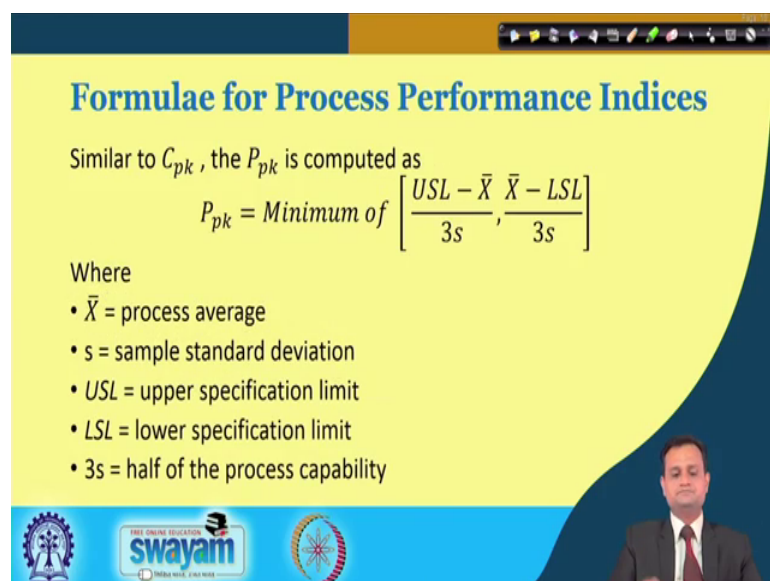
$$P_p = \frac{USL - LSL}{6s}$$

Where

- USL is upper specification limit
- LSL is lower specification limit
- $6s$ is the natural process variation

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Formulae for Process Performance Indices

Similar to C_{pk} , the P_{pk} is computed as

$$P_{pk} = \text{Minimum of } \left[\frac{USL - \bar{X}}{3s}, \frac{\bar{X} - LSL}{3s} \right]$$

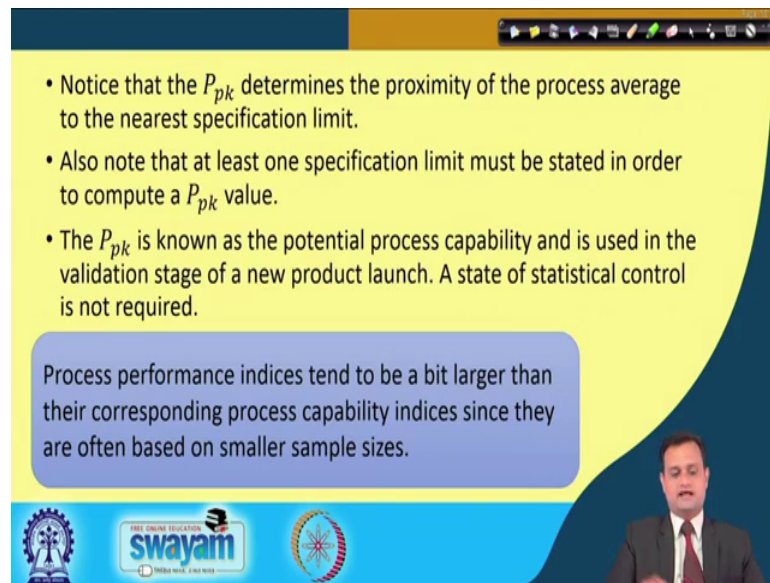
Where

- \bar{X} = process average
- s = sample standard deviation
- USL = upper specification limit
- LSL = lower specification limit
- $3s$ = half of the process capability

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Same way you can capture only thing you have to consider the long term process standard deviation. And a here you consider the halfway, but again it is long term. So, sample standard deviation, but for a reasonably long period of time.

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• Notice that the P_{pk} determines the proximity of the process average to the nearest specification limit.

• Also note that at least one specification limit must be stated in order to compute a P_{pk} value.

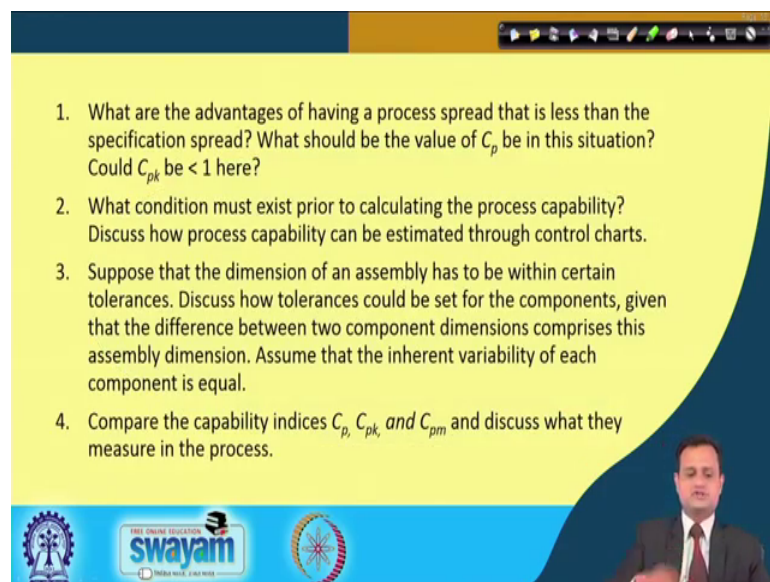
• The P_{pk} is known as the potential process capability and is used in the validation stage of a new product launch. A state of statistical control is not required.

Process performance indices tend to be a bit larger than their corresponding process capability indices since they are often based on smaller sample sizes.

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So, P_{pk} can determine the proximity of the process average to the nearest specification limit, and this tends to be a bit larger than their corresponding process capability indices, since they are often based on small sample sizes as I mentioned.

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1. What are the advantages of having a process spread that is less than the specification spread? What should be the value of C_p be in this situation? Could C_{pk} be < 1 here?
2. What condition must exist prior to calculating the process capability? Discuss how process capability can be estimated through control charts.
3. Suppose that the dimension of an assembly has to be within certain tolerances. Discuss how tolerances could be set for the components, given that the difference between two component dimensions comprises this assembly dimension. Assume that the inherent variability of each component is equal.
4. Compare the capability indices C_p , C_{pk} , and C_{pm} and discuss what they measure in the process.

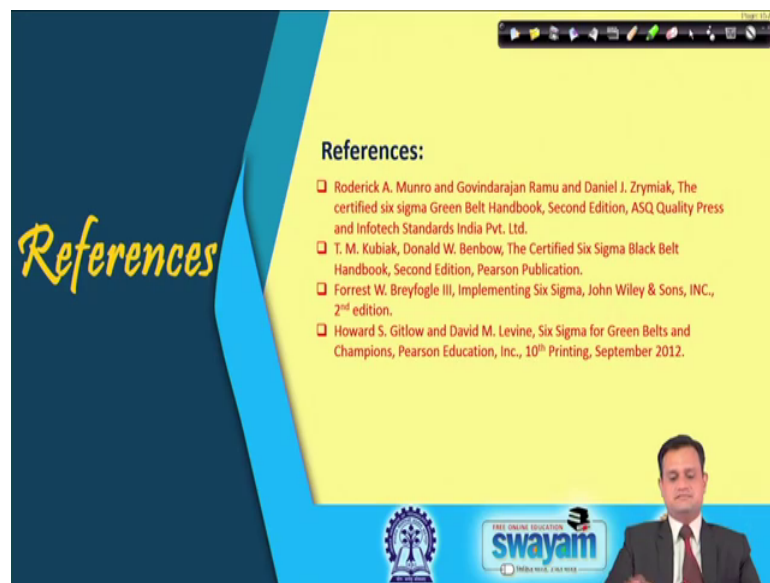
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So, just think it couple of questions for you to introspect, what are the advantages of having a process spread; that is less than the specifications spread, what should be the value of C_p be in this situation? Could C_{pk} be less than 1 here? What conditions must exist while we calculating the process capability discuss how process capability can be

estimated through process control charts. And suppose that the question 3, suppose that the dimension of an assembly has to be within certain tolerances.

So, please discuss how tolerances could be set for the components, given that the difference between two component dimensions comprises this assembly dimension, assume that inherent variability each component is equal. And finally, you compare the various process capability indices C_p , C_{pk} and C_{pm} .

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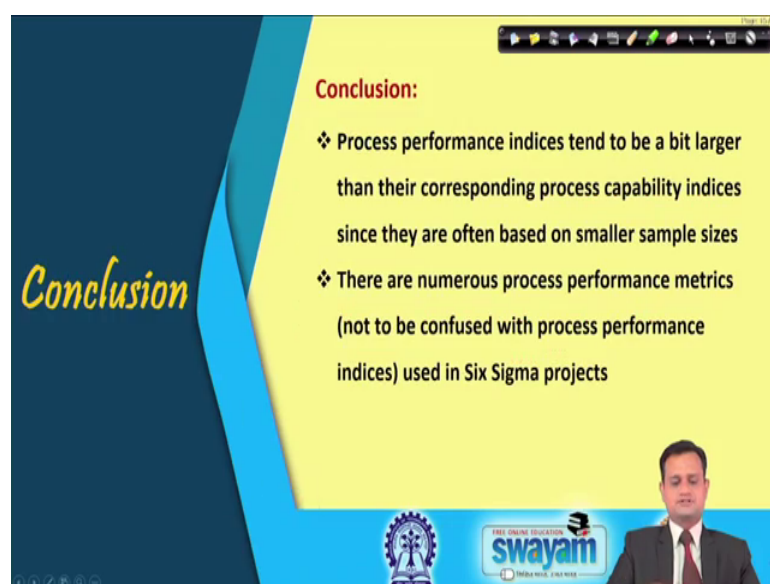


References:

- ❑ Roderick A. Munro and Govindarajan Ramu and Daniel J. Zrymiak, The certified six sigma Green Belt Handbook, Second Edition, ASQ Quality Press and Infotech Standards India Pvt. Ltd.
- ❑ T. M. Kubiak, Donald W. Benbow, The Certified Six Sigma Black Belt Handbook, Second Edition, Pearson Publication.
- ❑ Forrest W. Breyfogle III, Implementing Six Sigma, John Wiley & Sons, INC., 2nd edition.
- ❑ Howard S. Gitlow and David M. Levine, Six Sigma for Green Belts and Champions, Pearson Education, Inc., 10th Printing, September 2012.

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Conclusion:

- ❖ Process performance indices tend to be a bit larger than their corresponding process capability indices since they are often based on smaller sample sizes
- ❖ There are numerous process performance metrics (not to be confused with process performance indices) used in Six Sigma projects

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Please see these references to revise your concepts and to conclude process performance indices tend to be a bit larger than their corresponding process capability, since they are often based on smaller sample size.

Thank you very much for your presence and interest in learning this particular topic, and we have talked about various issues in the major phase of my DMAIC cycle. So, keep revising, keep noting down your queries, also refer the suggested text and be with me. Enjoy.