**Six Sigma**
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**Lecture - 25**
**Process Capability Analysis : Measures and Indices**

Hello friends, I invite you to the lecture 25, Process Capability Analysis: Measures and Indices and I would like to remind you that we are in the major phase of our DMAIC Six sigma journey. And last time we have talked about various concepts key concepts related to process capability analysis. This time we will go deeper and try to appreciate the importance of various measures and indices.

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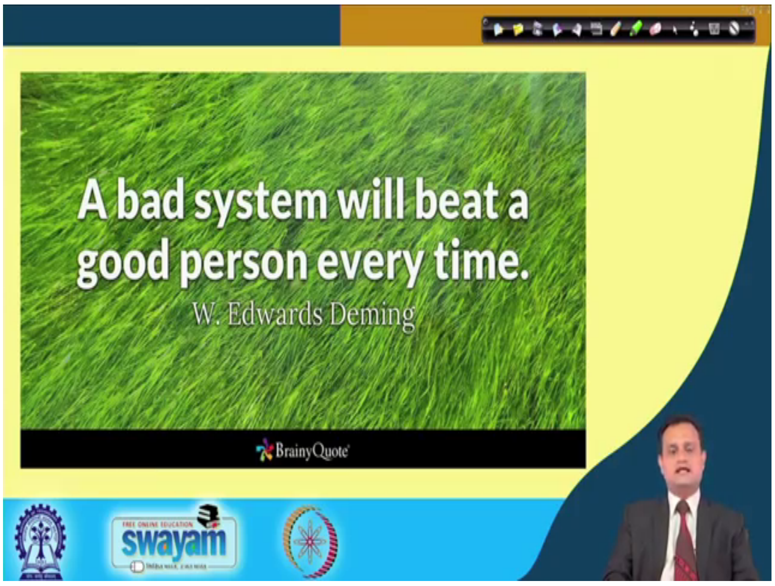


So once again I would like to emphasize that a bad system will beat a good person every time and it is necessary to see that, my processes are really capable enough to meet the customer expectations or not. So, processes which are not meeting the customer expectations, specifications, then these processes first needs to be corrected. And once you achieve the desired capability of the process, then only you can really produce the good quality products.

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So, if you just see the small recap, then we talked about need of process capability analysis, natural process limit, specification limit, control limits, natural tolerance limits, short term and long term process capability, differences between process performance metrics and process capability indices. So, many a times you get confused, we had seen DPMO DPU, how to find sigma level, all these are process performance metrics. when we talk about process capability indices, these are the aggregate measures basically represents the capability of the process and C p, C pm, C pk. These are the various measures that we will see in this particular lecture

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So, we would also keep revising couple of things which we discussed last time, but in addition to that, we will mainly focus on process capability and capability indices in this lecture. Process performance and performance indices, short term long term capabilities and related indices, capability related assumptions. Then conducting process capability study, natural process limit, specification limit and process performance metrics. So, these are the broad topics that we would like to see and simultaneously I would like to give you some recap of the last lecture, so that you can strengthen your understanding on process capability indices.

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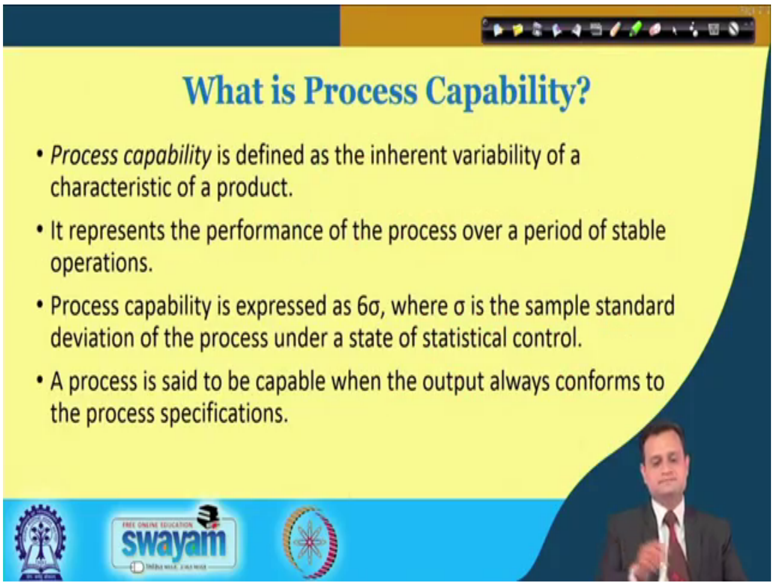


So, we have seen what is process capability? So, typically it expresses the capability of the process to meet customer specifications and to what extent my process is capable enough to meet the customer requirement. So, typically it is expressed as 6 sigma and where sigma is the sample standard deviation of the process which is under a state of control. So, please be reminded that I can only conduct the process capability study if my process is under the state of statistical control.

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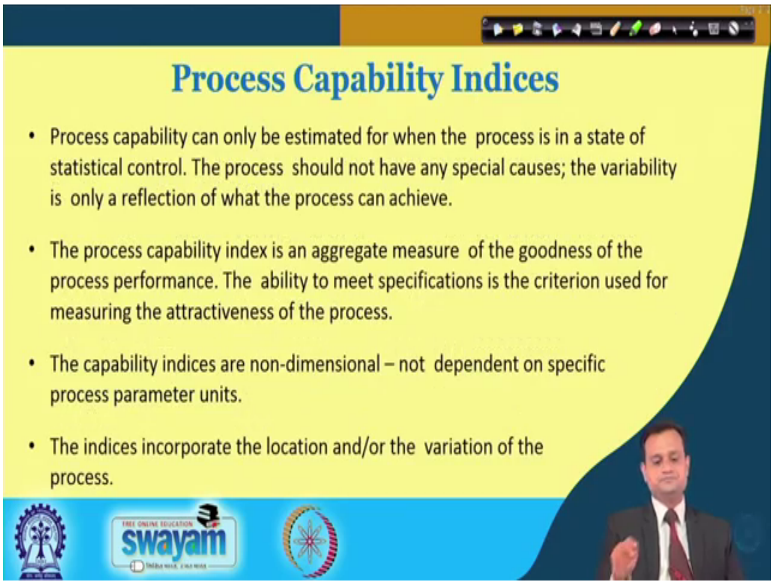


We have various process capability indices, but basically these are aggregate measures, and they typically, say check the goodness of the process performance and ability to meet the specifications. So, indices in corporate locations as well as variability both, and we will try to see it through different indices.

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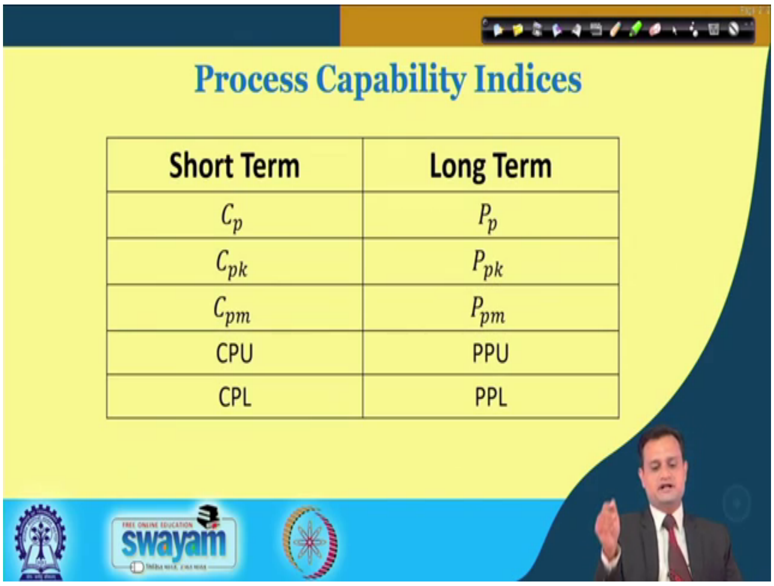


So, broadly I can classify the indices as short term indice, long term indice, C p C pk. C pm, CPU, CPL. These are the various short term indices and long term indices P p, P pk P pm.

So, when I says short term, I am specific on a particular sub group or a limited period of time, I am trying to capture the status of process. When I say long term, then for the entire horizon, overall a say working period, long span I am trying to absorb the process and then I am trying to say calculate the process capability to comment about the process performance.

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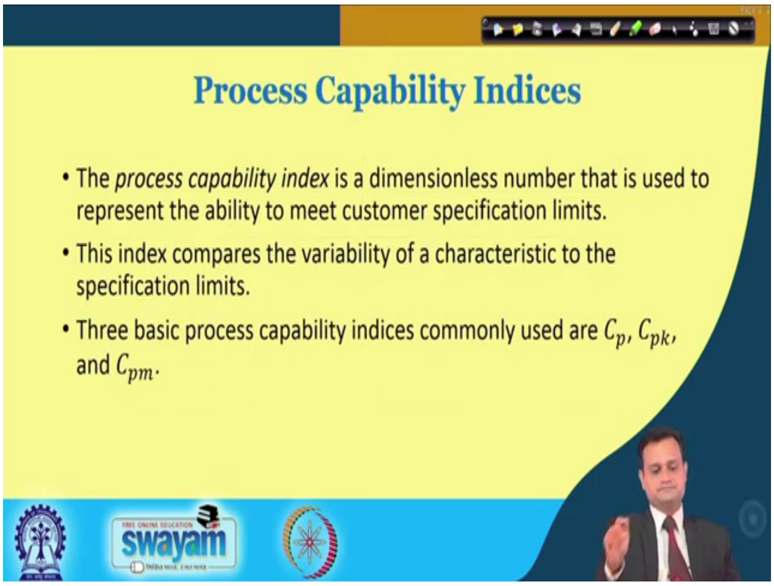


So, basically it is a dimensionless number and is used to represent the ability to meet customer specification limit and typically for short term as we have seen C p, C pk, C pm are the widely used process capability indices.

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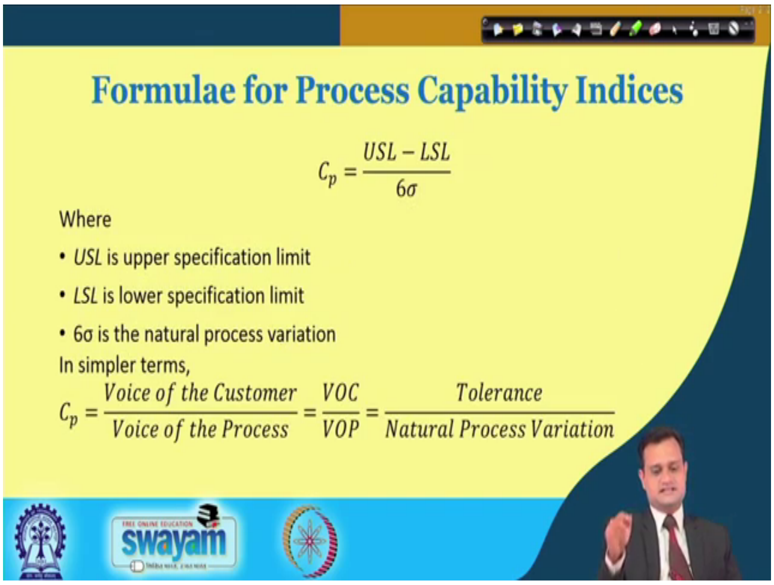


So, just see the expression for Cp. I am considering the overall band USL minus LSL divided by 6 sigma; that is the variability associated. So, when I say USL minus LSL by 6 sigma typically it tries to capture the voice of the customer and voice of the process. So, VOC divided by voice of process VOP, tolerances divided by natural process variation.

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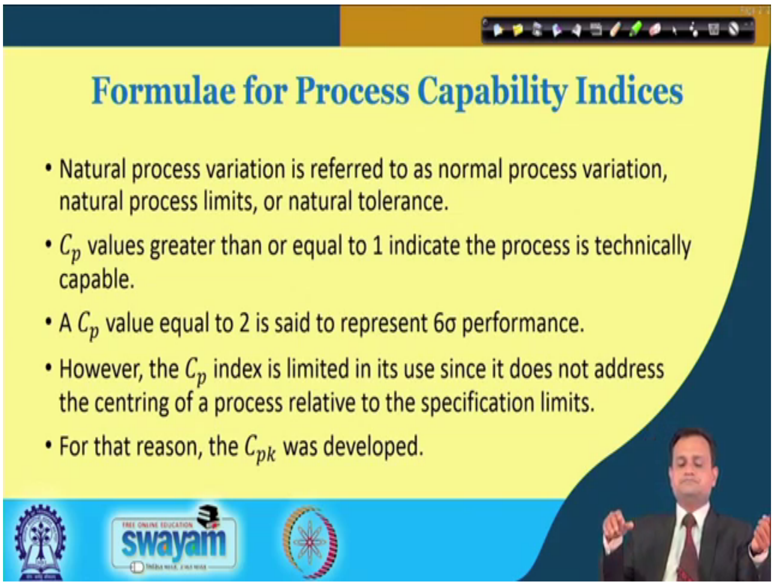


So, we have certain guidelines that C p value is greater than equal to 1, indicates the process is technically capable. When I say C p is equal to 2, then it represents the world class performance like 6 sigma performance, and C p value equal to 2 is said to represent typically your 6 sigma performance which we are aiming for and C p index is limited to use, since it does not address the center of a process. So, process has a center mean as well as the variability, but typically C p does not take into account the center or shift in the mean, because over a period of time there may be change in the setting and process may get shifted to the new mean. This part is not considered and that is why the new measure is developed that is C pk.

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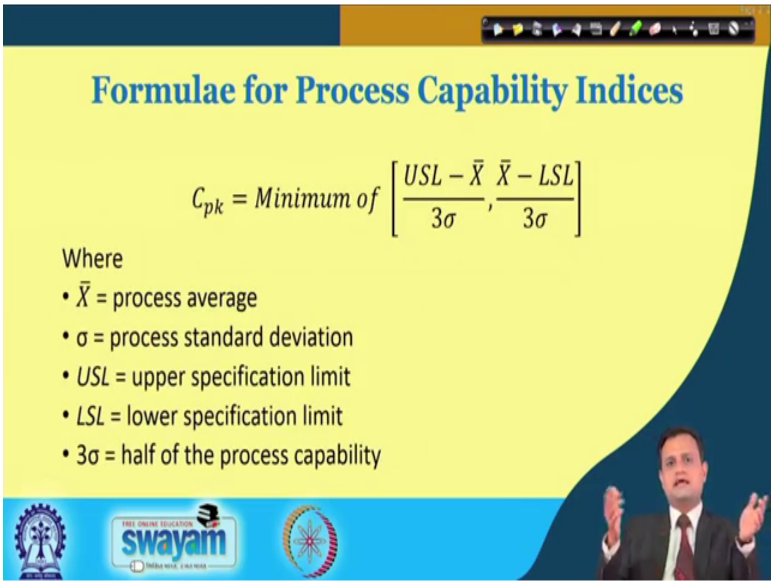


So, C pk is basically minimum of USL minus X bar divided by. So, X bar is your process average, USL is upper specification limit divided by 3 sigma and another term that we calculate is X bar minus LSL divided by 3 sigma. So, I will take minimum of these two, to check whether my process also meets the center requirement or not or it is now performing poorly, because there is shift in say center or mean value.

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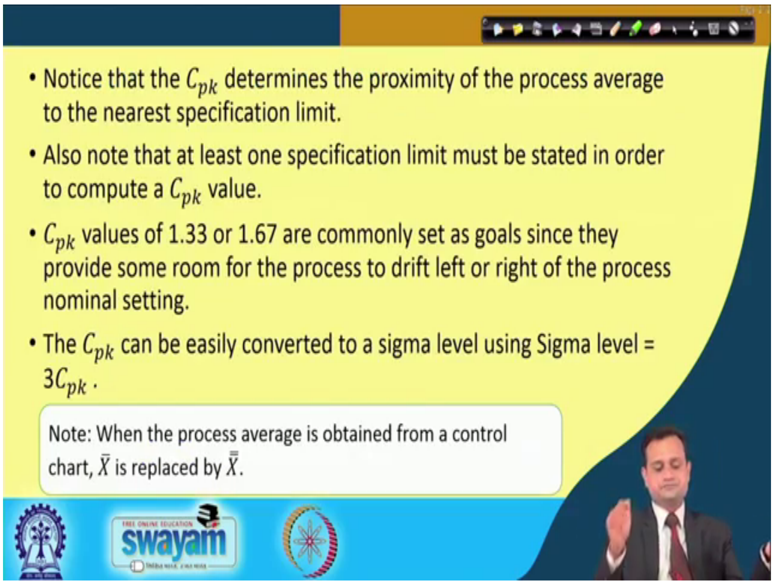


So, typically C pk determines the process, proximity of the process average to the nearest specification either lower or upper, and we can note that one specification limit must be stated in order to compute the C pk. Sometimes some products they have only one specification limit, so it is ok, but at least you should have one specification limit.

So, C pk can be easily converted to a sigma level using sigma level is equal to 3 into C pk and when the process average is obtained from control chart, X bar is typically replaced by X double bar.

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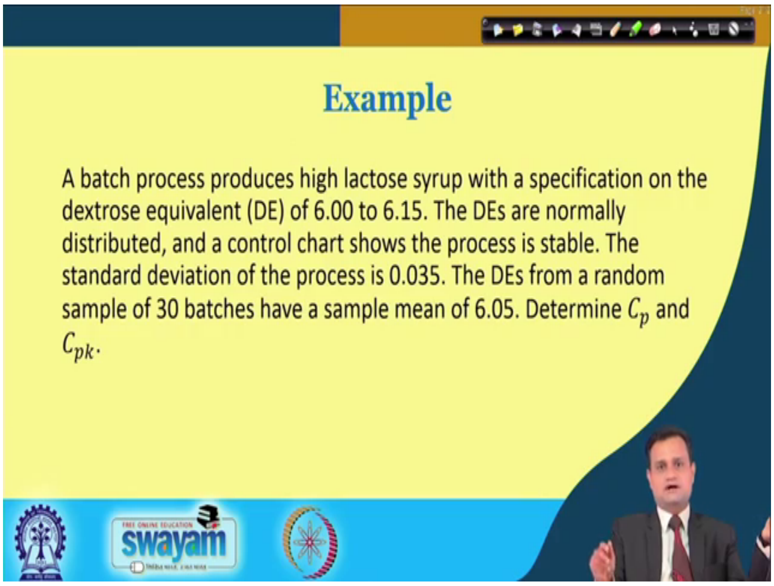

So, let us see the example to make the idea clear ah. Suppose you have a batch process, typically produces high lactose syrup with a specification on the dextrose equivalent; that is to 6 to 6.15 and the DEs are normally distributed. So, I assume that my production is normally distributed and a control chart shows the processes stable.

So, the first requirement for going for process capability is satisfied. Their standard deviation of the process is 0.035, and the DE is from the random sample of 30 batches have a sample mean of 6.05. Now I want to comment on the capability of this process, so let me find C p and C pk. So, these two measures they do not have any conflict with each other. In fact, they provide the more information, complemented to each other and I would like to find both Cp and C pk.

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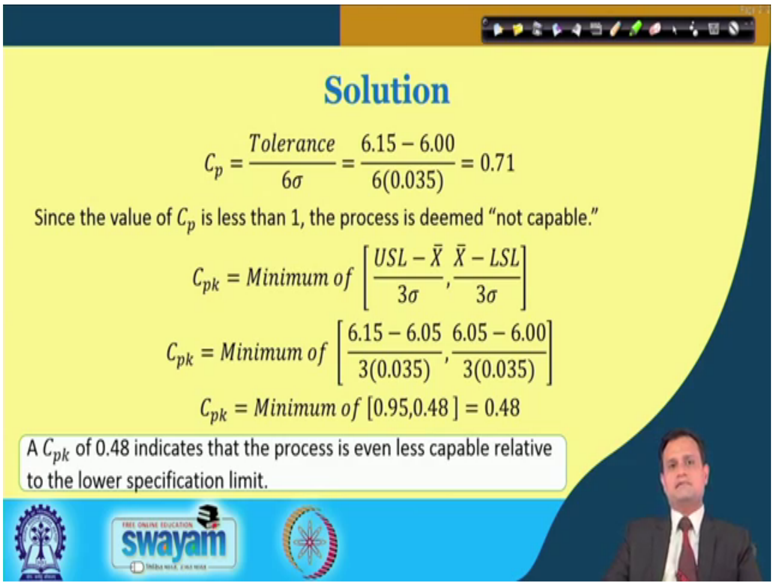


So, here my C p is tolerance divided by 6 sigma, you say it is voice of customer divided by voice of process or sometimes you say USL minus LSL divided by 6 sigma. So, here it comes out to be 0.71. So, this is the value I am referring. This is 0.71. Now you can see that C p is less than 1. So, when the C p is less than 1, my say threshold limit says that if it is equal to 1 or more than 1, technically my process is capable enough, but here my process is not capable.

Now, if I investigate further and find out the C pk, then I will take the minimum of these two with respect to, say upper and lower specification limit compared with the mean value and I will get 0.48. So, C pk 0.48 typically indicates that process is even less capable relative to the lower specification limit. So, number 1 conclusion; processes not capable and second process is even very much less capable compared to relative to the lower specification limit.

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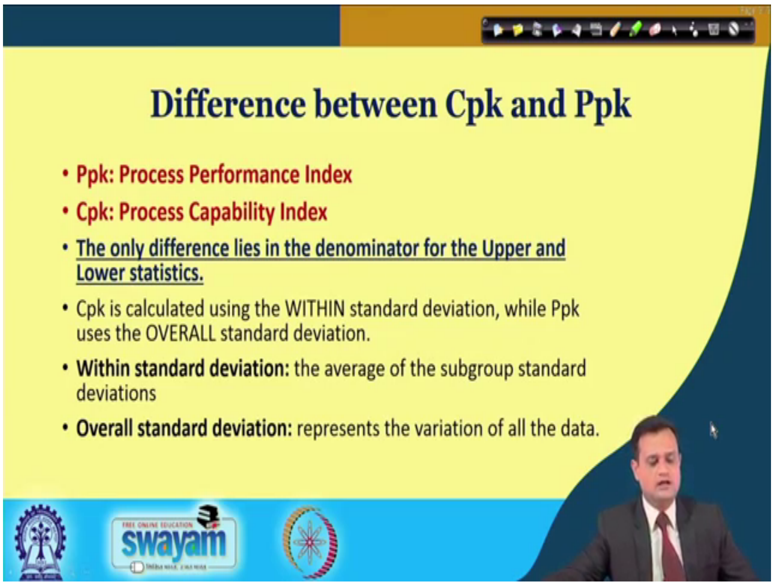


So, you have other measures like Pp P pk P pm. So, you would be wondering what is the difference. I have already highlighted that when I say C p C pk C pm, typically this particular measures they go by the subgroup standard deviation, within standard deviation and when I go by P p P pm, this is more pertaining to the long term process capability and it goes with, say variability of all the data or the process which is being absorbed for a long period of time and this is where C p C pk C pm are short term measures to investigate the process and P p, P pm, P pk are long term measures for investigating the capability of the process

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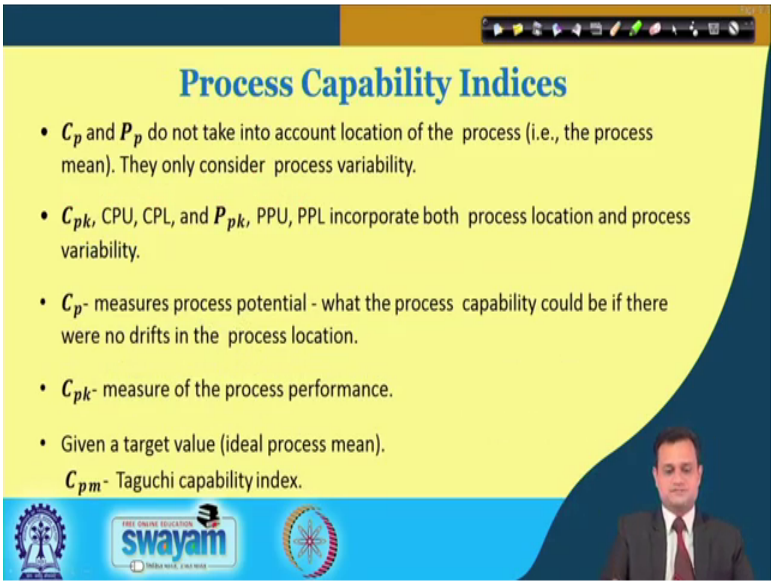


So, you have C p and P p, this do not take into account the shift in mean or location of the process. You have C pk, CPU, CPL, P pk, PPU, PPL short term and long term discovery respectively, they do consider the center as well as location of the process. And if my process, because of say usage wear and tear of some of the components or problem in the setting has shifted to the other mean, then that would be captured by this C pk P pk CPU CPL measures. C p is basically measures the process potential.

So, please try to focus on this particular word, that C p measure the process potential. So, what the process capability could be, if there were no drips in the process location. There is no shaped disturbance in the setting in that case, what is the capability of my process, what is the potential of my process. When you say C pk, typically it talks about the measures of, measure of the process performance. So, even if there is a shift, there is a change in settings, shift in the mean to what extent my process would be capable enough to produce the products which can meet the customer service specification.

When I talked about it, it is about process performance. And you can see the third one that is C pm. So, this is typically called Taguchi capability index. And here instead of I would say some central value or mean value, I will talk about the target value.

So, there is a target value which I am trying to heat as much as possible and to what extent my process is capable enough to meet this target value, when my interest of analysis is this, then I would make use of C pm; that is Taguchi capability index. So C p is USL minus LSL by 6 sigma, desirable is greater than equal to 1.

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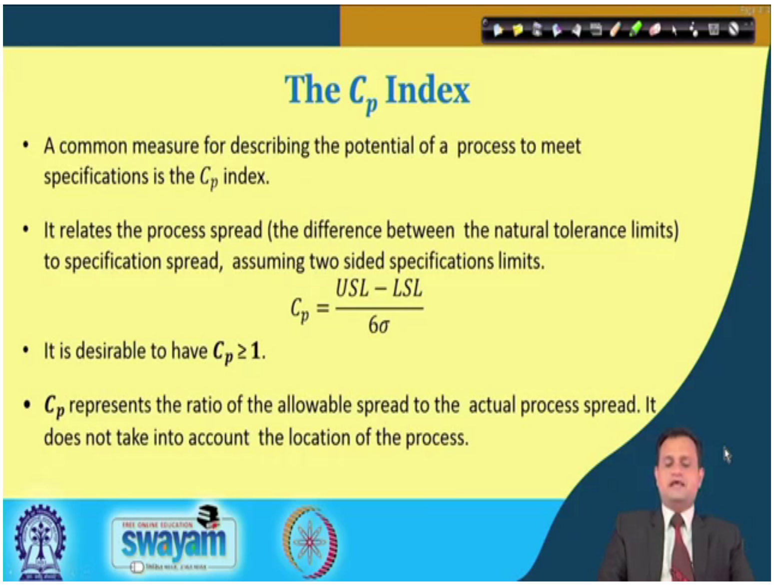


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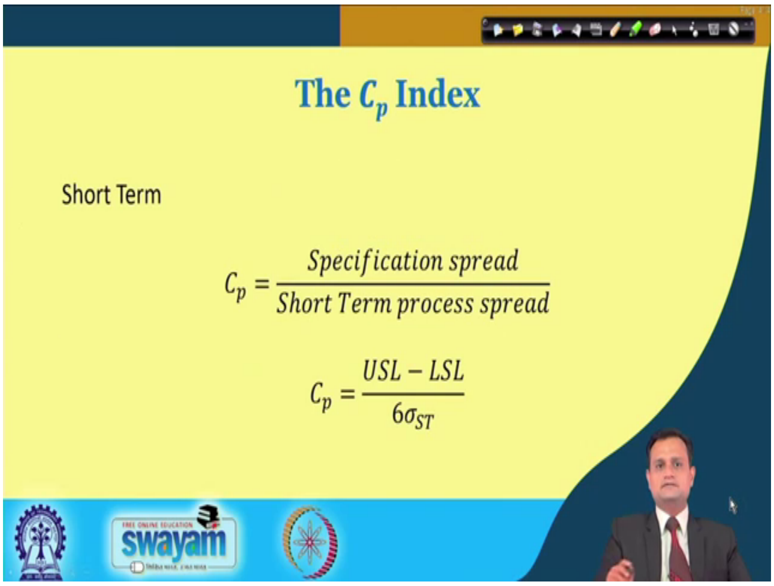


C p when you talk about long term, you have to consider a. You talk about the short term, you have to consider the short term process spreads sigma 6 sigma ST.

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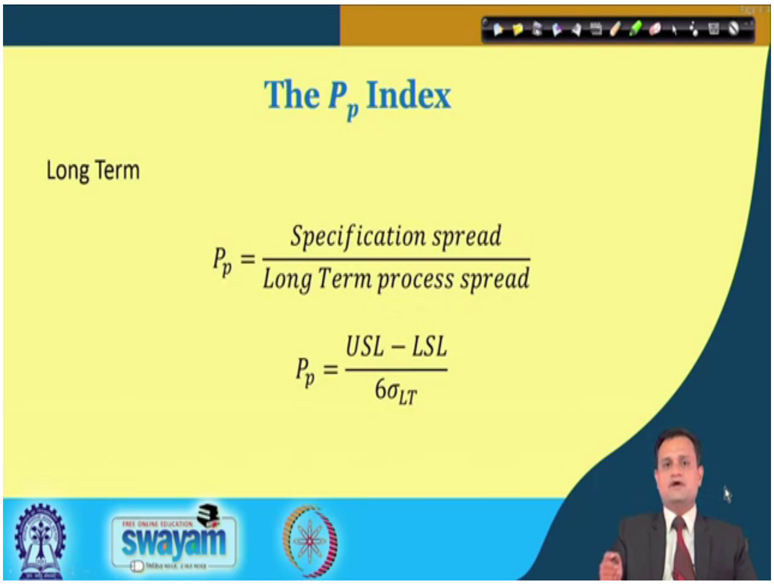


When you talk about the long term, you will consider the long term process spread 6 sigma LT; you are allowing the process, observing the process for a longer or extended period of time and then computing the standard deviation.

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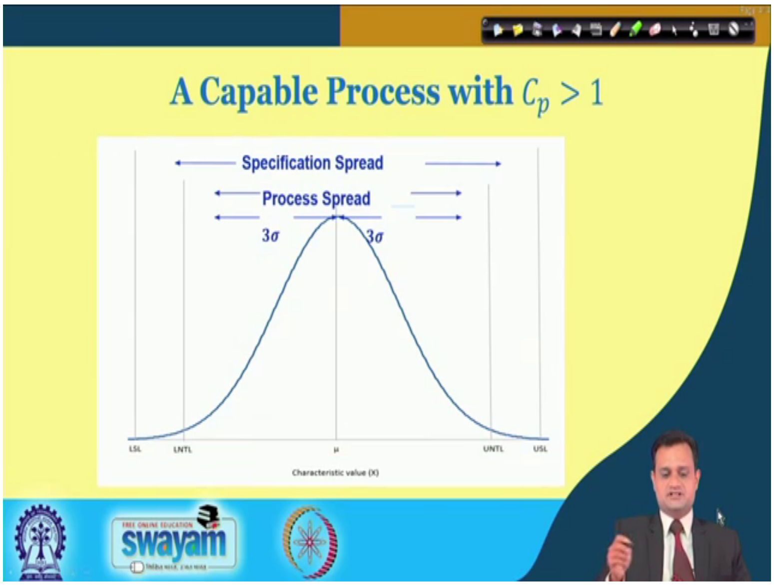


So, just see this and you will get an idea that here C p is greater than one, and now you can relate these with the understanding we evolved in the previous class. So, that time we just commented about the health of the process capability of the process in terms of upper and lower specification limit and tolerance limit, natural tolerance limit. Here I we are also relating that with the Cp value.

So, to remind you, tolerance limits basically decided by the process variability, capability of the process and specification limits are given by the customer. So, this is a process which is capable and we can very well see, that my upper natural tolerance limit and lower natural of tolerance limit, they are well within the upper and lower specification limit. So, my process is capable enough to meet or produce the products as per the specifications.

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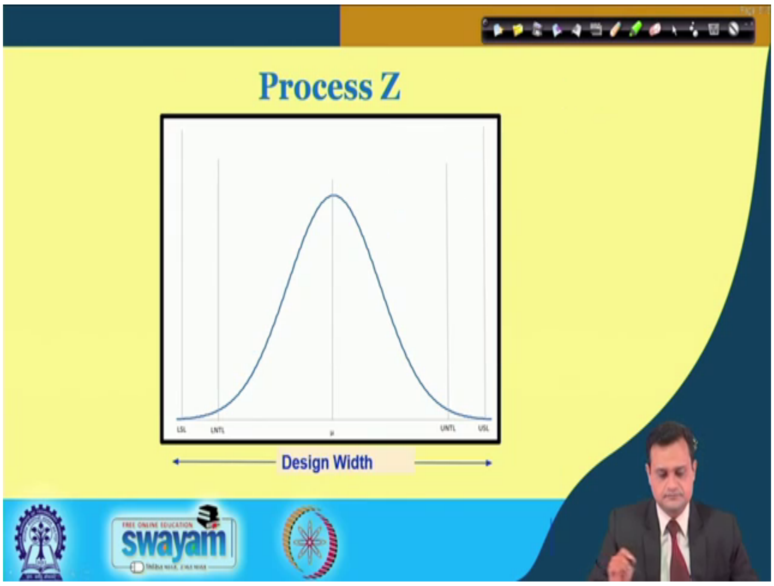


So, this is my design width, I have to design my product process within this particular range, and this is typically my process Z that. I call it Z because we are referring to the normal distribution.

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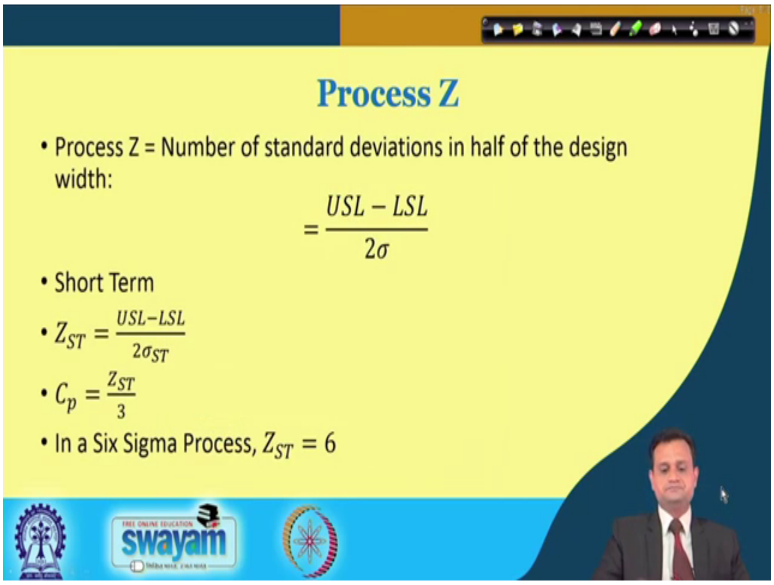


So, you can find process Z as USL minus LSL divided by 2 sigma. So, it is the number of standard deviation in half of the design width, and you can find out ZST, it is short term. So, USL minus LSL divided by 2 sigma ST short term and Cp is equal to Z ST divided by 3. It is a standard expression, because half of the width I am considering. So, in a 6 sigma process; obviously, my ZT ST would be 6.

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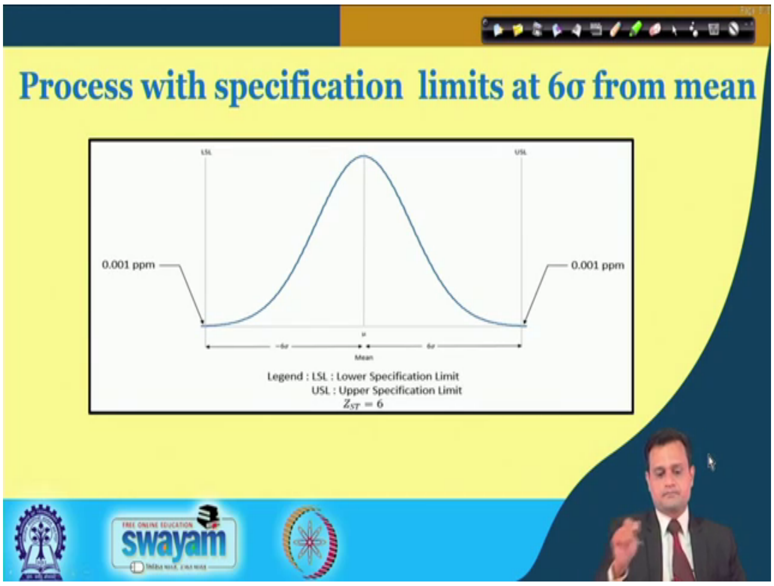


So, process with specification as a limit at 6 sigma, you can see here that my particular process has plus 6 sigma minus 6 sigma and the number of components I would be producing, which is defective is only 0.001 P pm on either side. And again I would say that 6 sigma means not larger, 6 sigma means a process which has extremely less variability, and hence it can give you the 6 sigma outcome; that is 3.4 say defectives per million, or here you can say 0.001 parts per million.

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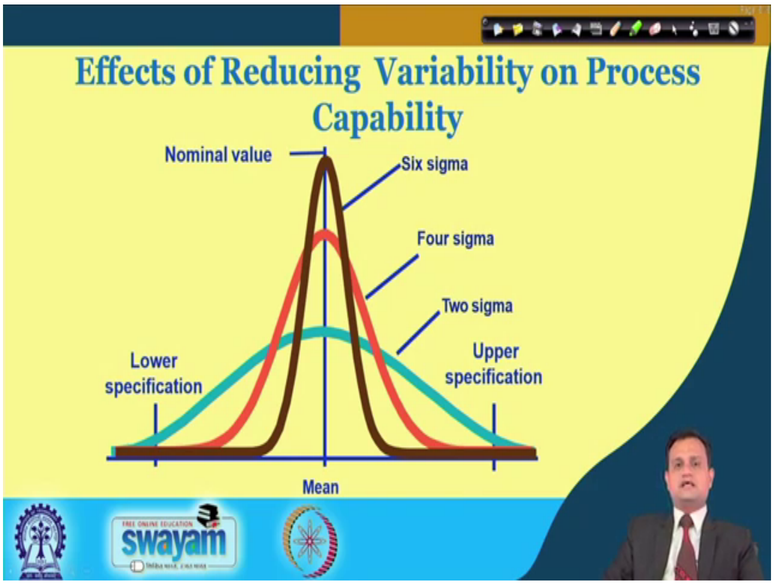


So, just see to remind you and as I mentioned that 6 sigma. This is my nominal value, this is my 6 sigma and this is my 6 sigma curve. So, I have extremely less variability. This is my upper specification, this is my lower specification, and you can see that this process, because it is part of, say well within the upper and lower, the probability that I would produce the defect you part is very very less. As you move to 4 sigma, then your quality level goes down 2 sigma, it further goes down and so on.

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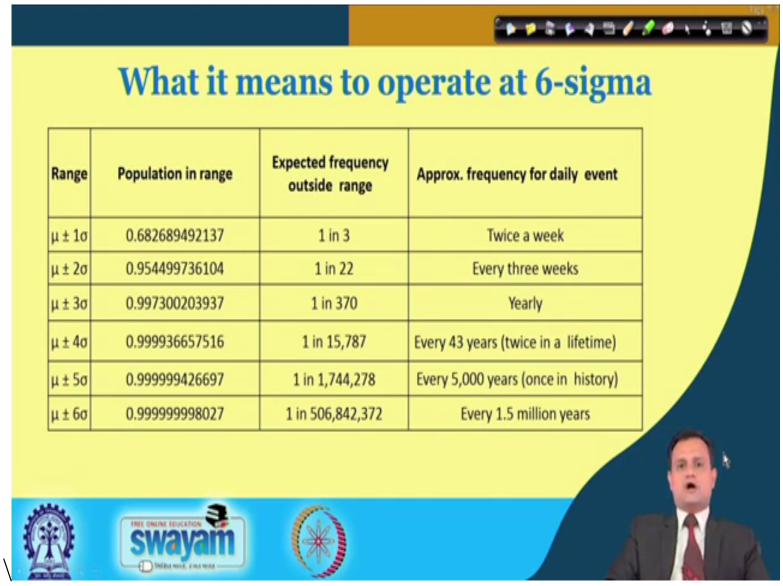


So, this is what we have seen, I would like to remind you again, that if my process is controlled within plus or minus 1 sigma, then expected frequency outside range 1 in 3, very very high, 1 in 3. If you see 2 sigma 1 in 22, if you see 3 sigma 1 in 370 and if you see 4 sigma 1 in 15787; if you see 5 sigma 1 in, say 1744278 and if you see 6 sigma, then the probability of producing defective component is very very less.

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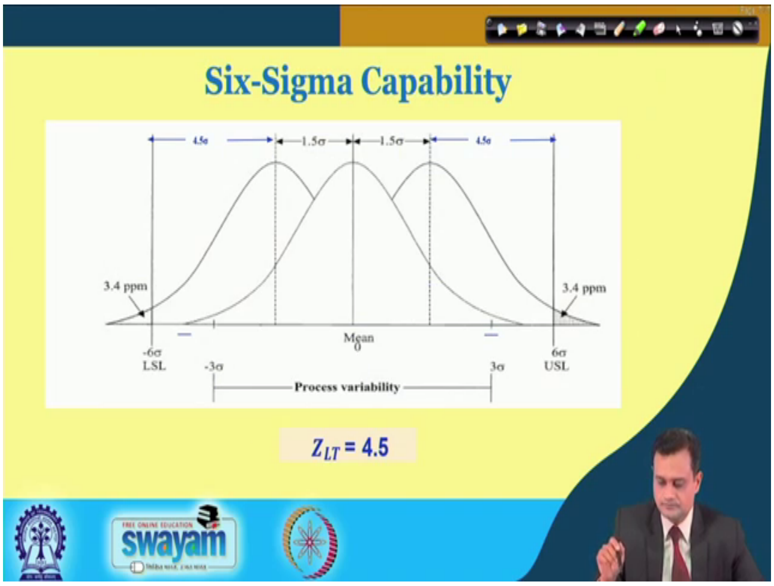


You can once again see that when I talk about the process capability, this is my process which is centered at mean, this is my process which is centered at mean and you can see that there could be a shift of 1.5 sigma, this side there could be a shift of 1.5 sigma this side, and even if there is a one 5 sigma shift I would only be producing 3.4 P pm on the either side. So, this is where I am trying to achieve the reduction in the process variability, but in order to do so, first I must have an idea what is the capability of my process and what are the measures necessary to improve the capability.

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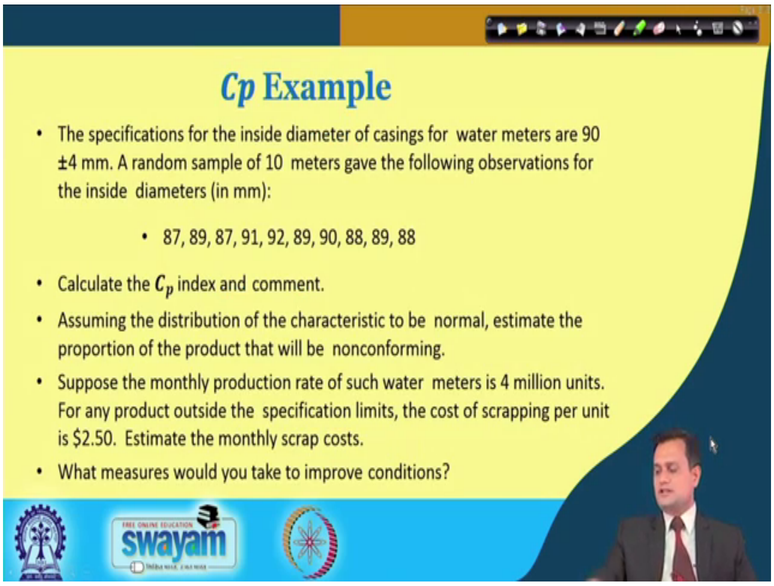


So, there is an example, a couple of examples I am including to help you to appreciate the concept in detail. So, let us say specification of the inside diameter of casing for water meter are 90 plus or minus 4 mm in a random sample of 10 meters gave the following observations. So, there is 87 89. So, diameter is measured in mm 87, 91 and so on. We are expected to compute the C p index and comment, assuming the distribution is normal, and suppose the monthly production rate of such water meter is 4 million units. For any product outside the specification limit, the cost of scrapping this particular meter water meter, is let us dollar 2.5, estimate the monthly scrap costs and what measures would you like to recommend to improve the condition.

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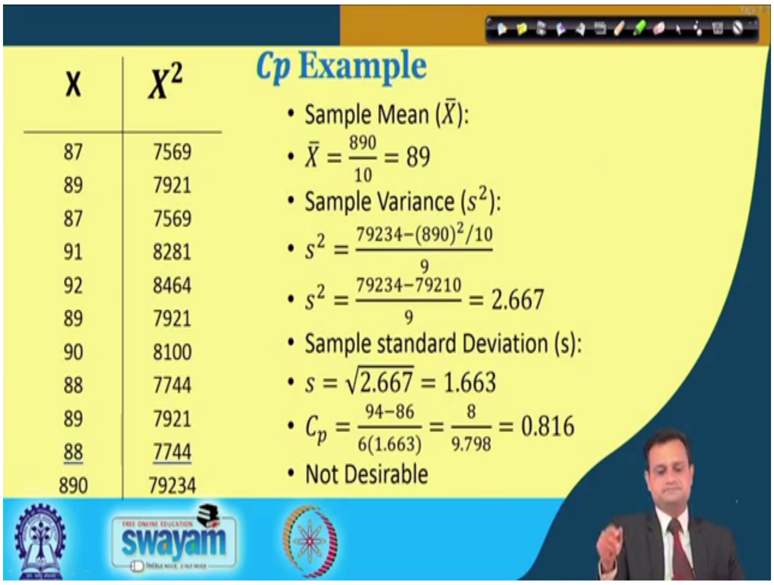


So, this is just the computation which can be easily followed. So, I have the X bar, I have taken the X square, I can find out the X bar. So, this is the total 890 divided by 10 is 89. I can find the sample variants, so I have 79234 then this is the value 890 whole square divided by 10 and divided by 9. So, my S square is 2.667. I can find the standard deviation as the square root and C p comes out to be 0.816. So, I would; obviously, say C p is less than one, so it is not desirable and my process is actually not capable.

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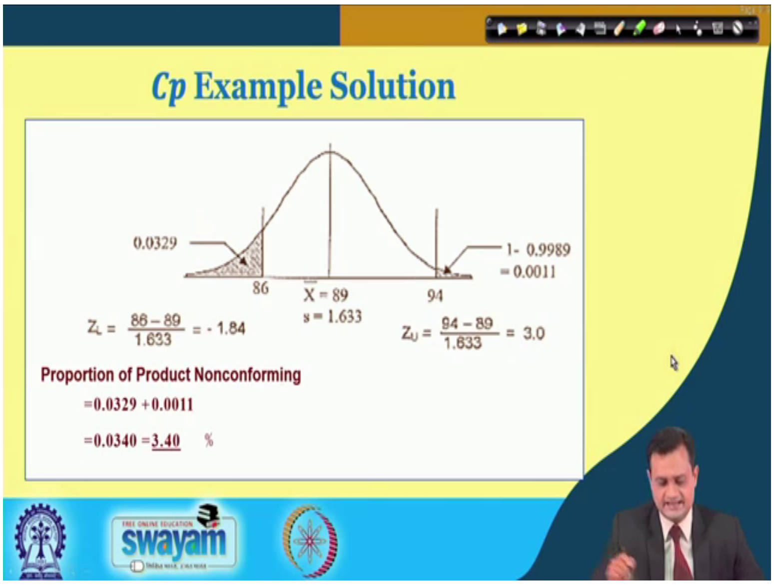

So, now, the issue is that, if it is not capable then how much amount of scrap I would be producing, non-conforming items I would be producing. So, here you can see that you have 86 and 94, because 90 plus or minus. If you just see the specifications then it is, this is 90 plus or minus 4 mm.

So, 86 to 94 is the specification range. And if I consider this, then this side is the region which shows me that 0.0329 will go out of the specification limit and this side says that 0.0011 will go out of the specification limit. So, typically I would have 0.0329 plus 0.0011 equal to 3, 0.0340 34 percent, as the defective product reject product.

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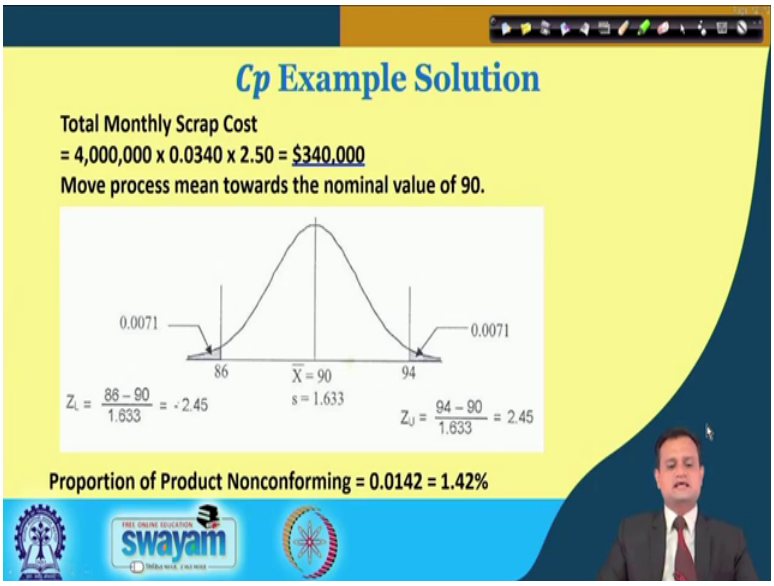


So, if I just convert this into the number, then you can say that 400 into, 04000; that is 4,000,000 into 0.0340 into 2.5 is the cost dollar. So, I would be almost losing dollar 340,000. So, it is not a small amount. So, proportion of product non-conforming, if you just move the process mean to, towards the value of 190, then again you will have some non-conforming, but this goes down and the proportion of product non-conforming is 0.0142. So, typically it is 0.0071 plus 0.0071. And if you do a little bit resetting, you will get 1.42 percent.

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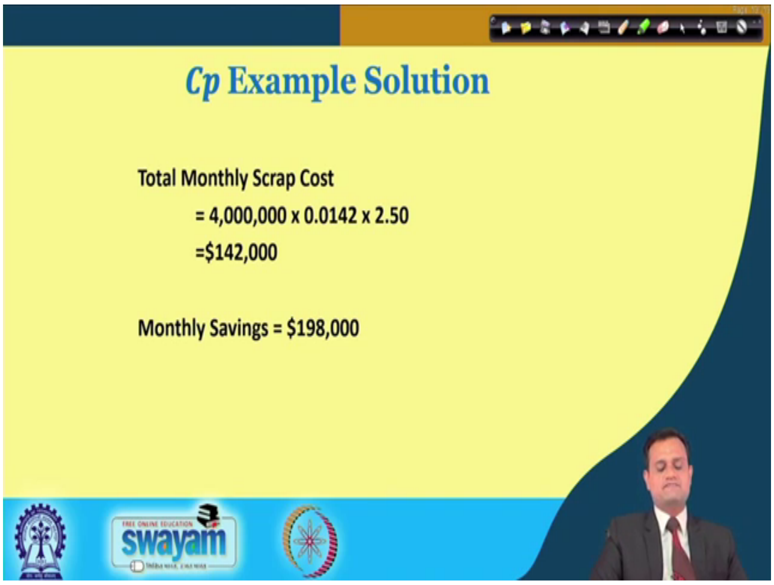


So if I consider this 1.42 percent then total monthly scrap will come down and it would be 142,000. So, compared to the previous one, compared to the previous one which was 340,000 Dollar, it has come down to the 142,000, so saving would be almost 198,000. So, just see the importance of, say knowing in the process capability and doing a little bit setting to the my process, and setting it at the right mean can really help me to reduce my say scrap and hence that could be huge saving in my, say rejecting rejected products scrap products and this is where it is beneficial.

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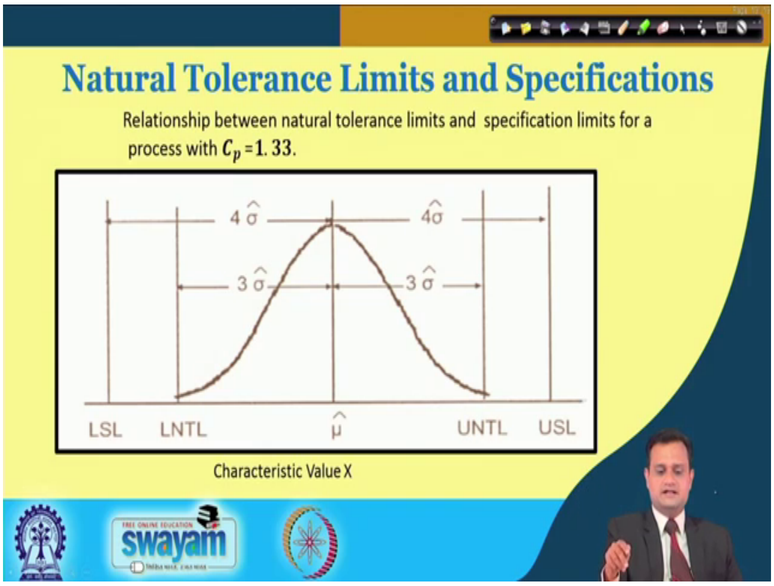

So, you have natural tolerance limit and specification limit just again to remind you, that natural tolerance limits are pertaining to the process capability and variability of the process where in your specification limit are basically given by the customer and we try to produce a product within this particular specification limits.

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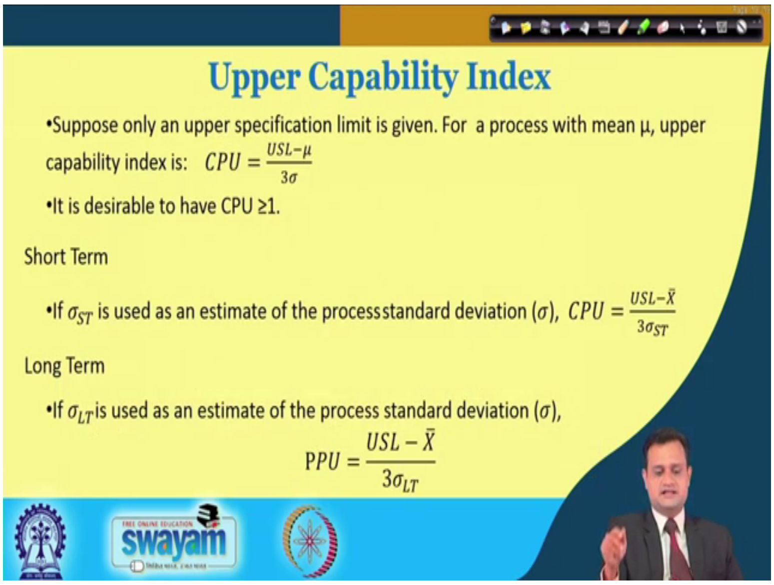


So, you have a couple of other indices derived indices; like upper capability index. So, CPU is basically USL, upper specification limit, I just consider half minus mu divided by 3 sigma, because it is half and the desirable is that CPU should be greater than or equal to 1.

Similar way short term I can find my CPU just by dividing it by sigma ST; that is short term and I can find long term PPU dividing it by sigma LT.

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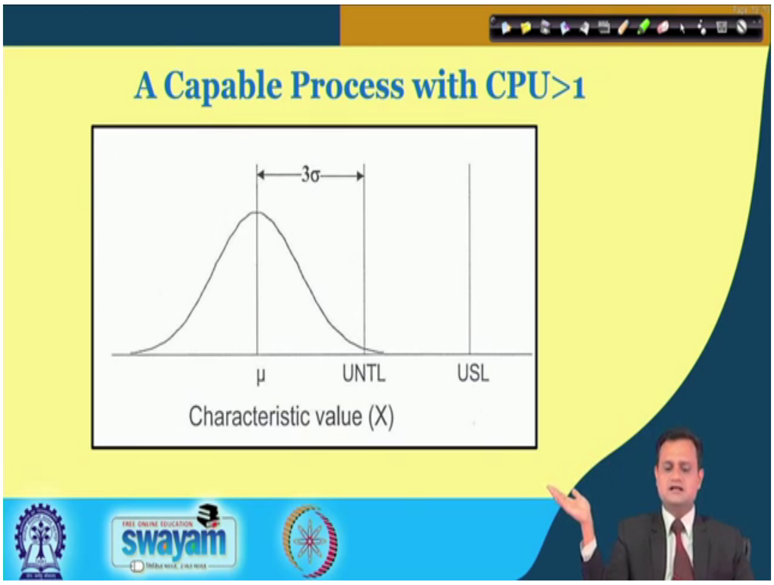


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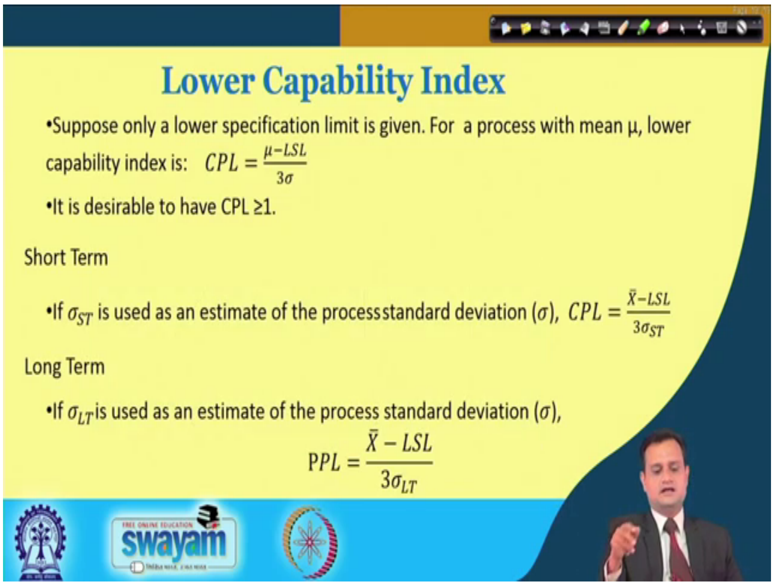


So, this is the process capable with respect to upper specification, CPU is greater than 1. And same way you define the lower, and you lower is for short term CPL, for long term it would be PPL.

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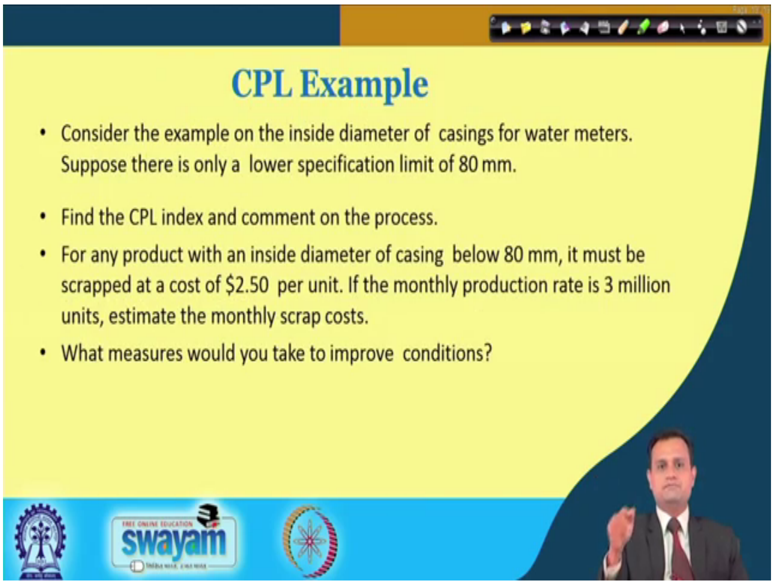


So, we can just see the example of a CPL; that is with respect to one specification limit, lower specification. So, consider the example on the inside diameter of casing of water meter, the same example. And suppose there is only one lower specification limit of 80 mm.

So, now find the CPL index and comment on the process. So, for any product within inside diameter of casing below 80 mm, it must be scrapped at a cost dollar 2.5 as we had considered previously and what measures would you like to consider.

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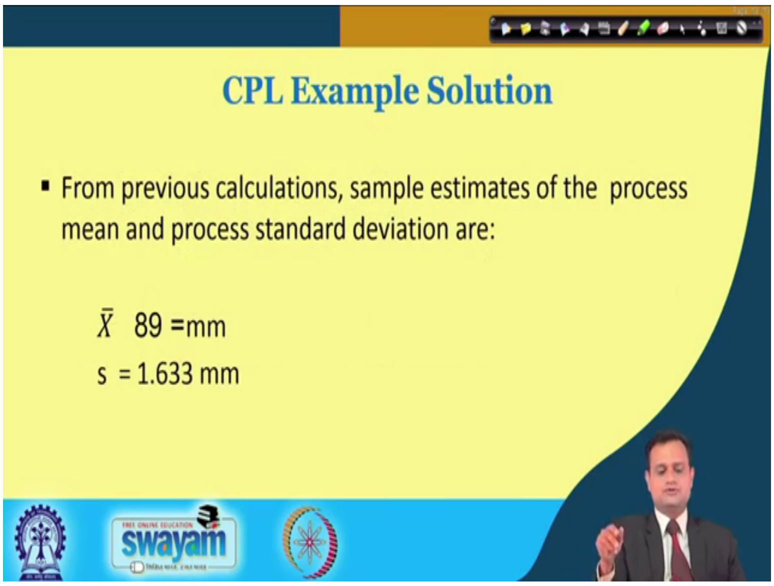

So, X bar is 89 mm s is 1.633. From the previous calculation I can consider. And just by putting this it is represented on a normal distribution, and I can put these values in the CPL expression, so it comes out to be 1.837.

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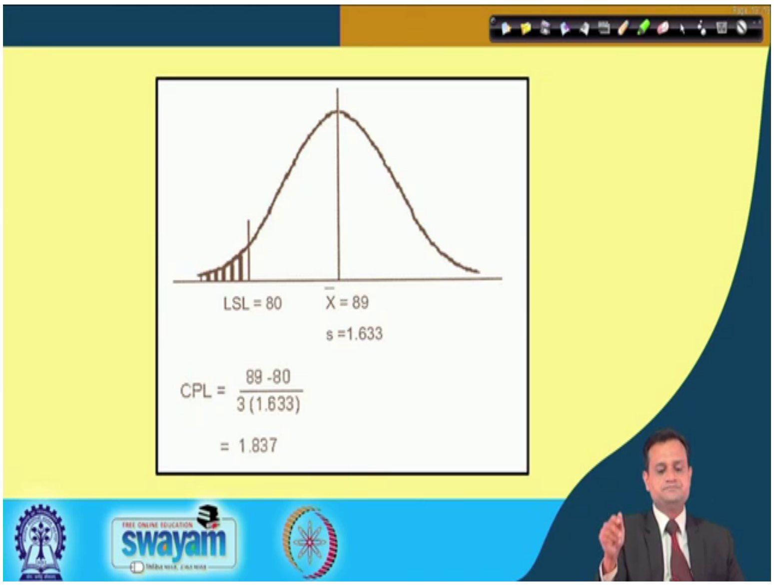


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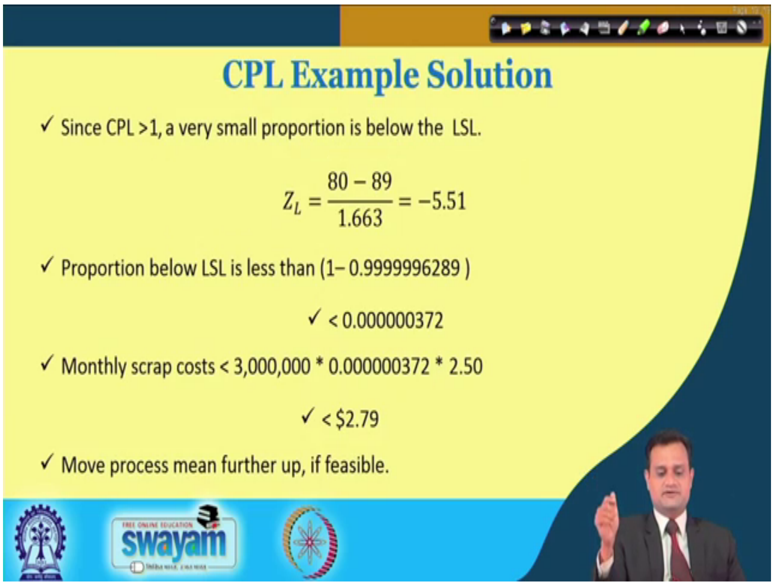


So, you have CPL greater than one. So, a very small proportion is below LSL lower limit, lower specification limit, and if you find a Z L. So, this comes out to be minus 5.51. So, proportion below your LSL is less than 0.000000372 and your monthly scrap would just be less than 2.79 dollar. So, I can say that more process mean further up if feasible. there is absolutely no problem so far the lower part is concerned, you have the flexibility to move the process up to adjust the process, in order to improve upon your scrap production and the cost saving.

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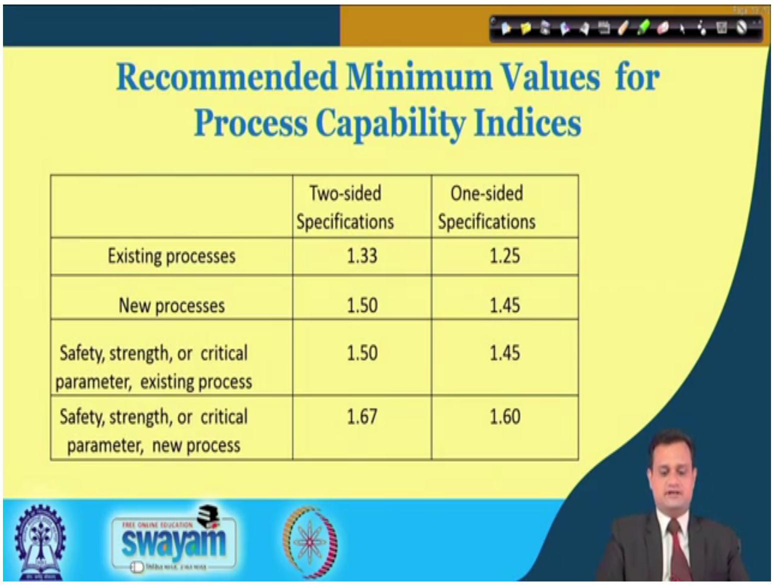


So, just see that recommended minimum value for process capability indices, when you consider two specification when you consider one specification. If you are talking about the existing process where the knowledge is available, about the process variability 1.33 for two sided and for one sided 1.25 is ok. When you say new process, where the process knowledge is less process is not stable, you may go for 1.5, 1.45. And similar way you can see the safety strength or critical parameter existing process 1.5 and when there is a concern of safety strength critical for new process, we are just putting the criteria a little bit tougher 1.67 and 1.6. So, these are the recommended, I must have greater than this values in order to accept my process.

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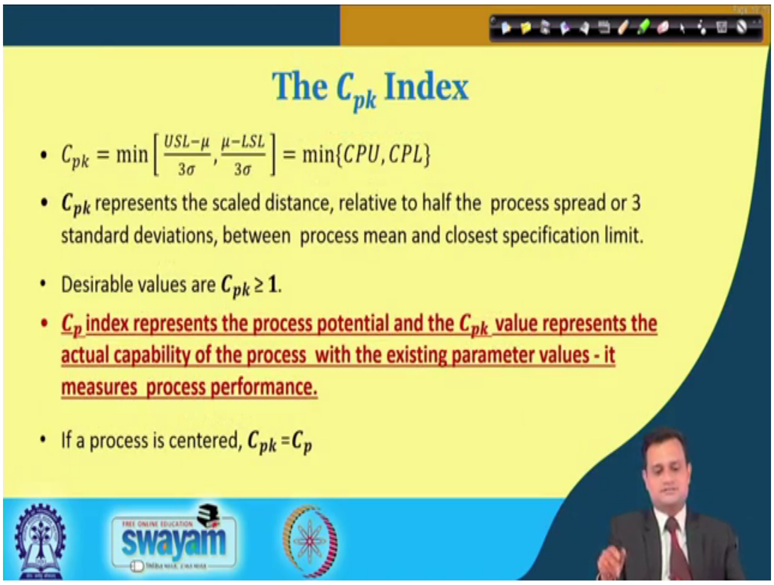


So, you have C pk index, already I express that it takes into account the mean value, your location part and tries to say that what is the performance of the process even if there is a shift in my process and also it considers the process variability through sigma. So, if a process is centered perfectly at the mean value, your C p is equal to C pk. So, a process is not capable.

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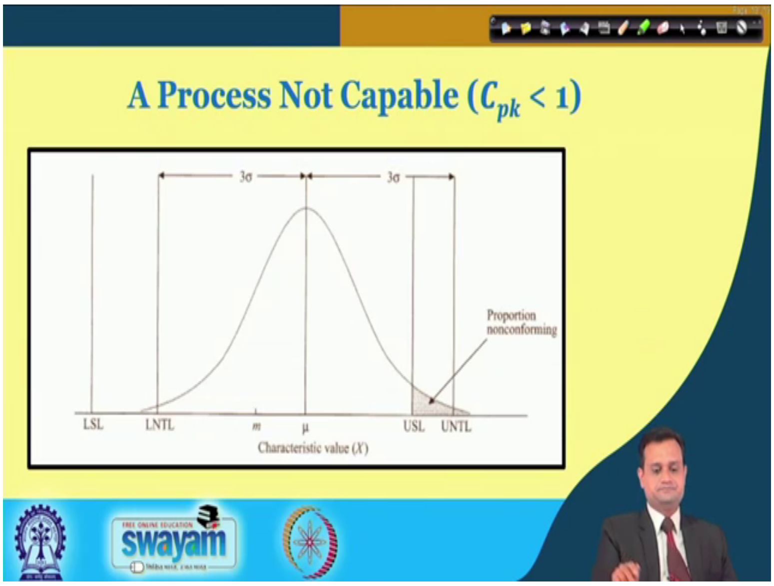

Now I am talking about C pk which is less than 1, and you can see that you are producing proportion non-conforming on this side, and this is where your C pk is less than 1.

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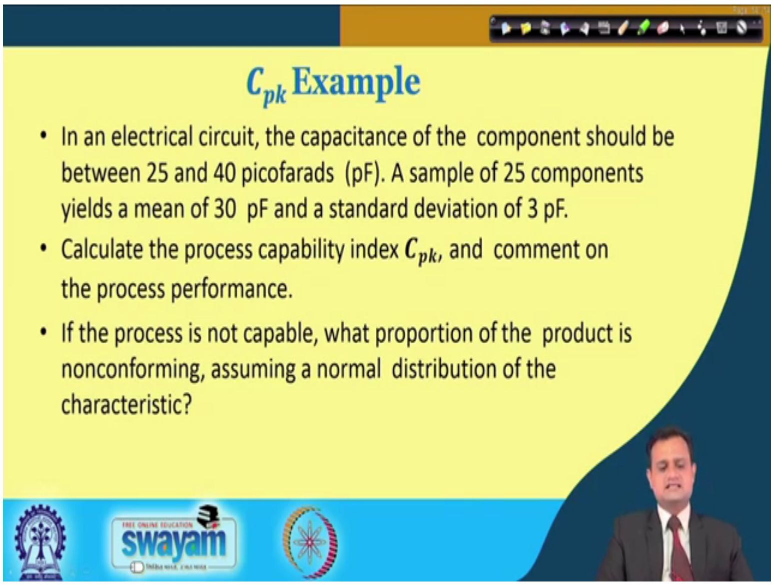


We can see the C pk example that you have an electric circuits. The capacitance of the components should be between 25 to 40 and a sample of 25 component yields a mean of 30 pF, standard deviation is 3 pF. Try to find out the C pk and comment on the process.

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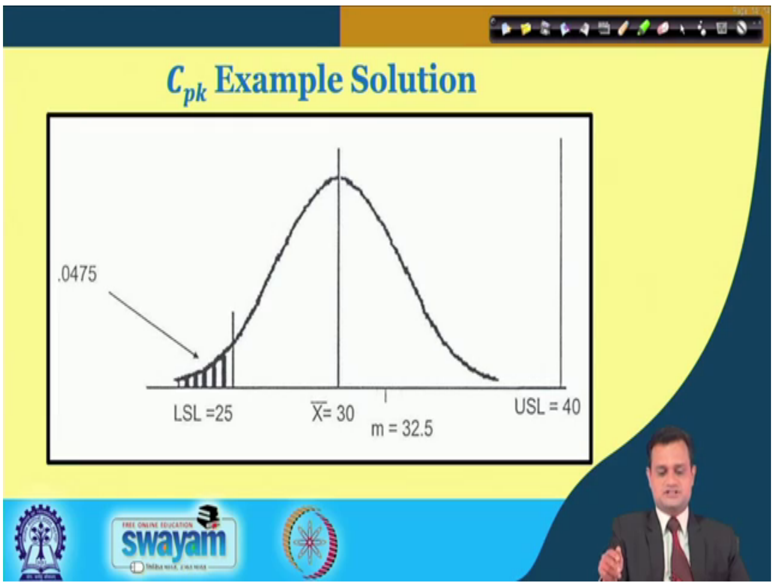

So, this C pk example I can just figure out through calculation; that LSL is equal to 25 USL is equal to 40, X bar is equal to 30. My m comes out to be 32.5, so 0.0475 is the rejection which I can show in the shaded region. So, Cpk is my minimum of these two and 0.556.

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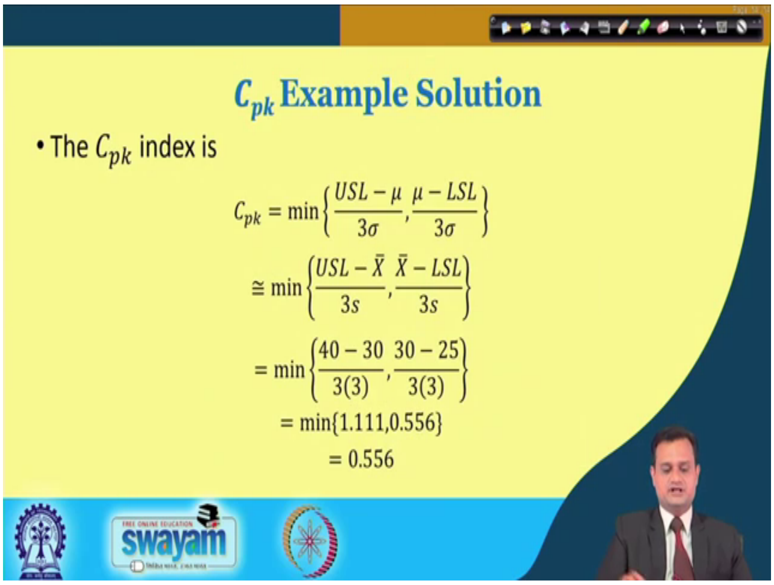


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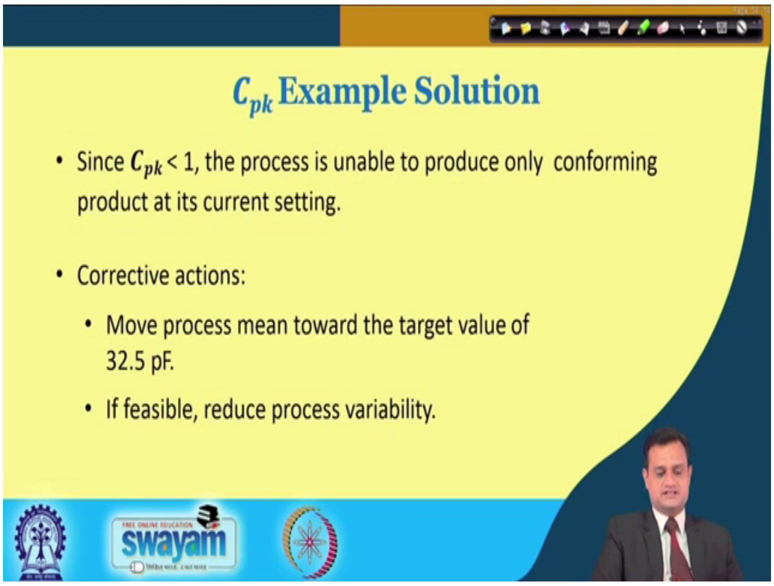


So, C pk is less than one, the process is unable to produce any conforming product and typically the corrective action is, more process mean towards target value; that is 32.5 and if feasible try to also reduce the process variability.

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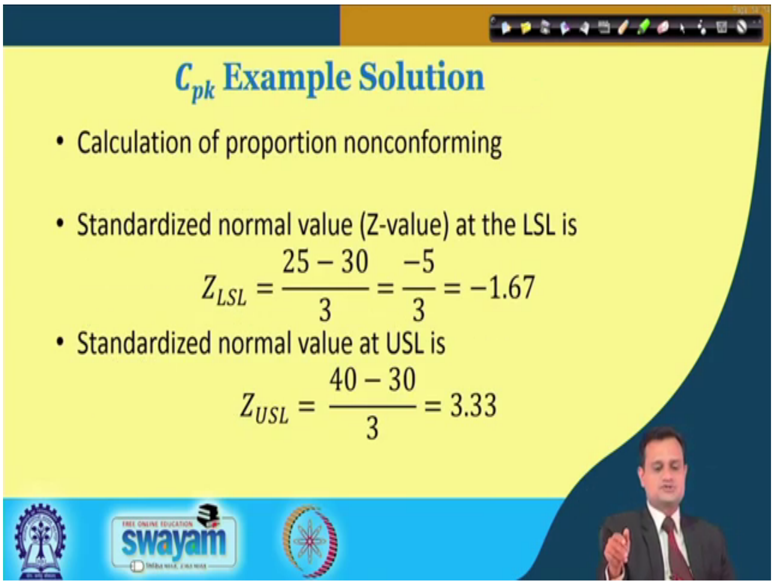


. So, you can further extend the calculation, to find the proportion non conforming; so, Z LSL 25 minus 30 divided by 3. This will give you minus 5 divided by 3 minus 1.67. Similar way you can find the Z USL 40 minus 30 by 3, this will give you 3.33.

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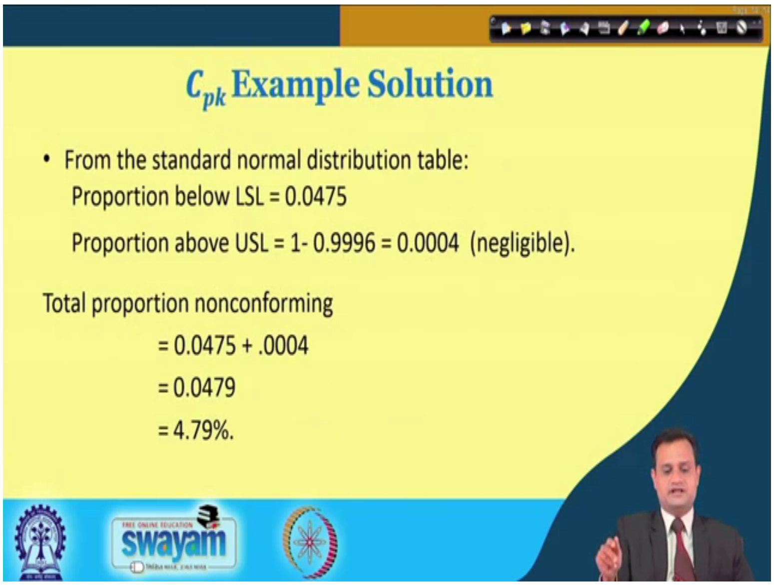


So, USL is the proportion below your USL is comes out to be 0.0475, when you use this Z table and refer the normal table, and USL is 0.0004 which is quite negligible. So, total proportion non-conforming on both the side would be 4.79 percent.

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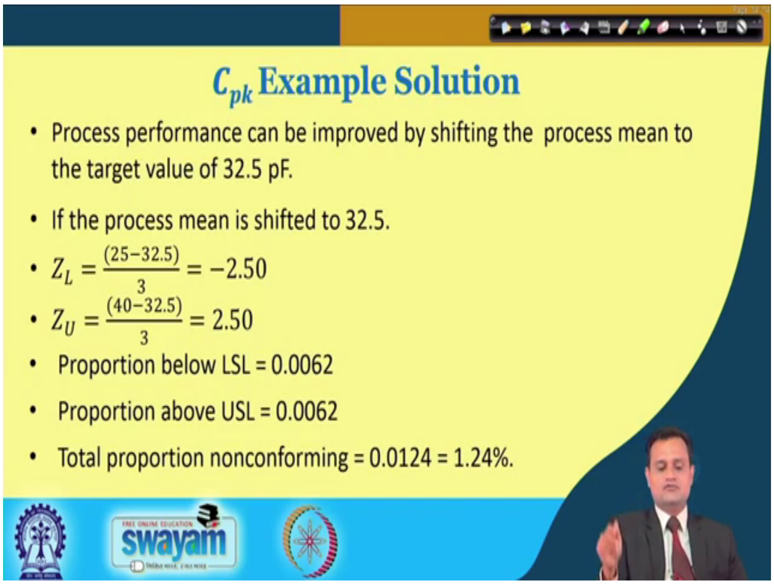


Now, process performance you can improve, by shifting the process as I mentioned and suppose I shift the process, let us into the target value 32.5 pF. So, in this case you once again calculate the Z L and Z U, you will get the proportion below LSL is 0.0062, above USL is 0.0062. So, proportion nonconforming would be only 1.24 percent and this would really improve your situation.

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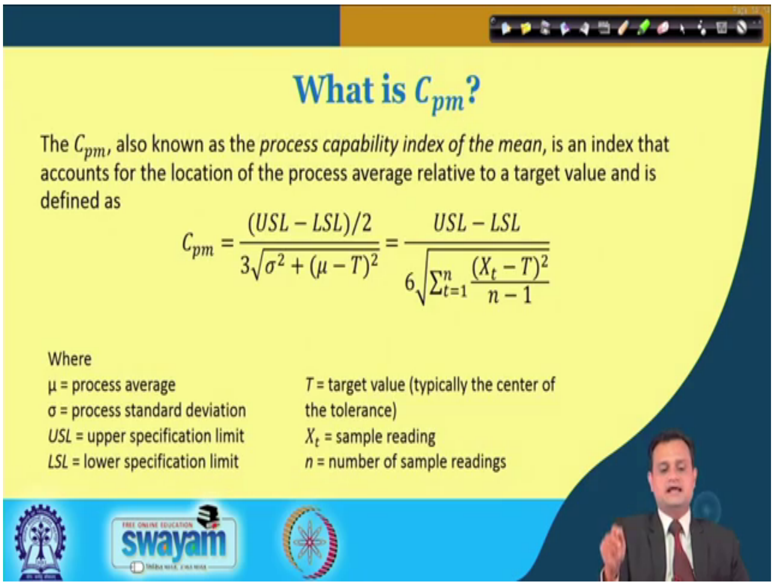


Now, you have another measure, this is C pm. So, typically if this measure is into say process capability index of the mean and that accounts for the process location target value and it is also called the Taguchi process capability index. So, I am just giving you the expression for your reference and you can plug in the values for a particular case we just did for one example; USL minus LSL divided by 2 3 into square root of sigma square plus mu minus T, T is the target value square and this will give you another measure that is C pm.

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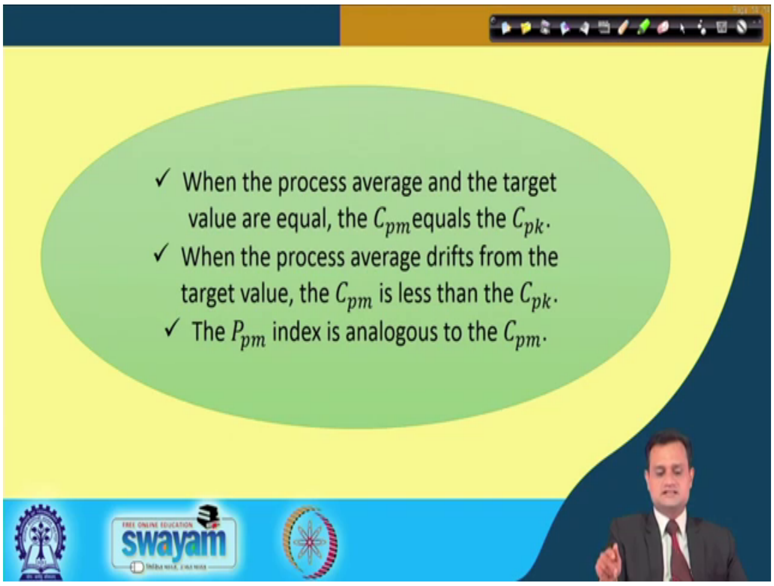


So, in summary we can say that when process average and target value are equal, C pm equal C pk. When the process average drips from the target value, pm is less than C pk and P pm index is analogous to C pm, only long term and short term difference is there

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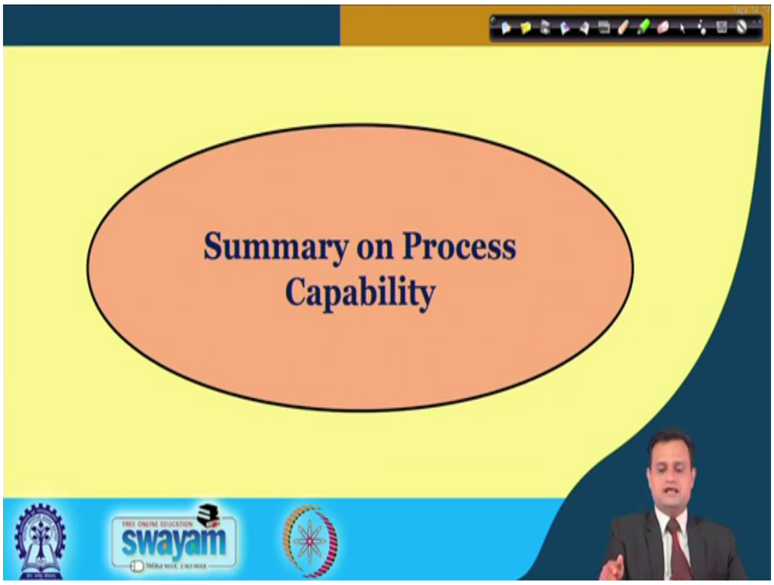


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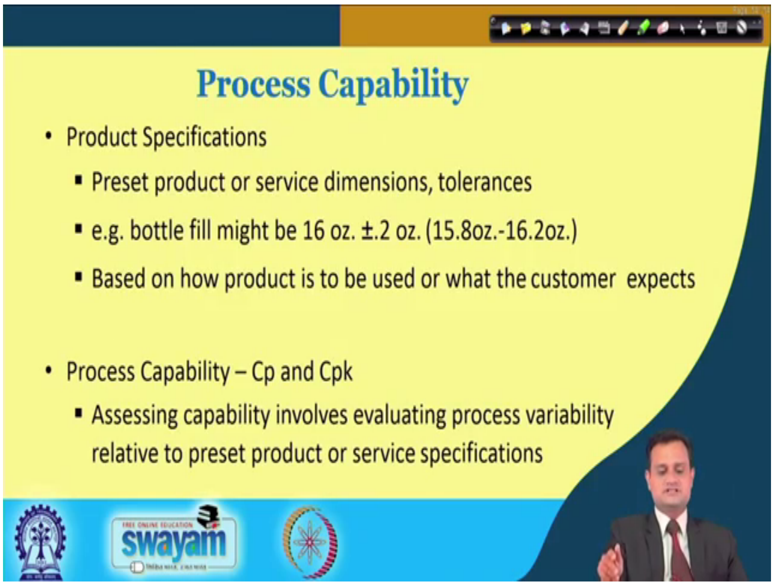


So, my summary on process capability is that, preset product or service dimensions tolerance; for example, bottle fill might be 16 ozone plus or minus 0.2, based on how product is to be used, based on the customer expectation, capture this data and find the C p and C pk.

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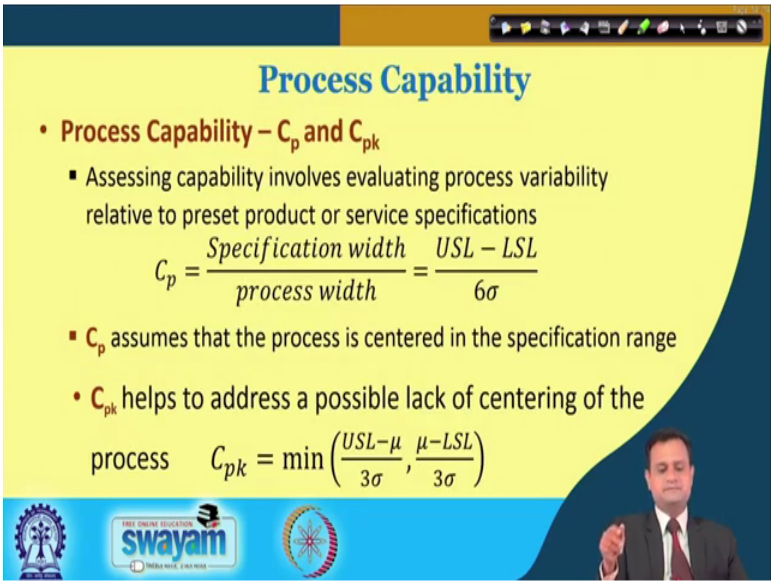


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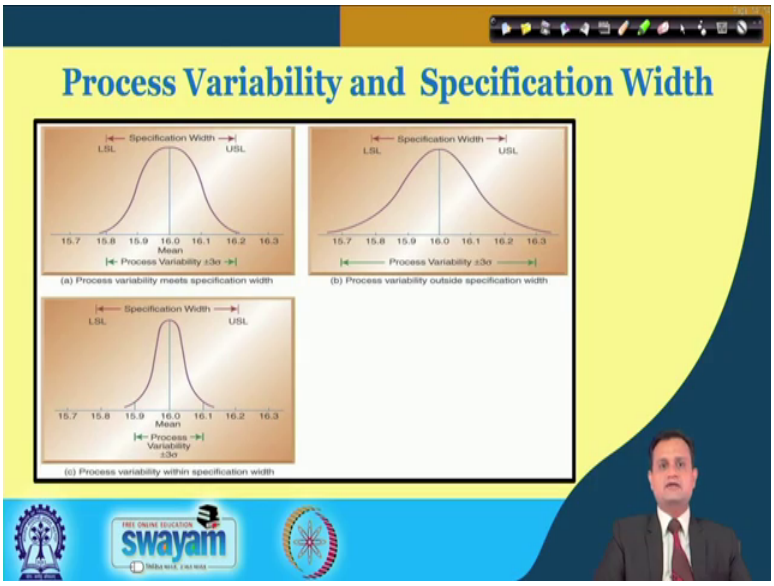


C p is the total ban USL minus LSL, C pk considers the mean value, and you can further see the three situations, three cases; a, your process variability meets the specifications and b, processed variability outside the specification and c the variability is extremely less process variability within the specification limit.

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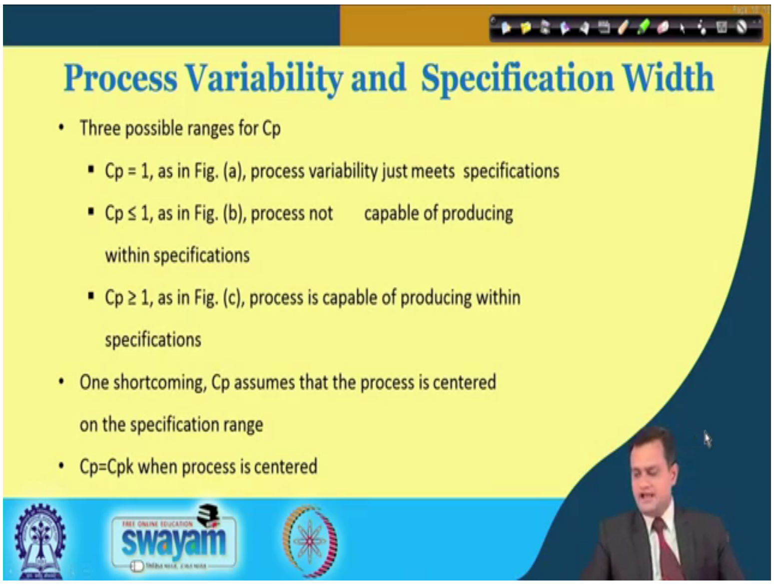


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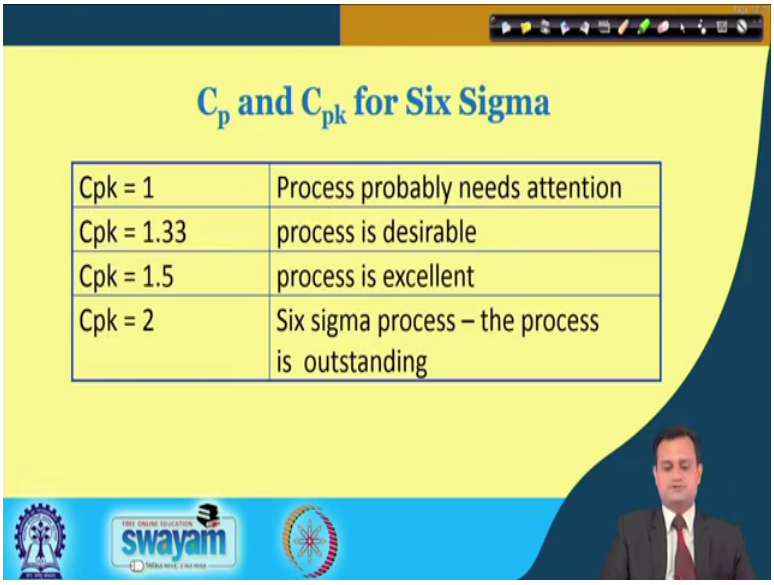


So, we have process variability and specification limit and we try to capture the health of the process. So, C p and C pk for 6 sigma, C pk should be targeted to one, it needs attention and C pk 1.33, it is a desirable; 1.5 excellent, 2 is basically 6 sigma process. So, we are targeting C pk 2 for 6 sigma 1 or less than 1 not capable, 1 demands immediate attention.

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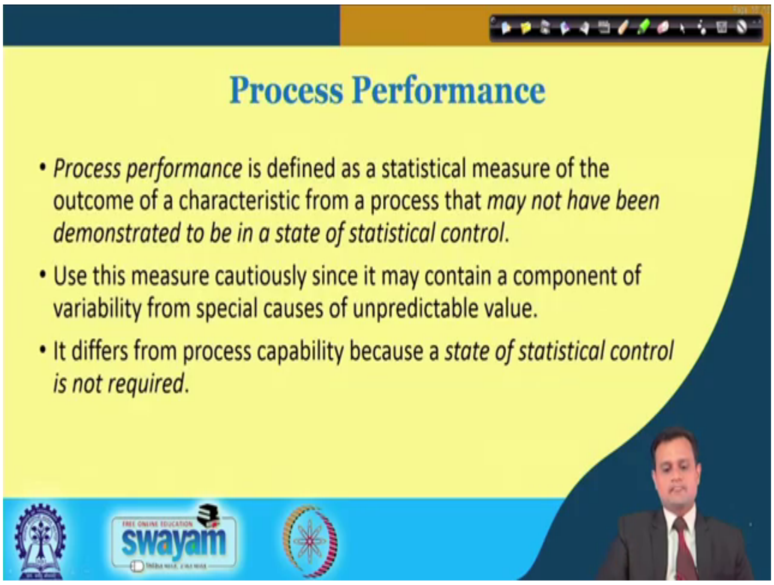


Now, to conclude with we have process performance and typically you try to capture the process performance of the process which is under statistical control for a long period of time, not by focusing on the sub group, as we did for C p C pk and other measures. So, P p P pk P pm are the measures which basically talks about process performance indices.

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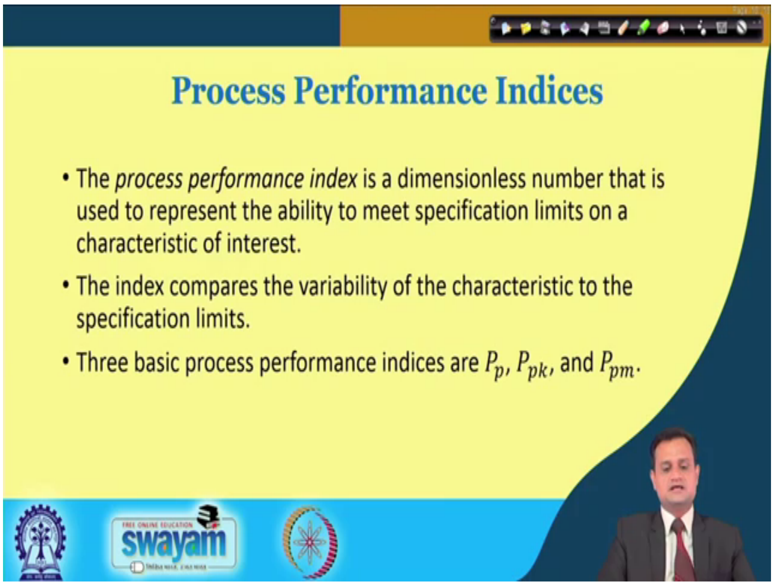

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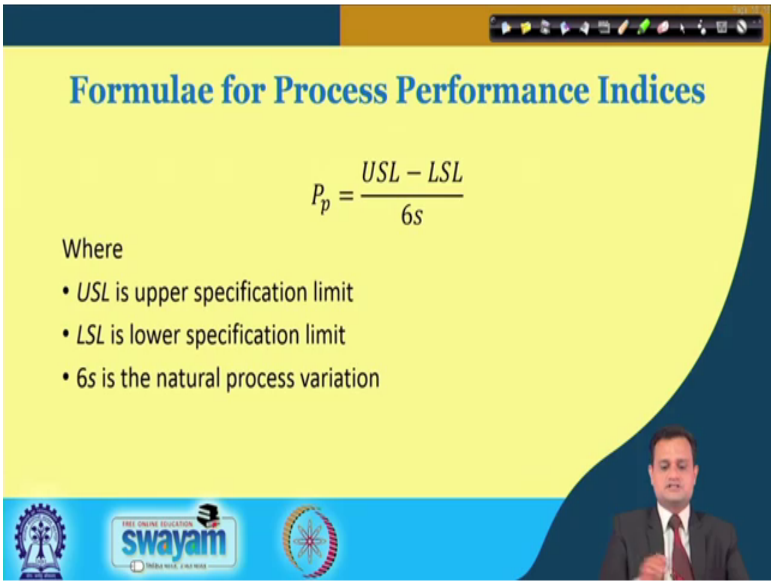


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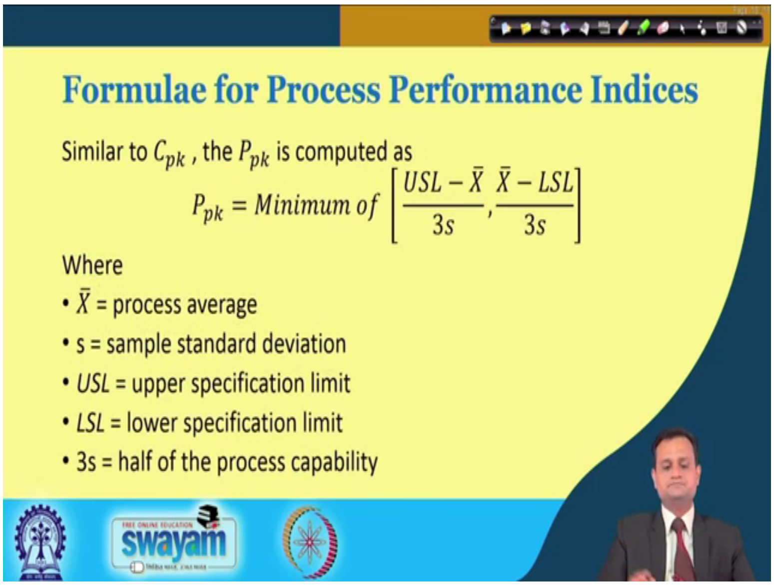


Same way you can capture only thing you have to consider the long term process standard deviation. And a here you consider the halfway, but again it is long term. So, sample standard deviation, but for a reasonably long period of time.

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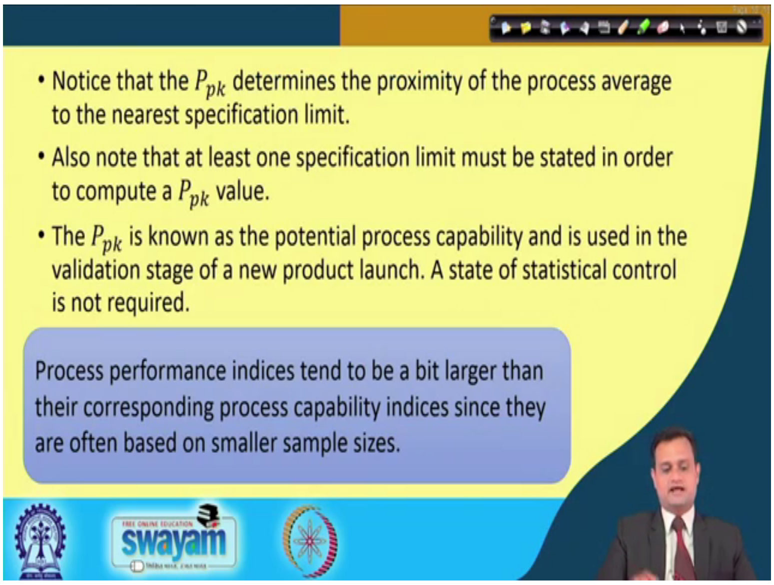


So, P pk can determines the proximity of the process average to the nearest specification limit, and this tend to be bit larger than their corresponding process capability indices, since they are often based on small sample sizes as I mentioned.

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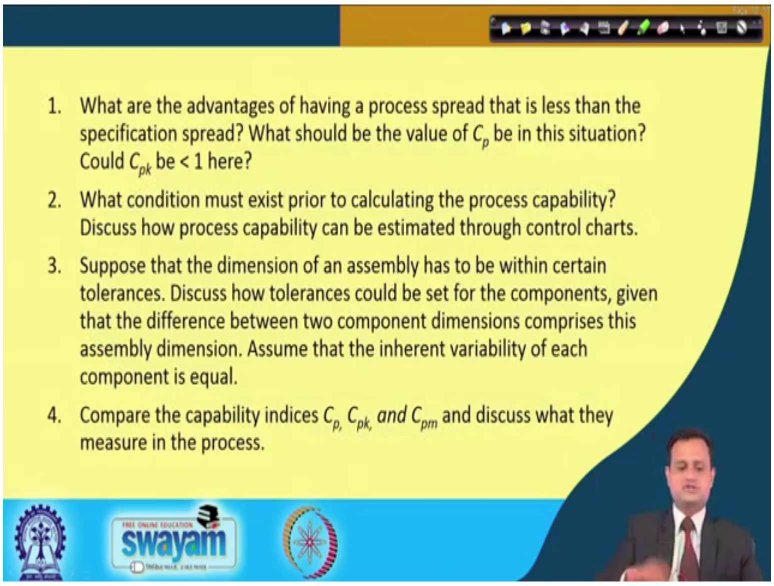


So, just think it couple of questions for you to introspect, what are the advantages of having a process spread; that is less than the specifications spread, what should be the value of C p be in this situation? Could C pk be less than 1 here? What conditions must exist while we calculating the process capability discuss how process capability can be estimated through process control charts. And suppose that the question 3, suppose that the dimension of an assembly has to be within certain tolerances.

So, please discuss how tolerances could be set for the components, given that the difference between two component dimensions comprises this assembly dimension, assume that inherent variability each component is equal. And finally, you compare the various process capability indices Cp Cpk and Cpm.

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Please see these references to revise your concepts and to conclude process performance indices tend to be a bit larger than their corresponding process capability, since they are often based on smaller sample size.

Thank you very much for your presence and interest in learning this particular topic, and we have talked about various issues in the major phase of my DMAIC cycle. So, keep revising, keep noting down your queries, also refer the suggested text and be with me. Enjoy.